Elementary Linear Algebra - MATH 2250 - Day 7

Name:

1. Mark the matrices in reduced row echelon form. If they are not in rref, explain why.

☑ $[1]$ ☑ $[0]$ ☑ $[1\ \ 0]$ ☑ $\begin{bmatrix}1 & 0\\0 & 0\end{bmatrix}$ ☑ $\begin{bmatrix}0 & 1\\0 & 0\end{bmatrix}$ ☐ $\begin{bmatrix}1 & 0 & 2\\1 & 1 & 2\end{bmatrix}$ ☐ $\begin{bmatrix}1 & 2 & 0\\0 & 1 & 0\end{bmatrix}$ ☐ $\begin{bmatrix}0 & 1 & 2\\1 & 0 & 0\end{bmatrix}$

nonzero below a leading 1.

nonzero above a leading 1

leading ones in wrong order.

2. Let $A = \begin{bmatrix} 2 & 0 & 2\\ 2 & 2 & 0\\ 4 & 4 & 0\\ 4 & 6 & -2 \end{bmatrix}$.

(a) Find the row echelon form of A.

$$A \to \begin{bmatrix} 2 & 0 & 2\\ 0 & 2 & -2\\ 0 & 4 & -4\\ 0 & 6 & -6 \end{bmatrix} \to \begin{bmatrix} 2 & 0 & 2\\ 0 & 2 & -2\\ 0 & 0 & 0\\ 0 & 0 & 0 \end{bmatrix}$$

(b) Find the reduced row echelon form of A.

$$\to \begin{bmatrix} 1 & 0 & 1\\ 0 & 1 & -1\\ 0 & 0 & 0\\ 0 & 0 & 0 \end{bmatrix}$$

(c) How many free variables are there in the system of linear equations $\begin{bmatrix} 2 & 0 & 2\\ 2 & 2 & 0\\ 4 & 4 & 0\\ 4 & 6 & -2 \end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\\x_4\end{bmatrix} = \begin{bmatrix}0\\0\\0\\0\end{bmatrix}$? What are those?

free col

1, it is x_3

(d) How many pivot variables are there in the above system? What are those?

2, they are x_1, x_2

(e) What is the nullspace matrix of A?

$$R = \text{rref}(A) = \begin{array}{c} \begin{array}{ccc} \text{I} & & \text{F} \end{array} \\ \begin{bmatrix} 1 & 0 & 1\\ 0 & 1 & -1\\ 0 & 0 & 0\\ 0 & 0 & 0 \end{bmatrix} \end{array} \rightsquigarrow N = \begin{bmatrix} -1\\ 1\\ 1 \end{bmatrix}$$

(f) Fill in the blank: The null space of A is a subspace of $\underline{\mathbb{R}^3}$.

(g) What is the nullspace of A?

$$\left\{ t \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}, t \in \mathbb{R} \right\}$$

(compare with previous worksheet)

(h) Check that the null space of A is actually a vector space. That is, for any real number c and vectors $\boldsymbol{v}$ and $\boldsymbol{w}$ that $A\boldsymbol{v} = \mathbf{0}$ and $A\boldsymbol{v} = \mathbf{0}$, then

- $A(v + w) = 0$, and
- $A(cv) = 0$.

$A(v+w) = Av + Aw = 0 + 0 = 0$

$A(cv) = c(Av) = c(0) = 0$

(i) Find a solution to $A\boldsymbol{x} = \begin{bmatrix} 2 \\ 0 \\ 0 \\ -2 \end{bmatrix}$, and call it v.

$v = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

(j) Take your favorite nonzero vector in the null space of A, and call it w. Then do the following multiplication: $A(v + w)$.

$w = \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$

$v + w = \begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix}$

$A \begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 0 \\ -2 \end{bmatrix}$

(k) How many solution are there to the equation $A\boldsymbol{x} = \begin{bmatrix} 2 \\ 0 \\ 0 \\ -2 \end{bmatrix}$?

infinitely many, because for every $w \in N(A)$ the vector $\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} + w$ is a solution to $Ax = \begin{bmatrix} 2 \\ 0 \\ 0 \\ -2 \end{bmatrix}$

(l) Take your favorite two vectors that solve the equation $A\boldsymbol{x} = \begin{bmatrix} 2 \\ 0 \\ 0 \\ -2 \end{bmatrix}$, and call them v and w. Does $v + w$ solve the equation? How about $2v$? How about $-3w$?

$v = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$, $w = \begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix}$ NO.

$A(v+w) \neq \begin{bmatrix} 2 \\ 0 \\ 0 \\ -2 \end{bmatrix}$

$A(2v) \neq \begin{bmatrix} 2 \\ 0 \\ 0 \\ -2 \end{bmatrix}$

$A(-3w) \neq \begin{bmatrix} 2 \\ 0 \\ 0 \\ -2 \end{bmatrix}$

(m) Does the zero vector solve the equation $A\boldsymbol{x} = \begin{bmatrix} 2 \\ 0 \\ 0 \\ -2 \end{bmatrix}$? Nope. $A0 = 0 \neq \begin{bmatrix} 2 \\ 0 \\ 0 \\ -2 \end{bmatrix}$

(n) Does the set of all solutions to the equation $A\boldsymbol{x} = \begin{bmatrix} 2 \\ 0 \\ 0 \\ -2 \end{bmatrix}$ form a vector space? Why?

{Nope. It doesn't contain zero.} ← this is enough for <u>not</u> being a V.S.

[Furthermore, it's not closed under addition and scalar multiplication.]

(o) (Optional) Find all the solution to $A\boldsymbol{x} = \begin{bmatrix} 2 \\ 0 \\ 0 \\ -2 \end{bmatrix}$.

$\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} + c \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$, $c \in \mathbb{R}$.