Elementary Linear Algebra - MATH 2250 - Day 20

Name:

1. Let us repeat a problem from previous worksheet: Using the cofactor formula evaluate the determinant of $A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$.

   Find A^{-1}.

   Recall that the cofactor $C_{ij} = (-1)^{i+j} \det M_{ij}$. Find all the cofactors of the matrix A and put them in a matrix C.

   Find AC^T.

   Compare C^T with A^{-1}.

2. Recall that if C is the cofactor matrix of A, then $AC^T = (\det A)\, I$. That is, for example, the first row of A times the first row of C is ____________, and the first row of A times the second row of C is ____________.

3. Is any row of C in the null space of A? Why?

4. Using Cramer's rule find the solution to $A\boldsymbol{x} = \boldsymbol{b}$, for $A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$, and $\boldsymbol{b} = (1, 0, 0)$.

5. Recall the formula for the cross product of $\boldsymbol{u} = (u_1, u_2, u_3)$ and $\boldsymbol{v} = (v_1, v_2, v_3)$ which is $\boldsymbol{u} \times \boldsymbol{v} = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix}$.

   Show that $\boldsymbol{w} = \boldsymbol{u} \times \boldsymbol{v}$ is perpendicular to $\boldsymbol{u}$ by calculating $\boldsymbol{w} \cdot \boldsymbol{u}$.

   Is $\boldsymbol{w}$ perpendicular to v, too? How do you know? (Explain using the properties of determinant)

6. Recall that $||\boldsymbol{u} \times \boldsymbol{v}|| = ||\boldsymbol{u}||\, ||\boldsymbol{v}||\, |\sin\theta|$. What is θ in terms of $\boldsymbol{u}$ and $\boldsymbol{v}$?

   Explain clearly when $||\boldsymbol{u} \times \boldsymbol{v}|| = 0$.

7. The are of a triangle with corners $(0, 0), (1, 1)$, and $(4, 2)$ is ________________ (give a number).

8. The are of a triangle with corners $(1, 1), (2, 2)$, and $(4, 2)$ is ________________ (give a number).