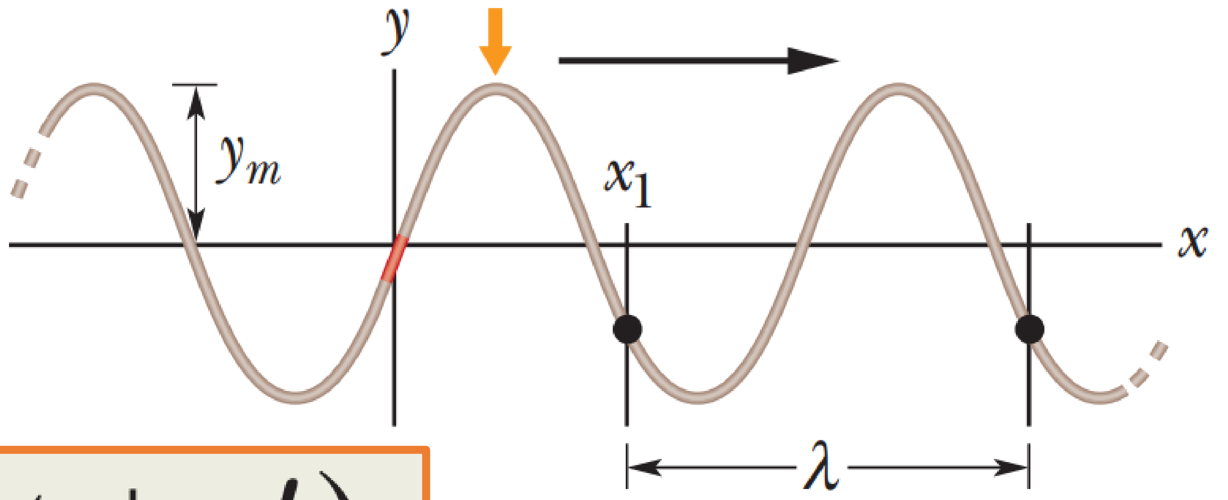
Lecture 15

Wave phenomenon

Wave



The generalized wave function

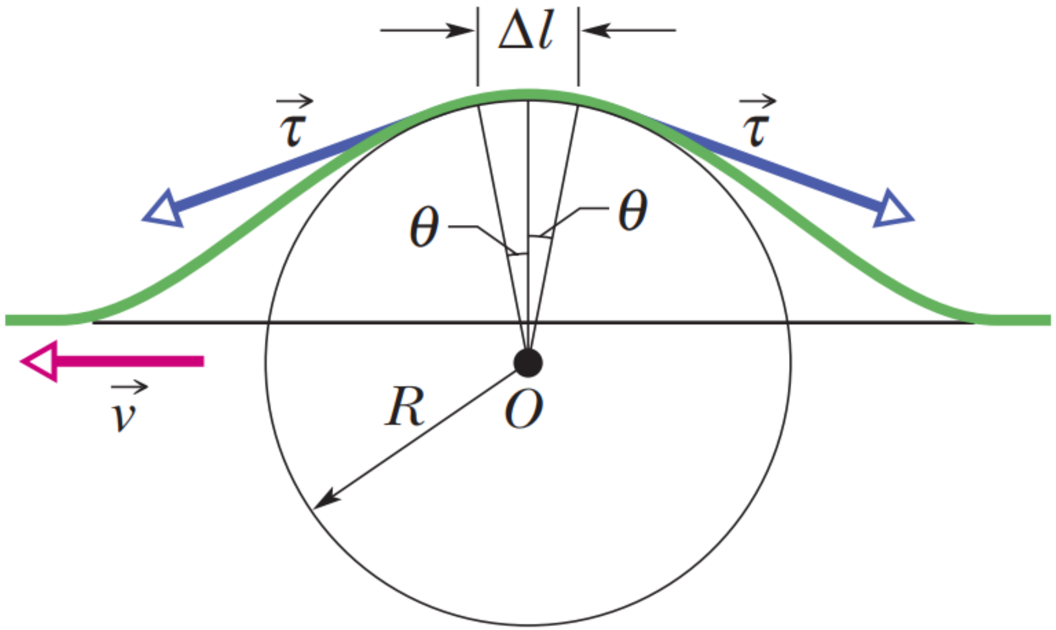
$$y = y_m \sin(kx \pm \omega t + \phi).$$

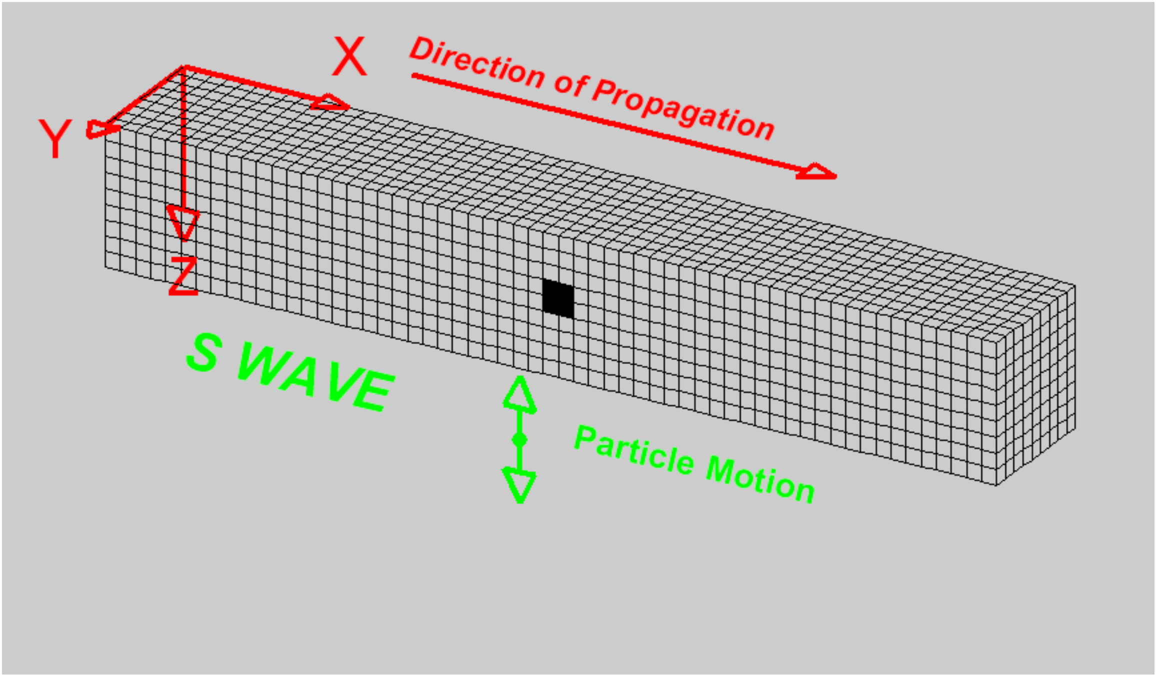
$$v = \frac{\omega}{k} = \frac{\lambda}{T} = \lambda f$$

