Watch What You Say

Although it has been many years, I still remember the story of Ricky as if it were only yesterday. I was a teacher consultant in a district where one of my roles was to support teachers in their classroom practice. A colleague had invited me to join one of her eighth-grade mathematics classes. The students were studying quadrilaterals, and our objective for the lesson was to discover what the students knew about quadrilateral properties. The lesson was informed by the van Hiele model of geometric thinking (Fuys, Geddes, and Tischler 1988). The van Hieles hypothesized that students' thinking begins at a visual level. At this level, the learner recognizes shapes by an overall gestalt based on visual clues rather than defining properties. From this level, the model suggests, learners move to a level of analysis that uses defining properties to categorize shapes and classes of shapes.

By Sally K. Roberts

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Our initial objective was to determine if the students were operating at a purely visual level of reasoning or if they were using properties to characterize specific quadrilaterals. We planned to build on this information during subsequent lessons to facilitate students' understanding of the logical interrelatedness of quadrilateral properties and the inclusionary nature of the quadrilateral "family tree" (Craine and Rubenstein 1993).

The Lesson

I began with a playful scenario, telling students that I needed help with my homework. Their task was to describe, over the phone, how to draw specific quadrilaterals. I started with a square. One student offered the following defining properties: "It has four equal sides." I drew a shape on the board (see **fig. 1**).

After a moment of surprise, the students began to chuckle. I knew I had their attention and they were game for the challenge. Another student spoke up and added, "It has four equal sides, and it is closed."

Again I drew a figure using all of the suggested properties (see **fig. 2**).

At this point, the students struggled a bit to find the words that would describe the missing property. Suggestions such as "it can't be slanty over" were common. I pretended not to understand what they were telling me until one student suggested that I needed to include the property of right angles. Aha, now I could draw the desired closed, equiangular, equilateral figure (see **fig. 3**). I was pleased to see that through this sequence, the students were beginning to think about what properties were necessary and sufficient to define a figure.

After drawing the square, I asked the students to describe a rectangle. They were on a roll, knowing they needed to include the properties "closed" and "right angles." At this point, Ricky chimed in: "It has two long sides and two short sides." With Ricky's suggestion, I knew I was facing a teaching dilemma. Ricky's image of a rectangle was based on his interpretation of the "perfect" or "protocol" rectangle (Hannibal 1999; Schifter 1999; Woleck 2003). Clearly, he was operating at a visual level of geometric thinking. I also realized that a better sequence would have been to begin with the least specific figure and move toward the square, with each figure building on previously defined properties. For example, in the most general case of a quadrilateral, the required properties are "four sides" and "closed." A parallelogram adds the property of "opposite sides congruent or parallel," followed by the rectangle on one branch, which adds the property of "equal angles," and the rhombus on the other branch, with "equal sides." Finally, the "marriage of the rhombus and the rectangle" produces a square that has all the given properties of its ancestors (see **fig. 4**).

However, I had started with the most specific case of a quadrilateral, and there was no turning back. I needed to find a way to respond to Ricky's suggestion that would help him unravel his misconceptions. In an effort to bring an authoritative voice to the conversation, I decided to evoke the word according to mathematicians. I told Ricky that it is possible for a rectangle to have two long sides and two short sides; however, "mathematicians don't have a rule that rectangles have to have this property." But Ricky was not about to let me slide on this one. He questioned, "You mean my kindergarten teacher lied to me?" As a former kindergarten teacher, I was cut to the quick. To add insult to injury, I was pretty sure that somewhere in my past I had offered the same property—"a rectangle has two long sides and two short sides"—as a defining property of a rectangle to a class of eager, unsuspecting kindergarteners. To Ricky, I quietly responded, "No, Ricky, I am sure your kindergarten teacher did not lie to you, but she may have misrepresented the truth."

I distinctly remember attending a professional development activity where the facilitator emphasized the fact that all squares were rectangles, but not all rectangles were squares. As I listened, this sounded more like double talk than important information. I had discounted the statement, convinced that the person leading the session was showing off his facility with language. I had thought, *why* is it important for students to understand that squares are a subset of rectangles, which are a subset of parallelograms, which are a subset of the larger set of quadrilaterals? It was not until later in my career, when I was teaching a geometry course for teacher candidates, that I began to better understand why this was indeed an important idea. Craine and Rubenstein (1993) make an excellent case for how

