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Estimation of Gas Leak Rates Through Very Small Orifices and Channels

by
Herbert J. Bomelburg

February 1977

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ESTIMATION OF GAS LEAK RATES THROUGH VERY SMALL ORIFICES AND CHANNELS

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Herbert J. Bomelburg

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Abstract

As a result of a literature search, equations have been compiled for estimating the flow rates of pure gases through very small orifices and capillaries. Such equations might be useful in establishing upper limits of leak rates from sealed PuO_2 containers under accident conditions.

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Nomenclature

A = area

c = velocity of sound

c_p = specific heat at const. pressure

c_v = specific heat at const. volume

d = diameter

f = friction factor

g = gravity constant

i = enthalpy

ℓ = length

L = flow rate in atm cm^3/sec

M = Mach number

m = molecular weight

n = polytropic exponent

p = pressure

Q = mass flow

R = gas constant = $82.1 \frac{cm^3 atm}{mol °}$

Re = Reynolds number

r = radius

T = temperature (absolute

v = velocity

α = area correction factor

γ = ratio of specific heats c_p/c_v

ρ = density

ψ = exit flow function as defined by eq. (9)

λ = mean free path

ε = roughness in capillary tubes

μ = absolute viscosity

ν = kinematic viscosity ($\nu = \mu/\rho$)

Subscripts

o	refers to stagnation conditions
e	refers to exit conditions
i	refers to inlet conditions
a	refers to ambient conditions
c	refers to critical conditions
u	refers to upstream conditions
d	refers to downstream conditions

ESTIMATION OF GAS LEAK RATES THROUGH VERY SMALL ORIFICES AND CHANNELS.

A. Introduction

Plutonium-oxide (PuO_2) is shipped with great precautions to ensure, that even under severe accident conditions, no harmful quantities of this powdery material can escape into the environment. The shipping container is a heavy, tightly sealed steel cylinder, which is designed to withstand high impact loads, which might be generated in an airplane crash.

Since, strictly speaking a seal is seldom perfectly tight, there is always the possibility that minute cracks may exist. Normally any leakage resulting from such cracks can be neglected (as long as the total leak rate is smaller than 10^{-7} st cm^3/sec), because under normal conditions the pressure inside the container is atmospheric. However, if the sealed container is heated up in a fire (which e.g. could result from an airplane crash) the temperature and pressure inside the container would rise and thus a net leakage out of the container would result.

It would be of vital interest to know what the resulting leak rate would be under such fire accident conditions. If the container's leak rate near ambient conditions had been determined beforehand in a static test, it appears possible that the accident leak rate could at least be conservatively estimated, if the internal temperature and pressure resulting from the accident would be known.

However, in order to arrive at such an estimate, the basic two-phase flow phenomena (gas flow with suspended solid PuO_2 particles) through various types of leak holes must be understood and be describable in a quantitative form. Unfortunately the relevant field of fluid mechanics is largely unknown and only little, if any, systematic research has been done in it. Therefore, initially a thorough literature search must be done in order to uncover any previous related work in this field. Then, as a further step towards an analytical treatment of the two-phase leakage flow of PuO_2 powder, a thorough understanding of the one-phase (i.e. purely gaseous) flow through narrow passageways of simple geometry is required. The most simple examples of this type of flow would be the flow of an ideal gas through small orifices and channels.

Since the size of the leak hole can vary substantially and since the pressure and temperature conditions upstream and downstream of the leak can also vary over a wide range, it must be expected that the flow rates for the cases under consideration here could well extend over 10 orders of magnitude. The character of the flow under such condition could change from molecular and transition type flow for the very low flow rates through extremely small leaks to turbulent choked flow for the higher flow rates through larger leaks. Unfortunately, it must therefore be expected that any analytical description of leakage flows would not be simple enough that it could be expressed by a single formula.

Within the overall scope of the above described leakage problem the objectives of the present report will thus be limited to:

1) Screening the existing literature for analytical treatment and experimental tests on gas flow through various types of narrow openings and passageways of well defined dimensions, which would be representative of an actual leak.

2) Deriving correlations for estimating gas flow through small orifices and channels and defining the limits of applicability for these correlations.

B. Literature Survey on Leakage Flows

Historically, leaks have mostly been investigated in connection with vacuum projects, since even a small leak is intolerable for maintaining a hard vacuum. An extensive literature dealing with vacuum leak problems has therefore been accumulated over the years. References 1, 2, 3, 4, 5, 6, 7 represent a small sample of this literature. It deals primarily with leak detection methods (References 8, 9, 10, 11, 12, 13, 14, 15) and methods of estimating and sealing leak flows for various types of gases.

Leaks from pressurized vessels (e.g. due to bad welds) have been investigated to a much lesser degree (e.g. Ref. 15).

Reference 5 presents a general overview of the various phenomena of leaking gases and liquids for a wide variety of technical applications. It points out that leak rates, within the range of practical interest, can range over eleven orders of magnitude (see Appendix).

The determination of the rate at which the gas escapes through a leak is primarily a fluid mechanics problem, which has been approached at various levels of sophistication, both experimentally (e.g. Refs. 2, 16) and theoretically (e.g. Refs. 17, 18, 19). Basically, the flow rate has to be determined as a function of the 1) prevailing upstream condition (characterized by at least 2 parameters) 2) prevailing downstream condition (characterized by at least 1 parameter) 3) geometry and conditions along the leak path (number of parameters could go to 8) 4) type of the leaking gas (characterized by at least 3 parameters). Because of the many possible parameter combinations a general treatment, covering all conceiveable cases, proves to be practically impossible. This is mainly due to the fact, that the leak path can assume an almost infinite variety of shapes. To make matters even worse, the geometric shape parameters are of overriding importance, as will be shown below. Therefore investigations reported in the literature have mainly dealt with idealized leak paths, in which the geometry and conditions along the leak paths can be described by very few (say 3 or less) parameters.

In particular the following groups of investigations, reported in the literature on practical leak problems, will be discussed here in greater detail, since they could be of some interest to the present problem.

a) vacuum leaks

b) leaks through mechanical seals

c) flow in capillary tubes for fluidic resistors

d) leakage of sealed containers for electronic circuits

a) <u>Vacuum Leaks</u>

In vacuum leaks the upstream pressure is usually ambient, whereas the downstream pressure is very low, usually < 1 torr, but quite often 10^{-6} torr or even less. Since a hard vacuum can rapidly be destroyed by even a very small leak, one usually has to deal with geometrically very small leak hole dimensions. For such reasons the mean free path of the molecules in the leaking gas is quite often larger than the leak diameter*, when assuming a single cylindrical hole shape. This, therefore, leads to a type of flow, which is described as molecular or transition type flow and which must be distinguished from the more conventional type of continuous flow, which is laminar at the lower velocities or turbulent at the higher velocities. For small volumetric leaks the flow normally remains laminar, because its Reynolds number $\left(Re=\frac{dv\rho}{\mu}\right)$ is normally <1000 for such leaks in which both velocity v and hole diameter d are small.

The formula, which is mostly quoted for the determination of flow rates into vacuum through tubular leaks, contains therefore two terms. One term accounts for the laminar (viscous) flow and the second term for the molecular flow. (see e.g. Ref. 20). This formula cannot be extended beyond the area of application for which it was originally derived (Ref. 17), since other phenomena such as choking may have to be considered also. This particular feature will be discussed at greater length in Section D of this report.