$$y(x,t) = h(kx \pm \omega t)$$

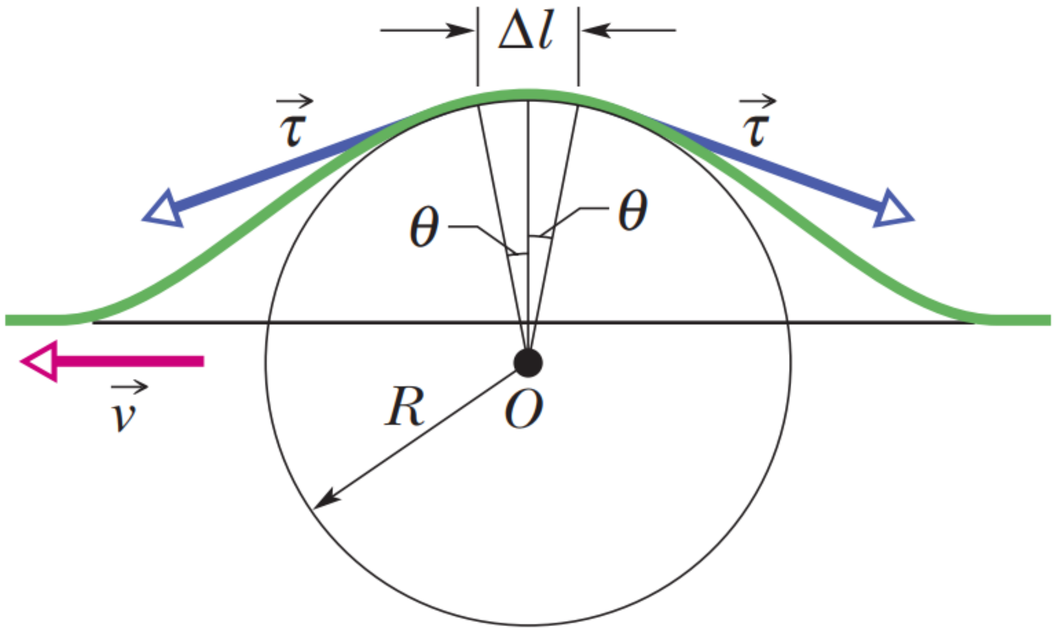
$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2} \quad \text{(wave equation)}$$

