Artificial Intelligence

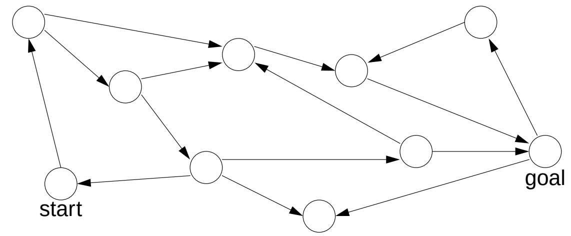
Probabilities

Nilsson - Chapter 19

Russell and Norvig - Chapter 13

this starts our excursion into
probabilistic knowledge representation and reasoning
and
probabilistic planning

search and planning - so far



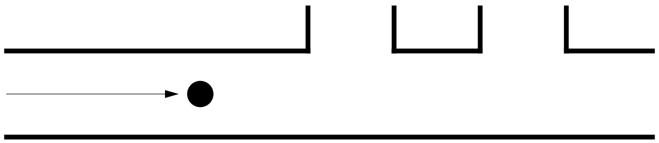
actions have deterministic effects
states are completely observable

a plan is a sequence of actions that can be executed blindly in the world

search and planning: robot navigation example (1)

noisy actuators
noisy sensors of limited range
uncertainty in the interpretation of the sensor data
map uncertainty
uncertainty about the (initial) location of the robot
uncertainty about the dynamic state of the environment

search and planning: robot navigation example (2)



search and planning: more realistic framework

actions have nondeterministic effects
states are not completely observable

need to distinguish between
actions that achieve a task
and actions that gather information

plans are no longer sequences of actions
and can no longer be executed blindly in the world

probabilities

we need to learn more about probabilities

- frequentist view
- objectivist view
- subjectivist view

notation

P(Var = value)
the probability that (random) variable VAR takes on the given value

P(STUDENTS = 38) = 0.93

P(propositional_sentence)
the probability that the given propositional sentence is true

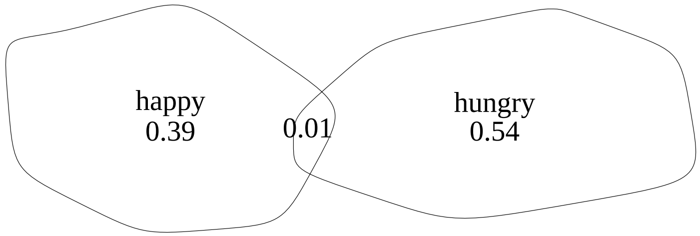
P(happy) = 0.40
P(happy AND NOT hungry) = 0.39

(Note: P(A, B) means P(A AND B).)

conditional probabilities

P(happy | hungry) = 0.02

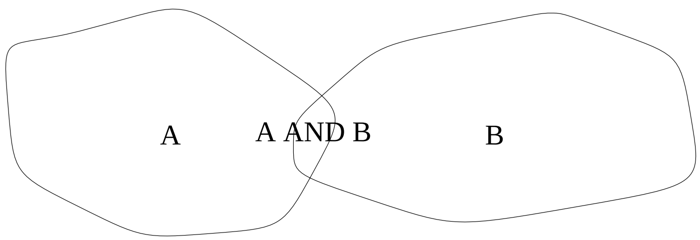
P(B | A) = P(A AND B) / P(A)
P(A AND B) = P(A) P(B | A)



axioms of probability

how to calculate with probabilities
can be derived from a few rules

1. 0 <= P(A) <= 1
2. P(true) = 1, P(false) = 0
3. P(A OR B) = P(A) + P(B) - P(A AND B)



example derivation

P(A) + P(NOT A) = 1

why is this so?

P(A OR NOT A) = P(A) + P(NOT A) - P(A AND NOT A)

P(true) = P(A) + P(NOT A) - P(false)

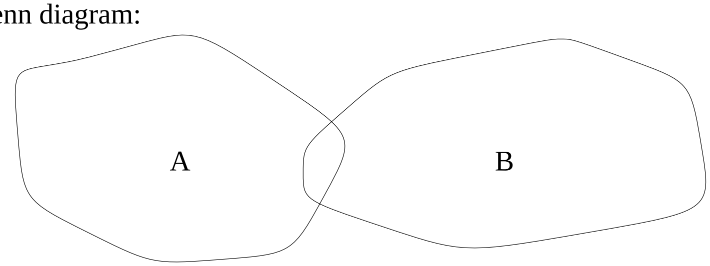
1 = P(A) + P(NOT A)

joint probability distribution

truth table:

happy	hungry	P
true	true	0.01
true	false	0.39
false	true	0.54
false	false	0.06

Venn diagram:



computing probabilities from the joint probability distribution

enumerate models and add their probability

P(happy OR (hungry EQUIV NOT happy)) = 0.94

marginalize:

P(NOT hungry) =
P(NOT hungry AND happy) + P(NOT hungry AND NOT happy) =
0.45

conditional:

P(NOT happy | hungry) = 0.54 / 0.55 = 0.98

conditional probabilities and Bayes rule (1)

P(A | B) = P(A) P(B | A) / P(B)

why is this so?