Figure 1

Four equal sides

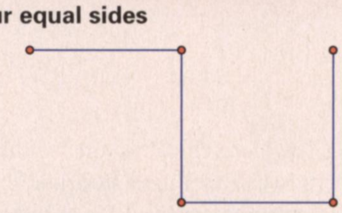


Figure 2

Four equal sides and closed

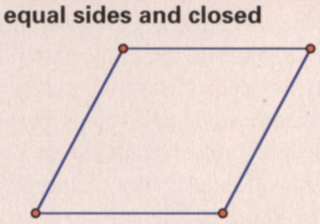


Figure 3

Square

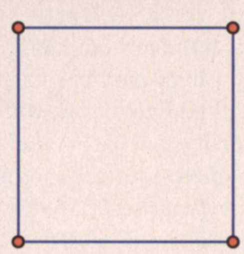
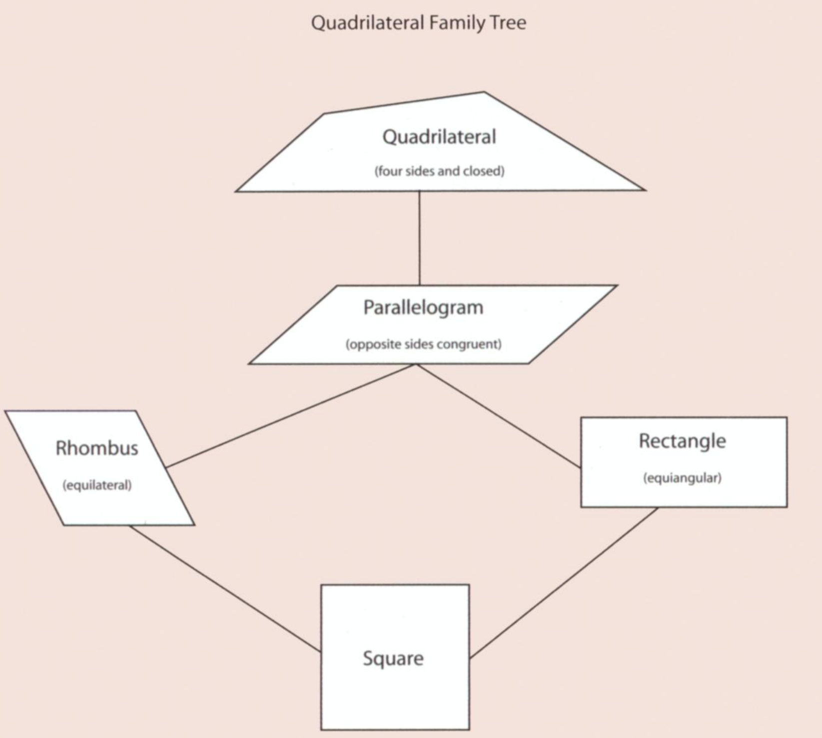


Figure 4

Quadrilateral Family Tree



understanding the hierarchical nature of the quadrilateral properties can facilitate students' learning, noting that the inclusive nature of the properties carries over to concepts of symmetry of figures, diagonals of quadrilaterals, area relationships, properties of midpoint quadrilaterals, and so forth (p. 32).

Students who are introduced to the interrelated and inclusive nature of the quadrilateral family tree early in their education will be better prepared to encounter more advanced topics in geometry. Understanding the inclusive nature of definitions benefits students and makes their learning not only holistic but also more efficient. Statements that are proved for the least specific member in the family tree hold true for all the family members that follow. For example, the properties of a parallelogram will also be true for a rectangle, rhombus, and square (Craine and Rubenstein 1993).

Although my career path has changed over the last two decades, and I now work with teacher candidates at a university, the story of Ricky comes back to haunt me (or, more precisely, inform me) about the importance of the powerful role of the teacher in helping students construct understanding. Ricky's kindergarten teacher made a lasting impression, as do so many early childhood educators. Young children are eager to learn; they worship their teachers and tend to believe all that they say. However, the willingness of students to hang on every word is inversely related to years in school. By third grade, students begin to suspect what adults say and are more willing to spar with them; the frequency of such behavior increases over time. A healthy dose of skepticism is an admirable quality in students, but teachers must watch what they say. In my days as a kindergarten teacher, I had little notion of the impact I had on framing students' future learning. I am sure I used the "two long sides, two short sides" property to define a rectangle with no thought of the possible repercussions. My words were reinforced by "cute" worksheets that asked students to color all the rectangles blue and all the squares yellow. Had I been true to the underlying mathematical concept, the squares would have ended up green as the students mixed blue and yellow.

