CS360 – Homework #9

Probabilistic Reasoning

1) Consider the following joint probability table

PASSEXAM	WILDPARTY	
t	t	0.1
t	f	0.2
f	t	0.3
f	f	0.4

Calculate:

P(PASSEXAM)

P($\neg$ WILDPARTY)

P(PASSEXAM $\vee$ WILDPARTY)

P(PASSEXAM $\rightarrow$ WILDPARTY)

P(PASSEXAM | WILDPARTY)

P(WILDPARTY | PASSEXAM)

2) Remember that X and Y are called independent iff (= if and only if) $\mathrm{P}(X|Y) = \mathrm{P}(X)$ for all values of X and Y (that is, iff $\mathrm{P}(X = a|Y = b) = \mathrm{P}(X = a)$ for all values a that X can take on and all values b that Y can take on). Call X and Y conditionally independent given Z iff $\mathrm{P}(X|Y, Z) = \mathrm{P}(X|Z)$ for all values of X, Y and Z. (Note: $\mathrm{P}(X|Y, Z)$ means $\mathrm{P}(X|Y \wedge Z)$.)

Consider any X, Y and Z. Prove or disprove:

a) If X and Y are independent, are then X and Y necessarily conditionally independent given Z? (Assume that all conditional probabilities are well-defined.)

b) If X and Y are conditionally independent given Z, are then X and Y necessarily independent? (Assume that all conditional probabilities are well-defined.)

3) Prove that $\mathrm{P}(X|Y, Z) = \mathrm{P}(X|Z)$ for all values of X, Y and Z iff $\mathrm{P}(Y|X, Z) = \mathrm{P}(Y|Z)$ for all values of X, Y and Z. (Assume that all conditional probabilities are well-defined.)

Prove that $\mathrm{P}(X|Y, Z) = \mathrm{P}(X|Z)$ for all values of X, Y and Z iff $\mathrm{P}(X \wedge Y|Z) = \mathrm{P}(X|Z)\ \mathrm{P}(Y|Z)$ for all values of X, Y and Z. (Assume that all conditional probabilities are well-defined.)

4) Assume that TEST1 and TEST2 are conditionally independent given FLU. What is the probability of FLU (being true) given TEST1 and TEST2 (being true) if

P(FLU) = 0.1

P(TEST1 | FLU) = 0.9

P(TEST1 | ¬ FLU) = 0.2

P(TEST2 | FLU) = 0.8

P(TEST2 | ¬ FLU) = 0.1

5) Is it possible that $\mathrm{P}(X|Y) + \mathrm{P}(X|\neg Y) \neq 1$? Why or why not?

Is it possible that $\mathrm{P}(X|Y) + \mathrm{P}(\neg X|Y) \neq 1$? Why or why not?

6) Sven rolled 1 fair die. What's the probability of him having rolled a 1 given that he rolled an odd number? What's the probability of him having rolled a 1 given that he rolled an even number?

Sven rolled 2 fair dice, a red one and a green one. What's the probability of his total score being 4? What's the probability of him having rolled a 2 with the red die given that his total score was 4 or higher?

7) Three random variables X, Y and Z are called mutually independent iff $\mathrm{P}(X \wedge Y \wedge Z) = \mathrm{P}(X)\ \mathrm{P}(Y)\ \mathrm{P}(Z)$ for all values of X, Y and Z. Assume that X and Y are independent, X and Z are independent and Y and Z are independent. Are X, Y and Z then necessarily mutually independent? Why or why not?

8) In the worst case, how many probabilities does one need to specify at most to specify the joint probability table of 10 Boolean random variables? How does the answer change if the 10 Boolean variables are mutually independent?

9) After your yearly checkup, the doctor has bad news and good news. The bad news is that you tested positive for a serious disease and that the test is 99% accurate (that is, the probability of testing positive given that you have the disease is 0.99 as is the probability of testing negative given that you dont have the disease). The good news is that this is a rare disease, striking only one in 10,000 people. Why is it good news that the disease is rare? What are the chances that you actually have the disease?