Elementary Linear Algebra - MATH 2250 - Day 1

Name:

Consider the following system.

$$\begin{cases} 2x + y = 5 \\ -x + 3y = 0 \end{cases}$$

1. Mark this as True or False: (Explain why)

   If the two lines $2x + y = 5$ and $-x + 3y = 0$ meet at a point (a, b), then (a, b) is a solution to the system.

2. Write the matrix form for the system.

3. Draw the row picture of the system.

4. Draw the column picture of the system.

5. Give an example of a linear combination of two vectors $\begin{bmatrix} 2 \\ -1 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 3 \end{bmatrix}$.

6. The linear combinations of $\boldsymbol{v} = (2, -1, 0)$ and $\boldsymbol{w} = (1, 3, 0)$ fill a plane. Describe that plane.

   Find a vector that is not a combination of $\boldsymbol{v}$ and bmw.

7. For $\boldsymbol{v} = (2, -1)$ describe all points $c\boldsymbol{v}$ with (1) whole numbers c (2) nonnegative $c \geq 0$.

8. Find two equations for the unknowns c and d so that the linear combination $c\boldsymbol{v} + d\boldsymbol{w}$ equals the vector $\boldsymbol{b}$:

$$\boldsymbol{v} = \begin{bmatrix} -2 \\ 1 \end{bmatrix} \qquad \boldsymbol{w} = \begin{bmatrix} 0 \\ -1 \end{bmatrix} \qquad \boldsymbol{b} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

9. For $\boldsymbol{v} = (1, 2)$ and $\boldsymbol{w} = (2, 1)$ test the Schwarz inequality on $\boldsymbol{v} \cdot \boldsymbol{w}$, and the triangle inequality of $||\boldsymbol{v} + \boldsymbol{w}||$.

   Find $\cos\theta$ for the angle between $\boldsymbol{v}$ and $\boldsymbol{w}$.

When will we have equality $|\boldsymbol{v} \cdot \boldsymbol{w}| = ||\boldsymbol{v}||||\boldsymbol{w}||$, and $||\boldsymbol{v} + \boldsymbol{w}|| = ||\boldsymbol{v}|| + ||\boldsymbol{w}||$?

10. Find a unit vector $\boldsymbol{u}$ in the direction of $\boldsymbol{v} = (12, 5)$.

11. Find **all** the unit vectors which are perpendicular to $\boldsymbol{v} = (12, 5)$.

12. Find a vector $x = (c, d)$ that has dot product $\boldsymbol{x} \cdot \boldsymbol{r} = 1$, and $\boldsymbol{x} \cdot \boldsymbol{s} = 0$ with the given vectors $\boldsymbol{r} = (1, 3)$ and $\boldsymbol{s} = (-3, 1)$.