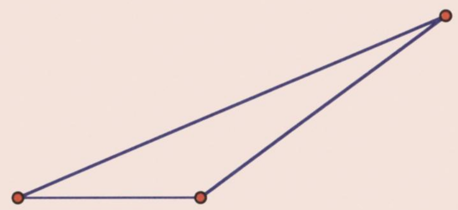
So, how can we avoid planting seeds of knowledge that must be unlearned or will be challenged by our colleagues later in our students' educational experiences? I can't help but think of the advice often offered to parents when their children raise questions about the facts of life: "Give accurate, age-appropriate information" (Talk with Your Kids 2006) or "Be brief. Don't go into a long explanation." "Be honest." (American Academy of Pediatrics 2000). Young children do not need an elaborate explanation of the inclusive nature of the quadrilateral family, but we might say, "Rectangles and squares belong to the same family," adding that "a square is a special rectangle." Oberdorf and Taylor-Cox (1999) caution that the practice of providing "incorrect or incomplete information…in hopes of re-teaching and altering paradigms later" has unfortunately become an accepted practice (p. 340). The case of Ricky demonstrates that the "undoing" of misconceptions is easier said than done. The challenge for teachers is to be true to the mathematics while presenting content in a meaningful way that is developmentally appropriate.

Facilitating Classroom Conversations

Classroom conversations among students, facilitated by the teacher, are opportunities to help students clarify their thinking and to introduce geometric terms. Young children often do not distinguish between integral attributes associated with shape properties (number of sides or angles) and

Figure 5

Scalene obtuse triangle



nonintegral attributes (size, orientation, or color). This stage of development fits with the visual level of thinking described in the van Hiele literature. The teacher's role is to help clarify these distinctions, thus moving students toward shape analysis that uses defining properties.

Choosing Models and Representations

It is important to pay attention to the models, representations, and materials we use in our classrooms. Hannibal (1999) noted in her research with three- to six-year-old children that children referred to how shapes varied from the "real" or "perfect" shape (p. 354). For example, if students only have an opportunity to see illustrations of isosceles or equilateral triangles, they generalize triangle to mean only these specific cases. When presented with an obtuse scalene triangle (see **fig. 5**), students often do not recognize that this as a "real" triangle (Woleck 2003).

Unfortunately, many of the examples that young children encounter in books, toys, or commercial manipulatives only offer examples of the prototypical image of a shape or the "perfect," most specific case. To help students focus on properties, rather than their natural gestalt about a figure, we must expose them to a variety of models that appear in different orientations. This encourages students to focus on properties, rather than using visual clues based on a single example. We must move away from static representations of figures on worksheets or in textbooks and instead allow students to construct their own representations. Student-generated or student-constructed models offer a way to introduce a variety of shapes and also serve to provide students with ownership of their developing thinking.

Selecting Appropriate Activities

Professional Standards for Teaching Mathematics (NCTM 1991) stresses the importance that knowledge of content plays in a teacher's preparation: "Teachers' comfort with, and confidence in, their own knowledge of mathematics affects both what they teach and how they teach it. Their conceptions of mathematics shape their choice of worthwhile mathematical tasks, the kinds of learning environments they create, and the discourse in their classrooms" (p. 132). It is critical to carefully orchestrate the activities we choose to explore with our students so that they follow a logical developmental sequence. Before definitions are introduced, students need to experience activities that develop their critical observation skills and focus on classification of figures based on properties or characteristics rather than "naming" figures.

Figure 6

Squares differ in orientation and size.

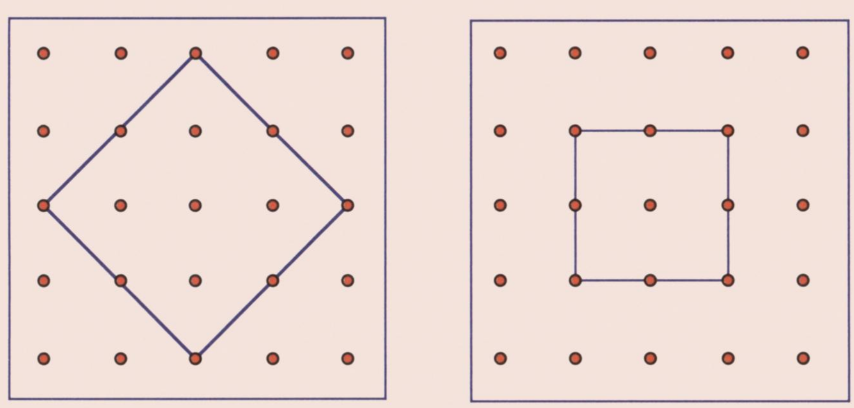
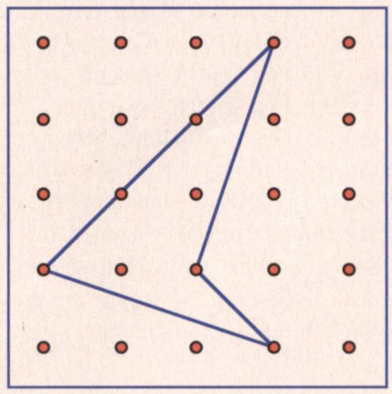


Figure 7

Concave scalene quadrilateral



Next Stop: The Classroom

Geoboards are the manipulative of choice for the following activities because they offer an excellent vehicle for helping students view a range of figures in a variety of orientations and yet provide constraints that ensure the integrity of the figure that freehand sketches do not offer.

