Elementary Linear Algebra - MATH 2250 - Day 9

Name:

1. Mark each of the followings as True or False. If a set is dependent, provide a nonzero combination of the vectors which gives the zero vector.

   $\boxed{\text{T}}$ $\boxed{\text{F}}$ The vectors $(1, 0)$ and $(0, 0)$ are dependent.

   $\boxed{\text{T}}$ $\boxed{\text{F}}$ The vectors $(1, 0)$ and $(0, 1)$ are dependent.

   $\boxed{\text{T}}$ $\boxed{\text{F}}$ The vectors $(1, 0)$ and $(0, 1)$ and $(\pi, -\pi)$ are dependent.

2. The vectors $v_1, v_2, \ldots, v_n$ are independent if ______________________________

   ____

3. Let A be an $m \times n$ matrix with rank n whose columns are linearly dependent, then $r \,\square\, n$. (Fill the box with the best choice from $<, =, >, \leq, \geq$.) Explain.

4. Give two distinct bases for $\mathbb{R}^3$.

5. Can the zero vector be in any basis? Could it be in any independent set?

6. The vectors $\boldsymbol{v} = (1, 0)$, $\boldsymbol{w} = (0, 1)$ and $\boldsymbol{u} = (\pi, \pi)$ are linearly dependent. To show this build a matrix $A = \left[\begin{array}{c|c|c} \boldsymbol{v} & \boldsymbol{w} & \boldsymbol{u} \end{array}\right]$, and show that its null space is nonzero.

7. The vectors $(1,0,0)$, $(0,1,0)$, and $(0,0,1)$ are linearly independent. To show this, we need to show that the only linear combination of them that is equal to the zero vector is the zero combination. That is, if $c_1\begin{bmatrix}1\\0\\0\end{bmatrix}+c_2\begin{bmatrix}0\\1\\0\end{bmatrix}+c_3\begin{bmatrix}0\\0\\1\end{bmatrix}=\mathbf{0}$, then $c_1, c_2, c{+}3=0$. Write a system of linear equations with variables c_1, c_2, c_3 corresponding to the above equation. Then find **all** the solutions to that system.

8. Show that the vectors $(1,-1,0)$, $(0,1,-1)$, and $(0,0,1)$ are linearly independent.

9. Are there 4 vectors in $\mathbb{R}^3$ that are linearly independent? Why?

10. Does 1 vector span $\mathbb{R}^3$? What the **maximum** dimension of a space that it could span?

11. Do 2 vectors span $\mathbb{R}^3$? What the **maximum** dimension of a space that they could span?

12. Do 3 vectors span $\mathbb{R}^3$? Could they span a plane? A line?

13. Do 4 vectors span $\mathbb{R}^3$? Could they span a 3-D space? A plane? A line?

14. Are vectors $(1,1,1)$, $(1,1,1)$, and $(1,2,3)$ linearly independent? Why?

15. Consider the matrix $A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 5 \end{bmatrix}$.

Do the columns of A span the columns space of A?

Are the columns of A linearly independent?

Find three different bases for the column space of A.

What is the rank of A?

What is the dimension of the column space of A?

Find two **more** bases for the column space of A.

Find a basis for the column space of A.

Find a nonzero linear combination of columns of A which gives the zero vector.

Find a nonzero linear combination of columns of A that is not a 'multiple' of the previous linear combination, but still gives the zero vector.

16. Assume that the rank of a matrix $A = \left[\begin{array}{c|c|c|c|c} A_1 & A_2 & A_3 & A_4 & A_5 \end{array}\right]$ is 3, and the vectors

$$A\begin{bmatrix}1\\0\\0\\0\\0\end{bmatrix}, A\begin{bmatrix}0\\1\\0\\0\\0\end{bmatrix}, \text{ and } A\begin{bmatrix}0\\0\\0\\0\\1\end{bmatrix}$$

are linearly independent. Find a basis for the column space of A. What is the dimension of $C(A)$?

Find a basis for the null space of A. What is the dimension of $N(A)$?

17. **(Important)** Let $A = \begin{bmatrix} 1 & 0 & 1 & 1 \\ -1 & 1 & 0 & 4 \\ 0 & -1 & -1 & -5 \end{bmatrix}$. Is $(0, 0, 1)$ in the column space of A? How do you know?

Find $R = \text{rref(A)}$. Is $(0, 0, 1)$ in the column space of R?

How are the column spaces of A and R related to each other?