

Elementary Linear Algebra - MATH 2250 - Quiz 14

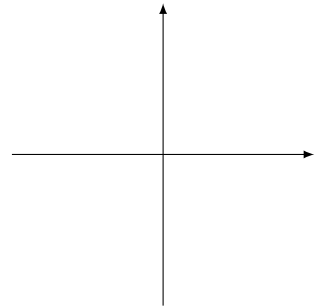
Name:

1. ☐ T ☐ F The projection of $2\mathbf{a}$ onto \mathbf{b} is equal to 2 times the projection of \mathbf{a} onto \mathbf{b} . Give a detailed example.

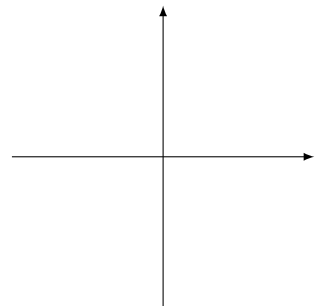
2. ☐ T ☐ F The projection of \mathbf{a} onto $2\mathbf{b}$ is equal to 2 times the projection of \mathbf{a} onto \mathbf{b} . Give a detailed example.

3. Let $\mathbf{a} = (1, 1, 1, 1)$ and $\mathbf{b} = (1, -1, 1, -1)$.

Find \mathbf{p} , the projection of \mathbf{a} onto \mathbf{b} , and draw all **three** vectors.



4. Let $\mathbf{a} = (1, 1, 1, 1)$ and $\mathbf{b} = (2, 2, 2, 2)$. Find \mathbf{q} , the projection of \mathbf{b} onto \mathbf{a} , and draw all **three** vectors.



5. If $\mathbf{b} = c\mathbf{a}$, for a real number c , then projection of \mathbf{b} onto \mathbf{a} is _____.

6. If $\mathbf{b} = c\mathbf{a}$, for a real number c , then projection of \mathbf{a} onto \mathbf{b} is _____.

7. Let $\mathbf{a} = (1, 2, 0, 2)$.

(a) Find $\mathbf{a}^T \mathbf{a}$. Is it nonzero?

(b) Find $\mathbf{a} \mathbf{a}^T$. What is its rank?

(c) Find the projection matrix P that projects every vector onto \mathbf{a} . What is its rank?

(d) Find the column space of P .

(e) Is P symmetric?

(f) If you find the vector $P\mathbf{b}$ for some vector \mathbf{b} , where do you expect it to live? Be as precise as possible.

(g) Let $\mathbf{b} = (2, 0, 3, 6)$. Find $\mathbf{a}^T \mathbf{b}$, and $\frac{\mathbf{a}^T \mathbf{b}}{\mathbf{a} \mathbf{a}^T}$.

(h) Find $P\mathbf{b}$.

(i) Note that $P\mathbf{b}$ is a multiple of \mathbf{a} . What multiple is it?

(j) Find $P^2\mathbf{b}$.

(k) What do you expect about $P^3\mathbf{b}$, $P^4\mathbf{b}$ etc?

(l) Is $P^2 = P$?

(m) What about P^3 , P^4 and P^5 ?

8. Consider the equation $A\mathbf{x} = \mathbf{b}$, where $A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 2 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 6 \\ 0 \\ 0 \end{bmatrix}$. Does it have a solution?

Find $B = A^T A$.

Find $\mathbf{p} = A^T \mathbf{b}$.

Solve the system $B\hat{\mathbf{x}} = \mathbf{p}$.

Is $(A^T A)^{-1}$ invertible? Why? Find its inverse.

Find $(A^T A)^{-1} A^T \mathbf{b}$, and compare it with $\hat{\mathbf{x}}$.

Find $A(A^T A)^{-1} A^T \mathbf{b}$, and compare it with \mathbf{p} .

Find $\mathbf{e} = \mathbf{b} - \mathbf{p}$.

Find $\mathbf{b} - A\hat{\mathbf{x}}$, and compare it with \mathbf{e} .

Find $A^T \mathbf{e}$. Is \mathbf{e} orthogonal to $C(A)$? Why?

9. What is the projection matrix for projection onto column space of A ? Is it symmetric? What is P^2 ?

10. Draw a picture similar to the one in page 221 of the book for Problem 8.