Elementary Linear Algebra - MATH 2250 - Day 16

Name:

1. $\boxed{\text{T}}\boxed{\text{F}}$ If $AB = I$ then $BA = I$.

2. $\boxed{\text{T}}\boxed{\text{F}}$ If Q is an orthonormal matrix, then $Q^TQ = I$ and $QQ^T = I$.

3. $\boxed{\text{T}}\boxed{\text{F}}$ If Q is an orthonormal matrix, then QQ^T is a projection matrix.

4. Assume that we start with independent vectors v_1, v_2, and v_3, and proceed with the Gram-Schmidt algorithm, and produce w_1, w_2, and w_3. What relations hold between w_1, w_2, and w_3?

5. Let $\boldsymbol{b} = (4, 0, 0, 0)$, $\boldsymbol{v} = (1, 1, 1, 1)$, and $\boldsymbol{w} = (1, -1, 1, -1)$. Find the projection of $\boldsymbol{b}$ onto $\boldsymbol{v}$ and call it $\boldsymbol{u}_1$. Find the projection of $\boldsymbol{b}$ onto $\boldsymbol{w}$ and call it $\boldsymbol{u}_2$. Find the projection of $\boldsymbol{b}$ onto the space spanned by $\boldsymbol{v}$ and $\boldsymbol{w}$,

   and call it $\boldsymbol{u}_3$. What is the relation between $\boldsymbol{u}_1, \boldsymbol{u}_2$, and $\boldsymbol{u}_3$.

6. Consider the vectors $\boldsymbol{a}_1 = (1, 1, 1, 1)$, $\boldsymbol{a}_2 = (1, 1, 1, 0)$, and $\boldsymbol{a}_3 = (1, 1, 0, 0)$. Proceed with Gram-Schmidt algorithm and produce 3 vectors $\boldsymbol{q}_1, \boldsymbol{q}_2$, and $\boldsymbol{q}_3$. Recall that in the QR-decomposition of a matrix A, Q is found by Gram-Schmidt algorithm and $R = Q^T A$. Let $A = \begin{bmatrix} \boldsymbol{a}_1 & \boldsymbol{a}_2 & \boldsymbol{a}_3 \end{bmatrix}$.

   Find the QR-decomposition of A.

7. Compare $C(A)$ and $C(Q)$.

8. Recall that if $A = QR$, where Q is orthnormal and R is upper-triangular, then instead of solving $A\boldsymbol{x} = \boldsymbol{b}$, one can easily solve $R\hat{\boldsymbol{x}} = Q^T\boldsymbol{b}$. Solve the equation $A\boldsymbol{x} = \boldsymbol{b}$, for A as above and $\boldsymbol{b} = (1, 0, 0, 0)$.