Elementary Linear Algebra - MATH 2250 - Day 22

Name:

1. Find the eigenvalues and the eigenvectors of a triangular matrix: Once with distinct eigenvalues and once with equal eigenvalues.

2. Let $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$. Find the eigenvalues of A and their corresponding eigenvectors. Call these eigenvalues λ_1 and λ_2, and the corresponding eigenvectors v_1 and v_2.

   Let $S = \left[\begin{array}{c|c} v_1 & v_2 \end{array}\right]$. Evaluate AS.

   Let $\Lambda = \text{diag}(\lambda_1, \lambda_2)$. Evaluate $S\Lambda$.

   Evaluate S^{-1}.

   Evaluate $S^{-1}AS$.

Evaluate $S\Lambda S^{-1}$.

Find the echelon form of A, and multiply its pivots.

Multiply the eigenvalues of A, and compare it with the product of the pivots.

Is it true that the pivots of A are the same as eigenvalues of A?

3. Using the same matrix A as in problem 1, recall that $A = S\Lambda S^{-1}$. Evaluate A^2, A^3, and A^4. What are the eigenvalues of A^2, A^3, A^4?

4. What are the eigenvalues of $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$. Is the matrix diagonalizable? What is the diagonlized A?

5. What are the eigenvalues of $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$.

What are the algebraic multiplicities of the eigenvalues of A?

What are the geometric multiplicities of the eigenvalues of A?

Is the matrix diagonalizable? Why?

6. Let $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 2 \\ 0 & 0 & 3 \end{bmatrix}$. Find λ and x such that $Ax = \lambda x$

Switch two rows of A to get B. Is it true that $Bx = \lambda x$?

What are all the eigenvalues of B?

7. If the all the eigenvalues of $A_{n\times n}$ are distinct, then the eigenvectors of A are ________________, and hence they form a ________________ for $\mathbb{R}^n$. That is, every vector $v \in \mathbb{R}^n$ can be written as a ________________ of the eigenvectors of A.

8. Is it true that if A is diagonalizable, then the eigenvalues of A are distinct? Check it for the identity matrix.

9. If A diagonalizable, is it true that the diagonalized A is the same as $\mathrm{rref}(A)$? Give an example.

10. Find the eigenvalues and eigenvectors of $A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$.