Elementary Linear Algebra - MATH 2250 - Day 21

Name:

1. $\boxed{\text{T}}\boxed{\text{F}}$ If λ is an eigenvalue of A and μ is an eigenvalue of B then $\lambda + \mu$ is an eigenvalue of $A + B$. Explain.

2. $\boxed{\text{T}}\boxed{\text{F}}$ If λ is an eigenvalue of A and μ is an eigenvalue of B then with the same eigenvector x, then $\lambda + \mu$ is an eigenvalue of $A + B$. Explain.

3. Let $\boldsymbol{x} = (2, 3, 1)$ be an eigenvector of A corresponding to the eigenvalue 3. Evaluate Ax.

4. The Fundamental Theorem of Algebra asserts that any polynomial of degree n has exactly n (complex) roots. How many eigenvalues does an $n \times n$ matrix have? Why?

5. If A is singular then one of its eigenvalues is ______.

6. If P is a nonzero projection matrix in $\mathbb{R}^3$, then two of its eigenvalues are ______, and ______.

7. If λ is an eigenvalue of A, then $A - \lambda I$ is a(n) ________________ matrix.

8. What is the sum of the eigenvalues of the $n \times n$ identity matrix?

9. What is the sum of the eigenvalues of $A = \operatorname{diag}(d_1, \ldots, d_n)$?

10. Let's find (guess?) all the eigenvalues of $A = \operatorname{diag}(d_1, \ldots, d_n)$. Let $\boldsymbol{e}_i$ be the vector with a 1 in its i-th position and 0's elsewhere, e.g. $\boldsymbol{e}_1 = (1, 0, 0, \ldots, 0)$ etc. What is $A\boldsymbol{e}_i$, for each i?

11. What is the trace (the sum of the eigenvalues) of $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 6 \end{bmatrix}$?

12. If λ is an eigenvalue of A, then $A - \lambda I$ is a(n) ________________ matrix.

13. Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$. What is the characteristic equation of A?

14. Let $A = \begin{bmatrix} 1 & 1 \\ 1 & 5 \end{bmatrix}$. What is the characteristic equation of A? What are all the eigenvalues of A?

15. Let $\boldsymbol{x}$ be an eigenvector of A for an eigenvalue λ. is $2\boldsymbol{x}$ an eigenvector of A? For what eigenvalue?

    What are all the eigenvectors of A for the eigenvalue λ? (*Agreement: we do not consider the zero vector, and eigenvalue for any eigenvalue, not even for the zero eigenvalue!*)

16. Find a matrix with eigenvalues $1, 2, 3$, and 4.

17. Let A be a matrix with an eigenvalue λ and the corresponding eigenvector $\boldsymbol{x}$. Let $B = 2A$, and evaluate $B\boldsymbol{x}$.

    What can you tell about the eigenvalue of B in terms of the eigenvalues of A?

18. Find an eigenvector for each of the eigenvalues of $A = \begin{bmatrix} 1 & 1 \\ 1 & 5 \end{bmatrix}$.

19. Let A be as in problem 11, and let $B = A + 2I$. What are the eigenvalues of B?

Find an eigenvector for each of the eigenvalues of B.

What relations hold between the eigenvalues and eigenvectors of A and B?

20. Find all the eigenvalues and their corresponding eigenvectors of $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$. (refer to problem 10)

21. Find all the eigenvalues and their corresponding eigenvectors of $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$. Do eigenvectors of the matrix from problem 20 work?

22. We are not going to prove this, but it is good to remember that

    (a) the eigenvalues of any symmetric matrix are ____________ numbers, and

    (b) the eigenvalues of any skew-symmetric matrix are ________________________ numbers.