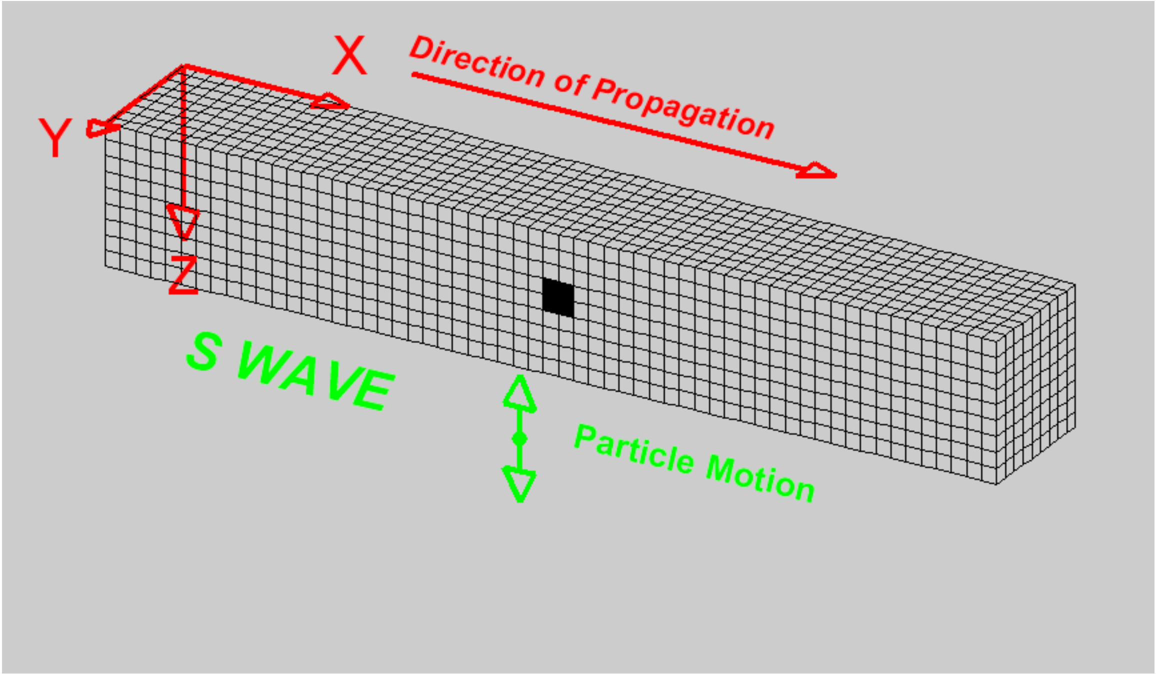
Wave in a string





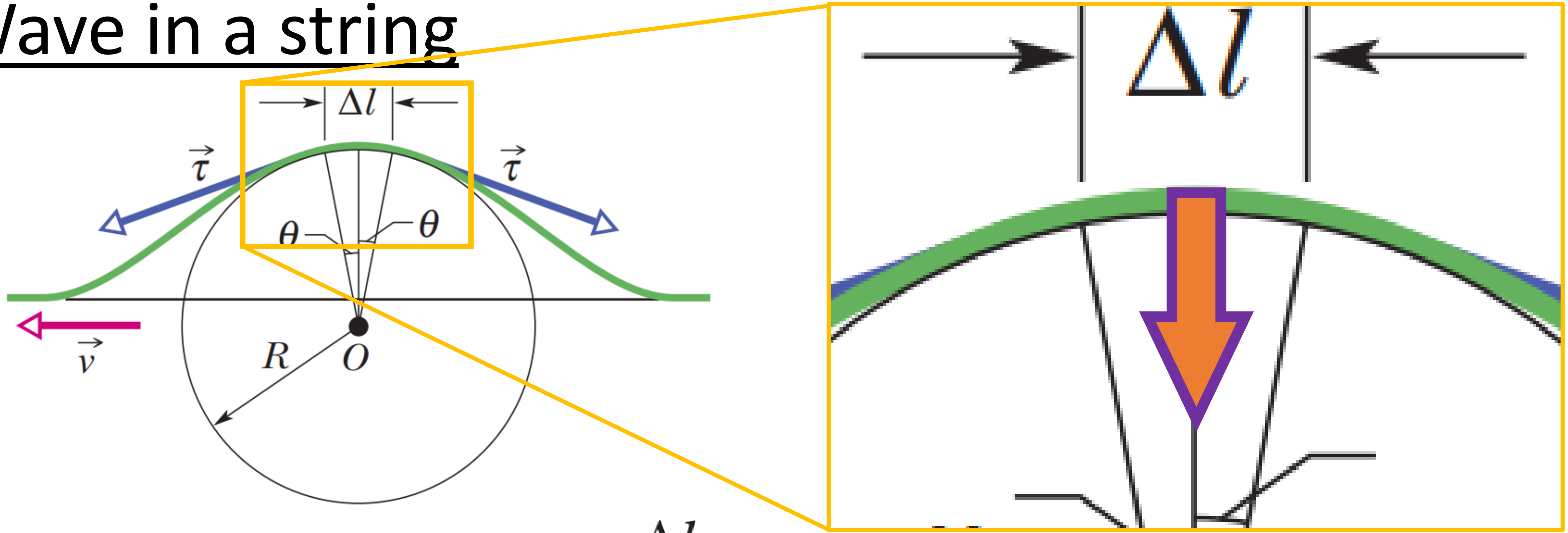
Wave in a string





$$F = 2(\tau \sin \theta) \approx \tau(2\theta) = \tau \frac{\Delta l}{R}$$

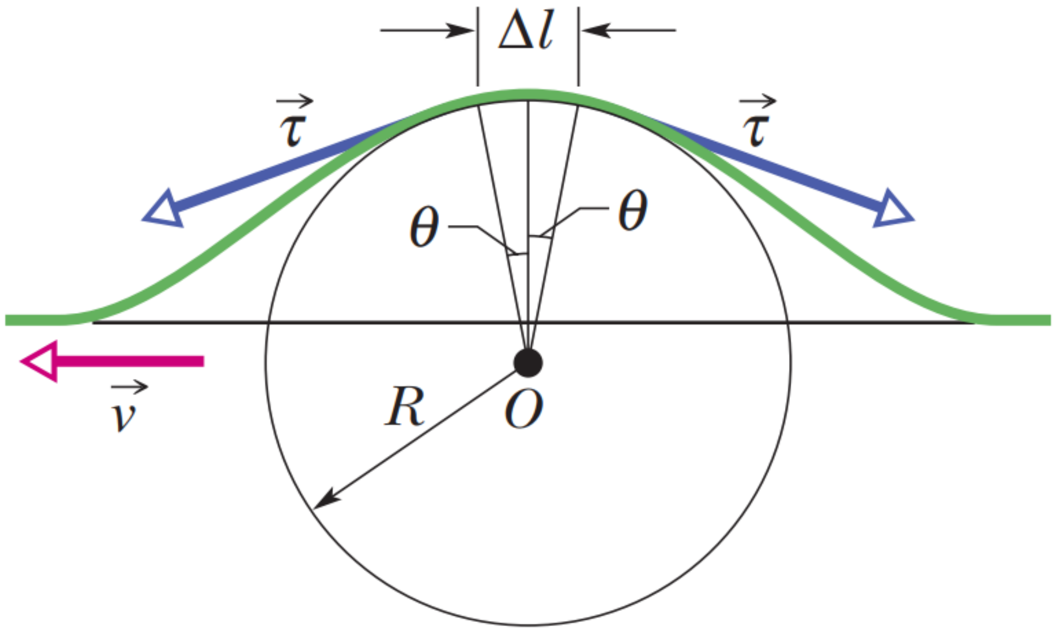
Wave in a string

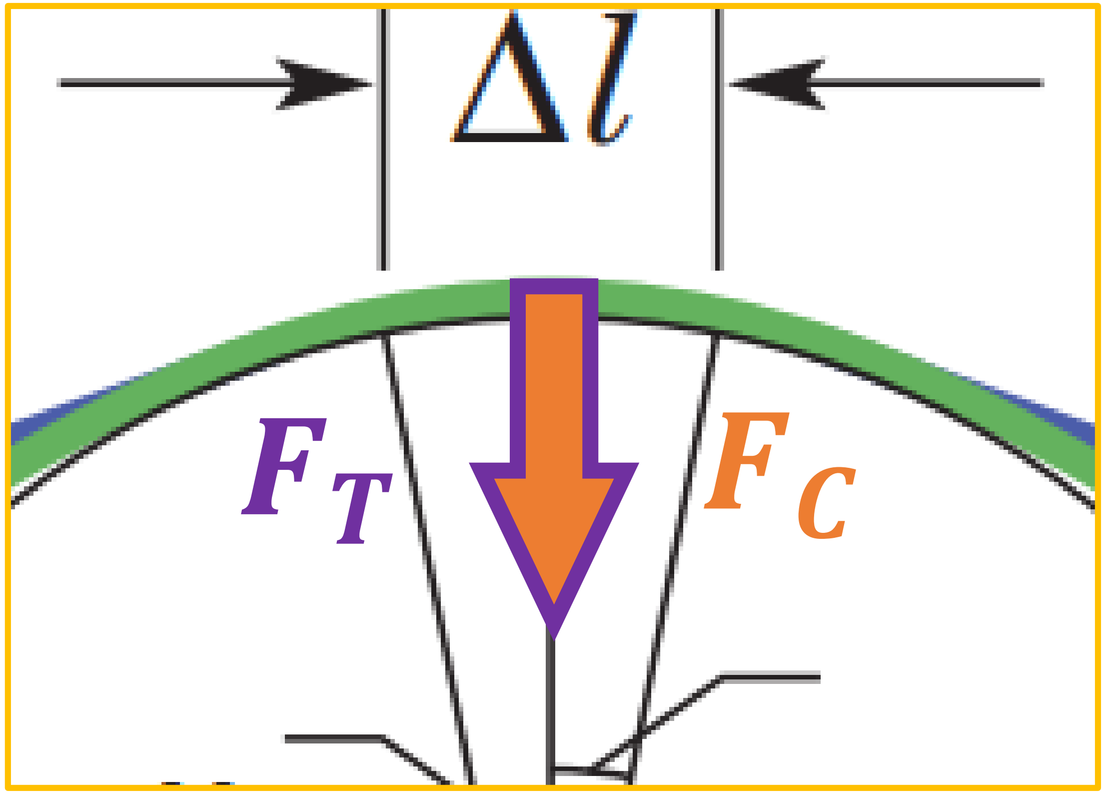


$$F = 2(\tau \sin \theta) \approx \tau(2\theta) = \tau \frac{\Delta l}{R}$$

