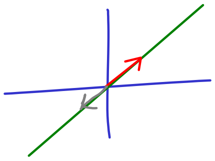
Elementary Linear Algebra - MATH 2250 - Day 6

Name:

1. Mark each of the followings as True or False (Explain why when True, or give an example when False).

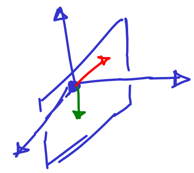
☑ T ☐ F The set of all vectors that lie on a line through origin form a vector space.



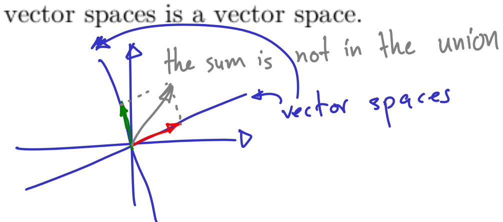
Any multiple of a vector on the line lies on the line.
Sum of any two vectors on the line, lies on the line.

☑ T ☐ F The set of all vectors that lie on a plane through origin form a vector space.

Same as above:

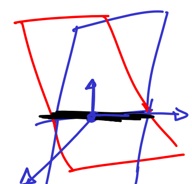


☐ T ☑ F The union of any two vector spaces is a vector space.



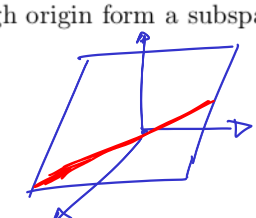
☑ T ☐ F The intersection of any two vector spaces is a vector space.

$v, w \in V \cap W \Rightarrow v + w \in V \cap W$
$cv \in V \cap W$

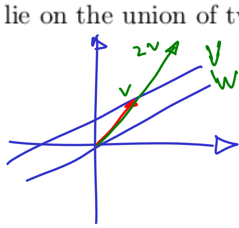


☐ T ☑ F The set of all vectors that lie on the union of two distinct lines through origin form a vector space.

☑ T ☐ F The set of all vectors that lie on a line through origin form a subspace of a plane that contains that line.

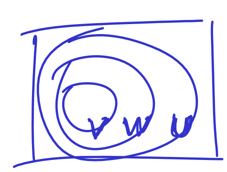


☐ T ☑ F The set of all vectors that lie on the union of two distinct parallel lines form a vector space.



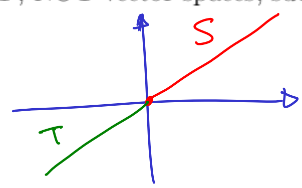
$v \in V \cup W$
but $2v \notin V \cup W$

☑ T ☐ F Let V be a subspace of W and W be a subspace of U. Then V is a subspace of U.



$V \subset U$
V is a vector space $\Big\} \longrightarrow V$ is a subspace of U.

$\boxed{\text{T}}$ $\boxed{\text{F}}$ (optional) There are sets S and T, NOT vector spaces, such that $S \cup T$ is a vector space.



Example: S, T two halves of a line.

2. Let $A = \begin{bmatrix} 2 & 0 & 2 \\ 2 & 2 & 0 \\ 4 & 4 & 0 \\ 4 & 6 & -2 \end{bmatrix}$.

(a) Fill in the blank: The column space of A (that is, $C(A)$) is a subspace of $\underline{\mathbb{R}^4}$.

(b) Is the columns space of A the whole space specified in part (a)? Why?

nope. Three vectors (columns) cannot "span" a 4-D space.

(c) Does the equation $A\boldsymbol{x} = \boldsymbol{b}$ have a solution for any right hand side $\boldsymbol{b}$? Explain.

Since the $C(A)$ is not the whole $\mathbb{R}^4$, then for some $b \in \mathbb{R}^4$ it doesn't have a solution.

(d) Does the equation $A\boldsymbol{x} = \boldsymbol{0}$ have a solution? Explain.

$Ax=0$ alway has a trivial solution, $x=0$.

(e) Does the equation $A\boldsymbol{x} = \begin{bmatrix} 2 \\ 2 \\ 4 \\ 4 \end{bmatrix}$ have a solution? Explain.

first col of A.

Yes, $x = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

(f) Does the equation $A\boldsymbol{x} = \begin{bmatrix} 1 \\ 1 \\ 2 \\ 2 \end{bmatrix}$ have a solution? Explain.

half of first col of A.

Yes, $x = \begin{bmatrix} 1/2 \\ 0 \\ 0 \end{bmatrix}$

(g) Does the equation $A\boldsymbol{x} = \begin{bmatrix} 0 \\ 2 \\ 4 \\ 6 \end{bmatrix}$ have a solution? Explain. second col of A.

Yes, $x = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

(h) Does the equation $A\boldsymbol{x} = \begin{bmatrix} 4 \\ 2 \\ 4 \\ 2 \end{bmatrix}$ have a solution? Explain.

(Solve)

Yes, $x = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

(i) Does the equation $A\boldsymbol{x} = \boldsymbol{b}$ have a solution for any $\boldsymbol{b}$ in the columns space of A? Explain.

Yes, if $b \in C(A)$, then $b = c_1 A_1 + c_2 A_2 + c_3 A_3$ for $c_i \in \mathbb{R}$ and A_i: col's of A.

$\Rightarrow A \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = b$.

(j) What are all the right hand sides $\boldsymbol{b}$ such that $A\boldsymbol{x} = \boldsymbol{b}$ has a solution?

All the vectors in the column space of A.

(k) Does the equation $A\boldsymbol{x} = \boldsymbol{0}$ have a nonzero solution? Explain.

Yes, solve: one solution is $x = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$

(l) What are all the solutions to the equation $A\boldsymbol{x} = \boldsymbol{0}$?

Solve by elimination: $\begin{bmatrix} 2 & 0 & 2 \\ 2 & 2 & 0 \\ 4 & 4 & 0 \\ 4 & 6 & -2 \end{bmatrix} \to \begin{bmatrix} 2 & 0 & 2 \\ 0 & 2 & -2 \\ 0 & 4 & -4 \\ 0 & 6 & -6 \end{bmatrix} \to \begin{bmatrix} 2 & 0 & 2 \\ 0 & 2 & -2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \to$

$2x_1 + \quad 2x_3 = 0$

$2x_2 - 2x_3 = 0$

free

$x_3 = t,\ t \in \mathbb{R} \Rightarrow x_1 = -t,\ x_2 = t \to$ all sol'ns: $\begin{bmatrix} -t \\ t \\ t \end{bmatrix},\ t \in \mathbb{R}$.

(m) What is the null space of A?

$\left\{ t \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix},\ t \in \mathbb{R} \right\}$