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*The Knudsen number (defined as the ratio of mean free path to the hole diameter) for such a case is >1.

b) Leaks Through Mechanical Seals

For obvious reasons the leakage of fluids (liquids as well as gases) through various types of seals, e.g. on rotating shafts, must be kept as low as possible. Therefore seals of this kind have been investigated by experimental as well as theoretical means. Of particular importance in this context are the labyrinth seals, which have one of the lowest leak rates among the noncontacting, rotating seals (Refs. 21-26). Their behavior clearly shows how the leakage rate is changed very significantly by judicious changes in the geometry of the leak paths. Generally, by making the path narrower and/or more tortuous the leak rate can be reduced drastically. In the labyrinth seal the cross-sectional area along the leak path is changed repeatedly by a large factor. Thereby pressure drops (or flow resistances) are generated by momentum losses (sometimes called "Borda" losses), which are usually much larger than the pure viscous losses which are due to wall friction. However, since the present investigation will be restricted to flow through small circular orifices and straight capillaries only, there will be no direct application of the various analytical approaches to labyrinth seal leakage, which has been treated in the literature.

c) Leakage From Sealed Containers for Electronic Circuits (Refs. 27-29)

On the surface it might appear, that the leakage from hermetically sealed containers of highly sensitive electronic circuits might have a bearing on the leak problem under consideration here.

However the pressure difference for the containers between the inside (usually an inert gas at ~1atm and ambient temperature) and the outside (air at ambient conditions) is normally quite small, so that compressibility effects need not be considered. For these and other reasons the leak rates are extremely low ($\lesssim 10^{-9}$ atm cm^3/sec)*, which obviously is of little interest in the present problem. The formulas cited in the literature (Refs. 27, 28) are basically those which are being used for vacuum work; but they have been amended by consideration of self diffusion effects (Ref. 30), which are totally unimportant in the present problem. The suggestion, put forward in Refs. 27 and 28, that in a leak path of varying diameter only the narrower sections need to be considered, will give wrong results for larger leak rates, since it ignores the Borda losses (discussed above), which originate at any sudden enlargement of the leakage paths.

d) Flow in Capillary Tubes in Fluidic Resistors. (Refs. 31, 32)

In fluidic control devices linear resistances are often generated by laminar flow through capillary tubes. For such reasons, flow phenomena in capillaries have been investigated recently in greater detail than before. Theoretically, if the flow resistance for small pressure differentials in a capillary is caused by laminar wall friction only (according to the Hagen-Poiseuille Law), then the flow resistance should be linear, i.e. the flow rate should

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*See Appendix for discussion of leak rate units.

increase linearly with the pressure drop. However, closer examinations have revealed, that certain deviations from the expected linear behavior are encountered, which are mainly due to entrance losses and compressibility effects. For the problem under consideration here the investigations in this field of fluidics are only of minor importance, since they are restricted to small pressure ratios (generally $\frac{p_u}{p_d} \stackrel{\sim}{<} 2$), whereas our problem is concerned with large pressure ratios ($\frac{p_u}{p_d} >> 1$).

C. <u>General Characteristics of Orifice and Pipe Flow as Described in the Literature</u>.

Since the search for references, directly applicable to the PuO_2 leakage problem, has turned out to be rather disappointing, the search was extended to the more general category of flow through regular orifices and tubes (or pipes). Here indeed a large body of published literature does exist, which might be pertinent and useful. But since it is quite impossible to review all papers which might have some direct or indirect bearing on the leakage problem, only a number of the more important and valuable papers were studied.

The main difference between the work reported in the literature and the present leakage problem lies in the diameter range of the orifices and tubes. As far as it could be determined, none of the past systematic flow tests were extended to diameters of less than $\sim$0.1 inch. However, it may be expected that the basic correlations, presented in the literature for standard orifice and pipe flow, can be extrapolated with reasonable confidence to smaller dimensions.

a) <u>Orifice Flow.</u>

For the practical cases of gas leakage from a PuO_2 container which are the subject of this investigation, the overall pressure ratios $\frac{p_u}{p_d}$ are practically always* large $\left(\frac{p_u}{p_d} >> 1\right)$. Therefore it is necessary that compressibility effects are fully taken into account. Such effects become important as soon as $\frac{p_u}{p_d}$ exceeds about 1.5. When $\frac{p_u}{p_d}$ exceeds a value of about 2, the flow already becomes choked†. Therefore, papers as e.g. Refs. 33 and 34 will be of significant value to this project. The tests reported in Ref. 33 cover sharp-edged orifices in the diameter range between 0.15" and 0.25" with upstream pressures as high as 100 psi. Temperature are ambient for all test conditions. The tests reported in Ref. 34 cover a similar parameter range. The purpose of these tests was to compare theoretically predicted flow rates with actually measured flow rates. Agreement was found to be well within a few percent or even better for certain cases. Such results lend support to the widely used practice of using orifice plates as flow measuring devices.

In this context it must be remarked, that the above mentioned close agreement between experiment and analysis for orifice flow is generally found only in the sharp-edged orifices, for which the

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*Those conditions, for which the pressure ratios are $\lesssim 2$, would not contribute much to the total leakage out of the sealed PuO_2 container, and therefore need not be considered here. For this reason Ref. 35 and similar references are of little practical value for this investigation.

†The phenomenon of choking is explained on pp 18-21.

thickness is small compared to the diameter. For thick orifices certain deviations must be expected, which are in the order of about 10% for the tests reported in Ref. 34. The PuO_2 container leak problem would entail test diameters, which are very much different from any previously tested conditions. They could be as small as a few micrometers ($1\mu m = 0.0001$ cm $= 3.9 \times 10^{-5}$in), with pressures as high as 1000 psi and temperatures as high as 1800°F. For such conditions no published test data could be found in the literature, which would even come close.

Extrapolation over several orders of magnitude is usually considered tentative. Therefore, even though no significant deviations are expected on theoretical grounds, tests should be performed to verify the analytically derived leakage rates for orifices of very small diameter (around 100μm).

b) Capillary Flow.

As in the case of orifice flow, the prevailing pressure ratio $\frac{p_u}{p_d}$ for capillary flow is practically always >>1, when the gas leakage out of a PuO_2 container is considered. Therefore compressibility effects must be taken fully into account here also.

Basically, a capillary is a smooth tube or pipe of small inside diameter (in this case in the order of .001"). It is therefore most likely, that the correlations which have been derived for ordinary pipe flow, are generally valid also for capillary flow. These correlations cannot totally be derived from basic principles however, since they contain an empirical (or semi-empirical) element, reflecting the friction which the flow encounters on the tube walls.

For incompressible pipe flow numerous tests have been performed to determine the friction factor for various fluids and flow conditions. The subject has now been thoroughly researched for pipe diameters which are of practical importance (Refs. 36, 37). The main results are the existence of two basically different flow regimes (laminar and turbulent) and the fact that the friction factor in both regimes is a function of only the Reynolds number and the roughness of the pipes' interior walls. As expected, no systematic attempts were discovered in the literature search dealing with capillary flow under conditions that would be of interest for this project. It can be assumed, however, that the general relations for pipe flow can be extrapolated to smooth capillary tubes.

For compressible flow through pipes the number of investigations is much smaller than for the incompressible case. The theoretical derivation of flow correlations becomes much more involved, and the flow rate usually cannot be expressed any more in explicit terms. As in the case of incompressible pipe flow, a friction factor must be determined empirically. However, as shown in Ref. 40, the friction factors for incompressible and compressible pipe flow are essentially the same, fortunately.

The most striking property of compressible flow is its capability of becoming choked, if a critical pressure ratio $\left(\frac{p_u}{p_d}\right)_{crit}$ is exceeded. This general phenomenon can be predicted on strictly theoretical grounds and has in fact been demonstrated experimentally many times. It must be expected, that the same choking phenomenon will also prevail in capillary flow even though no specific test reports towards this end were found in the literature search.*

The exact value of $\left(\frac{p_u}{p_d}\right)_{crit}$ at which the flow through tubes becomes choked is not as easily determined as it is in the case of orifice flow, because in capillary flow the pressure drop, caused by fluid friction along the tube walls, has to be taken into account. Such pressure drop is practically non-existent for flow through a sharp-edged orifice. In section D,c the flow relations for compressible capillary flow will be derived.

D. Basic Analytical Treatment of Leakage Flows.

In the following the correlations for incompressible and compressible flow out of a container through various types of leaks will be considered.