$$\frac{\tau \Delta l}{R} = (\mu \Delta l) \frac{v^2}{R}$$

Wave in a string





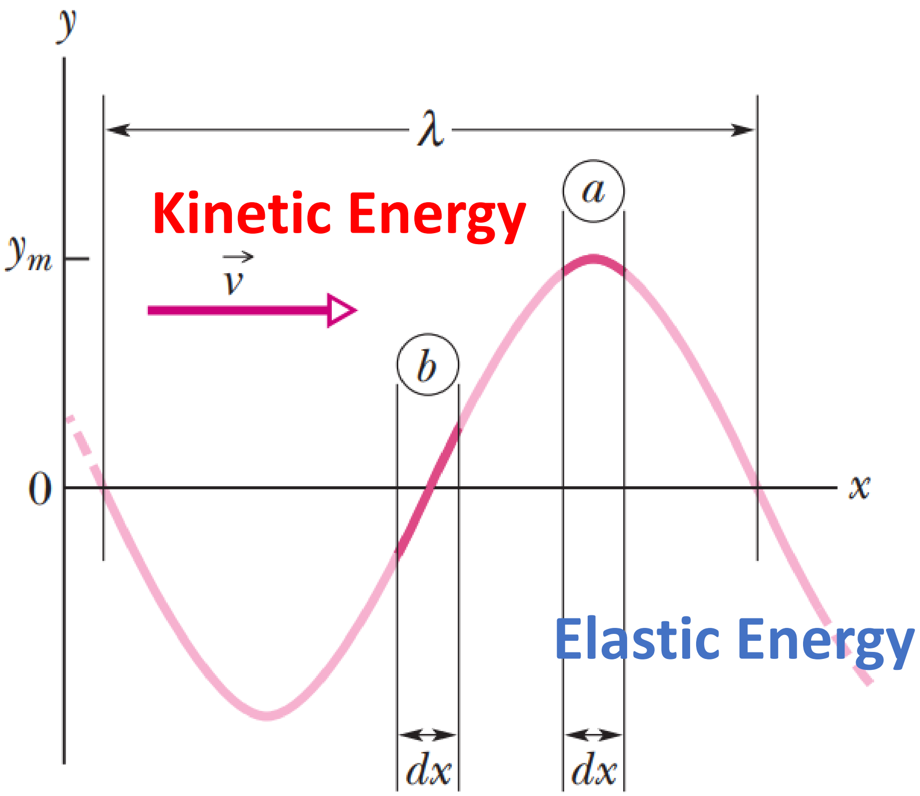
$$F = 2(\tau \sin\,\theta) \approx \tau(2\theta) = \tau \frac{\Delta l}{R}$$

$$\frac{\tau\,\Delta l}{R} = (\mu\,\Delta l)\,\frac{v^2}{R}$$

$$\boxed{v = \sqrt{\frac{\tau}{\mu}}}$$

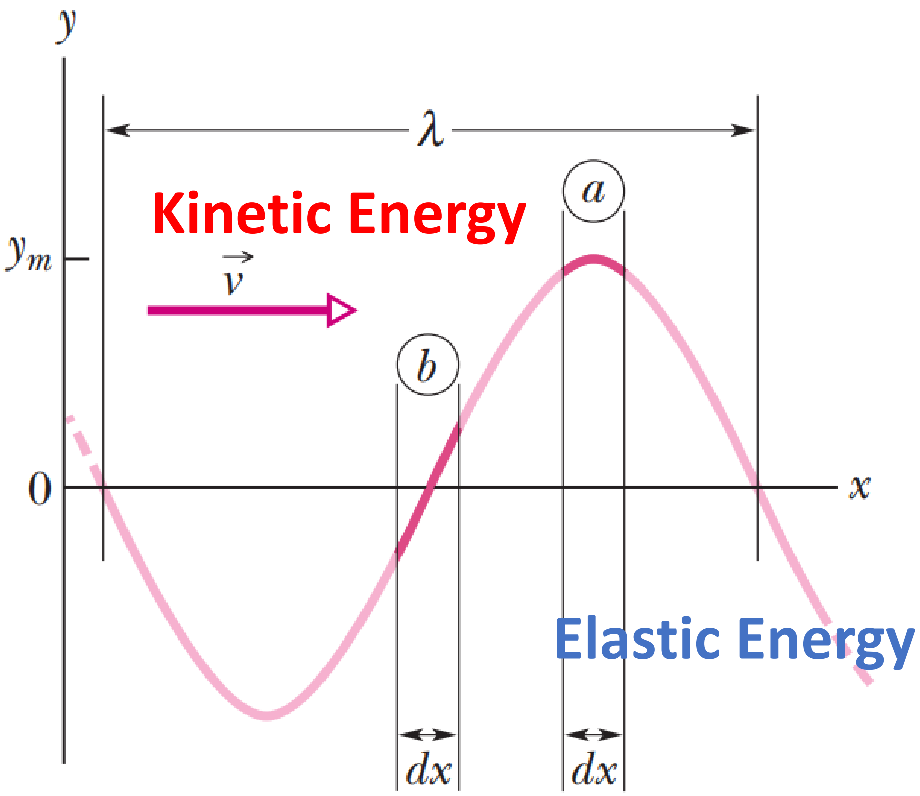
μ (dimension ML^{-1})
τ (dimension MLT^{-2})

