Elementary Linear Algebra - MATH 2250 - Day 11

Name:

1. Start with the vectors $\boldsymbol{v}_1 = (0, 2, 1)$ and $\boldsymbol{v}_2 = (2, 1, 0)$.

   (a) Are they linearly independent? Why?

   (b) Are they a basis for any space?

   Yes, let $A = \begin{bmatrix} 0 & 2 \\ 2 & 1 \\ 1 & 0 \end{bmatrix}$, they form a basis for $C(A)$. same spaces!

   or let $V = \{c_1 v_1 + c_2 v_2 : c_1, c_2 \in \mathbb{R}\}$, then $\{v_1, v_2\}$ is a basis for V.

   (c) What space V do they span?

   (d) What is the dimension of V?

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   (e) Which matrices A have V as their column space? any matrix of the form:

   $$\left[c_{11}v_1 + c_{12}v_2 \;\middle|\; c_{21}v_1 + c_{22}v_2 \;\middle|\; c_{31}v_1 + c_{32}v_2 \;\middle|\; \cdots \right]$$

   as long as two of the columns are lin. ind. for example if $\begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix}$ has full rank. In general if $\begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \\ c_{31} & c_{32} \\ \vdots & \vdots \end{bmatrix}$ has full col. rank.

(f) Which matrices A have V as their null space? Any $A = \left[A_1 \middle| A_2 \middle| A_3 \right]$ s.t.

$N(A) = \left\langle \begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} \right\rangle$

$$\begin{cases} 2A_2 + A_3 = 0 \rightarrow A_3 = -2A_2 \rightarrow A_3 = 4A_1 \\ \text{and} \\ 2A_1 + A_2 = 0 \rightarrow A_2 = -2A_1 \end{cases}$$

That is, $A = \left[v \middle| -2v \middle| 4v \right]$, where $v \neq 0$.

(g) Describe all vectors $\boldsymbol{v}_3$ that *complete* a basis $\boldsymbol{v}_1, \boldsymbol{v}_2, \boldsymbol{v}_3$ for $\mathbb{R}^3$.

Anything not in the plane spanned by v_1, v_2

i.e. anything not in the form $a v_1 + b v_2$.

2. **(Important)** Suppose $\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_n$ is a basis for $\mathbb{R}^n$ and the $n \times n$ matrix A is invertible. Show that $A\boldsymbol{v}_1, A\boldsymbol{v}_2, \ldots, A\boldsymbol{v}_n$ is also a basis for $\mathbb{R}^n$.

In the book.

3. Write a rank 3 matrix $A_{4\times 7}$ and find its four fundamental subspaces, by describing a basis for each of them.