As a first step, the equations for frictionless flow through circular orifices will be derived from basic principles. Then a correlation for flow in pipes with friction will be developed. In particular the expressions for choked conditions will be considered and discussed. The results will enable us to better judge under what conditions and with what accuracy the derived formulas can be

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*Refs. 16, 38 and 39 are examples of the type of papers which are available. They normally treat leakage flows from atmospheric conditions into vacuum, both theoretically and experimentally. Although they may provide valuable information on experimental techniques, particularly for very small leaks, their results are not directly applicable to the present problem.

extended to the cases of leak flow through small openings.

For the most common leak problems, in which atmospheric air leaks into an evacuated container, the leaks are usually in a size range where the flow is laminar, or where it may even reach into the molecular range. This generally simplifies the theoretical treatment. However, in the problems considered here, the gas is leaking from a pressurized container into atmospheric air. This means an increased density (ρ) of the flow and also a high exit velocity v_e. Therefore the Re number ($Re = \frac{\rho v_e d}{\mu}$) for the leakage flow will be much larger and may in fact reach into the turbulent regime for many actual leak conditions. This could be one of the reasons why the leak flows which are under study here, have hardly been treated so far in the literature.

Sometimes the word "viscous" flow is used in connection with leakage problems. This is meant to be a flow in which any contributions due to diffusion, molecular, or transition phenomena can be disregarded, because they are insignificantly small in comparison with the viscous friction phenomena. Both laminar and turbulent flows fall into the category of viscous flows.

These remarks pertain generally more to leak paths which resemble long capillary tubes rather than abrupt, sharp-edged orifices. But because of the simplicity of their theoretical as well as experimental treatment, orifice flows will be considered here nevertheless. They will serve as a guide to the more complex capillary flows.

a) Discussion of the Basic Problem

In its most simple form the leak problem can be described as follows:

A closed container (Figure 1) is filled with a perfect gas at a pressure and temperature p_o and T_o. The container wall of the thickness ℓ has a single smooth cylindrical hole of diameter d, connecting to the outside, which is assumed to be under ambient conditions, $p_a < p_o$ and $T_a < T_o$.

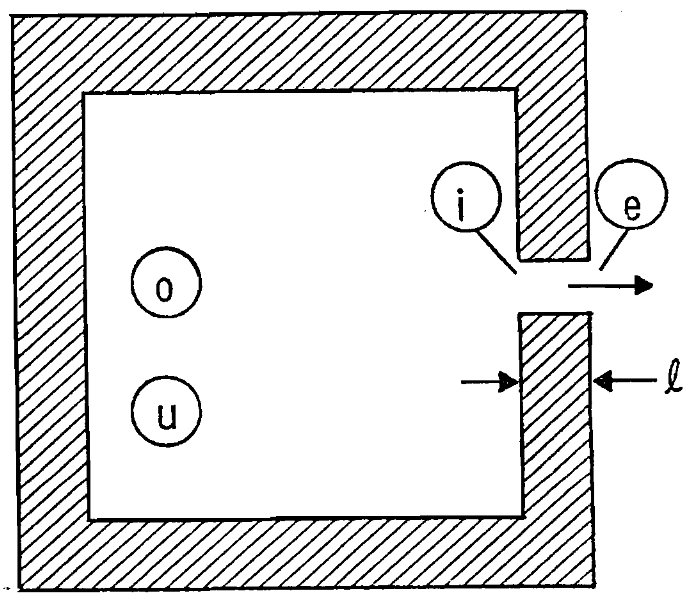

Figure 1. Identification of Flow Conditions by Subscripts. (If the upstream condition (u) is at zero velocity, it is characterized by subscript (o); if the downstream condition (d) is at ambient temp. and pressure it is characterized by subscript (a).)

The variable parameters in this case are ℓ, d, p_o and T_o. If $\ell \ll d$, the leak can be considered an orifice leak. If $\ell \gg d$, the leak can be considered a tubular leak. If ℓ is of the same order of magnitude as d ($\ell \approx d$) the opening is a "thick orifice" (Ref. 34) which will deviate in its behavior from that of a true orifice ($\ell << d$). The absolute value of d will always be considered large compared to the mean free path λ of the gas. This means that molecular and transition flow can be excluded from the considerations.

The pressure ratio p_o/p_a will primarily determine the flow character, e.g. whether the flow can be considered laminar or turbulent, compressible or incompressible, choked or non-choked etc. The temperature ratio T_o/T_a will primarily determine the viscosity of the gas. It is assumed that the container walls are always in thermal equilibrium with the gas inside the container, i.e., the walls are always at T_o.

For the purpose of the present report, the internal volume of the vessel may conveniently be considered infinite, since only stationary events will be treated initially.

It is also assumed that the cross sectional shape of the leaks is always circular and does not change in size along the leak path. The walls of the leak path are considered to be smooth. These are, of course, quite restrictive assumptions which may hardly ever hold true for a real leak path. But in order to keep the problem theoretically tractable, they have to be imposed.

b) <u>Flow Through Orifices</u>

It is assumed that a closed container is filled with an ideal gas at a pressure p_o, a density ρ_o and a temperature T_o. The container is large enough so that these conditions can be considered steady (or at least quasi-steady) even though some gas flows out through a circular orifice. The outside conditions (p_a, ρ_a, T_a) are ambient. Since it cannot be expected that the conditions in the exit plane are identical with the ambient conditions, the exit conditions will be identified as p_e, ρ_e and T_e. The flow areas in the exit plane is A_e and the velocity v_e.

In order to determine v_e, the energy equation for an ideal gas with constant specific heats will be utilized. The enthalpy difference $i_o - i_e$ is related to v_e in the following way:

(1) $$i_o - i_e = c_p(T_o - T_e) = \frac{v_e^2}{2g}$$

If the flow through the orifice is adiabatic and without friction the isentropic gas relation holds*:

(2) $$\frac{T_e}{T_o} = \left(\frac{p_e}{p_o}\right)^{(\gamma - 1)/\gamma}$$

in which γ is the ratio of specific heats $\frac{c_p}{c_v}$.

For an ideal gas: $T_o = \frac{p_o}{\rho_o R}$ and $\frac{c_p}{R} = \frac{\gamma}{\gamma - 1}$

We therefore obtain

(3) $$v_e = \sqrt{2g \frac{\gamma}{\gamma - 1} \frac{p_o}{\rho_o} \left[1 - \left(\frac{p_e}{p_o}\right)^{(\gamma - 1)/\gamma}\right]}$$

For <u>small</u> pressure differences across the orifice (i.e. $1 - \frac{p_o}{p_e} \ll 1$) the exit pressure p_e is practically equal to the ambient pressure ($p_e \approx p_a$) and the equation for v_e will reduce to the well known formula for incompressible orifice flow

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*For flow through an orifice friction and heat transfer can usually be neglected (which has been experimentally verified). This is not true, however, for capillary flow. See below.

(4) $$v_e = \sqrt{2g\,\frac{p_o - p_a}{\rho_o}}$$

Or if the mass flow $Q = A_e v_e \rho_e$ is expressed, we obtain by dropping the subscripts and introducing a correction factor α:

(5)* $$Q = \alpha A\sqrt{2g\rho\Delta p}$$

The factor α is in the order of one and will take care of two different approximations: 1) the real flow is never completely free of friction and 2) the geometric area is usually larger than the effective flow area (because of boundary layer effects). Therefore the factor α is usually smaller than 1 and must generally be determined empirically. It forms part of both the incompressible and compressible orifice flow equation.

