Elementary Linear Algebra - MATH 2250 - Day 15

Name:

1. If $\boldsymbol{b}$ is in the column space of A and P is the projection matrix onto the column space of A, then $P\boldsymbol{b} =$ b.
2. If $\boldsymbol{b}$ is perpendicular to the column space of A and P is the projection matrix onto the column space of A, then $P\boldsymbol{b} =$ 0.
3. Recall that a projection matrix P has two key properties: P is symmetric and $P^2 = P$. Check that if P is a projection matrix, then $I - P$ is a projection matrix.

$P^T = P$

① $(I-P)^T = I^T - P^T = I - P$ ✓

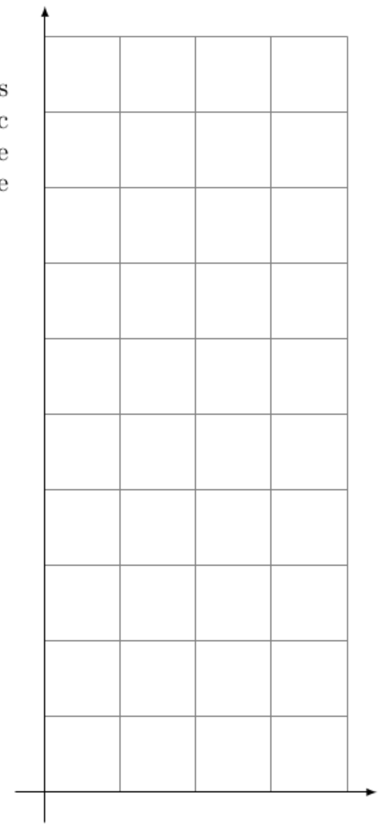
② $(I-P)^2 = (I-P)(I-P) = I^2 - IP - PI + P^2 = I - 2P + P = I - P$ ✓

in general

$(A-B)^2 \neq A^2 - 2AB + B^2$.

why?

4. Consider the 4 points $(1,1), (2,4), (3,9)$. Draw the three points in the xy-plane.



We want to find a line that the sum of the vertical distances of the above points from this line is the minimum possible. To do this we start with a parametric equation of such a line, that is, $y = Cx + D$. Then we write equations each time considering one of the points is on the line, for example, for the point $(1,1)$ we get the equation $1 = m \cdot 1 + b$. Write all the three equations.

Form the matrix equation $A \begin{bmatrix} C \\ D \end{bmatrix} = \boldsymbol{b}$ for the above system.

Does the system have a solution? Why?

Form the normal equations given by $A^T A\hat{x} = A^T\boldsymbol{b}$, and solve it for $\hat{C}$ and $\hat{D}$.

Draw the line $y = \hat{C}x + \hat{D}$.

Find P the projection matrix.

Find $\boldsymbol{p} = P\boldsymbol{b}$.

Check with Sage, mathematica, maple, matlab, Wolfram-α, or any other software/device.

Find the error vector $\boldsymbol{e} = \boldsymbol{b} - \boldsymbol{p}$.

Show $\boldsymbol{p}$ and $\boldsymbol{e}$ on the picture.

Evaluate $\boldsymbol{p} \cdot \boldsymbol{e}$.

Check that $\boldsymbol{e}$ is perpendicular to every column of A. What does it tell you about perpendicularity of $\boldsymbol{e}$ to the column space of A?

$\rightarrow$ for each v_i, col of A, $v \cdot e = 0$

$\Rightarrow$ for any $w \in C(A)$, $w \cdot e = w(c_1 v_1 + \cdots + c_n v_n)$

col's of A

$= c_1(w \cdot v_1) + c_2(w \cdot v_2) + \cdots + c_n(w \cdot v_n)$

$= c_1 \times 0 + c_2 \times 0 + \cdots + c_n \times 0 = 0.$

$\Rightarrow$ e is orthogonal to $C(A)$.