Energy Transportation



Energy Transportation

$$dK = \frac{1}{2}\, dm\, u^2$$



Energy Transportation

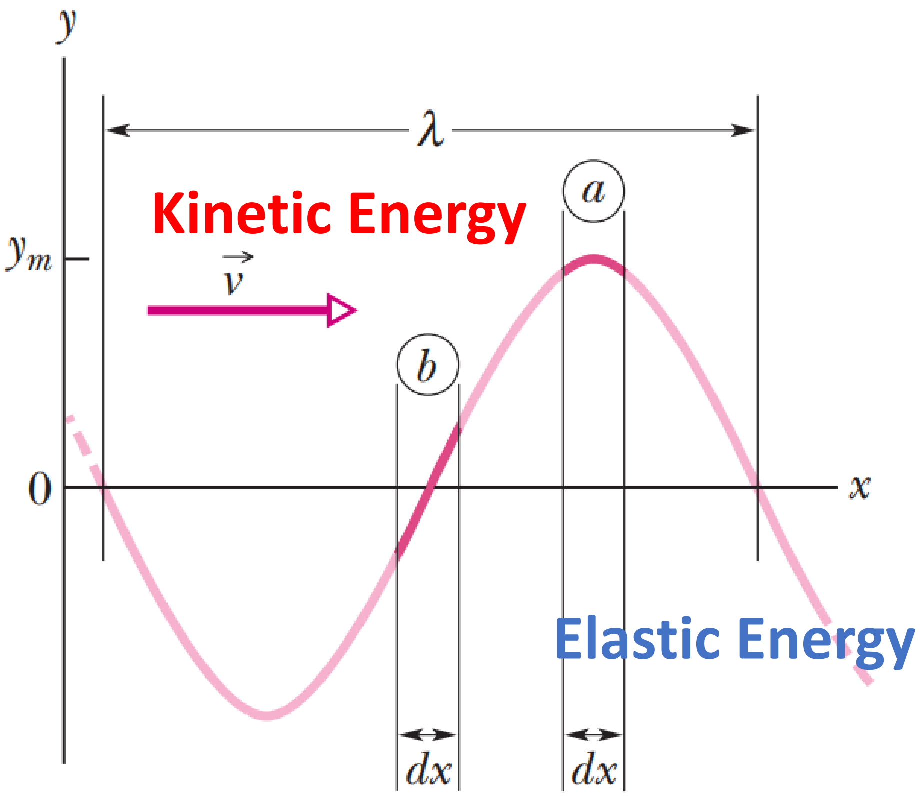
$$dK = \frac{1}{2}\, dm\, u^2$$

Using wave function $\boxed{y(x,t) = y_m \sin(kx - \omega t)}$

$$u = \frac{\partial y}{\partial t} = -\omega y_m \cos(kx - \omega t).$$

$$dK = \tfrac{1}{2}(\mu\, dx)(-\omega y_m)^2 \cos^2(kx - \omega t).$$

$$\frac{dK}{dt} = \tfrac{1}{2}\mu v \omega^2 y_m^2 \cos^2(kx - \omega t).$$



Energy Transportation

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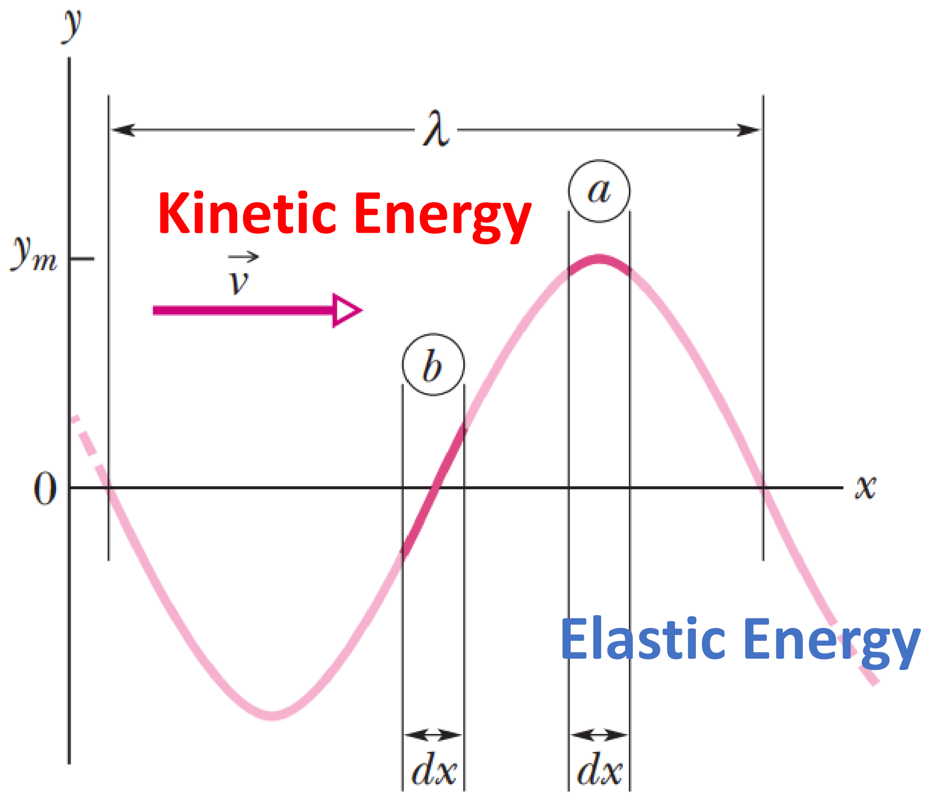
Using wave function $\boxed{y(x, t) = y_m \sin(kx - \omega t)}$



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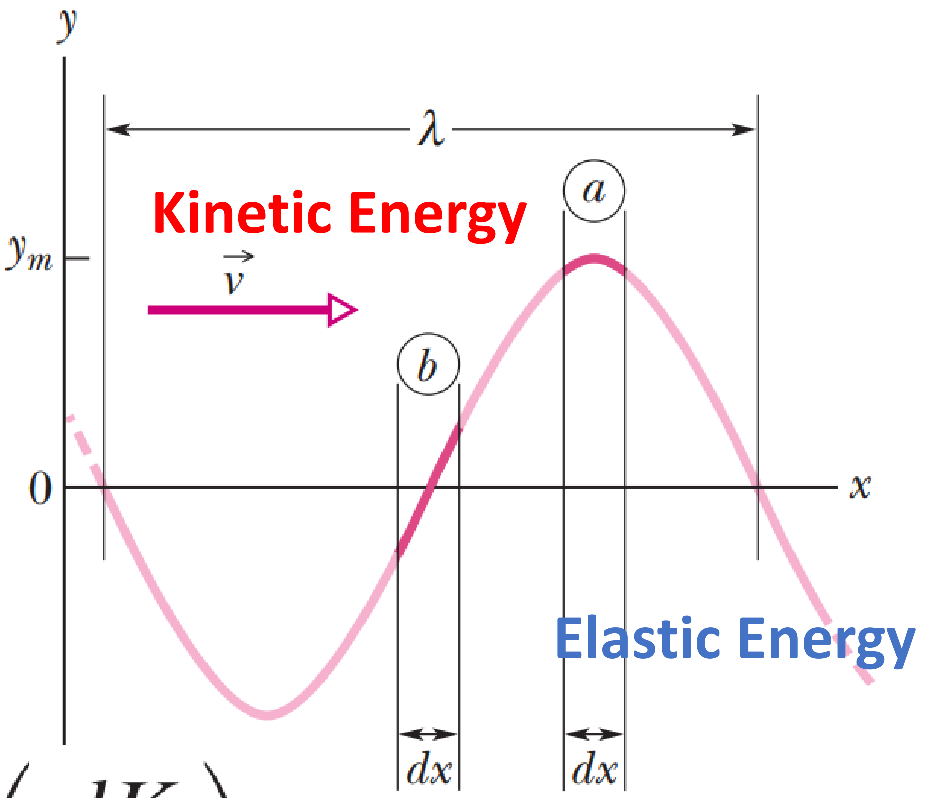
$$\frac{dK}{dt} = \tfrac{1}{2}\mu v \omega^2 y_m^2 \cos^2(kx - \omega t).$$

