Lecture 16

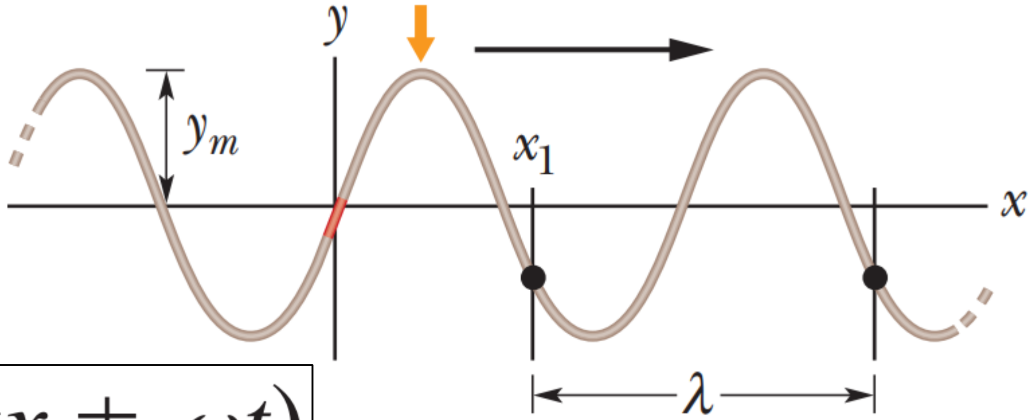
Wave Interference

And

Numerical problems

Wave

$$y = y_m \sin(kx \pm \omega t + \phi).$$



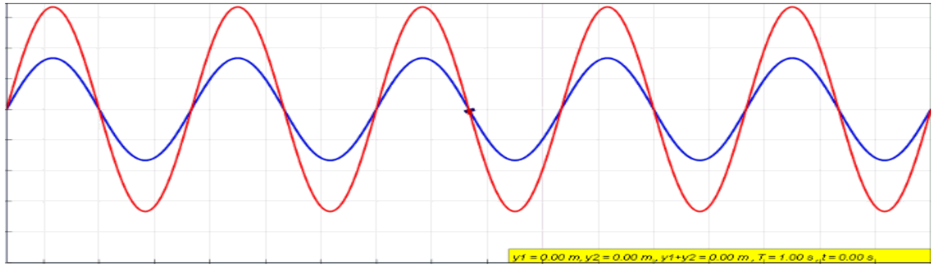
$$y(x,t) = h(kx \pm \omega t)$$

$$P_{\text{avg}} = \tfrac{1}{2}\mu v \omega^2 y_m^2 \quad \text{(average power).}$$

$$v = \frac{\omega}{k} = \frac{\lambda}{T} = \lambda f$$

$$y'(x,t) = [2y_m \cos \tfrac{1}{2}\phi] \sin(kx - \omega t + \tfrac{1}{2}\phi).$$

$$y'(x,t) = [2y_m \sin kx] \cos \omega t.$$



Sample Problem 16.01 Determining the quantities in an equation for a transverse wave

Homework

A string has linear density $\mu = 525$ g/m and is under tension $\tau = 45$ N. We send a sinusoidal wave with frequency $f = 120$ Hz and amplitude $y_m = 8.5$ mm along the string. At what average rate does the wave transport energy?

Two identical sinusoidal waves, moving in the same direction along a stretched string, interfere with each other. The amplitude y_m of each wave is 9.8 mm, and the phase difference ϕ between them is 100°.

(a) What is the amplitude y'_m of the resultant wave due to the interference, and what is the type of this interference?

Two identical sinusoidal waves, moving in the same direction along a stretched string, interfere with each other. The amplitude y_m of each wave is 9.8 mm, and the phase difference ϕ between them is 100°.

