Elementary Linear Algebra - MATH 2250 - Day 7

Name:

1. Mark the matrices in reduced row echelon form. If they are not in rref, explain why.

☐ $\begin{bmatrix} 1 \end{bmatrix}$ ☐ $\begin{bmatrix} 0 \end{bmatrix}$ ☐ $\begin{bmatrix} 1 & 0 \end{bmatrix}$ ☐ $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ ☐ $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ ☐ $\begin{bmatrix} 1 & 0 & 2 \\ 1 & 1 & 2 \end{bmatrix}$ ☐ $\begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \end{bmatrix}$ ☐ $\begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & 0 \end{bmatrix}$

2. Let $A = \begin{bmatrix} 2 & 0 & 2 \\ 2 & 2 & 0 \\ 4 & 4 & 0 \\ 4 & 6 & -2 \end{bmatrix}$.

   (a) Find the row echelon form of A.

   (b) Find the reduced row echelon form of A.

   (c) How many free variables are there in the system of linear equations $\begin{bmatrix} 2 & 0 & 2 \\ 2 & 2 & 0 \\ 4 & 4 & 0 \\ 4 & 6 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$? What are those?

   (d) How many pivot variables are there in the above system? What are those?

   (e) What is the nullspace matrix of A?

(f) Fill in the blank: The null space of A is a subspace of ______.

(g) What is the nullspace of A?

(h) Check that the null space of A is actually a vector space. That is, for any real number c and vectors $\boldsymbol{v}$ and $\boldsymbol{w}$ that $A\boldsymbol{v} = \mathbf{0}$ and $A\boldsymbol{v} = \mathbf{0}$, then

- $A(v + w) = 0$, and
- $A(cv) = 0$.

(i) Find a solution to $A\boldsymbol{x} = \begin{bmatrix} 2 \\ 0 \\ 0 \\ -2 \end{bmatrix}$, and call it v.

(j) Take your favorite nonzero vector in the null space of A, and call it w. Then do the following multiplication: $A(v + w)$.

(k) How many solution are there to the equation $A\boldsymbol{x} = \begin{bmatrix} 2 \\ 0 \\ 0 \\ -2 \end{bmatrix}$?

(l) Take your favorite two vectors that solve the equation $A\boldsymbol{x} = \begin{bmatrix} 2 \\ 0 \\ 0 \\ -2 \end{bmatrix}$, and call them v and w. Does $v + w$ solve the equation? How about $2v$? How about $-3w$?

(m) Does the zero vector solve the equation $A\boldsymbol{x} = \begin{bmatrix} 2 \\ 0 \\ 0 \\ -2 \end{bmatrix}$?

(n) Does the set of all solutions to the equation $A\boldsymbol{x} = \begin{bmatrix} 2 \\ 0 \\ 0 \\ -2 \end{bmatrix}$ form a vector space? Why?

(o) (Optional) Find all the solution to $A\boldsymbol{x} = \begin{bmatrix} 2 \\ 0 \\ 0 \\ -2 \end{bmatrix}$.