Elementary Linear Algebra - MATH 2250 - Exam 1

Name: _____

1. Describe geometrically (line, plane, or all of $\mathbb{R}^3$) all linear combinations of $\begin{bmatrix}1\\2\\3\end{bmatrix}$ and $\begin{bmatrix}3\\6\\9\end{bmatrix}$.

2. How long is the vector $\boldsymbol{v} = (1, 1, 1, 1, 1, 1, 1, 1, 1)$ in 9 dimensions?

3. Write down three 'independent' vectors $\boldsymbol{u}, \boldsymbol{v}$ and $\boldsymbol{w}$ in $\mathbb{R}^3$.

   Using the three vectors above as columns, form a matrix $A = \left[\begin{array}{c|c|c} \boldsymbol{u} & \boldsymbol{v} & \boldsymbol{w} \end{array}\right]$, and find its LU-decomposition.

4. Solve the system of linear equations

$$\begin{cases} x + 2y + 4z = 18 \\ -2x + 5y + z = 5 \\ -4x + y + 2z = 0 \end{cases}$$

5. Find the inverse of $A = \begin{bmatrix} 3 & -1 & -1 \\ -1 & 3 & -1 \\ -1 & -1 & 3 \end{bmatrix}$.

6. Describe the column space and the null space of A **and** the complete solution to $A\boldsymbol{x} = \boldsymbol{b}$ for

$$A = \begin{bmatrix} 2 & 4 & 6 & 4 \\ 2 & 5 & 7 & 6 \\ 2 & 3 & 5 & 2 \end{bmatrix}, \qquad \boldsymbol{b} = \begin{bmatrix} 4 \\ 3 \\ 5 \end{bmatrix}.$$

7. Answer ONLY ONE of the following questions:

   - A man is three times as old as his son was at the time when the father was twice as old as his son will be two years from now. Find the present age of each person if the sum of their ages is 55.
   - A movie star, unwilling to give his age, posed the following riddle to a gossip columnist. "9 years ago, I was 15 times as old as my daughter. Now I am 6 times as old as she is." How old are the star and his daughter?