Elementary Linear Algebra - MATH 2250 - Day 3

Name:

1. Let the subscripts denote the number of rows and columns of each matrix, respectively. Mark all the matrix products that are defined. In each case that the product is defined write down the number of rows and columns of the product.
   - ☐ $A_{2\times 3}B_{3\times 2}$
   - ☐ $A_{3\times 3}B_{2\times 2}$
   - ☐ $A_{3\times 3}B_{3\times 3}$
   - ☐ $A_{2\times 1}B_{2\times 1}$
   - ☐ $A_{1\times 2}B_{2\times 1}$

2. Find the inverse of A using Gauss-Jordan method. Show steps AND check your solution!

$$A = \begin{bmatrix} 1 & 2 \\ 2 & 5 \end{bmatrix}$$

3. Find the inverse of A using Gauss-Jordan method. Show steps AND check your solution!

$$A = \begin{bmatrix} 2 & 4 \\ 4 & 5 \end{bmatrix}$$

4. This afternoon, read the worked examples $2.4A$ and $2.4B$ in the book (page 72).

5. For the following matrices, by giving conditions of the entries p, q, r, z explain when does $AB = BA$? When does $BC = CB$? When does A times BC equal AB times C?

$$A = \begin{bmatrix} p & 0 \\ q & r \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \qquad C = \begin{bmatrix} 0 & z \\ 0 & 0 \end{bmatrix}$$

6. True or False. Give a specific example when false:

   (a) If columns of 1 and 3 of B are the same, so are columns 1 and 3 of AB.

   (b) If rows 1 and 3 of B are the same, so are rows 1 and 3 of BA.

   (c) If rows 1 and 3 of A are the same, so are rows 1 and 3 of ABC.

   (d) $(AB)^2 = A^2B^2$.

   (e) $(A + B)^2 = A^2 + 2AB + B^2$.

   (f) If $AB = B$ then $A = I$.

   (g) If A^2 is defined then A is necessarily square.

   (h) If AB and BA are defined then A and B are square.

   (i) If AB and BA are defined then AB and BA are square.

   (j) $(AB)^{-1} = A^{-1}B^{-1}$

7. What is the determinant of the matrix of coefficients for the following system of linear equations?

$$\begin{cases} x + 2y = 5 \\ -2x + 3y = 0 \end{cases}$$

What does the determinant tell you about the existence of a solution to the above system?

8. Recall that if A is invertible, then the system $A\boldsymbol{x} = \boldsymbol{b}$ has a solution for any right hand side $\boldsymbol{b}$. What is that solution $\boldsymbol{x}$ in terms of A and $\boldsymbol{b}$?

In problem 2 you found the inverse of a matrix. Use that inverse to solve each of the following systems of equations:

$$\begin{cases} x + 2y = 1 \\ 2x + 5y = 0 \end{cases} \qquad \begin{cases} x + 2y = 0 \\ 2x + 5y = 1 \end{cases} \qquad \begin{cases} x + 2y = 1 \\ 2x + 5y = 1 \end{cases}$$

9. Recall that when you multiplied the following equations by $1, 1, -1$ and added them you got $0 = 1$.

$$\begin{aligned} x + y + z &= 2 \\ x + 2y + z &= 3 \\ 2x + 3y + 2z &= 4 \end{aligned}$$

Let A be the matrix of coefficients for the above system. Does there exist a nonzero vector $\boldsymbol{v} = (x, y, z)$ such that $A\boldsymbol{v} = \mathbf{0}$? What is it?

Is A invertible? Why?

10. Let A be a square matrix (of any size) with a zero column. Is A invertible? Why?

11. Let A be a square matrix (of any size) with a repeated column. Is A invertible? Why?

12. Find the inverse of this lower triangular matrix, using Gauss-Jordan method: $\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix}$

Do you expect the inverse of any lower triangular matrix to be lower triangular? Why?