$$\left(\frac{dK}{dt}\right)_{\text{avg}} = \frac{1}{4}\mu v \omega^2 y_m^2.$$

Energy Transportation

$$dK = \frac{1}{2}\, dm\, u^2$$

Using wave function $\boxed{y(x,t) = y_m \sin(kx - \omega t)}$



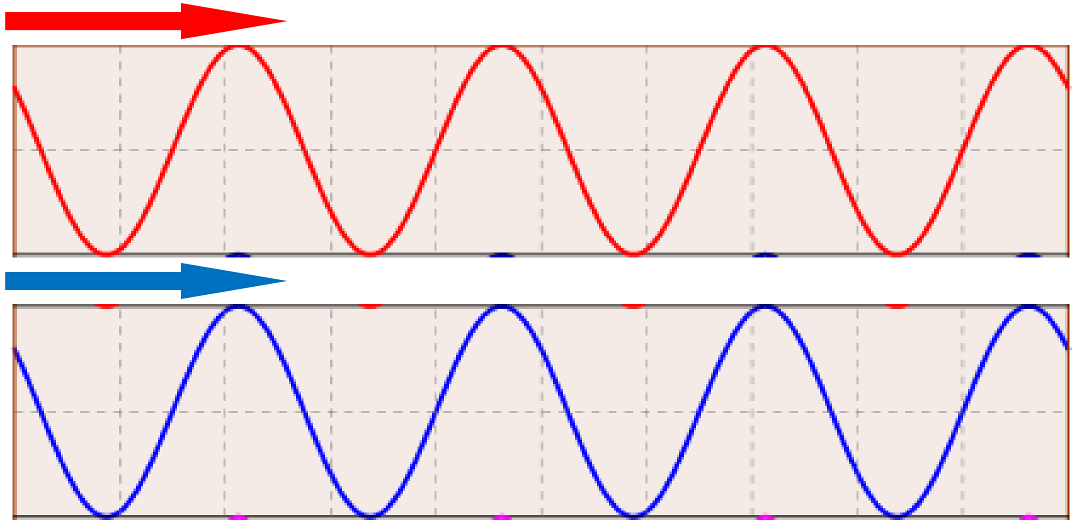
$$\left(\frac{dK}{dt}\right)_{\text{avg}} = \frac{1}{4}\mu v \omega^2 y_m^2.$$

$$P_{\text{avg}} = 2\left(\frac{dK}{dt}\right)_{\text{avg}}$$

$$\boxed{P_{\text{avg}} = \frac{1}{2}\mu v \omega^2 y_m^2 \quad \text{(average power).}}$$

Wave Interference

$$y_1(x,t) = y_m \sin(kx - \omega t)$$
$$y_2(x,t) = y_m \sin(kx - \omega t + \phi).$$



$$y'(x,t) = y_1(x,t) + y_2(x,t).$$

Go to simulation

$$y'(x,t) = [2y_m \cos \tfrac{1}{2}\phi] \sin(kx - \omega t + \tfrac{1}{2}\phi).$$

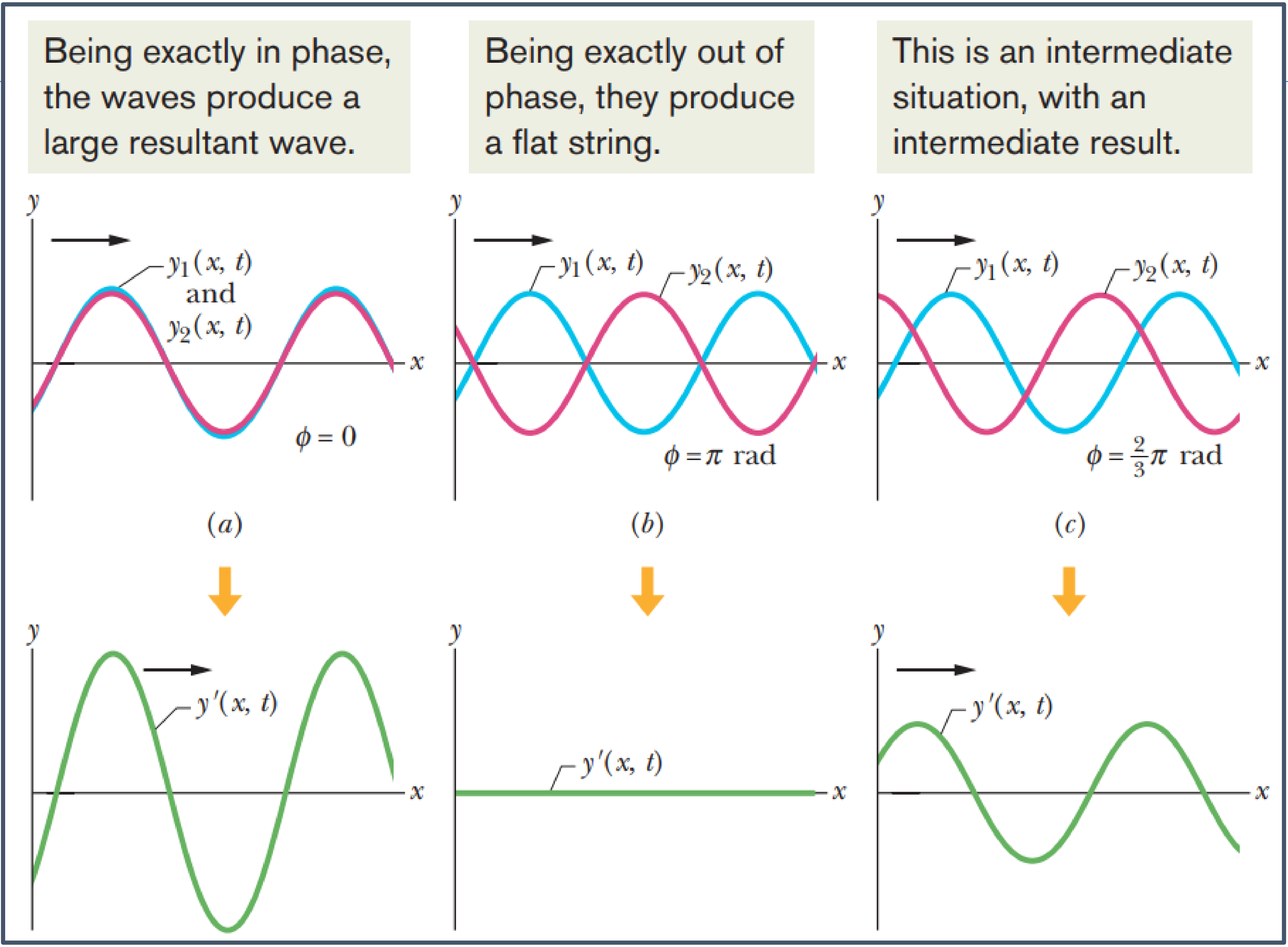
Overlapping waves algebraically add to produce a **resultant wave** (or **net wave**).

