Elementary Linear Algebra - MATH 2250 - Day 13

Name:

1. Mark each of the followings as True or False. In each case draw some pictures to clarify your answer.
   $\boxed{\text{T}}$ $\boxed{\text{F}}$ The xy-plane and the yz-plane are perpendicular to each other as vector spaces.

   $\boxed{\text{T}}$ $\boxed{\text{F}}$ The xy-plane and the z-axis are perpendicular to each other as vector spaces.

   $\boxed{\text{T}}$ $\boxed{\text{F}}$ The zero vector is perpendicular to any vector.

   $\boxed{\text{T}}$ $\boxed{\text{F}}$ Two planes through the origin are perpendicular to each other.

   $\boxed{\text{T}}$ $\boxed{\text{F}}$ Two planes through the origin could be perpendicular to each other.

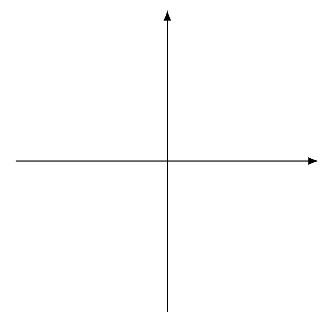
   $\boxed{\text{T}}$ $\boxed{\text{F}}$ A lines in the plane through the origin is perpendicular to that plane.

2. Let A be a matrix. Find the intersection of the null space of A and the row space of A.

3. Let A be a matrix. Find the intersection of the left null space of A and the column space of A.

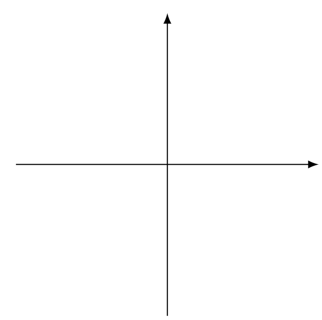
4. Give an example of two matrices A and B such that $AB = I$ but $BA \neq I$.

5. Consider the matrix $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$. "Draw" the row space and the column space of A in one system of coordinates, $\mathbb{R}^2$.



What is the angle between the two spaces?

"Draw" the column space and the left null space of A in one system of coordinates, $\mathbb{R}^2$.



What is the angle between the two spaces?

6. Find a vector orthogonal to $v = (2, 2, -1)$.

7. Build a matrix whose row space has the basis $v = (2, 2, -1)$, and call it A.

Find the null space of A Find the dot product of any vector in null space of A with v.

Find another matrix whose row space has the basis $v = (2, 2, -1)$, and call it B. Then find the null space of B.

Find the dot product of any vector in null space of A with v.

Find **all** the vectors that are orthogonal to v.

8. Is it true that if x is orthogonal to v and to w, then x is perpendicular to $v + w$? Why?

   Is it true that if x is orthogonal to v, then x is perpendicular to cv, for any real number c? Why?

   Is the set of all vectors perpendicular to v a subspace?

9. Is there a matrix with 5 columns whose row space and null space have the same dimension? If yes, give an example, if no, why?

10. Is there a matrix with 4 columns whose row space and null space have the same dimension? If yes, give an example, if no, why?

11. Recall that each column of AB is a linear combination of the columns of A. Then, $\dim C(AB) \square \dim C(A)$. That is, column rank AB $\square$ column rank A.

12. Recall that each row of AB is a linear combination of the rows of B. Then, $\dim R(AB) \square \dim R(A)$. That is, row rank AB $\square$ row rank B.

13. Recall that for any matrix X, $\dim R(X) = \dim C(X)$ (why?). Then, row rank of X $\square$ column rank of X.

14. Using the results from problems 11–13,

$$\text{rank(AB)} \square \text{rank(A)},$$

and

$$\text{rank(AB)} \square \text{rank(B)}.$$

15. Give an example of two matrices A and B such that rank(AB) = rank(A) = rank(B).

16. Give an example of two matrices A and B such that rank(AB) < rank(A) **AND** rank(AB) < rank(B).