1 Ball and Plate System

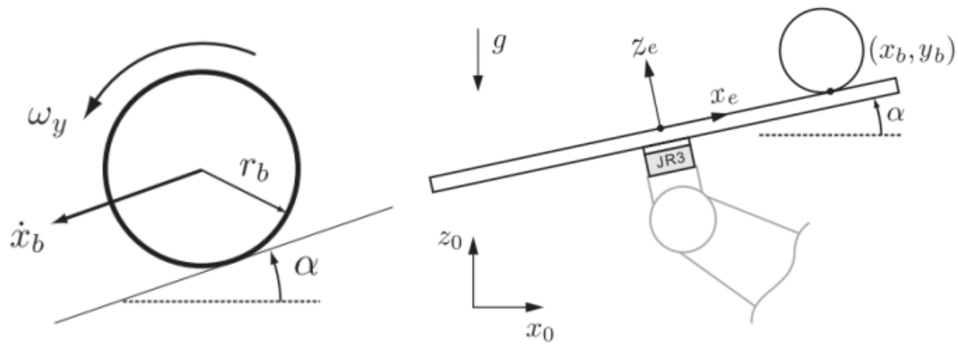

Figure 1: Side view of Ball and Plate System

We assume ball's position on the plate to be x_b and y_b and α and β the inclination of the plate. It is important to note that we assume the center of x y coordinates be at center of plate. m_b is mass of the ball and I_b is moment of inertia of the ball.

Parameter	Description	Value
m_b	Mass of the ball	$0.11kg$
r_b	Radius of the ball	$0.02m$
S_p	Dimension of the plate	$1.0 \times 1.0m^2$
v_{max}	Maximum velocity of the ball	$4mm/s$
m_p	Mass of the plate	$0.1kg$
I_p	Mass moment of inertia of the plate	$0.5kg.m^2$
I_b	Mass moment of inertia of the ball	$1.76 \times 10^{-5}kg.m^2$
g	Gravitational acceleration	9.81

$$m_b[\frac{5}{7}\ddot{x}_b - (x_b\dot{\alpha}^2 + y_b\dot{\alpha}\dot{\beta}) + g\sin\alpha] = 0$$
$$m_b[\frac{5}{7}\ddot{y}_b - (y_b\dot{\beta}^2 + x_b\dot{\alpha}\dot{\beta}) + g\sin\beta] = 0$$

We can linearise above equation by assuming:

- Small angle of inclination for the plate(up to $\pm 5°$): $\alpha << 1$ and $\beta << 1$, thus $\sin\alpha \approx \alpha$, $\sin\beta \approx \beta$
- Slow rate of change for the plate: $\dot{\alpha} << 0$ and $\dot{\beta} << 0$, thus $\dot{\alpha}\dot{\beta} \approx 0$, $\dot{\alpha}^2 \approx 0$, $\dot{\beta}^2 \approx 0$

$$\frac{5}{7}\ddot{x}_b + g\alpha = 0$$

$$\frac{5}{7}\ddot{y}_b + g\beta = 0$$