Elementary Linear Algebra - MATH 2250 - Day 6

Name:

1. Mark each of the followings as True or False (Explain why when True, or give an example when False).

   [T] [F] The set of all vectors that lie on a line through origin form a vector space.

   [T] [F] The set of all vectors that lie on a plane through origin form a vector space.

   [T] [F] The union of any two vector spaces is a vector space.

   [T] [F] The intersection of any two vector spaces is a vector space.

   [T] [F] The set of all vectors that lie on the union of two distinct lines through origin form a vector space.

   [T] [F] The set of all vectors that lie on a line through origin form a subspace of a plane that contains that line.

   [T] [F] The set of all vectors that lie on the union of two distinct parallel lines form a vector space.

   [T] [F] Let V be a subspace of W and W be a subspace of U. Then V is a subspace of U.

T F (optional) There are sets S and T, NOT vector spaces, such that $S \cup T$ is a vector space.

2. Let $A = \begin{bmatrix} 2 & 0 & 2 \\ 2 & 2 & 0 \\ 4 & 4 & 0 \\ 4 & 6 & -2 \end{bmatrix}$.

(a) Fill in the blank: The column space of A (that is, $C(A)$) is a subspace of _____.

(b) Is the columns space of A the whole space specified in part (a)? Why?

(c) Does the equation $A\boldsymbol{x} = \boldsymbol{b}$ have a solution for any right hand side $\boldsymbol{b}$? Explain.

(d) Does the equation $A\boldsymbol{x} = \boldsymbol{0}$ have a solution? Explain.

(e) Does the equation $A\boldsymbol{x} = \begin{bmatrix} 2 \\ 2 \\ 4 \\ 4 \end{bmatrix}$ have a solution? Explain.

(f) Does the equation $A\boldsymbol{x} = \begin{bmatrix} 1 \\ 1 \\ 2 \\ 2 \end{bmatrix}$ have a solution? Explain.

(g) Does the equation $A\boldsymbol{x} = \begin{bmatrix} 0 \\ 2 \\ 4 \\ 6 \end{bmatrix}$ have a solution? Explain.

(h) Does the equation $A\boldsymbol{x} = \begin{bmatrix} 4 \\ 2 \\ 4 \\ 2 \end{bmatrix}$ have a solution? Explain.

(i) Does the equation $A\boldsymbol{x} = \boldsymbol{b}$ have a solution for any $\boldsymbol{b}$ in the columns space of A? Explain.

(j) What are all the right hand sides $\boldsymbol{b}$ such that $A\boldsymbol{x} = \boldsymbol{b}$ has a solution?

(k) Does the equation $A\boldsymbol{x} = \boldsymbol{0}$ have a nonzero solution? Explain.

(l) What are all the solutions to the equation $A\boldsymbol{x} = \boldsymbol{0}$?

(m) What is the null space of A?