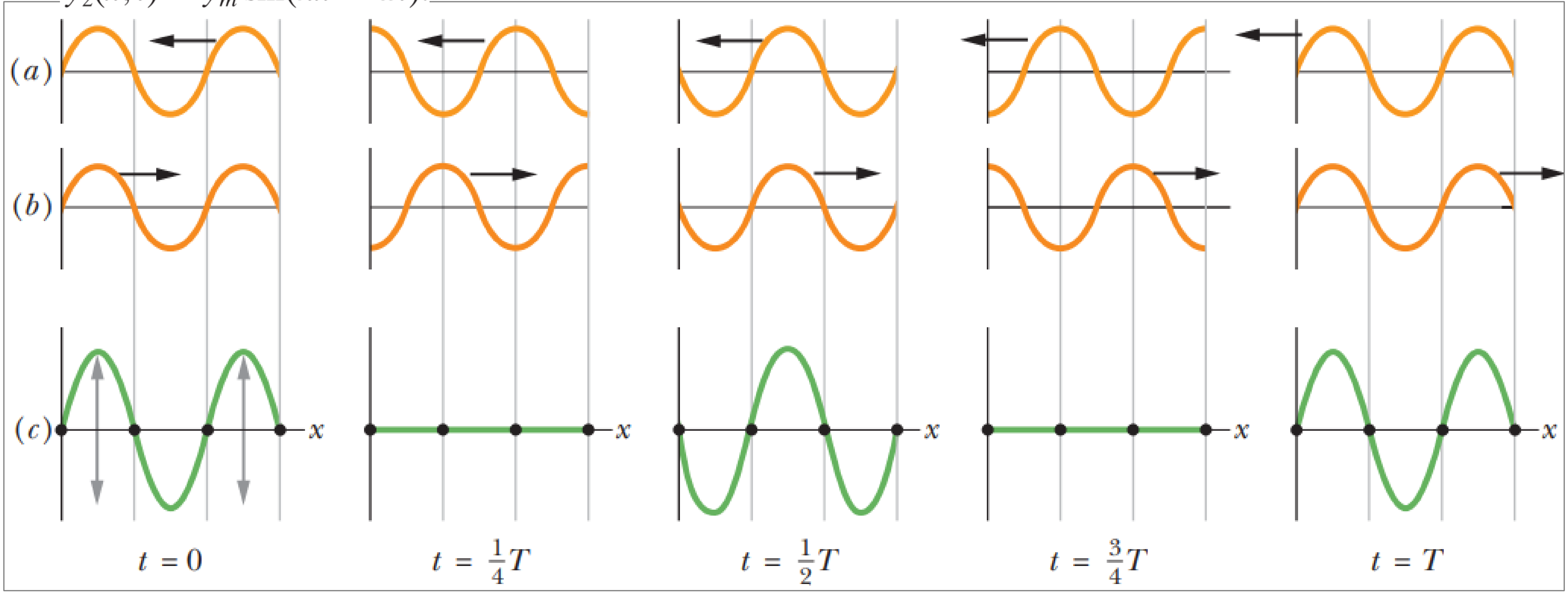
(b) What phase difference, in radians and wavelengths, will give the resultant wave an amplitude of 4.9 mm?

