CS360 – Homework #2

Entailment and Resolution

1) Solve the following problems:

(a) Is it possible that propositional sentence P entails propositional sentence Q and that it also entails sentence $\neg Q$? Explain (for example, give an example if possible).

Yes. If P is unsatisfiable then there is no interpretation that satisfies P. Therefore all the interpretations that satisfy P also satisfy both Q and $\neg Q$.

(b) Is it possible that propositional sentence P entails propositional sentence Q but does not entail sentence $\neg Q$. Explain (for example, give an example if possible).

Yes. An example would be $P \Leftrightarrow Q$, where P is satisfiable.

(c) Is it possible that propositional sentence P entails propositional sentence Q and sentence $\neg P$ also entails sentence Q? Explain (for example, give an example if possible).

Yes. If Q is a tautology then it is entailed by any propositional sentence P.

2) How can you use resolution to show that a propositional sentence is unsatisfiable?

We can conclude that a propositional sentence is unsatisfiable if we can use resolution to derive the empty clause.

3) Using resolution, show that $(P \Rightarrow Q) \wedge (\neg P \Rightarrow Q)$ entails Q.

We start by adding the negation of Q and converting to CNF, to obtain $(\neg P \vee Q) \wedge (P \vee Q) \wedge (\neg Q)$. We then perform resolution on clauses $(\neg P \vee Q)$ and $(P \vee Q)$ to derive the clause (Q). A second resolution on clauses $(\neg Q)$ and (Q) derives the empty clause and proves that $(P \Rightarrow Q) \wedge (\neg P \Rightarrow Q)$ entails Q.

First Order Logic

4) Translate the following English sentences to first-order logic using the following predicates: Owns(x,y), Dog(x), Cat(x), Cute(x), and Scary(x). For example, $Owns(x, y)$ means that object x owns object y:

(a) Joe has a cute dog.
$\exists X$ (Owns(Joe, X) $\wedge$ Dog(X))

(b) All of Joe's dogs are cute.
$\forall X$ ((Owns(Joe, X) $\wedge$ Dog(X)) $\Rightarrow$ Cute(X))

(c) Unless Joe owns a dog, he is scary.
$\neg$($\exists X$ (Owns(Joe, X) $\wedge$ Dog(X)) $\Rightarrow$ Scary(Joe)

(d) Either Joe has at least one cat and at least one dog or he is scary (but not both at the same time).
($\exists X$ (Owns(Joe, X) $\wedge$ Dog(X))) $\wedge$($\exists Y$ (Owns(Joe, Y) $\wedge$ Cat(Y))) $\Leftrightarrow \neg$ Scary(Joe).

(e) Not all dogs are both scary and cute.
$\exists X$ (Dog(X) $\wedge\neg$ (Scary(X) $\wedge$ Cute(X)))

5) Translate the following sentences in first-order logic to English. $\mathit{Apple}(x)$ means that object x is an apple, $\mathit{Red}(x)$ means that object s is red, $\mathit{Loves}(x, y)$ means that person x loves person y:

(a) $\forall x\ (\mathit{Apple}(x) \Rightarrow \mathit{Red}(x))$
All apples are red

(b) $\forall x\ \exists y\ \mathit{Loves}(x, y)$
Every person has some person he loves

(c) $\exists y\ \forall x\ \mathit{Loves}(x, y)$
There is a single person whom everybody loves

6) Specify what a grandmother is, using the predicates $IsGrandMotherOf$, $IsMotherOf$ and $IsFatherOf$. $IsGrandMotherOf(x, y)$ means that person x is the grandmother of person y, $IsMotherOf(x, y)$ means that person x is the mother of person y, and $IsFatherOf(x, y)$ means that person x is the father of person y. Define additional predicates if needed.

$\exists z\ (\text{IsMotherOf}(x, z) \wedge (\text{IsMotherOf}(z, y) \vee \text{IsFatherOf}(z, y))) \iff \text{IsGrandMotherOf}(x, y)$

7) For each of the following sentences in first-order logic, specify whether it is valid, satisfiable, and/or unsatisfiable:

(a) $P(a) \Rightarrow \forall x P(x)$
Satisfiable.

(b) $P(a) \Rightarrow \forall x \neg P(x)$
Satisfiable.

(c) $P(a) \Rightarrow \exists x P(x)$
Valid.

(d) $P(a) \Rightarrow \exists x \neg P(x)$
Satisfiable.