# Oscillatory Motion

when the pivot and center of mass on same axis.

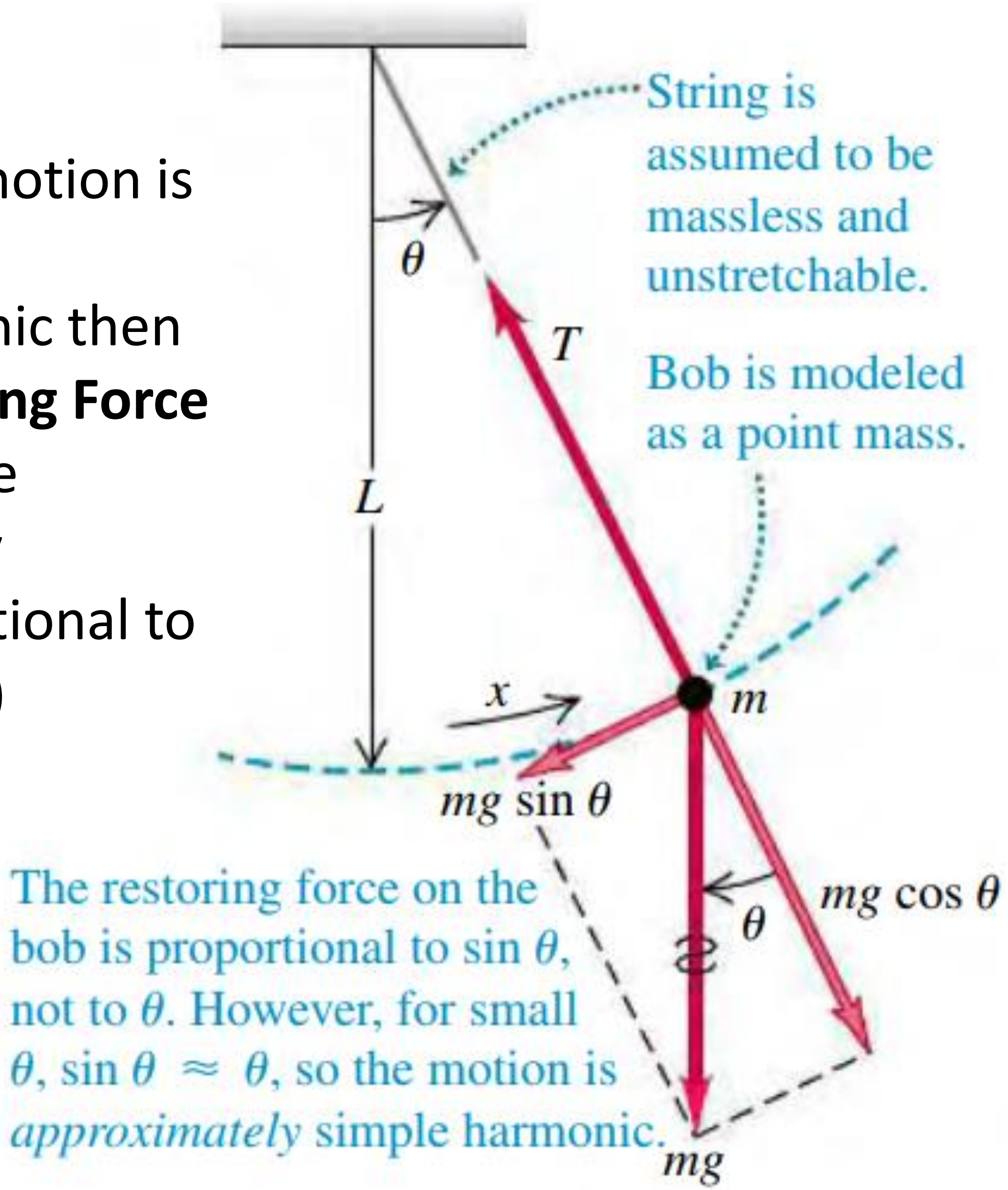
oscillations under gravitational force

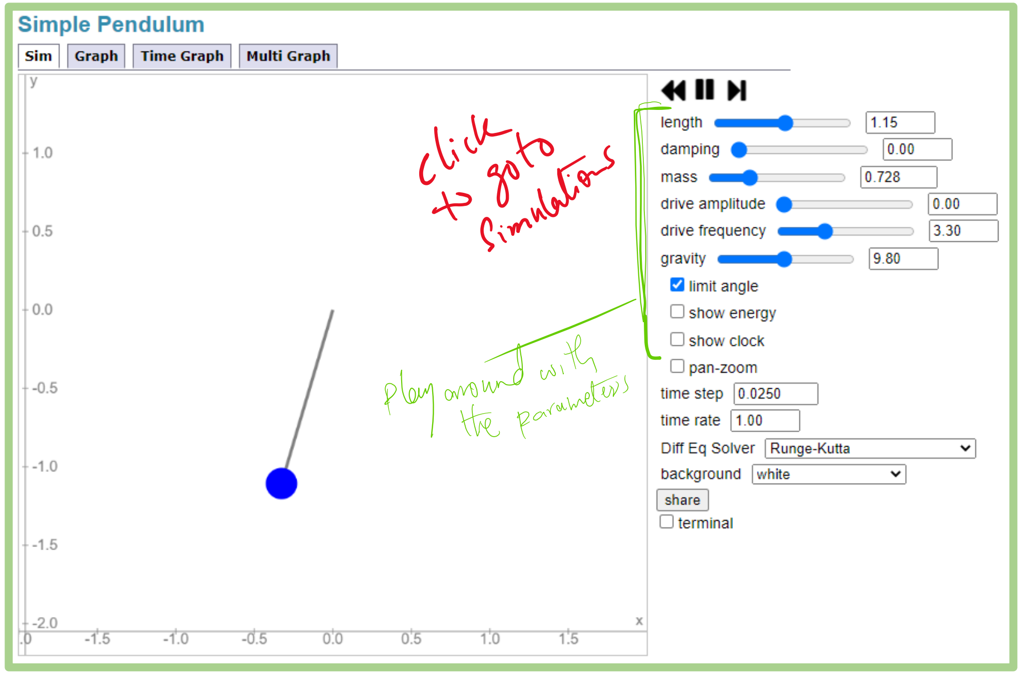
# Simple Pendulum

$$F_\theta = -mg\sin\theta$$

Force in $\hat{\theta}$ direction

If the motion is simple harmonic then **Restoring Force** must