Elementary Linear Algebra - MATH 2250 - Day 18

Name:

Use the 'big formula' to answer the following questions:

1. $\begin{vmatrix} 0 & a & 0 \\ 0 & 0 & b \\ c & 0 & 0 \end{vmatrix} =$

2. $\begin{vmatrix} 0 & a & d \\ 0 & 0 & b \\ c & 0 & 0 \end{vmatrix} =$

3. $\begin{vmatrix} 0 & a & d \\ 0 & e & b \\ c & 0 & 0 \end{vmatrix} =$

4. $\begin{vmatrix} 0 & 0 & 0 & a \\ 0 & 0 & b & 0 \\ 0 & c & 0 & 0 \\ d & 0 & 0 & 0 \end{vmatrix} =$

5. $\begin{vmatrix} 0 & 0 & e & a \\ 0 & f & b & 0 \\ g & c & 0 & 0 \\ d & 0 & 0 & h \end{vmatrix} =$

6. How many 4×4 permutation matrices are there? What are their determinants?

7. How many terms are in the 'big formula' for the determinant of a 4×4 matrix? What are their 'signs'?

8. Let's go back to the pivot formula for determinant. Recall that if elimination turns A into U with $PA = LU$, where P is a permutation matrix, L is a lower triangular matrix and U is an upper triangular matrix with $d_1, d_2, \ldots, d_n$ in pivot positions, then $\det(L) =$ ______, $\det(P) =$ ______, and $\det(U) =$ __________________. So,

$$\det(A) = \underline{\hspace{6cm}}.$$

9. Using the big formula find $\begin{vmatrix} a & b & c & d \\ 0 & e & f & g \\ 0 & 0 & h & i \\ 0 & 0 & 0 & j \end{vmatrix} =$

10. Using the big formula find $\begin{vmatrix} 1 & 0 & a & 0 \\ 0 & 1 & b & 0 \\ 0 & 0 & c & 0 \\ 0 & 0 & d & 1 \end{vmatrix} =$

11. $\begin{vmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{vmatrix} =$

12. $\begin{vmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{vmatrix} =$

13. Evaluate the followings using the big formula:

$\begin{vmatrix} a & b \\ c & d \end{vmatrix} =$

$\begin{vmatrix} e & f \\ g & h \end{vmatrix} =$

$\begin{vmatrix} a & b & 0 & 0 \\ c & d & 0 & 0 \\ 0 & 0 & e & f \\ 0 & 0 & g & h \end{vmatrix} =$

14. Using the big formula evaluate $\begin{vmatrix} 2 & -1 \\ -1 & 2 \end{vmatrix} =$

15. Using the big formula evaluate $\begin{vmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{vmatrix} =$

16. Using the cofactor formula evaluate the determinant of $A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$.

Find A^{-1}.

Recall that the cofactor $C_{ij} = (-1)^{i+j} \det M_{ij}$. Find all the cofactors of the matrix A and put them in a matrix C.

Find AC^T.

Compare C^T with A^{-1}.

17. Using the cofactor formula evaluate $\begin{vmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{vmatrix} =$

18. What formula would you use to evaluate the determinant $\begin{vmatrix} 2 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 \\ 1 & 1 & 2 & 1 \\ 1 & 1 & 1 & 2 \end{vmatrix}$?

Evaluate it.

19. Recall the big formula for the determinant of an $n \times n$ matrix:

$$\det(A) = \sum_{\text{all } n! \text{ permutations}} (\det P)\, a_{1\alpha} a_{2\beta} \cdots a_{n\omega}.$$

Using this formula, explain if you multiply each a_{ij} by the fraction $\frac{i}{j}$, why is $\det(A)$ unchanged?

20. Use cofactor formula to evaluate $\begin{vmatrix} a & b & c & d \\ e & 0 & 0 & 0 \\ f & 0 & 0 & 0 \\ g & 0 & 0 & 0 \end{vmatrix} =$

21. What is the rank of the matrix in problem 20?

22. Let the 4×4 Vandermonde matrix be $V_4 = \begin{bmatrix} 1 & a & a^2 & a^3 \\ 1 & b & b^2 & b^3 \\ 1 & c & c^2 & c^3 \\ 1 & x & x^2 & x^3 \end{bmatrix}$. Explain why the determinant of V_4 contains x^3, but not x^4 or x^5.

The determinant is zero at $x =$ ______, ______, and ______. The cofactor of x^3 is $|V_3| = (b-a)(c-a)(c-b)$. Then $|V_4| =$ ______________________________.