Name:

1. Let A, B, and C be invertible, and A^{-1}, B^{-1}, and C^{-1} be their inverses, respectively. What is the inverse of ABC, in terms of A^{-1}, B^{-1}, and C^{-1}?

   $(ABC)^{-1} =$

   Check your solution.

2. Let
   $$M = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -3 & 1 \end{bmatrix}.$$

   What is the inverse of M? (Hint: note that M is the product of three elementary matrices.) How can you

   check your solution without finding M? Check your solution.

3. How many 4×4 permutation matrices are there?

4. How many 5×5 permutation matrices are there? Explain.

5. If $A = LU$ is the LU-decomposition of A, for a lower triangular matrix L and an upper triangular matrix U, then to solve $A\boldsymbol{x} = \boldsymbol{b}$, one can solve $LU\boldsymbol{x} = \boldsymbol{b}$, by solving $L\boldsymbol{y} = \boldsymbol{b}$ first, and then $U\boldsymbol{x} = \boldsymbol{y}$. Solve the matrix equation $A\boldsymbol{x} = \boldsymbol{b}$, using LU-decomposition of A when

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 \\ 1 & 3 & 6 & 10 \\ 1 & 4 & 10 & 20 \end{bmatrix}, \qquad \boldsymbol{b} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

6. Find the LU-decomposition of the matrix

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix}$$

What do you expect about the LU-decomposition of a lower-triangular matrix?

What about the LU-decomposition of an upper-triangular matrix?

7. Find the LU-decomposition of $A = \begin{bmatrix} 1 & 2 & 0 & 0 \\ 2 & 3 & 1 & 0 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 3 & 4 \end{bmatrix}$.

8. Find the inverse of

$$A = \begin{bmatrix} 1 & -1 & 1 & -1 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Solve $Ax = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$

9. Find A^2, A^3, A^4, and A^5 for $A = \begin{bmatrix} 0 & -2 & 0 & 0 \\ 0 & 0 & -2 & 0 \\ 0 & 0 & 0 & -2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$.

10. What rows or columns or matrices do you multiply to find

    (a) the third column of AB?

    (b) the first row of AB?

    (c) the entry in row 3, column 4 of AB

    (d) the entry in row 1, column 1 of CDE?

11. Compute:

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} =$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} =$$

$$\begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} =$$