**Name:**

1. Let $A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 \\ -2 & -3 & -4 & -5 \end{bmatrix}$.

   Find all the right hand sides $b$ such that $Ax = b$ has a solution (the solvability condition).

2. Fill in the blank: For $Ax = b$ to have a solution, If a combination of rows of $A$ gives the zero row, then _____ of entries of $b$ must be _____ .

3. what is rank of $A$?

4. Find all the solutions to $Ax = 0$, for $A$ given in problem 1 (that is, find the null space of $A$).

5. Using the results of problem 1, does the equation $Ax = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$ have a solution?

6. Find a particular solution to $Ax = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$ by letting the free variables equal to zero (that is, a $x_{\text{particular}}$).

7. Find all the solutions to $A\boldsymbol{x} = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$ (that is, the $x_{\text{complete}}$).

8. Find all the solutions to $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$.
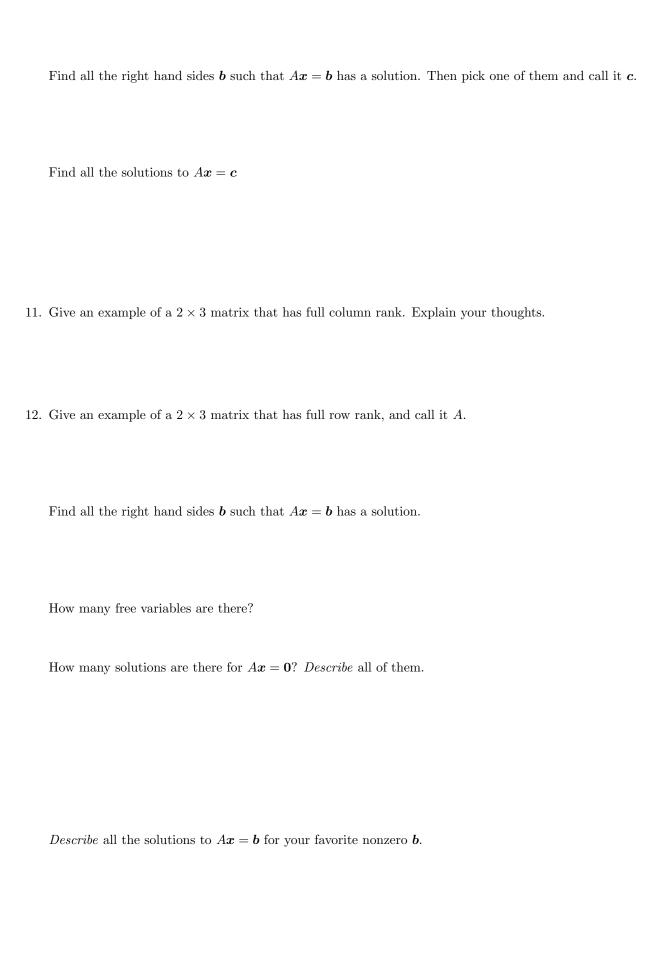
9. If an $m \times n$ matrix $A$ has full column rank, then how many free variables are there? why?

What is $N(A)$, the null space of $A$? Explain

How many solution are there for $A\boldsymbol{x} = \boldsymbol{b}$?

10. Give an example of a $3 \times 2$ matrix that has full column rank, and call it $A$.

What is the rref of $A$?

Find all the right hand sides $\boldsymbol{b}$ such that $A\boldsymbol{x} = \boldsymbol{b}$ has a solution. Then pick one of them and call it $\boldsymbol{c}$.

Find all the solutions to $A\boldsymbol{x} = \boldsymbol{c}$

11. Give an example of a $2 \times 3$ matrix that has full column rank. Explain your thoughts.

12. Give an example of a $2 \times 3$ matrix that has full row rank, and call it $A$.

Find all the right hand sides $\boldsymbol{b}$ such that $A\boldsymbol{x} = \boldsymbol{b}$ has a solution.

How many free variables are there?

How many solutions are there for $A\boldsymbol{x} = \boldsymbol{0}$? *Describe* all of them.

*Describe* all the solutions to $A\boldsymbol{x} = \boldsymbol{b}$ for your favorite nonzero $\boldsymbol{b}$.

13. Let $A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 \\ 1 & 3 & 6 & 10 \\ 1 & 4 & 10 & 20 \end{bmatrix}$. Find the reduced row echelon form for $A$.

What is the rank of $A$.

Is $A$ invertible? Explain.

How many free variables are there for $A$?

What is the null space of $A$?

Find all the right hand sides $b$ such that $Ax = b$ has a solution.

How many solutions are there for $Ax = b$ for any $b$?

14. Before watching the next video, go back and watch the last 5 minutes of the previous video (Lecture 8) and make sure that you understand it well. Write what's on the last board below.

15.         'Here lies Diophantus,'
the wonder behold.

Through art algebraic,
the stone tells how old:

'God gave him his boyhood
one-sixth of his life,

One twelfth more as youth
while whiskers grew rife;

And then yet one-seventh
ere marriage begun;

In five years there came
a bouncing new son.

Alas, the dear child of master
and sage After attaining half the measure of his father's life
chill fate took him.

After consoling his fate
by the science of numbers for four years,
he ended his life.'

Stated in prose, the poem says that Diophantus's youth lasts 1/6 of his life. He grew a beard after 1/12 more of his life. After 1/7 more of his life, Diophantus married. Five years later, he had a son. The son lived exactly half as long as his father, and Diophantus died just four years after his son's death. All of this totals the years Diophantus lived. [1]

How many year Diophantus and his son lived, each?

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[1] Adapted from: Weisstein, Eric W. "Diophantus's Riddle." From MathWorld–A Wolfram Web Resource. `http://mathworld.wolfram.com/DiophantussRiddle.html`.