P(A | B) = P(A AND B) / P(B)
P(B | A) = P(A AND B) / P(A) and thus P(A AND B) = P(A) P(B | A)

conditional probabilities and Bayes rule (2)

posterior / prior / normalizing factor (no need to compute it)

P(model | data) = P(model) P(data | model) / P(data)

for example

speech recognition
P(word | utterance) = P(word) P(utterance | word) / P(utterance)

diagnosis
P(disease | symptoms) = P(disease) P(symptoms | disease) / P(symptoms)

causal information

conditional probabilities and Bayes rule (3)

posterior / prior / normalizing factor (no need to compute it explicitly)

P(model | data) = P(model) P(data | model) / P(data)

P(model1 | data) = 0.56 / P(data) = 0.56 / 0.84 = 0.67
P(model2 | data) = 0.23 / P(data) = 0.23 / 0.84 = 0.27
P(model3 | data) = 0.05 / P(data) = 0.05 / 0.84 = 0.06

0.84 = P(data) 1.00

Bayes rule: problem

You are a witness of a night-time hit-and-run accident involving a taxi in Athens. All taxis in Athens are either blue or green. You swear, under oath, that the taxi was blue. Extensive testing shows that under the dim lighting conditions, discrimination between blue and green is 75% reliable. Calculate the most likely color for the taxi, given that 9 out of 10 Athenian taxis are green.

(Russell and Norvig, Exercise 14.11)

Bayes rule: solution

P(tg) = 0.90
P(tb) = 1 - P(tg) = 0.10
P(yg | tg) = 0.75
P(yb | tg) = 1 - P(yg | tg) = 0.25
P(yb | tb) = 0.75
P(yg | tb) = 1 - P(yb | tb) = 0.25

tg = taxi was green
tb = taxi was blue
yg = you saw green
yb = you saw blue

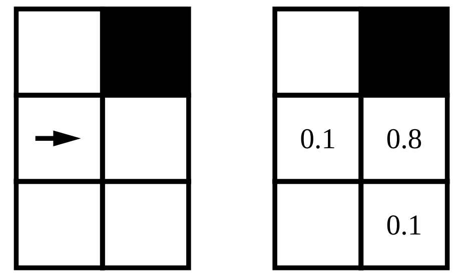
P(tb | yb)
= P(tb AND yb) / P(yb)
= P(tb AND yb) / (P(tb AND yb) + P(tg AND yb))
= P(yb | tb) P(tb) / (P(yb | tb) P(tb) + P(yb | tg) P(tg))
= 0.75 0.10 / (0.75 0.01 + 0.25 0.90) = 0.25

P(tg | yb) = 1 - P(tb | yb) = 0.75

The taxi was most likely green.

Bayes rule: robot-navigation problem

Lets assume that, whenever a robot moves east, it succeeds with probability 0.8, it strays to the left with probability 0.1, and it strays to the right with probability 0.1. Instead of moving to a blocked cell, the robot does not move.



before   afterwards

This exercise might look artificial but we will see later that one can indeed base a very robust robot navigation architecture exactly on this principle.

Bayes rule: robot-navigation problem

the robot moves east →

before

0.1	0.2	0.0
0.1	0.1	0.3
0.1	0.1	0.0

afterwards

???	???	???
???	???	???
???	???	???

Bayes rule: solution

the robot moves east →

before

0.1	0.2	0.0
0.1	0.1	0.3
0.1	0.1	0.0

afterwards

0.01	0.11	0.17
0.00	0.10	0.41
0.01	0.10	0.09

warning: I have not carefully checked this - if you find a problem, please let me know!

Bayes rule: robot-navigation problem

Lets assume that, whenever there is a wall to the east of the robot, the robot sees it with probability 0.7 (and does not see it with probability 0.3). Whenever there is no wall to the east of the robot, the robot sees no wall with probability 0.8 (and a wall with probability 0.2).

Bayes rule: robot-navigation problem

the robot sees a wall to the east

before

0.01	0.11	0.17
0.00	0.10	0.41
0.01	0.10	0.09

afterwards

???	???	???
???	???	???
???	???	???

Bayes rule: solution

the robot sees a wall to the east

before

0.01	0.11	0.17
0.00	0.10	0.41
0.01	0.10	0.09

afterwards (rounded)

0.004	0.041	0.222
0.000	0.037	0.536
0.004	0.037	0.118

warning: I have not carefully checked this - if you find a problem, please let me know!

conditional independence (1)

powerful tool to reduce complexity

$$P(A \mid B) = P(A)$$

datum B does not change probability of model A

entails

$$P(A \text{ AND } B) = P(A \mid B)\, P(B) = P(A)\, P(B)$$
$$P(B \mid A) = P(A \text{ AND } B) / P(A) = P(A)\, P(B) / P(A) = P(B)$$

happy	green t-shirt	P
true	true	0.08
true	false	0.32
false	true	0.12
false	false	0.48

→

happy	P
true	0.40
false	0.60

green t-shirt	P
true	0.20
false	0.80

conditional independence (2)

probabilistic vacuum: dirt is removed with probability 0.80
what is the probability remained dirty during 4 sucks?

P(d1 AND d2 AND d3 AND d4)
= P(d1) P(d2 | d1) P(d3 | d1 AND d2) P(d4 | d1 AND d2 AND d3)

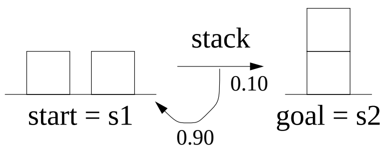
we don't want to assume
= P(d1) P(d2) P(d3) P(d4)

instead, we often assume (= Markov property)
= P(d1) P(d2 | d1) P(d3 | d2) P(d4 | d3)

conditional independence (3)

probabilistic blocksworld

here:
actions have probabilistic effects
states are completely observable



stack action fails 90 % of the time

P(s2 at time t+1 | s1 and “stack action” at time t) = 0.10
P(s2 at time t+1 | s2 and “stack action” at time t) = 1.00