Equation (3) must therefore be written:

(6) $$v_e = \sqrt{2g\,\frac{\gamma}{\gamma-1}\,\frac{p_o}{\rho_o}\left[1-\left(\frac{p_e}{p_o}\right)^{(\gamma-1)/\gamma}\right]}$$

By using the adiabatic relation $\frac{\rho_e}{\rho_o} = \left(\frac{p_e}{p_o}\right)^{1/\gamma}$ the mass flow is then expressed by

(7) $$Q = \alpha A\left(\frac{p_e}{p_o}\right)^{1/\gamma}\sqrt{\frac{\gamma}{\gamma-1}\left[1-\left(\frac{p_e}{p_o}\right)^{(\gamma-1)/\gamma}\right]}\sqrt{2gp_o\rho_o}$$

This equation has general validity not only in the cross-section at the exit (identified by subscript 3), but at any cross sectional area within the flow (as long as the velocity vector is normal to

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*This equation for incompressible orifice flow has little significance for this investigation and will therefore not be used any further.

this area). By dropping subscript e we can write in abbreviated form:

$$Q = \alpha A\psi\sqrt{2gp_o\rho_o} \tag{8}$$

$$\text{with } \psi = \left(\frac{p}{p_o}\right)^{1/\gamma}\sqrt{\frac{\gamma}{\gamma-1}\left[1-\left(\frac{p}{p_o}\right)^{(\gamma-1)/\gamma}\right]} = \sqrt{\frac{\gamma}{\gamma-1}}\sqrt{\left(\frac{p}{p_o}\right)^{2/\gamma}-\left(\frac{p}{p_o}\right)^{(\gamma+1)/\gamma}} \tag{9}$$

In this form ψ contains the dependence of the flow on γ and on the pressure ratio, whereas the rest of the equation depends on the conditions inside the container. If ψ is drawn up as function of p/p_o for a given γ (see Figure 2), a curve with a distinct maximum results. This maximum occurs for

$$\frac{p}{p_o} = \left(\frac{2}{\gamma+1}\right)^{\gamma/(\gamma-1)} = \frac{p_c}{p_o} \tag{10}$$

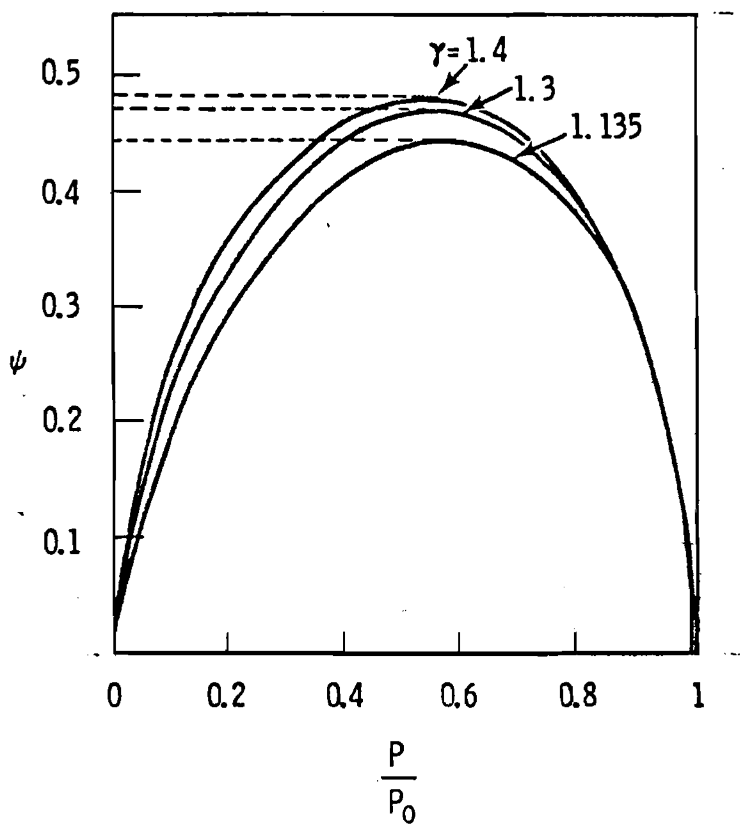

Figure 2.

and has the value of

$$\psi_{max} = \left(\frac{2}{\gamma+1}\right)^{1/(\gamma-1)} \sqrt{\frac{\gamma}{\gamma+1}} \tag{11}$$

This means that for the same pressure ratio (i.e. the same ψ) the outflow of a certain gas depends only on the stagnation conditions in the vessel. With $p/\rho = RT$ (for ideal gases) the mass flow Q is

$$Q = \alpha A \psi p_o \sqrt{\frac{2g}{RT_o}} \tag{12}$$

and with the same p_o the mass flow decreases with temperature proportional to $1/\sqrt{T_o}$.

Since for steady flow $A\psi$ = constant will hold for all cross sectional areas, the function ψ must increase as the flow approaches the exit or orifice of the container (see Figure 1). But according to Figure 2, ψ will increase with decreasing p (p_o is assumed to be constant) only until it reaches its maximum ψ_{max}, no matter how far the pressure p is lowered beyond this point.

For air with a $\gamma = 1.4$ the maximum value of ψ is $\psi_{max} = 0.484$ at a critical pressure ratio of $p_c/p_o = 0.53$. If p/p_o is decreased below this value, ψ will remain constant at its maximum value, and the pressure at the exit will start to rise above the ambient pressure p_a in such a way, that the p_e/p_o ratio remains at its critical value p_c/p_o.

This means as long as $p_a \geq p_c$, the exit pressure p_e essentially equals p_a and equation (6) becomes

$$v = \sqrt{2g\frac{\gamma}{\gamma-1}\,\frac{p_o}{\rho_o}\left[1-\left(\frac{p_a}{p_o}\right)^{(\gamma-1)/\gamma}\right]} \tag{13}$$

and equation (7) becomes

$$(14)\qquad Q = \alpha A\sqrt{2g\,\frac{\gamma}{\gamma-1}\,p_o\rho_o\left[\left(\frac{p_a}{p_o}\right)^{2/\gamma}-\left(\frac{p_a}{p_o}\right)^{(\gamma+1)/\gamma}\right]}$$

If $p_a = p_c$, the critical exit velocity v_c becomes

$$(15)\qquad v_c = \sqrt{2g\,\frac{\gamma}{\gamma+1}\,\frac{p_o}{\rho_o}}$$

Or, if we assume adiabatic flow with $p_o/\rho_o^{\gamma} = p_c/\rho_c^{\gamma}$, this equation becomes

$$(16)\qquad v_c = \sqrt{g\gamma\,\frac{p_c}{\rho_c}}$$

This particular velocity turns out to be equal to the sonic velocity in the exit flow. Thus it is shown that, when the flow reaches critical conditions at the exit, its exit velocity equals the sonic velocity; or in other words: for critical or choked flows the Mach number equals one.

If $p_a < p_c$, the exit velocity within the orifice (or throat) will remain sonic ($v_e = v_c$). The mass flow for such conditions will be

$$(17)\qquad \boxed{Q = \alpha A\psi_{max}\sqrt{2gp_o\rho_o} = \alpha A\left(\frac{2}{\gamma+1}\right)^{1/(\gamma-1)}\sqrt{\frac{\gamma}{\gamma+1}}\sqrt{2gp_o\rho_o}}$$

For one particular case of leakage out of the pressurized vessel ($p_o \gtrsim 1000$ psi) the mass flow rate can be computed with this last equation, since practically always $p_a < p_c$. The flow rate depends on the type of gas since γ and thus ψ_{max} may be different for different gases. For air ($\gamma = 1.4$) ψ_{max} is 0.484 and for helium ($\gamma=1.66$) ψ_{max} is 0.515.