Wave Interference

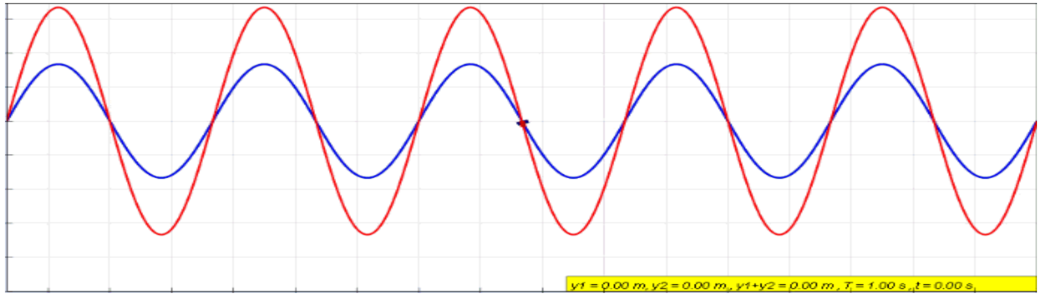
$$y_1(x, t) = y_m \sin(kx - \omega t)$$

$$y_2(x, t) = y_m \sin(kx + \omega t).$$

$$\boxed{y'(x, t) = [2y_m \sin kx] \cos \omega t.}$$

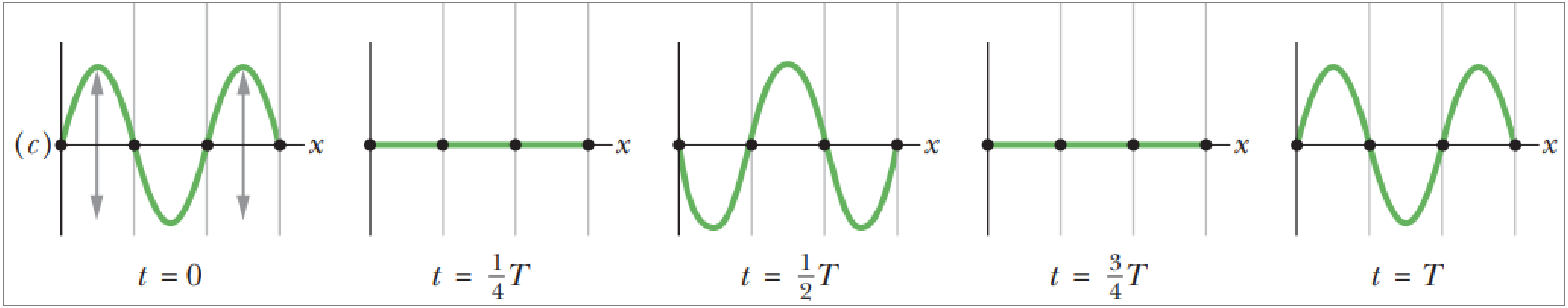


Points of Zero Amplitude



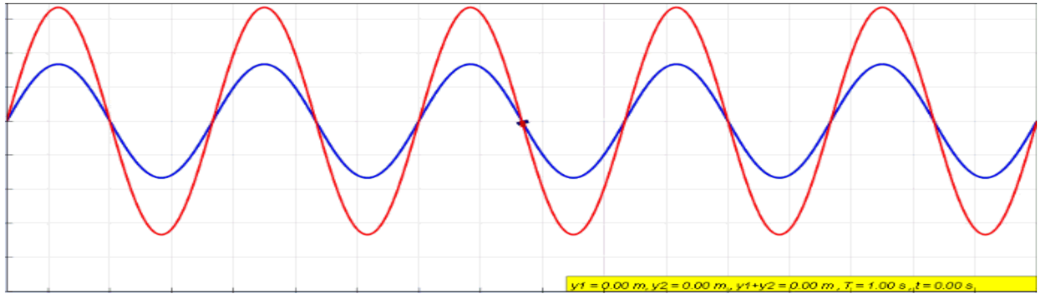
$$kx = n\pi, \qquad \text{for } n = 0, 1, 2, \ldots.$$

$$x = n\frac{\lambda}{2}, \qquad \text{for } n = 0, 1, 2, \ldots \qquad \text{(nodes)},$$