$$\sin\alpha + \sin\beta = 2 \sin \tfrac{1}{2}(\alpha + \beta) \cos \tfrac{1}{2}(\alpha - \beta)$$

Wave Interference

$$y'(x,t) = 2y_m \sin(kx - \omega t) \qquad (\phi = 0).$$
$$y'(x,t) = 0 \qquad (\phi = \pi \, \text{rad}).$$

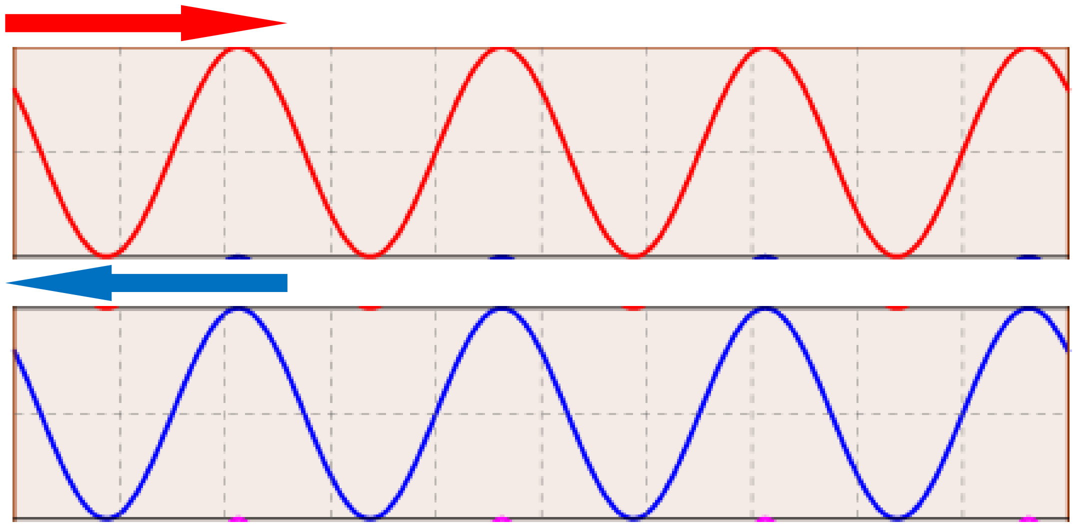
$$\overbrace{y'(x,t)}^{\text{Displacement}} = \underbrace{\left[2y_m \cos \tfrac{1}{2}\phi\right]}_{\substack{\text{Magnitude}\\ \text{gives}\\ \text{amplitude}}} \underbrace{\sin\left(kx - \omega t + \tfrac{1}{2}\phi\right)}_{\substack{\text{Oscillating}\\ \text{term}}}$$


Being exactly in phase, the waves produce a large resultant wave.
Being exactly out of phase, they produce a flat string.
This is an intermediate situation, with an intermediate result.
y
x
$y_1(x, t)$ and $y_2(x, t)$
$\phi = 0$
$y_1(x, t)$
$y_2(x, t)$
$\phi = \pi$ rad
$\phi = \frac{2}{3}\pi$ rad
(a)
(b)
(c)
$y'(x, t)$

Wave Interference

$$y_1(x, t) = y_m \sin(kx - \omega t)$$

$$y_2(x, t) = y_m \sin(kx + \omega t).$$



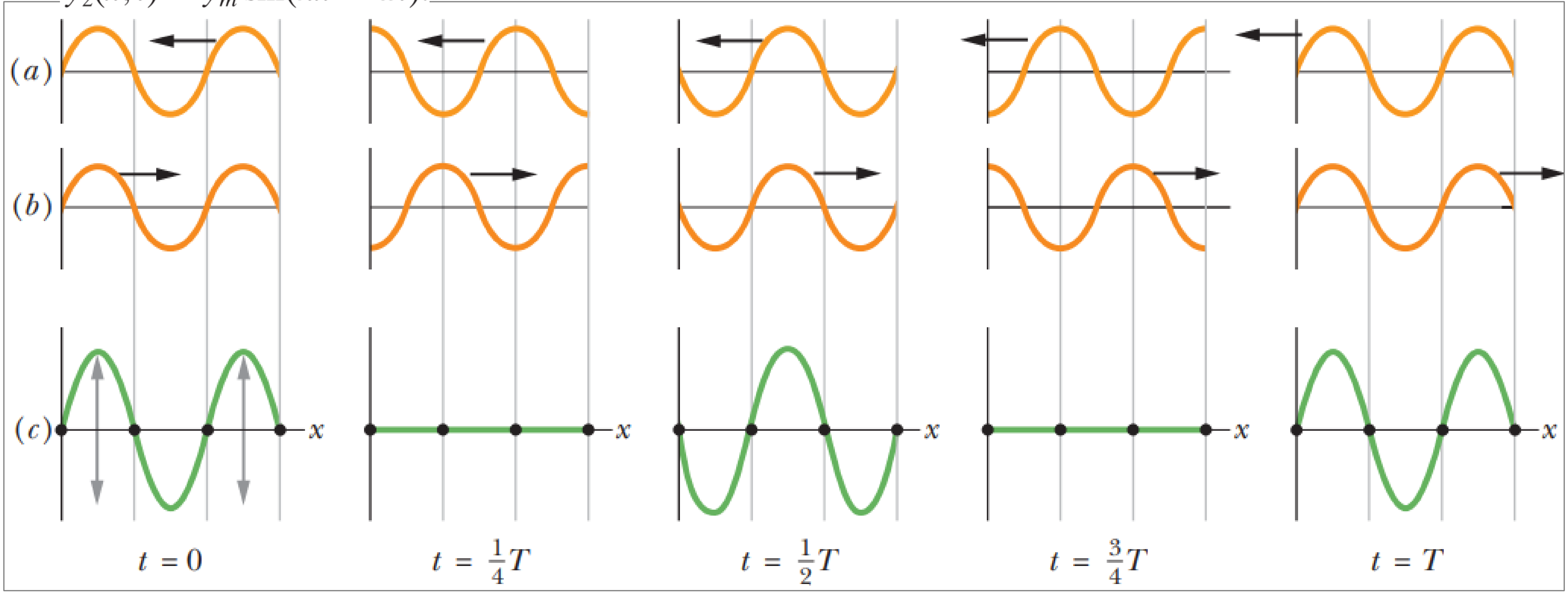
$$\boxed{y'(x, t) = [2y_m \sin kx] \cos \omega t.}$$