Numerical example:

Compute the leakage flow through a 100 μm diam. orifice out of a large vessel, containing helium at p_o = 1000 psi (≈ 70 atm) and T_o = 1000°F ≈ 810 K, into ambient air. The pressure ratio $p_o/p_a >> 2$, therefore according to equation (17) the mass flow Q is

$$Q = \alpha A \psi_{max} \sqrt{2gp_o\rho_o}$$

ψ_{max} = 0.515 for helium

$A = \frac{\pi}{4} \times 10^{-8} m^2$

For lack of accurate empirical data on α, it is assumed that $\alpha = 1$.

p_o = 70 atm = 70 kg/cm^2

$g \approx 10$ m/sec^2

ρ_o must be derived from T_o = 810 K with the aid of the ideal gas equation $\rho_o = p_o/RT_o$. The individual gas constant R for helium is 0.0205 ℓ atm/gram°K. Thus

$$\rho_o = \frac{70 \text{ atm gram°}}{0.0205 \text{ ℓ atm } 810°} = 4.21 \text{ gram/ℓ}$$

$$Q = 0.515 \times \frac{\pi}{4} 10^{-8} m^2 \sqrt{\frac{20 \text{ m}}{\sec^2} \frac{70 \text{ kg } 4.2\text{gram}}{\text{cm}^2 \text{ ℓ}}}$$

$$= 0.031 \text{ gram/sec}$$

Since the helium density at standard atmospheric conditions is ρ_a = 0.178 gram/ℓ, the above flow rate can also be expressed in standard cm^3/sec.

$$Q = 0.031 \text{ gram/sec} = 173 \text{ st. cm}^3\text{/sec}$$

If a leak rate of 10^{-7} st cm^3/sec is considered negligible (i.e. with such a low leak rate the vessel is considered leak tight for all practical purposes), it means that the just computed leak rate is 9 orders of magnitude larger than the allowable limit.

c) Flow in Capillary Tubes

As mentioned before, the gas pressures in the PuO_2 leak problem are usually so high that any contributions due to molecular flow can be entirely disregarded. For capillary flow, therefore, only viscous (i.e. laminar and turbulent) compressible and incompressible flows need to be considered. Among these the laminar incompressible flow is the most simple to treat analytically, followed by the turbulent incompressible flow. These two cases will therefore be treated first.

1) Laminar incompressible flow through capillary tubes.

The Hagen-Poiseuille flow equation determines the laminar flow velocity v as function of the prevailing pressure difference Δp.

$$v = \frac{\Delta p \cdot r^2}{8\mu\ell} \tag{18}$$

where r = radius of the capillary tube

ℓ = length of the capillary tube

μ = viscosity of the fluid.

v is considered an average velocity over the cross-sectional area πr^2 of the circular tube. The mass Q flow through the tube is therefore

$$Q = \rho v \pi r^2 = \frac{\rho \pi r^4 \Delta p}{8\ \mu\ell} = \frac{\rho d^4 \pi}{128\mu\ell} \quad (p_u - p_d) \tag{19}$$

where d = diameter of the tube

p_u = pressure at upstream end of tube

p_d = pressure at downstream end of tube

Since for gases the density is proportional to the pressure, the density at the tube inlet would be larger than at the tube outlet. Therefore a medium density is usually used in the Hagen-Poiseuille flow equation for gases.

This medium density is derived from the isothermal gas equation $\rho = p/RT$, where T is the absolute temperature and R the universal gas constant. With $1/RT$ = constant, the medium density $\bar{\rho}$ is therefore

(20) $$\bar{\rho} = \text{const}\ \bar{p} = \text{const}\ \frac{p_u + p_d}{2}$$

By substituting into equation (19) we arrive at

(21) $$Q = \text{const}\ \frac{d^4 \pi}{256\ \mu \ell}\ (p_u^2 - p_d^2)$$

For 0°C with the mass flow Q rate in gram/sec, the pressures in atmopheres, the diameter d and length ℓ of the capillary tube in cm and the viscosity μ of the gas in centipoise the equation becomes

(22) $$\boxed{Q = 54.8\ m\ \frac{d^4}{\mu \ell}\ (p_u^2 - p_d^2)}$$

where m is the molecular weight of the gas in grams.

Since 1 mol (m gram) of any ideal gas at ambient conditions takes up a volume of 22400 cm^3, the volumetric flow rate in standard cm^3/sec can be obtained by multiplying this equation by 22400 cm^3/ m gram. In such a form it is quoted in Ref. (20) as

(23) $$L = 3810\ \frac{d^3}{\ell} \cdot 323\ \frac{d}{\mu}\ (p_u^2 - p_d^2)$$

or $$L = 1.23 \times 10^6\ \frac{d^4}{\mu \ell}(p_u^2 - p_d^2)$$

with L in atm cm^3/sec

μ in centipoise

d and ℓ in cm

p in atm

The Hagen-Poiseuille equation for laminar flow through pipes has been verified to be correct as long as the velocity profile in the pipe is parabolic and the velocity itself remains small (say less than 30%) of the local sound velocity. This means that entrance effects must be negligible and the pressure ratio p_u/p_d must not be larger than about 1.5. For larger pressure ratios* compressibility effects must be taken fully into account. The fact that in the above Hagen-Poiseuille equation for gas flows a medium density is introduced, merely represents a linear approximation of the compressibility effect which is sufficient for small Mach number flows (say for $M < 0.3$) only. It is unfortunate that hardly ever a warning is attached to the Hagen-Poiseuille flow equation that it becomes less and less accurate with increasing flow velocities, even if the flow remains laminar, as it would e.g. in narrow capillaries.

2) Turbulent incompressible flow through capillary tubes

If the Reynolds number $Re = vd\rho/\mu$ of the flow through a pipe (that includes also capillary tubes) becomes larger than about 2000 the laminar flow breaks up into turbulent flow and the Hagen-Poiseuille

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*In the present investigation of the PuO_2 leak problem the pressure ratios are practically always larger than 1.5.

equation can no longer be applied. For such cases equations with empirical constants in the form of dimensionless friction factors f must be used instead (see Figure 3). In numerous tests it has been shown that the pressure drop for flow of the medium velocity v through a pipe (diameter d and length ℓ) can coveniently be expressed as follows:

(24) $$p_u - p_d = \Delta p = f \frac{\ell}{d} \frac{\rho}{2} v^2$$

The mass flow $Q = \rho v A$ can then be determined with the aid of this expression by

(25) $$Q = \rho \frac{\pi d^2}{4} \sqrt{\frac{2d\Delta p}{f\,\ell\rho}} = \frac{\pi}{4} \sqrt{\frac{2d^5\rho(p_u - p_d)}{f\ell}}$$

With $f = \frac{64}{Re}$ for laminar flow this equation converts to the Hagen-Poiseuille equation (19).

As in the case for laminar flow (see above) the medium density could again be expressed by

(20) $$\bar{\rho} = \text{const} \times \frac{p_u + p_d}{2} \qquad \text{with const} = 1/RT = \frac{m}{22.4\ \ell\ \text{atm}} \text{ for } 0°C$$

Thus we arrive at

(26) $$Q = \sqrt{\text{const}}\ \frac{\pi}{4} \sqrt{\frac{d^5}{f\ell} \left(p_u^2 - p_d^2\right)}$$

If d and ℓ are given in cm and p in atm, Q will be obtained at 0°C in gram/sec by the following equation:

(27) $$\boxed{Q\ (\text{gram/sec}) = 5.25 \sqrt{\frac{d^5 m}{f\ell} \left(p_u^2 - p_d^2\right)}}$$ m = mol. weight of gas in grams

This is the equivalent turbulent version of equation (22).