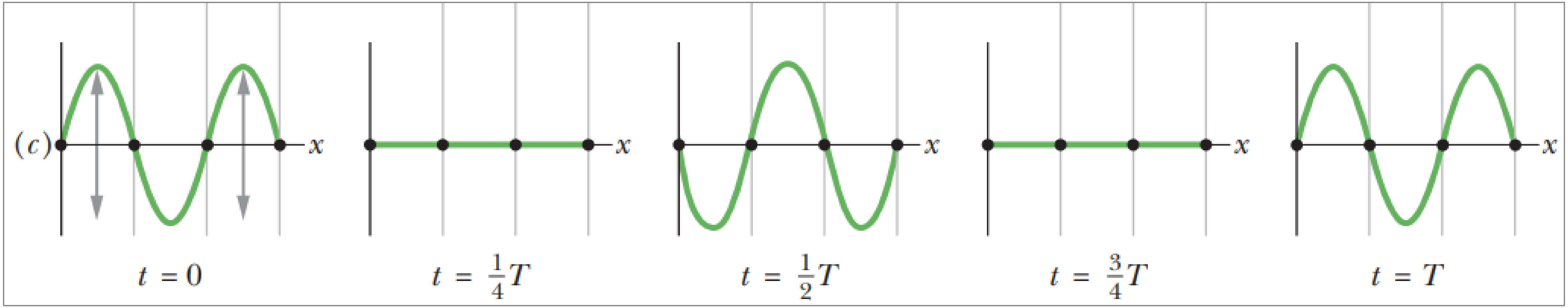


Points of Maximum Amplitude

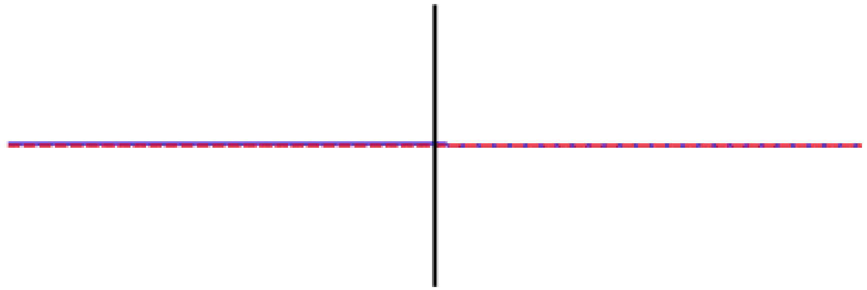
$$kx = \left(n + \tfrac{1}{2}\right)\pi, \qquad \text{for } n = 0, 1, 2, \ldots$$

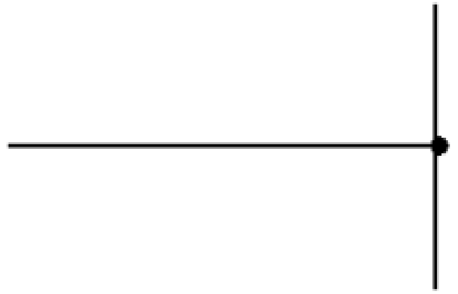


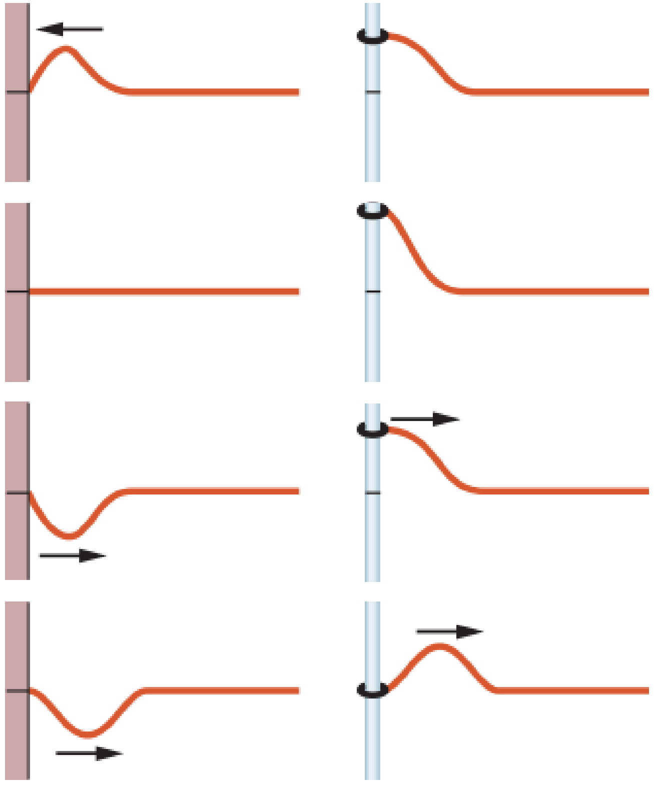
$$x = \left(n + \frac{1}{2}\right)\frac{\lambda}{2}, \qquad \text{for } n = 0, 1, 2, \ldots \qquad \text{(antinodes)},$$