Wave Interference

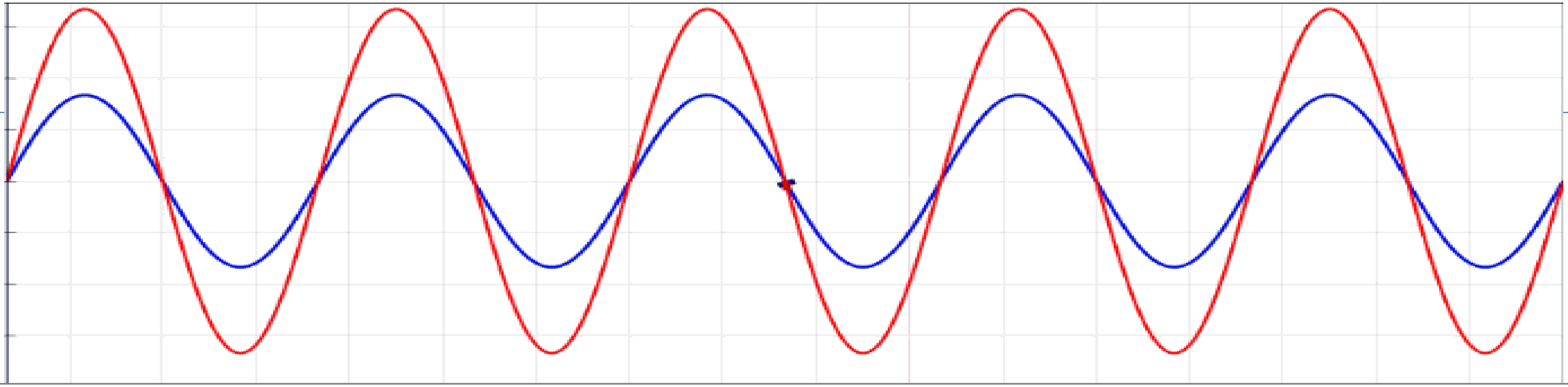
$$y_1(x, t) = y_m \sin(kx - \omega t)$$

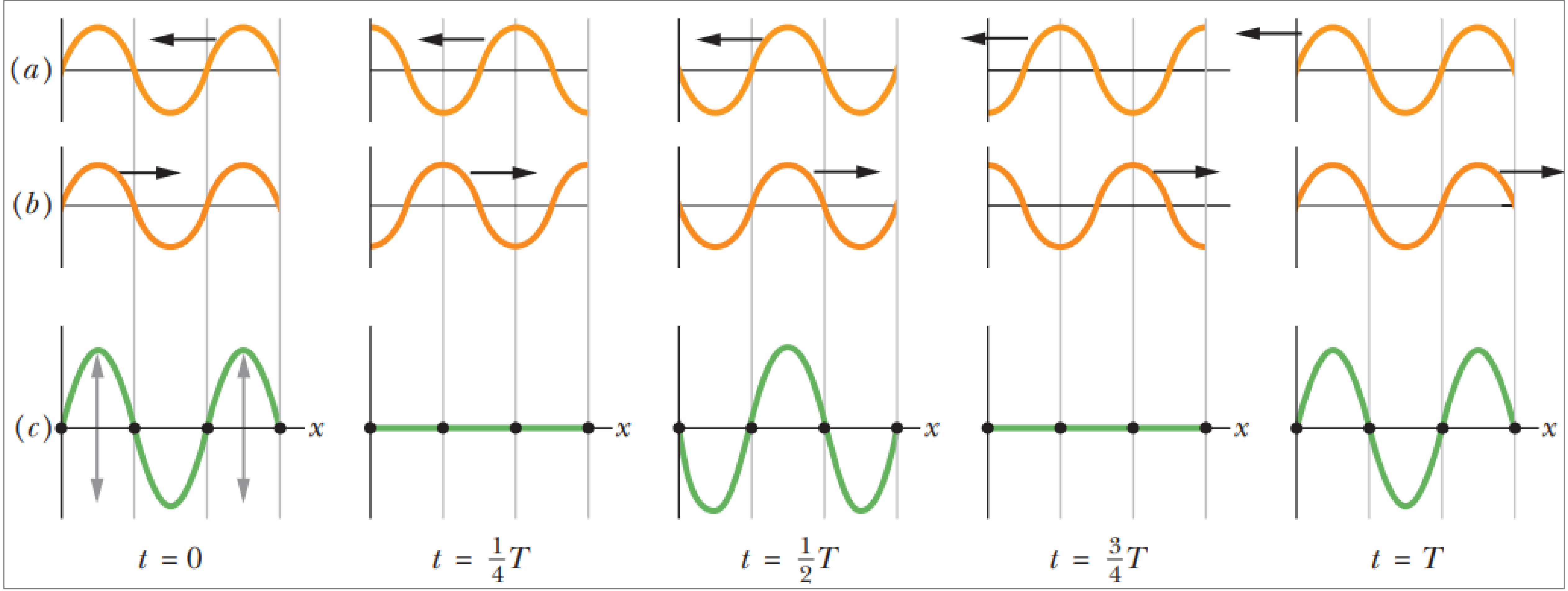
$$y_2(x, t) = y_m \sin(kx + \omega t).$$

$$\boxed{y'(x, t) = [2y_m \sin kx] \cos \omega t.}$$



Lecture 14





Wave Interference

Checkpoint 5

Two waves with the same amplitude and wavelength interfere in three different situations to produce resultant waves with the following equations:

(1) $y'(x, t) = 4 \sin(5x - 4t)$

(2) $y'(x, t) = 4 \sin(5x) \cos(4t)$

(3) $y'(x, t) = 4 \sin(5x + 4t)$

In which situation are the two combining waves traveling (a) toward positive x, (b) toward negative x, and (c) in opposite directions?