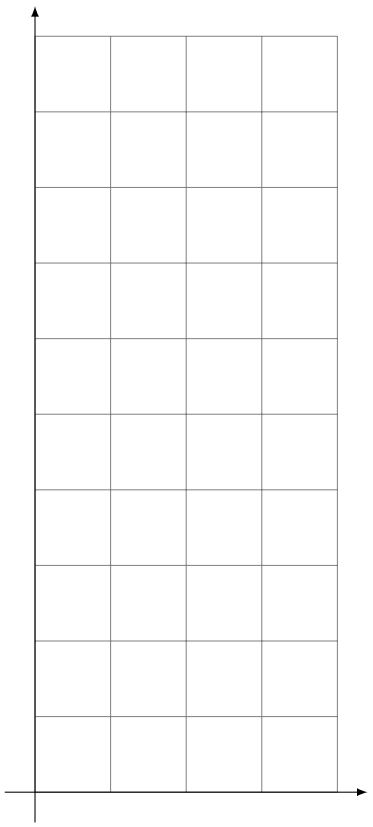
Elementary Linear Algebra - MATH 2250 - Day 15

Name:

1. If $\boldsymbol{b}$ is in the column space of A and P is the projection matrix onto the column space of A, then $P\boldsymbol{b} =$
2. If $\boldsymbol{b}$ is perpendicular to the column space of A and P is the projection matrix onto the column space of A, then $P\boldsymbol{b} =$
3. Recall that a projection matrix P has two key properties: P is symmetric and $P^2 = P$. Check that if P is a projection matrix, then $I - P$ is a projection matrix.

4. Consider the 4 points $(1, 1), (2, 4), (3, 9)$. Draw the three points in the xy-plane.



We want to find a line that the sum of the vertical distances of the above points from this line is the minimum possible. To do this we start with a parametric equation of such a line, that is, $y = Cx + D$. Then we write equations each time considering one of the points is on the line, for example, for the point $(1, 1)$ we get the equation $1 = m \cdot 1 + b$. Write all the three equations.

Form the matrix equation $A \begin{bmatrix} C \\ D \end{bmatrix} = \boldsymbol{b}$ for the above system.

Does the system have a solution? Why?

Form the normal equations given by $A^T A\hat{x} = A^T \boldsymbol{b}$, and solve it for $\hat{C}$ and $\hat{D}$.

Draw the line $y = \hat{C}x + \hat{D}$.

Find P the projection matrix.

Find $\boldsymbol{p} = P\boldsymbol{b}$.

Find the error vector $\boldsymbol{e} = \boldsymbol{b} - \boldsymbol{p}$.

Show $\boldsymbol{p}$ and $\boldsymbol{e}$ on the picture.
Evaluate $\boldsymbol{p} \cdot \boldsymbol{e}$.

Check that $\boldsymbol{e}$ is perpendicular to every column of A. What does it tell you about perpendicularity of $\boldsymbol{e}$ to the column space of A?