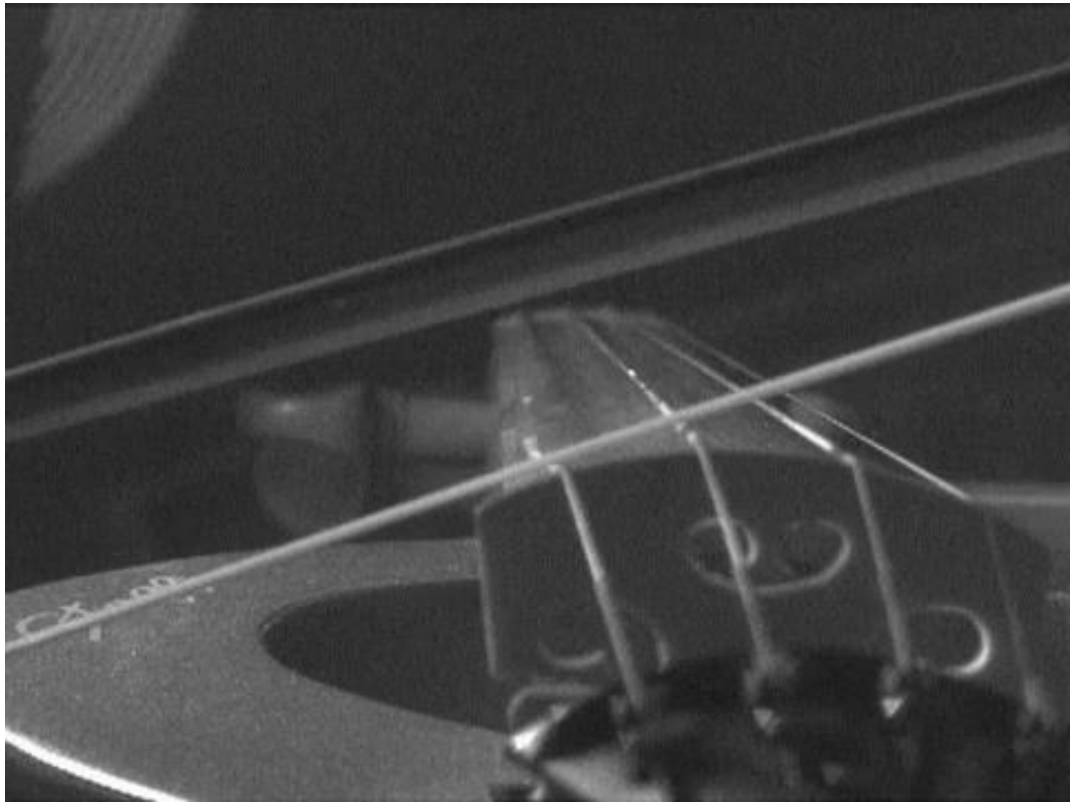


Wave Reflections and Harmonics

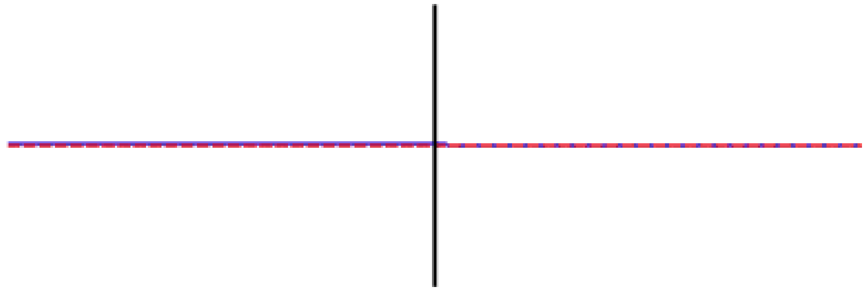


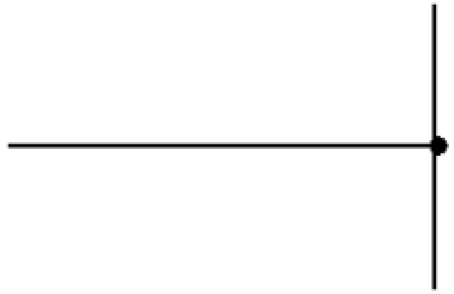


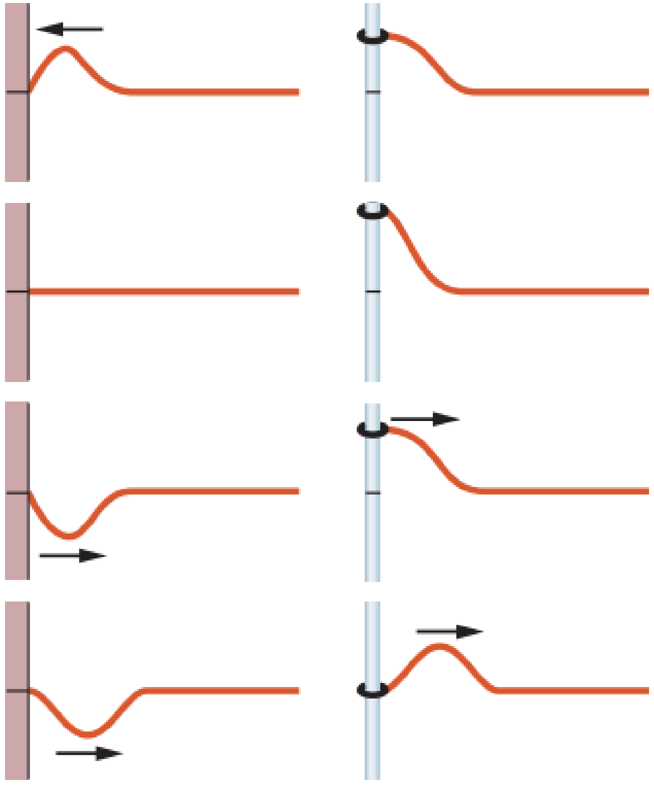


