Fick's Laws

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1 Fick's First Law

- an analogy with the Fourier's laws for heat transfer
- rate of transfer of the diffusing substance through unit area of a section is proportional to the concentration gradient measured normal to the section

$$F = D\frac{\partial C}{\partial x} \tag{1}$$

- defines mass flux with respect to distance
- first law for cartesian co-ordinates
  - $$J = -A * D * \frac{\partial C}{\partial z}$$
  - J is flux per unit area

- A is area accross which diffusion occurs
- C is concentration
- z is distance
- D is diffusion coefficient

2 Fick's Second Law

2.1 Derivation

- let's assume an isotropic medium
- consider the following volume - a rectangular parallepiped with sides 2dx, 2dy, and 2dz
- let concentration at the centre be C

  K+———————————+K' / // *2dz* | / | / | / | L+——————————+L' |———>

  4dydz(Fx+dFx/dxdx)

  ———>| | | |

4dydz(Fx+dFx/dx dx)| C | | | 2dy| J+——————————|—+J'

I+——————————+I'

- rate at which diffusing substance enters the control volume is given as

$$4dydz(F_x - \frac{\partial F_x}{\partial x}dx) \tag{2}$$

- rate at which diffusing substance exits the control volume is given as

$$4dydz(F_x + \frac{\partial F_x}{\partial x}dx) \tag{3}$$

- performing mass balance, net accumulation in the x direction is given as

$$-8dxdydz\frac{\partial F_x}{\partial x} \tag{4}$$

- similarly in y and z directions

$$-8dxdydz\frac{\partial F_y}{\partial y} \tag{5}$$

$$-8dxdydz\frac{\partial F_z}{\partial z} \tag{6}$$

- since average concentration is given as C, the rate at which diffusing amount changes is given as volume * concentration difference

$$8dxdydz\frac{\partial C}{\partial t} \tag{7}$$

- equating the above equations
- a more general conservation equation that gives change in concentration with time
  -
    $$\frac{\partial C}{\partial t} = D * \frac{\partial^2 C}{\partial r^2} = D(\frac{\partial^2 C}{\partial z^2} + \frac{1}{A} + \frac{\partial A}{\partial z}\frac{\partial C}{\partial z})$$
  - t is time
- For a non-linear isotherm [?]
  -
    $$\frac{\partial}{\partial z}(D\frac{\partial C}{\partial z}) = \frac{\partial C}{\partial t}$$

3 Anisotropic Media

4 Analogy with Heat Flow

5 Diffusion Constant

- much less sensitive to temperature than the rate constant of a chemical reaction or other phenomena
- also much less sensitive to the solute being studied, diffusion constant in a given system for a variety of solutes often fall within a factor of 10
- also much less sensitive to solute concentration, although there are exceptions
  - smaller solutes may show concentration dependent diffusion