Elementary Linear Algebra - MATH 2250 - Exam 3

Please read and sign (papers without printed name and signature will not be graded):
"On my honor, I have neither given nor received unauthorized aid in doing this assignment."

Print name: ______________________________ Sign: ______________________________

1. Let $A = \begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$.

   (a) What are the eigenvalues of A? Explain.

   (b) What is the rank of A? Explain.

   (c) Compute in simplest form e^{tA}.

2. Consider the matrix $A = \begin{bmatrix} x & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ with parameter x in the $(1,1)$ position.

(a) Specify all numbers x, if any, for which A is positive definite. Explain.

(b) Specify all numbers x, for which e^A is positive definite. Explain.

3. If A is symmetric, which of these four matrices are necessary positive definite. Explain.

$$A^3, (A^2 + I)^{-1}, A + I, e^A.$$

4. P is a 3×3 permutation matrix. List all the possible eigenvalues of P.

5. We are told that A is 2×2, symmetric, and Markov, and one of the real eigenvalues is y with $-1 < y < 1$.

   (a) What is the matrix A in terms of y?

   (b) Compute the eigenvectors of A.

   (c) What is A^{2014} in simplest form?

6. (*Optional: extra credit*) Suppose C is $n \times n$ and positive definite. If A is $n \times m$ and $M = A^T CA$ is not positive definite, find the smallest eigenvalues of M. Explain.