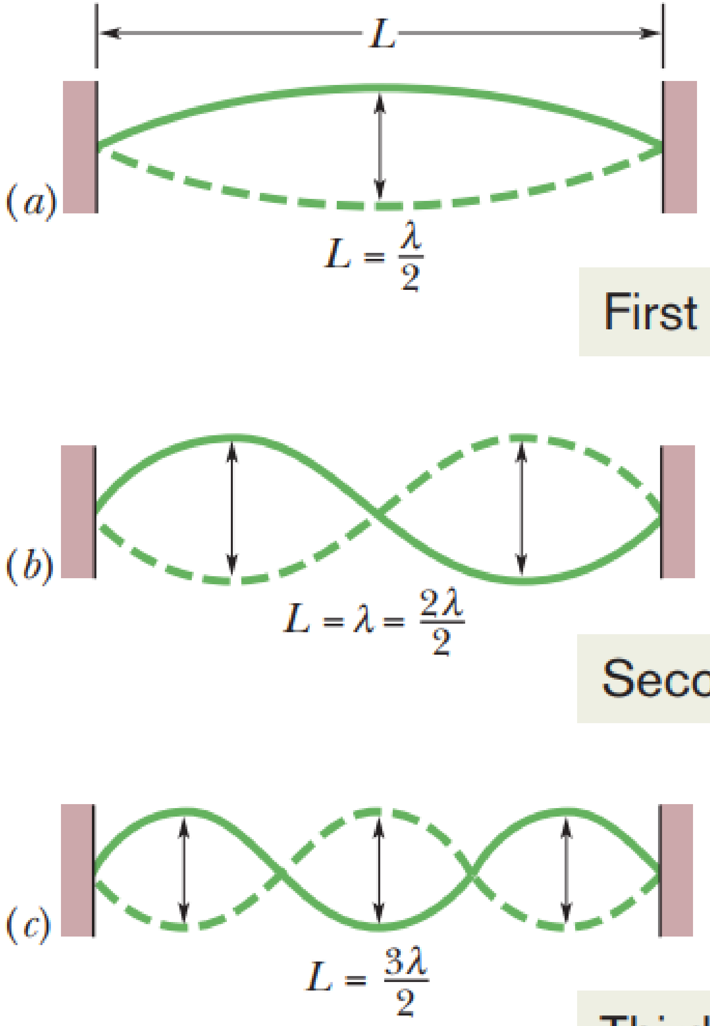


Wave Reflections and Harmonics









First harmonic

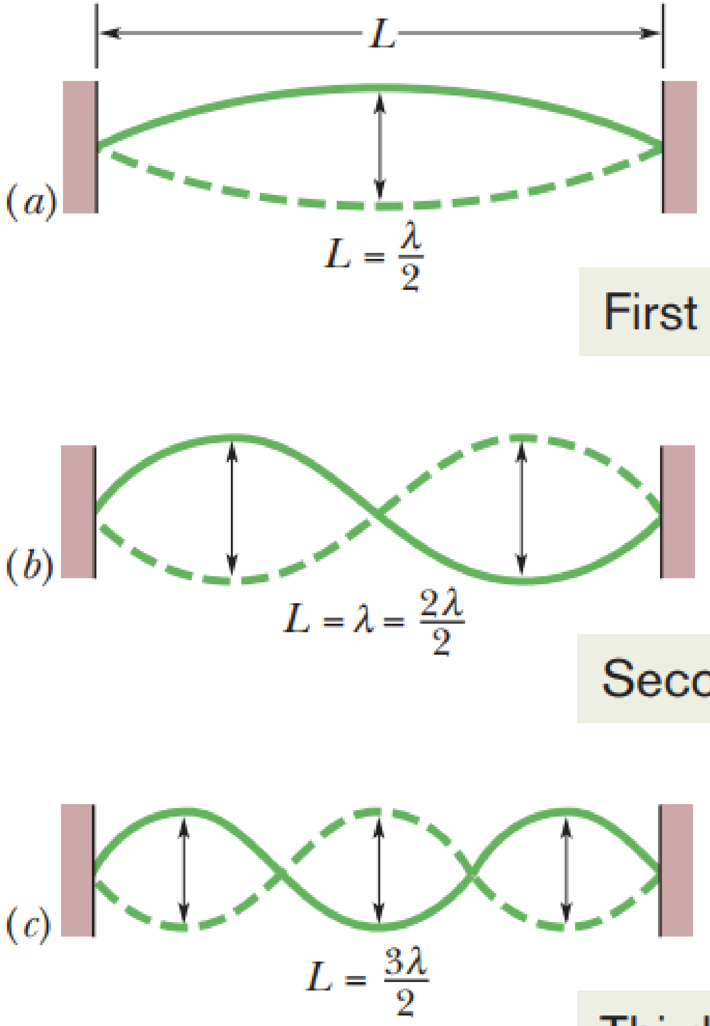
Second harmonic

Third harmonic

Wave Reflections and Harmonics

$$\lambda = \frac{2L}{n}, \qquad \text{for } n = 1, 2, 3, \ldots .$$

$$f = \frac{v}{\lambda} = n\frac{v}{2L}, \qquad \text{for } n = 1, 2, 3, \ldots .$$