By multiplying this equation with 22400 cm^3/mol we arrive at

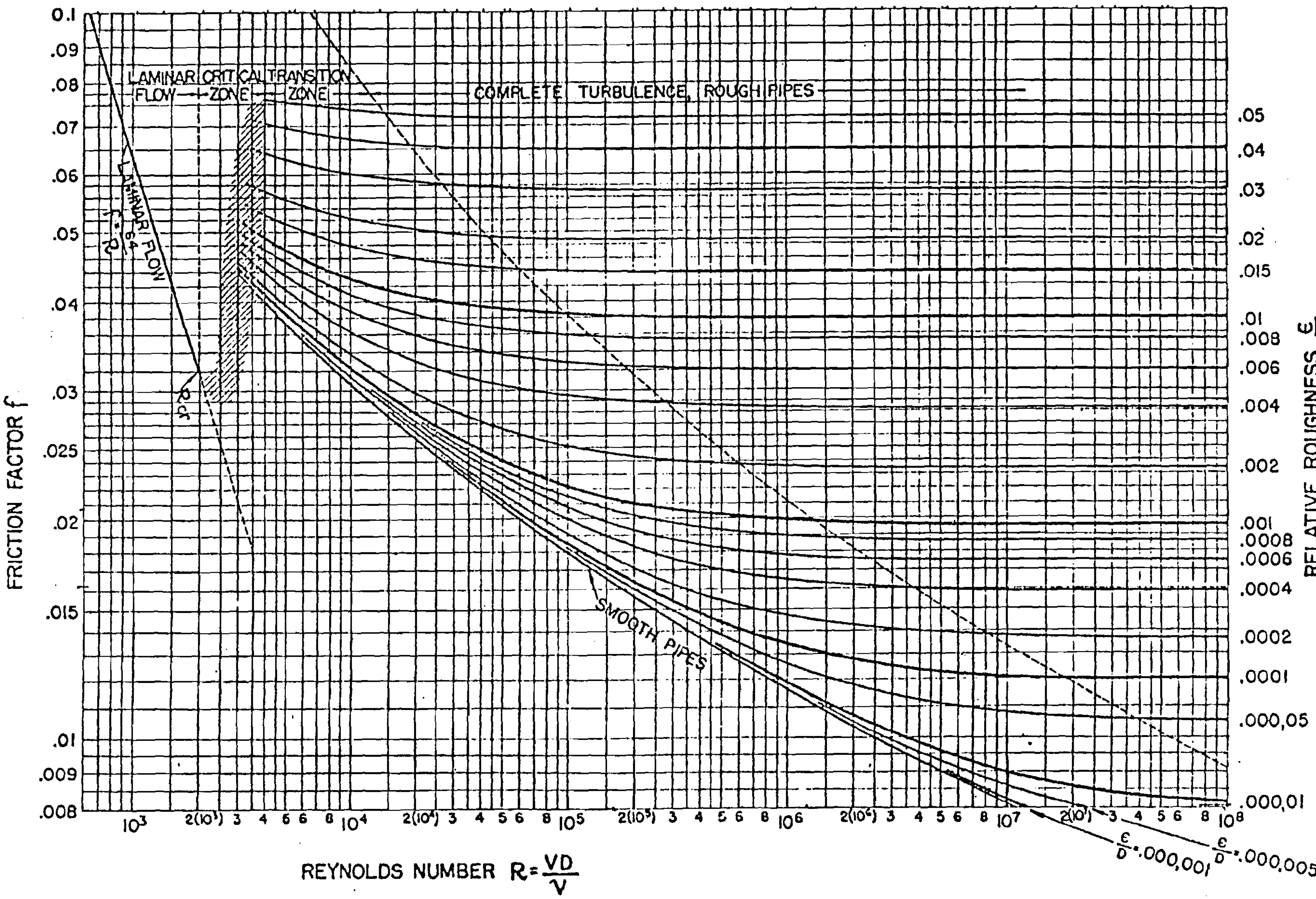

LAMINAR FLOW
CRITICAL ZONE
TRANSITION ZONE
COMPLETE TURBULENCE, ROUGH PIPES
LAMINAR FLOW
$f = \frac{64}{R}$
R_{cr}
SMOOTH PIPES
FRICTION FACTOR f
RELATIVE ROUGHNESS $\frac{\epsilon}{D}$
REYNOLDS NUMBER $R = \frac{VD}{\nu}$
$\frac{\epsilon}{D} = .000,001$
$\frac{\epsilon}{D} = .000,005$
0.1
.09
.08
.07
.06
.05
.04
.03
.025
.02
.015
.01
.009
.008
.05
.04
.03
.02
.015
.01
.008
.006
.004
.002
.001
.0008
.0006
.0004
.0002
.0001
.000,05
.000,01
10^3
$2(10^3)$
10^4
$2(10^4)$
10^5
$2(10^5)$
10^6
$2(10^6)$
10^7
$2(10^7)$
10^8

Figure 3.

$$(28) \quad Q\ (\text{st cm}^3/\text{sec}) = .118 \times 10^6 \sqrt{\frac{d^5}{f \ell m} \left(p_u^2 - p_d^2\right)}$$

with:

d and ℓ in cm

p in atm

m in gram

Unfortunately it must be expected that in most cases of leak flow out of highly pressurized container the relatively simple equations (23) and (28) for incompressible laminar and turbulent flow are seldom applicable. Rather the relations for compressible flow, to be treated in the next section, are more appropriate.

3) Compressible flow through capillary tubes

Generally compressibility effects become important (density changes become greater than 10%), if the gas velocity approaches about 1/2 of its sonic velocity, i.e. if the flow Mach number approaches 0.5. For such cases gas dynamics equations must be substituted for the more simple equations of ordinary incompressible fluid mechanics.

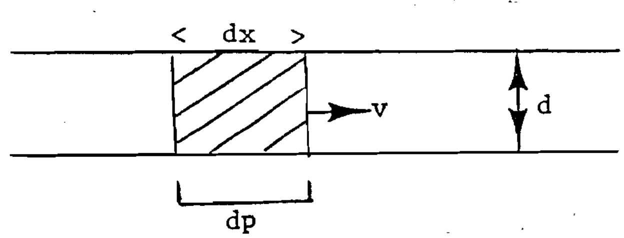


Figure 4.

The equation for the pressure drop for compressible gas flow through a tube can be derived as follows:

Under the assumption that the flow is steady and Newtonian in character and that gravitational forces are negligible, the pressure drop dp through a flow element of the length dx is given by an acceleration and by a friction term (Figure 4).

(29) $$dp = dp_a + dp_f$$

The acceleration term can be determined by the energy equation (usually called Bernoulli's equation in fluid dynamics) which states that

(30) $$p + \frac{\rho}{2} v^2 = \text{const}$$

or in differential form:

(31) $$dp + \rho v dv = 0$$

Therefore the acceleration term in equation (29) can be substituted by:

(32) $$dp_a = -\rho v dv = -\rho v^2 \frac{dv}{v}$$

The pressure loss due to friction cannot be derived from a similarly fundamental equation, but must be expressed with the aid of an empirically found relation.

When it is assumed that dp_f is proportional to the dynamic pressure $\frac{\rho v^2}{2}$ of the flow, to the length of the tube section dx and inversely proportional to its diameter d, then a proportionality factor f can be defined by

(33) $$dp_f = f \frac{1}{d} \frac{\rho v^2}{2} dx$$

f is the common friction factor for pipes, which is usually cited in engineering handbooks.

Figure 3 shows f as function of Re (Ref. 36).

Thus the equation for the pressure loss in pipe flow can be written in the following form

$$-\frac{dp}{\rho} = v^2\frac{dv}{v} + f\,\frac{v^2}{2d}\,dx \tag{34}$$

Other basically equivalent forms of this equation can be found in the literature.

In order to make the equation more adaptable to the leakage problem it can be modified in the following way:

Because of the small cross-section of a capillary leak hole, it can be assumed that the flow is essentially isothermal, i.e. it is always in thermal equilibrium with the walls of the tube. For such isothermal flow the continuity equation is applicable at any point along the constant area tube. With pv = const or $\frac{dp}{p} + \frac{dv}{v} = 0$ it can be written as:

$$dp = -\frac{dv}{v}\,p \tag{35}$$

At this point it is convenient to introduce the Mach number M, since the flow is considered to be compressible. For isothermal flow M is defined as (Refs. 18, 19):

$$M = \frac{v}{c} = \frac{v}{\sqrt{\frac{p}{\rho}}} = \frac{v}{\sqrt{RT}} \tag{36}$$

Hence

$$\frac{dM}{M} = \frac{dv}{v} \tag{37}$$

It is generally assumed (see Ref. 40) that the friction factor depends on the Reynolds number only, and that it is essentially independent of Mach number. Since in the case of isothermal flow, considered here, Re is constant, the equation (34) can be integrated.

