## Elementary Linear Algebra - MATH 2250 - Day 7

## Name:

Mark the matrices in reduced row echelon form. If they are not in rref, explain why.

- 2. Let  $A = \begin{bmatrix} 2 & 0 & 2 \\ 2 & 2 & 0 \\ 4 & 4 & 0 \\ 4 & 6 & -2 \end{bmatrix}$ .
  - (a) Find the row echelon form of A.

(b) Find the reduced row echelon form of A.

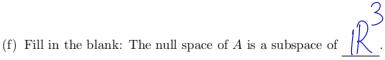
(c) How many free variables are there in the system of linear equations  $\begin{bmatrix} 2 & 0 & 2 \\ 2 & 2 & 0 \\ 4 & 4 & 0 \\ 4 & 6 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$ ? What are those?

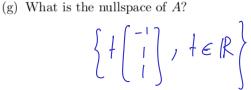
$$\mathbf{s} \begin{bmatrix} 2 & 0 & 2 \\ 2 & 2 & 0 \\ 4 & 4 & 0 \\ 4 & 6 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \blacksquare \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$
? What

(d) How many pivot variables are there in the above system? What are those?

(e) What is the nullspace matrix of  $\underline{A}$ 

$$R = \text{(ref(A) = } \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 - 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \sim P N = \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$$





(compare with previous worksheet)

(h) Check that the null space of A is actually a vector space. That is, for any real number c and vectors vand w that Av = 0 and Av = 0, then

• 
$$A(v+w)=0$$
, and

• 
$$A(v+w)=0$$
, and  $A(v+w)=0$  and  $A$ 

(i) Find a solution to 
$$Ax = \begin{bmatrix} 2 \\ 0 \\ 0 \\ -2 \end{bmatrix}$$
, and call it  $v$ .  $\bigvee = \begin{bmatrix} 0 \\ 6 \\ 1 \end{bmatrix}$ 

$$V+\omega = \begin{bmatrix} -1\\1\\2 \end{bmatrix}$$

$$A \begin{bmatrix} -1\\1\\2 \end{bmatrix} = \begin{bmatrix} 2\\0\\0\\-2 \end{bmatrix}$$

(k) How many solution are there to the equation 
$$Ax = \begin{bmatrix} 2 \\ 0 \\ 0 \\ -2 \end{bmatrix}$$
?

infinitely many, because for every 
$$w \in N(A)$$
  
the vector  $\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} + w$  is a solution to  $A \times = \begin{bmatrix} 2 \\ 6 \\ 0 \\ -2 \end{bmatrix}$ 

(l) Take your favorite two vectors that solve the equation  $Ax = \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}$ , and call them v and w. Does v + wsolve the equation? How about 2v? How about -3w?

$$V = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad \omega = \begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix}$$

$$A(2v) \neq \begin{bmatrix} 2 \\ 0 \\ -2 \end{bmatrix}$$

$$A(-3\omega) \neq \begin{bmatrix} 2 \\ 0 \\ -2 \end{bmatrix}$$

- $A(2v) \neq \begin{bmatrix} 2 \\ 0 \\ -2 \end{bmatrix}$   $A(-3w) \xrightarrow{} \neq \begin{bmatrix} 2 \\ 0 \\ -2 \end{bmatrix}$ (m) Does the zero vector solve the equation  $Ax = \begin{bmatrix} 2 \\ 0 \\ 0 \\ -2 \end{bmatrix}$ ? Nope.  $AO = O \neq \begin{bmatrix} 2 \\ 0 \\ 6 \\ -2 \end{bmatrix}$
- (n) Does the set of all solutions to the equation  $Ax = \begin{bmatrix} 2 \\ 0 \\ 0 \\ -2 \end{bmatrix}$  form a vector space? Why?

(o) (Optional) Find all the solution to  $Ax = \begin{bmatrix} 2 \\ 0 \\ 0 \\ -2 \end{bmatrix}$ .

$$\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} + C \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$
,  $C \in \mathbb{R}$ .