First harmonic

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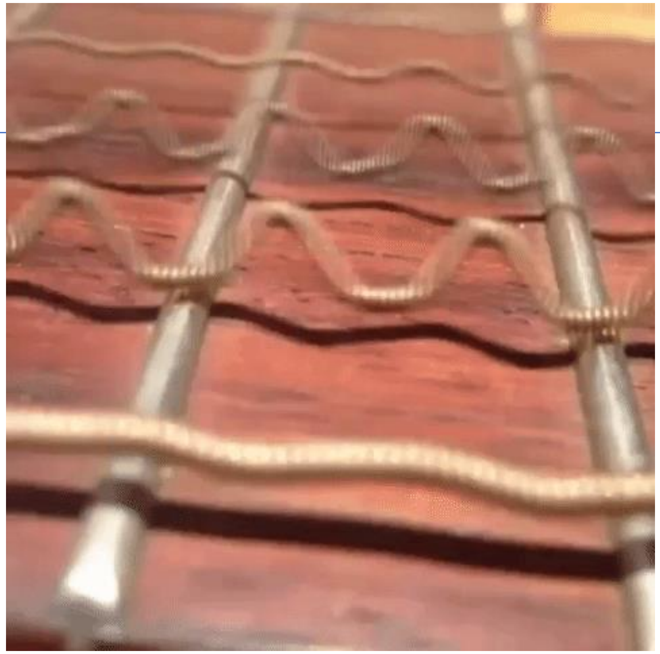
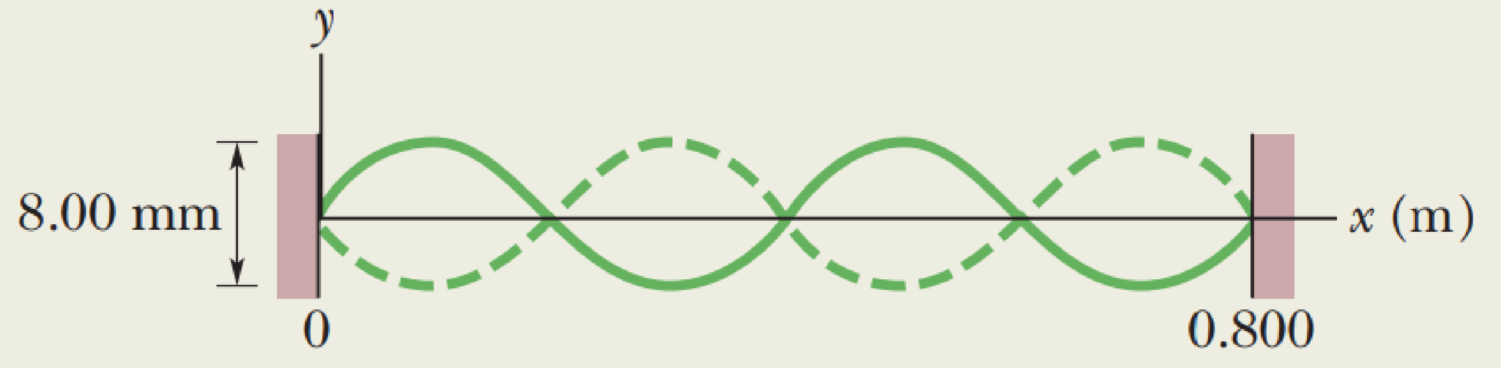
Wave Reflections and Harmonics



$$\lambda = \frac{2L}{n}, \qquad \text{for } n = 1, 2, 3, \ldots .$$

$$f = \frac{v}{\lambda} = n\frac{v}{2L}, \qquad \text{for } n = 1, 2,$$

Figure 16-23 shows resonant oscillation of a string of mass $m = 2.500$ g and length $L = 0.800$ m and that is under tension $\tau = 325.0$ N. What is the wavelength λ of the transverse waves producing the standing wave pattern, and what is the harmonic number n? What is the frequency f of the transverse waves and of the oscillations of the moving string elements? What is the maximum magnitude of the transverse velocity u_m of the element oscillating at coordinate $x = 0.180$ m? At what point during the element's oscillation is the transverse velocity maximum?



Lecture 16