(38) $$v^2 \frac{dv}{v} + \frac{dp}{\rho} + f \frac{v^2}{2d} dx = 0$$

After division by v^2

(39) $$\frac{dv}{v} + \frac{dp}{\rho v^2} + \frac{f}{2d} dx = 0$$

Since $\frac{dv}{v} = \frac{dM}{M}$ (see above)

(40) $$\int_i^e \frac{dM}{M} + \int_i^e \frac{dp}{\rho v^2} + \frac{f}{2d}\int_i^e dx = 0$$

After carrying out the integration:

(41) $$\ln M \Big|_{M_i}^{M_e} + \frac{1}{2M^2} \Big|_{M_i}^{M_e} + \frac{f\ell}{2d} = 0$$

As it can be assumed that for most practical cases the flow will be choked at the exit (i.e. $M_e = 1$) one obtains:

(42) $$\boxed{\frac{f}{2d} \ell = 1/2\left(\frac{1}{M_i^2} - 1\right) + \ln M_i}$$

It has to be remembered that this equation is valid only for isothermal flow. If the flow is polytropic the right side of the equation must be multiplied by $\frac{2}{n+1}$, where n is the polytropic exponent which can vary between 1 and γ. For $n = \gamma$ the flow is adiabatic.

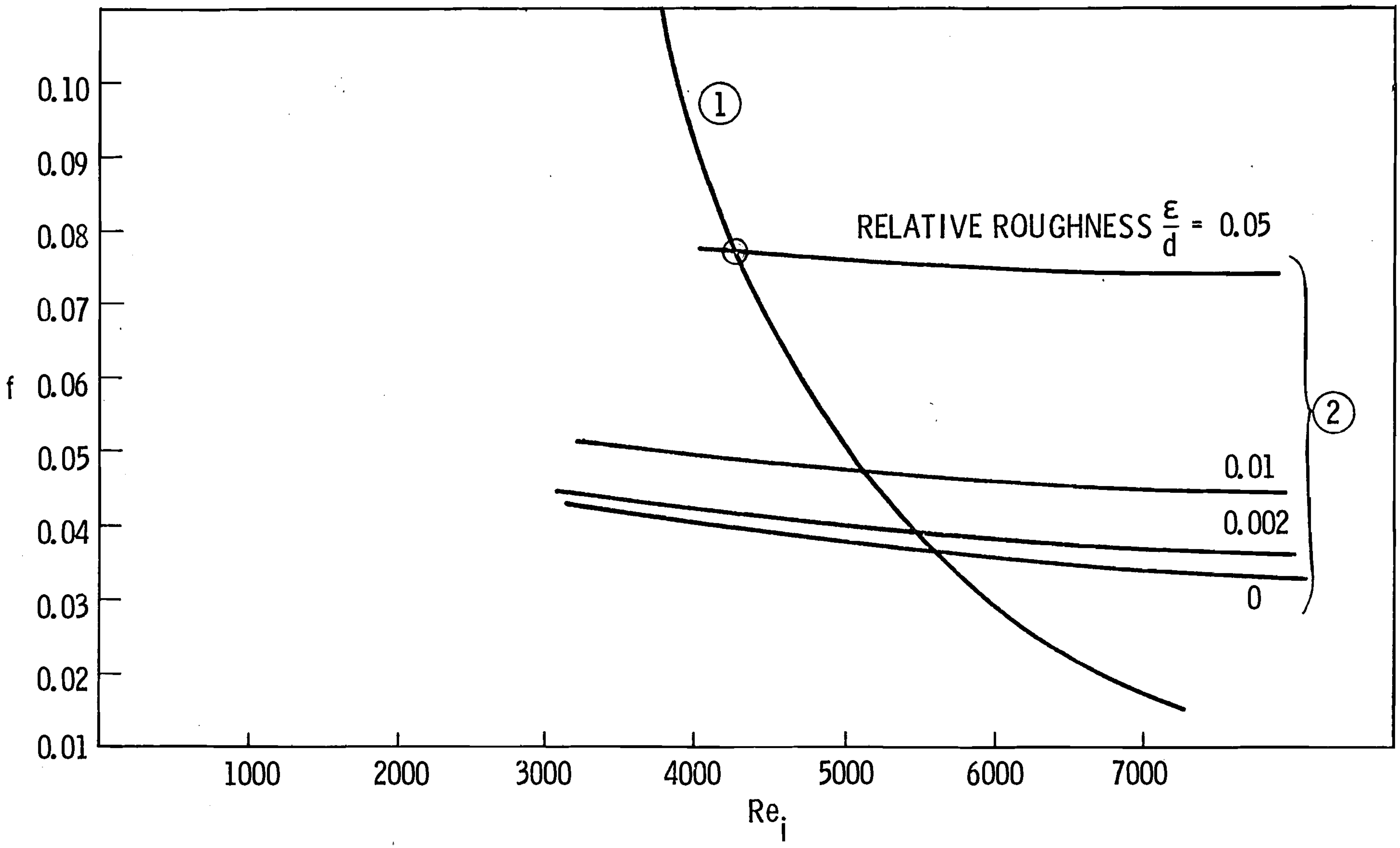
Since the friction factor f is not a constant but a function of velocity, an additional relationship is required before equation (42) can be solved. For laminar flow f can be expressed explicitely as a function of Re number: $f = \frac{64}{Re}$, but for turbulent flow empirical relationships (as shown in Figure 3) must be used. In either case, equation (42) remains a transcedental equation which must be solved numerically e.g. graphically*, by iteration or computer.

The solution of equation (42) is obtained most conveniently by graphical means.

For given conditions of p_o, T_o, d, ℓ and type of gas the relationship $f(M_i)$, as expressed by equation (42), can be drawn as a single curve (1) in Figure 5. Instead of using M_i directly in the abscissa, the variable $Re_i = \frac{M_i c_i d}{\nu}$ is chosen. Drawn into the same figure is a second set of curves (2) which represent the empirical relationships f(Re) as taken from Figure 3. The intersecting points of the curves represent solutions of equation (42) for the selected conditions (Helium at p_o = 70 atm, T_o = 1000°F (538°C), d = 100 μm, $\frac{\varepsilon}{d}$ = .05, ℓ = 1 cm). In this particular case it amounts to: f = .077 and Re ≈ 4300 with the Mach number M_i at the inlet of the capillary being ≈ 0.30. The inlet velocity v_i would be 387 m/sec. The mass flow Q (g/sec) could be computed from $Q = \rho_i v_i \frac{d^2}{4} \pi$. For the given conditions one obtains: ($\rho_i \approx \rho_o$, within 1%) Q = .0128 gram/sec or 71 st cm^3/sec.

These figures are based on the assumption that the internal relative roughness $\frac{\varepsilon}{d}$ of the capillary is about 0.05, i.e. irregularities of the wall amount to 5 μm in the 100 μm diameter capillary. With a perfectly smooth capillary (f ≈ 0.036 and Re ≈ 5600) the flow rate would be about 30% larger, i.e. about 93 st cm^3/sec. This would be an upper limit for the flow. It

*Examples of graphical solutions for compressible pipe flow are given in Refs. 41 and 42. However, since the graphs are intended to be very general, it takes considerable effort to become sufficiently familiar with them, before they can be used with confidence.



f
0.10
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
1000
2000
3000
4000
5000
6000
7000
Re$_i$
1
2
RELATIVE ROUGHNESS $\frac{\varepsilon}{d}$ = 0.05
0.01
0.002
0

FIGURE 5.

is more likely, however, that the flow would be even smaller than the 71 st cm^3/sec computed above, since it would be rather difficult in praxis to produce an internal smoothness as good as 5 μm. Quite apart from a manufacturing viewpoint it might be even harder to ascertain by measurements what the actual smoothness really is. Thus it must be expected that the reliability of any numerical flow rate predictions for capillaries is usually affected by a rather high error margin. This situation is further aggravated by the fact that it is even quite difficult to fabricate or accurately measure capillary tubes of a specified constant diameter. But since the diameter enters in the above equation with its fourth power, a small uncertainty in the diameter (say of 15%), would cause a rather significant uncertainty (75%) in the computed flow rate.

