Elementary Linear Algebra - MATH 2250 - Day 16

Name:

1. T ~~F~~ If $AB = I$ then $BA = I$.

$A = \begin{bmatrix} 1 & 0 \end{bmatrix}$ $\to AB = [1] = I$

$B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $BA = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \neq I$

2. T ~~F~~ If Q is an orthonormal matrix, then $\underbrace{Q^TQ = I}_{T}$ and $\underbrace{QQ^T = I}_{F}$.

ex.

3. ~~T~~ F If Q is an orthonormal matrix, then QQ^T is a projection matrix.

① $(QQ^T)^T = QQ^T$, ② $(QQ^T)^2 = Q\underbrace{Q^TQ}_{=I}Q^T = QQ^T$ ✓

4. Assume that we start with independent vectors v_1, v_2, and v_3, and proceed with the Gram-Schmidt algorithm, and produce w_1, w_2, and w_3. What relations hold between w_1, w_2, and w_3?

$\begin{cases} w_1 \perp w_2 \\ w_2 \perp w_3 \\ w_1 \perp w_3 \end{cases}$, $\begin{cases} \|w_1\| = 1 \\ \|w_2\| = 1 \\ \|w_3\| = 1 \end{cases}$

i.e. w_i's are mutually orthogonal, and length of each of them is 1.

5. Let $\boldsymbol{b} = (4, 0, 0, 0)$, $\boldsymbol{v} = (1, 1, 1, 1)$, and $\boldsymbol{w} = (1, -1, 1, -1)$. Find the projection of $\boldsymbol{b}$ onto $\boldsymbol{v}$ and call it $\boldsymbol{u}_1$. Find the projection of $\boldsymbol{b}$ onto $\boldsymbol{w}$ and call it $\boldsymbol{u}_2$. Find the projection of $\boldsymbol{b}$ onto the space spanned by $\boldsymbol{v}$ and $\boldsymbol{w}$,

$A^TA = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$

$(A^TA)^{-1} = \frac{1}{4}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\boxed{\text{check with some software!}}$

$A = \begin{bmatrix} 1 & 1 \\ 1 & -1 \\ 1 & +1 \\ 1 & 1 \end{bmatrix}$, $P = \frac{1}{4}\begin{bmatrix} 1 & 1 \\ 1 & -1 \\ 1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 2 & 0 & 2 & 0 \\ 0 & 2 & 0 & 2 \\ 2 & 0 & 2 & 0 \\ 0 & 2 & 0 & 2 \end{bmatrix}$

$u_3 = Pb = \frac{1}{8}\begin{bmatrix} 4 \\ 0 \\ 4 \\ 0 \end{bmatrix} = \begin{bmatrix} 1/2 \\ 0 \\ 1/2 \\ 0 \end{bmatrix}$

and call it $\boldsymbol{u}_3$. What is the relation between $\boldsymbol{u}_1, \boldsymbol{u}_2$, and $\boldsymbol{u}_3$.

$u_3 = \frac{1}{4}(u_1 + u_2)$

6. Consider the vectors $\boldsymbol{a}_1 = (1,1,1,1)$, $\boldsymbol{a}_2 = (1,1,1,0)$, and $\boldsymbol{a}_3 = (1,1,0,0)$. Proceed with Gram-Schmidt algorithm and produce 3 vectors $\boldsymbol{q}_1, \boldsymbol{q}_2$, and $\boldsymbol{q}_3$. Recall that in the QR-decomposition of a matrix A, Q is

Check with some software.

found by Gram-Schmidt algorithm and $R = Q^T A$. Let $A = \begin{bmatrix} \boldsymbol{a}_1 & \boldsymbol{a}_2 & \boldsymbol{a}_3 \end{bmatrix}$.

Find the QR-decomposition of A.

7. Compare $C(A)$ and $C(Q)$.

$C(A) = C(Q)$. (why?)

8. Recall that if $A = QR$, where Q is orthnormal and R is upper-triangular, then instead of solving $A\boldsymbol{x} = \boldsymbol{b}$, one can easily solve $R\hat{\boldsymbol{x}} = Q^T\boldsymbol{b}$. Solve the equation $A\boldsymbol{x} = \boldsymbol{b}$, for A as above and $\boldsymbol{b} = (1,0,0,0)$.

Check with some software.