# MATH 2200–Section 02–Homework 1*
# Fall 2012

**Please provide the details of your work for each problem. All problems are partial credit.**

**1.** (*1 point*) Suppose you drive the first half of a 100 mile trip at 50 mph and then drive the second half at 70 mph. What is your average speed for the trip? You don't need to "round" your answer. [Note that the answer is not 60 mph. If instead of distances, the trip times were the same, the answer would be 60 mph.]

**2.** (*2 points*) Suppose a particle is moving on the $x$-axis and its corresponding time-location graph is a straight line. If at times $t = 0$ and $t = 2$ it has positions $x = 2$ and $x = -1$, respectively, then find the average speed of the particle over the time interval $[0, 2]$, and its instantaneous speed at $t = 1$.

**3.** (*1 point*) Let $f : \mathbb{R} - \{0\} \to \mathbb{R}$ be defined by $f(x) = \frac{|x|}{x}$. Find $\lim_{x\to 0} f^2(x)$ or prove that it doesn't exist, where $f^2 = f \times f$ is the product of $f$ with itself. [We've proved that $\lim_{x\to 0} f(x)$ doesn't exist by considering the left and right limits at zero. Thus the moral is that the limit of the product of two functions, none of which having a limit at a particular point, might exist!]

**4.** (*2 points*) For what values of $c$ is the function $f$ continuous over $\mathbb{R}$?

$$f(x) = \begin{cases} -\frac{\sin(cx)}{2x} & \text{if } x < 0 \\ cx + 1 & \text{if } x \geq 0. \end{cases}$$

**5.** (*4 points*) Determine

1. $\lim_{x\to 1^+} \frac{1-\sqrt{x}}{|1-x|}$. (Hint: It might help to discard the absolute value sign at the beginning.)
2. $\lim_{x\to -3} \frac{\frac{1}{3}+\frac{1}{x}}{3+x}$. (Hint: Apply a little algebraic message and write it in a more familiar form.)
3. $\lim_{x\to a} f(x)$ if $|f(x)| \leq g(x)$ for all $x$, where $\lim_{x\to a} g(x) = 0$. (Hint: You need to introduce another function to make a "double" inequality for applying the squeeze theorem.)
4. $\lim_{x\to 0} \cos(x + \sin(x))$. (Hint: Remember that the cosine function is continuous, and you can transfer the limit inside it.)

*Submit on Monday, September 17 in class.

The following problem(s) are optional.

**6.** (*2 points*) (Optional) Is the following statement true? If yes, write down your reasoning, stating the results you're using. If no, give an explicit counterexample by defining $f, g$ for which the statement fails to hold.
"Suppose $f, g$ are any two functions with the property that $f(x) < g(x)$ for any real $x$, and $\lim_{x \to a} f(x), \lim_{x \to a} g(x)$ exist for some real number $a$. Then $\lim_{x \to a} f(x) < \lim_{x \to a} g(x)$." [First try to visualize the setting.]

**7.** (*2 points*) (Optional) Is the following statement true? If yes, write down your reasoning, stating the results you're using. If no, give an explicit counterexample by defining $f, g$ for which the statement fails to hold.
"Suppose $f$ is a function with the property that $\lim_{x \to 0} f(x) = 0$. Then $\lim_{x \to 0} f(x)g(x) = 0$ for any function $g$." (Hint: It might be helpful to think about some combination of functions defined by $x, x^2, 1/x, 1/x^2, \ldots$).

**8.** (*3 points*) (Optional) Find the domains of the functions given in parts $1, 2$, and $4$ of Problem 5 above. that is,

$$f(x) = \frac{1 - \sqrt{x}}{|1 - x|}, \quad g(x) = \frac{\frac{1}{3} + \frac{1}{x}}{3 + x}, \quad h(x) = \cos(x + \sin(x)).$$

Carefully justify your answers.