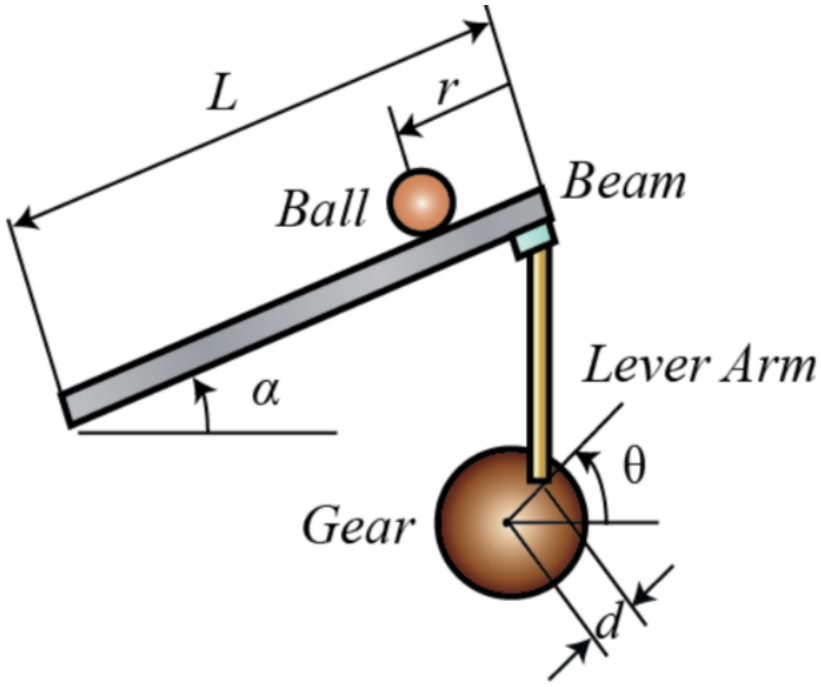
Ball and Beam System

A ball is placed on a beam, see figure below, where it is allowed to roll with 1 degree of freedom along the length of the beam. A lever arm is attached to the beam at one end and a servo gear at the other. As the servo gear turns by an angle θ , the lever changes the angle of the beam by α. When the angle is changed from the horizontal position, gravity causes the ball to roll along the beam. Design a controller which should be able to manipulate the ball position.



The second derivative of the input angle α actually affects the second derivative of r. However, we will ignore this contribution. The Lagrangian equation of motion for the ball is then given by the following.

$$0 = (\frac{J}{R^2} + m)\ddot{r} + mg\sin\alpha - mr\dot{\alpha}^2$$

The equation which relates the beam angle to the angle of the gear can be approximated as linear by the equation below:

$$\alpha = \frac{d}{L}\theta$$

Substituting this into the previous equation, we get:

$$(\frac{J}{R^2} + m)\ddot{r} = -mg\frac{d}{L}\theta$$

System Parameters

mass of the ball(m) = 0.11 kg

radius of the ball(R) = 0.015 m

lever arm offset(d) = 0.03 m

gravitational acceleration(g) = 9.8 m/s^2

length of the beam(L) = 1.0 m

ball's moment of inertia(J) = 9.99*10^{-6}kg.m^2

ball position coordinate = r

beam angle co-ordinate = α

servo gear angle = θ

desired position = 0.5 m

Make sure that the tuning parameters you choose give the system a rise time of about 3 seconds and maximum overshoot of less than 5%.