Elementary Linear Algebra - MATH 2250 - Day 11

Name:

1. Start with the vectors $\boldsymbol{v}_1 = (0, 2, 1)$ and $\boldsymbol{v}_2 = (2, 1, 0)$.

   (a) Are they linearly independent? Why?

   (b) Are they a basis for any space?

   (c) What space V do they span?

   (d) What is the dimension of V?

   (e) Which matrices A have V as their column space?

(f) Which matrices A have V as their null space?

(g) Describe all vectors $\boldsymbol{v}_3$ that *complete* a basis $\boldsymbol{v}_1, \boldsymbol{v}_2, \boldsymbol{v}_3$ for $\mathbb{R}^3$.

2. **(Important)** Suppose $\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_n$ is a basis for $\mathbb{R}^n$ and the $n \times n$ matrix A is invertible. Show that $A\boldsymbol{v}_1, A\boldsymbol{v}_2, \ldots, A\boldsymbol{v}_n$ is also a basis for $\mathbb{R}^n$.

3. Write a rank 3 matrix $A_{4\times 7}$ and find its four fundamental subspaces, by describing a basis for each of them.