Elementary Linear Algebra - MATH 2250 - Day 2

Name:

Consider the following system and answer the following questions.

$$\begin{cases} x + 2y = 5 \\ -2x + 3y = 0 \end{cases}$$

1. $\boxed{\text{T}}$ $\boxed{\text{F}}$ A pivot can be any number. What couldn't it be?

   $\boxed{\text{T}}$ $\boxed{\text{F}}$ for two matrices A and B always $AB = BA$. Give an example.

2. What are the two pivots of the above system after elimination? Show steps.

3. Does the elimination process for the system above fail or succeed? Why?

4. Write down the augmented matrix for the above system and solve the system, using forward elimination and back substitution.

5. Find the elementary matrix $E_{3,1}$ that satisfies the following matrix multiplication:

$$\begin{bmatrix} \quad & \quad & \quad & \quad \\ & & & \\ & & & \\ & & & \end{bmatrix} \begin{bmatrix} -2 & 5 & 3 & 9 \\ 0 & 6 & -1 & 2 \\ 2 & 3 & 0 & 1 \\ 1 & 0 & -6 & 7 \end{bmatrix} = \begin{bmatrix} -2 & 5 & 3 & 9 \\ 0 & 6 & -1 & 2 \\ 0 & 8 & 3 & 10 \\ 1 & 0 & -6 & 7 \end{bmatrix}$$

6. What is the inverse of the matrix $E_{3,1}$ you found in the previous problem?

7. What is the $(3,2)$-entry of the matrix M?

$$M = \begin{bmatrix} -2 & 5 & 3 & 9 \\ 0 & -6 & -1 & 2 \\ 2 & 3 & 0 & 1 \\ 1 & 0 & -6 & 7 \end{bmatrix} \begin{bmatrix} -2 & 5 & 3 & 9 \\ 0 & -6 & -1 & 2 \\ 2 & 3 & 0 & 1 \\ 1 & 0 & -6 & 7 \end{bmatrix}$$

8. Do the following multiplications:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 1 & & -1 & 0 \\ 1 & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} -2 & 5 & 3 & 9 \\ 0 & -6 & -1 & 2 \\ 2 & 3 & 0 & 1 \\ 1 & 0 & -6 & 7 \end{bmatrix} =$$

$$\begin{bmatrix} -2 & 5 & 3 & 9 \\ 0 & -6 & -1 & 2 \\ 2 & 3 & 0 & 1 \\ 1 & 0 & -6 & 7 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 1 & & -1 & 0 \\ 1 & 0 & 0 & -1 \end{bmatrix} =$$

9. If the the columns of a matrix A lie in a plane, then they can be combined into $A\boldsymbol{x} = \mathbf{0}$, and then each row has $\boldsymbol{r} \cdot \boldsymbol{x} = \mathbf{0}$.

$$\begin{bmatrix} \\ \boldsymbol{a}_1 & \boldsymbol{a}_2 & \boldsymbol{a}_3 \\ \\ \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \qquad \text{and by rows:} \quad \begin{bmatrix} \boldsymbol{r}_1 \cdot \boldsymbol{x} \\ \boldsymbol{r}_2 \cdot \boldsymbol{x} \\ \boldsymbol{r}_3 \cdot \boldsymbol{x} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

The three rows also lie in a plane. Why is that plane perpendicular to $\boldsymbol{x}$?

10. This system has no solution. The planes in the row picture don't meet at a point.

$$\begin{aligned} x + y + z &= 2 \\ x + 2y + z &= 3 \\ 2x + 3y + 2z &= 4 \end{aligned} \qquad \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 2 & 3 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix} = b$$

(a) Multiply the equations by $1, 1, -1$ and add to get $0 = 1$. No solution. Are any two of the planes parallel? What are the equations of planes parallel to $x + y + z = 2$?

(b) Take the dot product of each column of A (and also $\boldsymbol{b}$) with $y = (1, 1, -1)$. How do those dot product show that the system $A\boldsymbol{x} = \boldsymbol{b}$ has no solution?

(c) Find three right side vectors $\boldsymbol{b}^*$ and $\boldsymbol{b}^{**}$ and $\boldsymbol{b}^{***}$ tht do allow solutions.

11. Find the matrix P that multiplies (x, y, z) to give (y, z, x).

12. Find the matrix Q that multiplies (y, z, x) to bring back (x, y, z).

13. What 2×2 matrix R rotates every vector by $90°$? (R times $\begin{bmatrix} x \\ y \end{bmatrix}$ is $\begin{bmatrix} y \\ -x \end{bmatrix}$.)

14. Draw the row and columns pictures for the equations $x - 2y = 0$, $y + x = 6$.