Elementary Linear Algebra - MATH 2250 - Day 20

Name:

1. Let us repeat a problem from previous worksheet: Using the cofactor formula evaluate the determinant of $A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$.

   Find A^{-1}.

   Recall that the cofactor $C_{ij} = (-1)^{i+j} \det M_{ij}$. Find all the cofactors of the matrix A and put them in a matrix C.

   Find AC^T.

   $AC^T = \det(A)\, I$

   Compare C^T with A^{-1}.

   $C^T = \frac{1}{\det A} A^{-1}$

2. Recall that if C is the cofactor matrix of A, then $AC^T = (\det A)\, I$. That is, for example, the first row of A times the first row of C is $\underline{\ \det(A)\ }$, and the first row of A times the second row of C is $\underline{\ \ 0\ \ }$.

3. Is any row of C in the null space of A? Why?

if row i of C is in $N(A)$ then $A(C_i)^T = 0$

But $A(C_i)^T = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ \det(A) \\ 0 \\ \vdots \\ 0 \end{bmatrix} \leftarrow i\text{th row.}$

4. Using Cramer's rule find the solution to $A\boldsymbol{x} = \boldsymbol{b}$, for $A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$, and $\boldsymbol{b} = (1, 0, 0)$.

5. Recall the formula for the cross product of $\boldsymbol{u} = (u_1, u_2, u_3)$ and $\boldsymbol{v} = (v_1, v_2, v_3)$ which is $\boldsymbol{u} \times \boldsymbol{v} = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix}$.

Show that $\boldsymbol{w} = \boldsymbol{u} \times \boldsymbol{v}$ is perpendicular to $\boldsymbol{u}$ by calculating $\boldsymbol{w} \cdot \boldsymbol{u}$.

$w \cdot u = (u \times v) \cdot u = (u \times v)^T u = \begin{bmatrix} u_2v_3 - u_3v_2 & u_3v_1 - u_1v_3 & u_1v_2 - u_2v_1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$

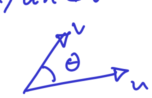
$= (\cancel{u_1u_2v_3} - \cancel{u_1u_3v_2}) + (\cancel{u_2u_3v_1} - \cancel{u_2u_1v_3}) + (\cancel{u_3u_1v_2} - \cancel{u_3u_2v_1}) = 0$

Is $\boldsymbol{w}$ perpendicular to v, too? How do you know? (Explain using the properties of determinant)

Yes, $u \times v = -(v \times u) \Rightarrow (u \times v) \cdot v = -(v \times u) \cdot v = -0 = 0$

exchange the last two rows

6. Recall that $||\boldsymbol{u} \times \boldsymbol{v}|| = ||\boldsymbol{u}||\, ||\boldsymbol{v}||\, |\sin\theta|$. What is θ in terms of $\boldsymbol{u}$ and $\boldsymbol{v}$? The angle between u, and v



Explain clearly when $||\boldsymbol{u} \times \boldsymbol{v}|| = 0$.

when $u = 0$ or $v = 0$, or when $\sin\theta = 0$,

that is when $\theta = 0$, or π $\Rightarrow$ i.e. when u and v are parallel.

7. The area of a triangle with corners $(0, 0), (1, 1)$, and $(4, 2)$ is $\underline{\qquad 1 \qquad}$ (give a number). $\left\| \begin{matrix} 1 & 1 \\ 4 & 2 \end{matrix} \right\| = |-2| = 2$

8. The area of a triangle with corners $(1, 1), (2, 2)$, and $(4, 2)$ is $\underline{\qquad 1 \qquad}$ (give a number).

$(0,0) \quad (1,1) \quad (3,1)$ $\qquad \left\| \begin{matrix} 1 & 1 \\ 3 & 1 \end{matrix} \right\| = |-2| = 2$