Shapes alike and different: Becoming careful observers

To begin, students compare four-sided shapes they have created on their geoboards. They determine how the shapes are alike and how they are different. During whole-class discussions, the teacher has the opportunity to help students distinguish between integral and nonintegral properties by asking questions. For example: "Are quadrilaterals that vary in size or orientation really different?" (See **fig. 6.**) This is also an excellent time to begin a class list of properties that will help students use more precise geometric language and can be posted in the classroom for future reference.

Once is never enough: Sorting and re-sorting shapes

Building on the previous activity, students create as many different, closed, four-sided figures as possible. If your students are having trouble getting started, introduce a concave, scalene quadrilateral to the class (see **fig. 7**) and ask if it fits the definition of *closed* and *four-sided.*

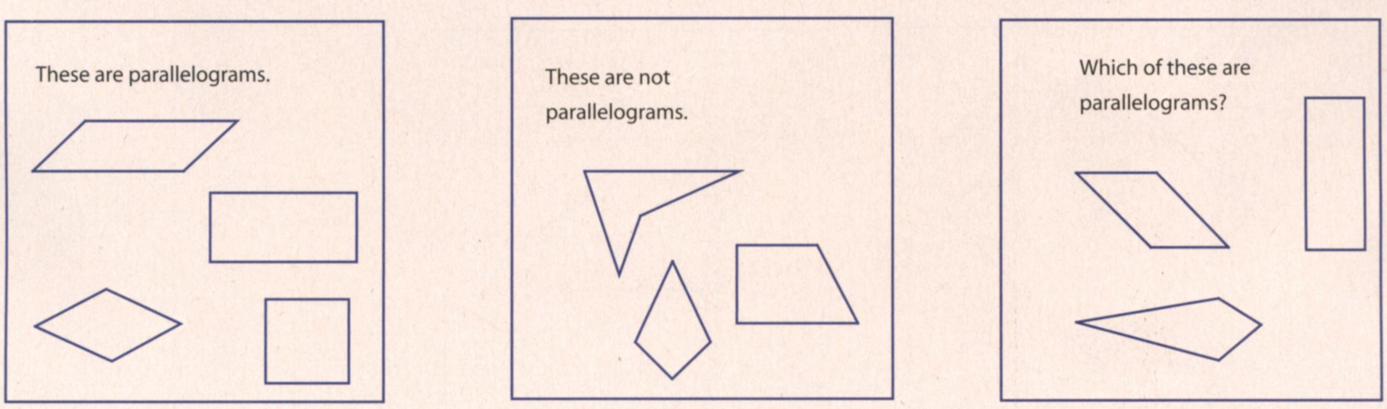
This will surely get the students thinking. Working in groups, students can sort the shapes they have created into categories (equal sides, unequal sides, and so forth). After students sort their quadrilaterals, encourage them to continue sorting and re-sorting their shapes using different attributes. This pushes students to look beyond the obvious and to hone their observation skills. They can copy figures represented on a geoboard to geopaper, cut them out, and laminate them for future sorting activities or learning centers.

These are/These are not: Comparing and contrasting examples and nonexamples

Opportunities for students to explore both examples and nonexamples provide yet another setting to cultivate students' observation skills. This can be done by placing shapes on cards labeled

Figure 8

These are/These are not ...



"These are________" and "These are not_______" (see **fig. 8**).

As new shapes are added, students determine where each shape belongs (Fuys, Geddes, and Tischler 1988, p. 27). When young learners experience a variety of opportunities to sort, classify, and re-sort shapes, they begin to recognize that some figures have more "special" properties than others. As figures cluster based on common properties, students begin to identify figures by their properties rather than just their visual appearance.

Implications for Classroom Teachers

Professional Standards for Teaching Mathematics (NCTM 1991) exposes that "central to the preparation of teaching mathematics is the development of a deep understanding of the mathematics of the school curriculum and how it fits within the discipline of mathematics. At all levels, teachers need to see the 'big' picture of mathematics across the elementary, middle, and high school years" and have a mental roadmap of the curriculum (p. 134). When teacher candidates ask, "Why do I need to know this? I am only going to teach elementary school mathematics," I use the story of Ricky to make the point that teachers need to know not only where our students have been and where they are in terms of their learning but also where their current learning will lead them in the future.

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