Elementary Linear Algebra - MATH 2250 - Exam 2

Please read and sign (papers without printed name and signature will not be graded):
"On my honor, I have neither given nor received unauthorized aid in doing this assignment."

Print name: ______________________________ Sign: ______________________________

1. Which of the following (if any) are subspaces. For any that are **not** subspaces give an example of how they violate a property of subspaces.

   (a) Given a 3×5 matrix with full row rank, the set of all solutions to $A\boldsymbol{x} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$.

   (b) All 3×5 matrices with $\begin{bmatrix} -3 \\ 2 \\ 1 \end{bmatrix}$ in their column space.

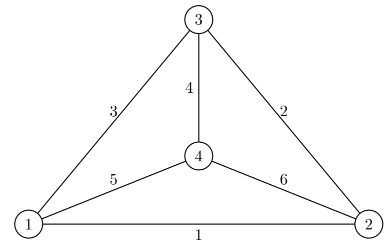
   (c) All 5×3 matrices with $(2, 1, 3)$ in their column space.

   (d) All vectors $\boldsymbol{x}$ with $||\boldsymbol{x} - \boldsymbol{y}|| = ||\boldsymbol{y}||$, for some given fixed vector $\boldsymbol{y} \neq \mathbf{0}$.

2. (a) Find the matrix P that projects every vector $\boldsymbol{b}$ in $\mathbb{R}^3$ onto the line in the direction of $(1, 2, 3)$.

   (b) Describe the Four fundamental subspaces of P by providing a basis for each of them.

3. Write down the 6×4 incidence matrix A of this graph. What is the dimension of the column space $C(A)$? Describe the null space $N(A)$.



4. (a) Consider the following data:

Year	US Population (million)
1900	70
1920	100
1940	130
1980	230

Suppose the population growth is linear, and you want to fit the best line $y = Cx + D$ to these values, where $x = 0$ represents the year 1900. What is the matrix A in the system $A \begin{bmatrix} C \\ D \end{bmatrix} = \boldsymbol{b}$? Find the best $\hat{C}, \hat{D}$, and the heights p_1, p_2, p_3, p_4 of that line $y = \hat{C}x + \hat{D}$ at years $1900, 1920, 1940$, and 1980. What is the error vector $\boldsymbol{e}$? Show by numbers that e is perpendicular to $C(A)$.

(b) What is your estimate for the population in year 1960? 2000? 2020? 3000?

5. Start with the two vectors (columns of A):

$$\boldsymbol{a}_1 = \begin{bmatrix} \sin\theta \\ 0 \\ \cos\theta \end{bmatrix} \text{ and } \boldsymbol{a}_2 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}.$$

(a) With $\boldsymbol{q}_1 = \boldsymbol{a}_1$ find an orthonormal basis $\boldsymbol{q}_1, \boldsymbol{q}_2$ for the space spanned by $\boldsymbol{a}_1$ and $\boldsymbol{a}_2$ (column space of A).

(b) What shape is the matrix R in $A = QR$ and why is $R = Q^T A$ (Here Q has columns $\boldsymbol{q}_1, \boldsymbol{q}_2$)? Compute R.

(c) Find the projection matrices P_A and P_Q onto the column spaces of A and Q.