**Elementary Linear Algebra - MATH 2250 - Day 26**

**Name:**

1. What is the length of a complex vector $v = (v_1, \ldots, v_n)$?

2. What is a unitary matrix?

3. The $n$-th roots of unity are all the (complex) numbers $z$ that $z^n = 1$. That is, all the solutions to the equation $z^n - 1 = 0$. First, recall that if $z^n = 1$, then $|z| = 1$, that is $z$ lives on the unit circle. Also, note that any point on the unit circle can be written as $\cos(t) + \mathrm{i}\sin(\mathrm{t})$, for some $0 \le t < 2\pi$. Recall that $\cos(t) + \mathrm{i}\sin(\mathrm{t}) = \mathrm{e}^{\mathrm{it}}$. That is $z = e^{\mathrm{it}}$, and $z^n = 1$ if $(e^{\mathrm{it}})^n = 1$. But
$$1 = e^{2\pi}.$$
That is
$$z^n = (e^{\mathrm{it}})^n = 1 = e^{2\pi}.$$
Hence
$$z = e^{\mathrm{it}} = \underline{\qquad}.$$
The number $z = \underline{\qquad}$ is called the *primitive* $n$-th root of unity, and us usually denoted by $\omega$ (read: omega). All the $n$-th roots of unity are powers of $\omega$: $1, \omega, \omega^2, \ldots, \omega^{n-1}$.

Find all the 4-th roots of unity: $\omega^0, \omega, \omega^2, \omega^3$, and draw them in a complex plane.

Check that $(\omega^i)^4 = 1$, for each $i = 1, \ldots, 3$.

4. Write $F_4$, the $4 \times 4$ Fourier matrix.

5. Find $F_4^{-1}$.

6. Find $F_2$ and $F_2^{-1}$.

7. Let $\omega$ be the primitive 4-th root of unity, and $D = \begin{bmatrix} 1 & 0 \\ 0 & \omega \end{bmatrix}$. Evaluate the matrix

$$\left[\begin{array}{c|c} I_2 & D \\ \hline I_2 & -D \end{array}\right] \left[\begin{array}{c|c} F_2 & O \\ \hline O & F_2 \end{array}\right] \left[\begin{array}{cc|cc} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \hline 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right].$$

Compare this with $F_4$.