Therefore a much larger discrepancy between measured experimental values and their theoretical predictions must be expected for capillary flows than for flow through small orifices of the same diameter. An uncertainty factor in the neighborhood of 2 appears not to be unreasonable at all.

E. Application to PuO_2 Leak Problems.

This report has derived various equations for leak flow through small passageways. Which of these equations is applicable for determining the gas leak rate of the PuO_2 container in a fire, depends mainly on:

a) the internal conditions in the container (type of gas, its pressure and temperature)

b) the geometry (size and shape) of the leak.

The internal conditions, which develop during a fire can best be determined by tests. Presumably this has been done, since it has been established that peak pressures are expected to be around 1000 psi and peak temperatures around 1700°F. The composition of the gas is known to be a mixture of helium and water vapor.

The geometry of the leak has a decisive influence on the leak rate since the flow may increase under certain conditions with the fourth power of the diameter. A direct measurement of the size and shape of the leak hole is practically impossible. Therefore it must be determined indirectly by the following procedure: If the leak rate is measured under well known internal and external conditions, the diameter of the leak hole could be computed with equation (17), if it is assumed that the leak hole is a sharp-edged circular orifice. Or, if it is assumed that leak were a straight, smooth capillary tube of a certain length, then the diameter of the tube could be determined with the aid of equations (22) or (27). Which of these equations is applicable in a particular case, depends on the Mach and Reynolds numbers of the flow. E.g. if the incompressible flow equation is used first and yields an exit flow velocity with $M > 0.5$, the compressible flow equation should be used instead.

Since by such an approach under certain arbitrary assumptions the diameter of the leak hole becomes known, it is then possible to determine analytically, what the leak rate would be under the assumed fire conditions, provided the leak hole size and shape does not change with temperature.

Admittedly, the arbitrariness of the assumptions in regards to the leak hole geometry is a serious drawback in this approach. The only definitive assertion, which can be made, is a statement on the maximum possible leak rate, which would result if the leak were assumed to be an orifice. One could then state: Under no circumstances will the leakage from the container be larger than the rate computed for an orifice leak. (For the same internal conditions, the leak rate through an orifice is always larger than the leak rate through a capillary of the same diameter, since the capillary generates frictional losses, which are not present in strict orifice flow).

This kind of approach may not be entirely satisfactory, however, since it could easily overestimate the actual leakage flow, possibly by one or two orders of magnitude. In order to improve on the approach, it appears possible to take flow measurements at various conditions (e.g. by changing the Δp across the leak hole), so that a whole series of test points could be generated through which a curve could then be drawn. This would allow to make a more reliable extrapolation of the leak flow rate to the anticipated fire conditions.

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APPENDIX

The general topic of leaks and leak testing is cluttered with different nomenclature and sets of units. The most important term involved is leakage, or mass flow rate, and has the dimensions of density times volume divided by time. Usually the units of atmospheric cm^3 per second (atm cm^3/sec) are being used. Note, that this is not a volumetric flow. The volumetric flow past a point is sometimes called the speed and its units are volume divided by time (cm^3/sec). The following table contains conversion factors for leakage or flow rates.

Table 1. LEAKAGE RATE (FLOW) CONVERSION FACTORS

TO CONVERT FROM	TO	MULTIPLY BY
Atm cc/sec	Micron liters/sec Micron cu ft/hr Torr liters/sec	7.60×10^2 9.66×10^4 7.60×10^{-1}
Micron liters/sec	Atm cc/sec Micron cu ft/hr Torr liters/sec	1.32×10^{-3} 1.27×10^2 1.00×10^{-3}
Micron cu ft/hr	Atm cc/sec Micron liters/sec Torr liters/sec	1.04×10^{-5} 7.87×10^{-3} 7.87×10^{-6}
Torr liters/sec	Atm cc/sec Micron liters/sec Micron cu ft/hr	1.32 1.00×10^3 1.27×10^5

In order to provide a better insight into the quantities involved when leakage rates are expressed in atm cm^3/sec, Table 2 is presented below. The table gives a vivid impression of the wide range (in this case 13 orders of magnitude) over which leak rates may have significance. For the particular application discussed in this report, it has been decided that any leakage below 10^{-7} atm cm^3/sec can be disregarded.

Table 2. LEAK RATE COMPARISON

Air				R-12* at Standard Conditions	
Atm cc/sec	Cu. in/day	Time to fill one cu. in.	Micron cu. ft/hr	Oz/year	Time for one lb to leak (yrs)
10	53,000	1.63 sec.	9.66×10^5	60,000	.0003
5	26,500	3.26 sec.	4.83×10^5	33,000	.0006
1	5,300	16.3 sec.	9.66×10^4	6,000	.0029
.5	2,650	32.6 sec.	4.83×10^4	3,300	.0058
1×10^{-1}	530	163 sec.	9660	600	.0288
5×10^{-2}	265	326 sec.	4830	330	.058
1.8×10^{-2}	100	14.4 min.	1720	100	0.16
1×10^{-2}	53	23.83 min.	966	60	.288
5×10^{-3}	26.5	54.33 min.	483	33	.58
1.8×10^{-3}	10	144 min.	172	10	1.6
1×10^{-3}	5.3	238.3 min.	96.6	6	2.88
5×10^{-4}	2.65	543.3 min.	48.3	3.3	5.8
1.8×10^{-4}	1	1 day	17.2	1	16
1×10^{-4}	.53	39.7 hrs	9.66	0.6	28.8
9×10^{-5}	0.5	2 days	8.6	0.5	32
5×10^{-5}	.26	3.77 days	4.83	.33	58
1.8×10^{-5}	0.1	10 days	1.72	0.1	160
1×10^{-5}	.053	16.54 days	.966	0.06	288
5×10^{-6}	.026	37.7 days	.083	0.033	580
1.8×10^{-6}	0.01	100 days	0.17	0.01	1600
1×10^{-6}	5.3×10^{-3}	165 days	.097	6×10^{-3}	2880
5×10^{-7}	2.6×10^{-3}	377 days	.048	3.3×10^{-3}	5800
1×10^{-7}	5.3×10^{-4}	4.517 yrs	.010	6×10^{-4}	2.88×10^4
5×10^{-8}	2.6×10^{-4}	10.32 yrs	.005	3.3×10^{-4}	5.8×10^4
1×10^{-8}	5.3×10^{-5}	45.17 yrs	0.001	6×10^{-5}	2.88×10^5
5×10^{-9}	2.6×10^{-5}	103.2 yrs	5×10^{-4}	3.3×10^{-5}	5.8×10^5
1×10^{-9}	5.3×10^{-6}	451.7 yrs	1×10^{-4}	6×10^{-6}	2.88×10^6
5×10^{-10}	2.6×10^{-6}	1032 yrs	5×10^{-5}	3.3×10^{-6}	5.8×10^6
1×10^{-10}	5.3×10^{-7}	4517 yrs	1×10^{-5}	6×10^{-7}	2.88×10^7
5×10^{-11}	2.6×10^{-7}	10320 yrs	5×10^{-6}	3.3×10^{-7}	5.8×10^7
1×10^{-11}	5.3×10^{-8}	45170 yrs	1×10^{-6}	6×10^{-8}	2.88×10^8
5×10^{-12}	2.6×10^{-8}	103200 yrs	5×10^{-7}	3.3×10^{-8}	5.8×10^8
1×10^{-12}	5.3×10^{-9}	451700 yrs	1×10^{-7}	6×10^{-9}	2.88×10^9

* Refrigerant 12 (Freon 12)

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