Elementary Linear Algebra - MATH 2250 - Day 17

Name:

1. $\boxed{\text{T}}$ $\boxed{\text{F}}$ $\det(A+B) = \det(A) + \det(B)$, for any two matrices A and B.
   $\boxed{\text{T}}$ $\boxed{\text{F}}$ $\det(cA) = c\det(A)$, for any real number c and any matrix A.
   $\boxed{\text{T}}$ $\boxed{\text{F}}$ $\det(A) = 0$, if and only if $A = O$, the zero matrix.
   $\boxed{\text{T}}$ $\boxed{\text{F}}$ $\det(P) = 1$, for any permutation matrix P.
   $\boxed{\text{T}}$ $\boxed{\text{F}}$ $\det(A) = \det(R)$, where R is the reduced row echelon form of A, with no row-exchanges.
   $\boxed{\text{T}}$ $\boxed{\text{F}}$ $\det(AB) = \det(A)\det(B)$, for any two matrices A and B.
   $\boxed{\text{T}}$ $\boxed{\text{F}}$ $\det(A^{-1}) = \det(A)$.
   $\boxed{\text{T}}$ $\boxed{\text{F}}$ $\det(I + A) = 1 + \det(A)$.
   $\boxed{\text{T}}$ $\boxed{\text{F}}$ $\det(ABC) = \det(A)\det(B)\det(C)$.
   $\boxed{\text{T}}$ $\boxed{\text{F}}$ $\det(2A) = 2\det(A)$.
   $\boxed{\text{T}}$ $\boxed{\text{F}}$ $\det(AB) = \det(BA)$.
   $\boxed{\text{T}}$ $\boxed{\text{F}}$ $\det(A^n) = \det(A)^n$
   $\boxed{\text{T}}$ $\boxed{\text{F}}$ $\det(A) = \det(A^T)$.
   $\boxed{\text{T}}$ $\boxed{\text{F}}$ If A is not invertible, then AB is not invertible
   $\boxed{\text{T}}$ $\boxed{\text{F}}$ The determinant of A is always the product of its pivots.
   $\boxed{\text{T}}$ $\boxed{\text{F}}$ $\det(A - B) = \det(A) - \det(B)$.
   $\boxed{\text{T}}$ $\boxed{\text{F}}$ $\det(cA) = c^n\det(A)$, for any $n \times n$ matrix A. Why?

2. What are the main three properties of the determinant? Mention briefly.

3. If $\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = 1$ then $\begin{vmatrix} a & b & c \\ -d & -e & -f \\ g & h & i \end{vmatrix} =$

4. If $\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = 1$, and $\begin{vmatrix} a & b & c \\ d & e & f \\ g' & h' & i' \end{vmatrix} = 2$, then $\begin{vmatrix} a & b & c \\ d & e & f \\ g+g' & h+h' & i+i' \end{vmatrix} =$

5. $\begin{vmatrix} a & b & c \\ 0 & 0 & 0 \\ g & h & i \end{vmatrix} =$

6. $\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = 1$

7. If U is the matrix obtained from A by reducing it, $\det(A) = \pm\det(U)$. Explain when it is $+$ and when it is $-$.

8. $\begin{vmatrix} a & b & c & d \\ 0 & e & f & g \\ 0 & 0 & i & j \\ 0 & 0 & 0 & k \end{vmatrix} =$

9. $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & 2 \\ 1 & 2 & 3 \end{vmatrix} =$

10. If $\det(A_{3\times 3}) = 2$, then $\det(2A) =$

11. $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix} =$

12. $\begin{vmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{vmatrix} =$

13. $\begin{vmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{vmatrix} =$

14. Let $A = LU$ be the LU-decomposition of A. Then $\det(A) =$

15. $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 3 \end{vmatrix} =$

16. $\begin{vmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{vmatrix} =$

17. We want to show that for ant orthogonal matrix Q $\det(Q) = \pm 1$. In order to show this, recall that

$$Q^T Q = \underline{\qquad}.$$

Also, recall that

$$\det(AB) = \underline{\qquad\qquad\qquad}.$$

Then

$$\det(Q^T Q) = \det(\underline{\qquad})\det(\underline{\qquad}).$$

Furthermore, recall that

$$\det(A^T) = \underline{\qquad\qquad},$$

and

$$\det(I) = \underline{\qquad}.$$

Hence

$$\det(Q^T Q) = \underline{\qquad\qquad}.$$

18. Let $PAP^{-1} = \Lambda = \operatorname{diag}(1, 2, -2, 3)$ be a diagonal matrix with $1, 2, -2$ and 3 on the main diagonal. Find $\det(A)$.

Find $\det(A^2)$.

Find $\det(A^n)$, for $n \geq 1$.

19. The commutator of A and B is $[A, B] = AB - BA$. Find $\det([A, B])$.

20. From $A = \begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix}$, find $A - xI$, where x is a real number. Which two numbers x lead to $\det(A - xI) = 0$?