Elementary Linear Algebra - MATH 2250 - Day 24

Name:

1. What is a Markov matrix?

2. If A is a Markov matrix, then the largest eigenvalue of A is _____.

3. Find all the eigenvalues of $A = \begin{bmatrix} 0.3 & 0.7 \\ 0.7 & 0.3 \end{bmatrix}$.

   Find all the eigenvectors of A.

4. Find all the eigenvalues of $A = \begin{bmatrix} 0.3 & 0.7 & 0 \\ 0.7 & 0.3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

   Find all the eigenvectors of A.

5. Is $A - I$ for the matrix A in problem 3 singular?

6. In order to find the eigenvalues of the transpose of a matrix A we start with the polynomial equation

$$\det(A^T - \lambda I) = 0.$$

Recall that $\det(A^T) = \det(\_\_\_\_\_)$. Then $\det(A^T - \lambda I) = \_\_\_\_\_ = 0$. Thus the eigenvalues of A^T are the same as the eigenvalues of $\_\_\_\_\_$.

7. [T] [F] If the rows of a square matrix are linearly dependent, then the matrix is singular. Why?

8. Is $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ a Markov matrix? How many of its eigenvalues have absolute value equal to 1?

9. ("Everybody moves") Start with three groups of people, and at each time step, half of group 1 goes to group 2 and the other half goes to group 3. The other groups also split in half and move. Write down the matrix A that represents one step move, that is, $A \begin{bmatrix} p_1 \\ p_2 \\ p_3 \end{bmatrix}$ represents the population after one time step.

$$A = \begin{bmatrix} 0 & 1/2 & 1/2 \\ & & \\ & & \end{bmatrix}$$

Find A^2.

Find the eigenvalues of A and A^2.

Find an eigenvector x_1 (the steady state) of A.

Start with population $u_0 = (8, 16, 32)$, evaluate the states u_1, u_2, and u_3.

What is the sum of each vector u_i? How do you explain this in terms of the population?

What is the population of each group (approximately) after 10000 time steps? Why?

10. (Perron-Frobenius Theorem) Let A be a matrix with all positive entries, and λ be the maximum eigenvalue of A with corresponding eigenvector x. Then $\lambda \square\ 0$, and all numbers in x are _____.

11. For two function f and g then the inner product of f and g is

$$(f, g) = \int_a^b \qquad\qquad\qquad dx$$

Then the length squared of f is

$$||f||^2 = \int$$