MATH 2200-02 – Homework 2*
Fall 2012

Please provide the details of your work for each problem. All problems are partial credit.

1. (*2 points*) Find the derivative of the function f given by $f(x) = x + \sqrt{x}$ using the <u>definition</u> of derivative. Also state the domain of the function f and the domain of its derivative.

2. (*1 point*) Find an equation of the tangent line to the curve $y = x\sqrt{x}$ that is parallel to the line $y = 1 + 3x$.

3. (*1 point*) We've seen that the function $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = |x|$ is continuous on $\mathbb{R}$, but is not differentiable at exactly <u>one</u> point, namely $x = 0$. Define a function $g : \mathbb{R} \to \mathbb{R}$, explicitly, such that g is continuous on $\mathbb{R}$ but is not differentiable at exactly <u>two</u> points. (Hint: First try to visualize the graph of such a function, keeping in mind that differentiability fails at "sharp corners" might also help.)

4. (*2 points*) In what follows, each limit represents the derivative of some function f at some point $x = a$. State such an f and a in each case, and find the value of the limit without calculating the limit directly, but using the derivative formulas which you've learned.
Example: Given $\lim_{h\to 0} \frac{(1+h)^{10}-1}{h}$, we can see that $f(x) = x^{10}$, and $a = 1$, since $\frac{f(1+h)-f(1)}{h} = \frac{(1+h)^{10}-1}{h}$. So, the limit in question is in fact $f'(1)$. But $f'(x) = 10x^9$, thus $\lim_{h\to 0} \frac{(1+h)^{10}-1}{h} = f'(1) = 10$. (Note: You might come up with different functions and different points for the same limit, but it won't affect the value of the limit.)

1. $\lim_{h\to 0} \frac{\sqrt[3]{8+h}-2}{h}$.
2. $\lim_{h\to 0} \frac{\cos(\pi+h)+1}{h}$.

5. (*4 points*)

- Find the following limits, or show that they don't exist:
    1. $\lim_{x\to 1} \frac{\sin(x-1)}{x^2+x-2}$. (Hint: Recall that $\lim_{t\to 0} \sin t/t = 1$.)
    2. $\lim_{\theta\to 0} \frac{\cos\theta - 1}{\sin\theta}$. (Hint: Divide both numerator and denominator by θ and write the limit of quotient as the quotient of limits.)
- Find the derivative of the following functions:
    1. $y = x\sec(\sqrt{x})$.
    2. $y = \sqrt{x + \sqrt{x}}$.

*Submit on Friday, September 28 in class.

The following problem(s) are optional.

6. (*2 points*) (Optional)

1. Find equations of both lines through the point $(2, -3)$ that are tangent to the parabola $y = x^2 + x$.
2. Show that there is no line through the point $(2, 7)$ that is tangent to the parabola. (You may want to draw a diagram to see why.)

7. (*2 points*) (Optional Reading assignment) We have looked at $\lim_{x\to 0} \frac{\sin x}{x} = 1$ from different 'angles' in class, and we quickly went through a proof which is, not surprisingly, of geometric flavor. There is also one proof given on pages pages 66–67 of Prof. Strang's *Calculus* available here. You'll receive 2 bonus points by including a line in your paper indicating that you've read these pages. This will show you, at the very least, why we measure x in radians; another reason is given in exercise 87 in your textbook: What is $\frac{d}{d\theta} \sin\theta$ if θ is measured in degrees?