be directly proportional to $x$ (or $\theta$)



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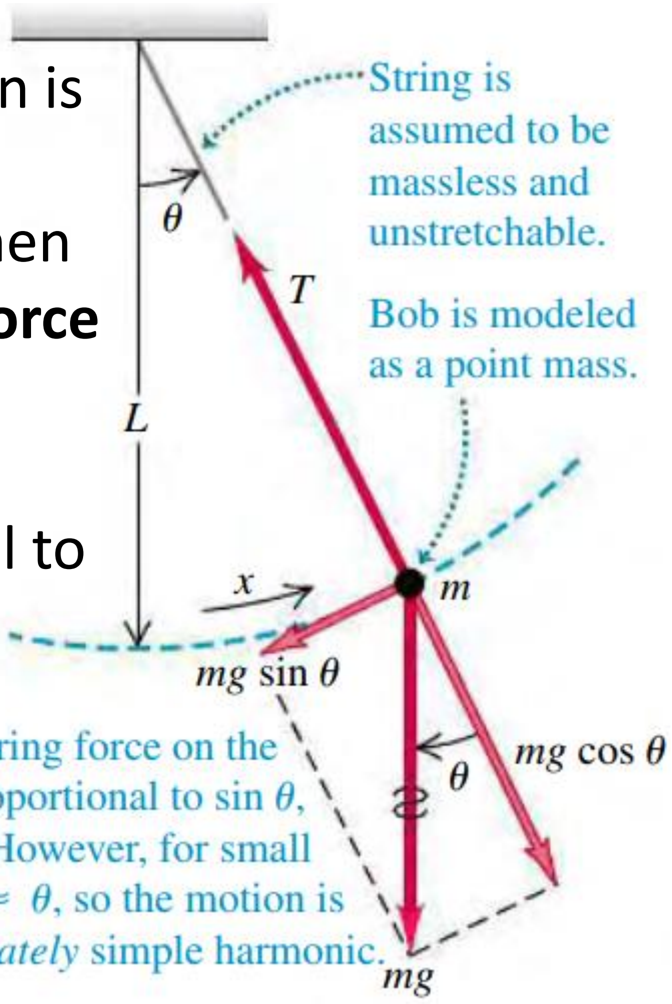



The restoring force on the bob is proportional to $\sin\theta$, not to $\theta$. However, for small $\theta$, $\sin\theta \approx \theta$, so the motion is *approximately* simple harmonic.

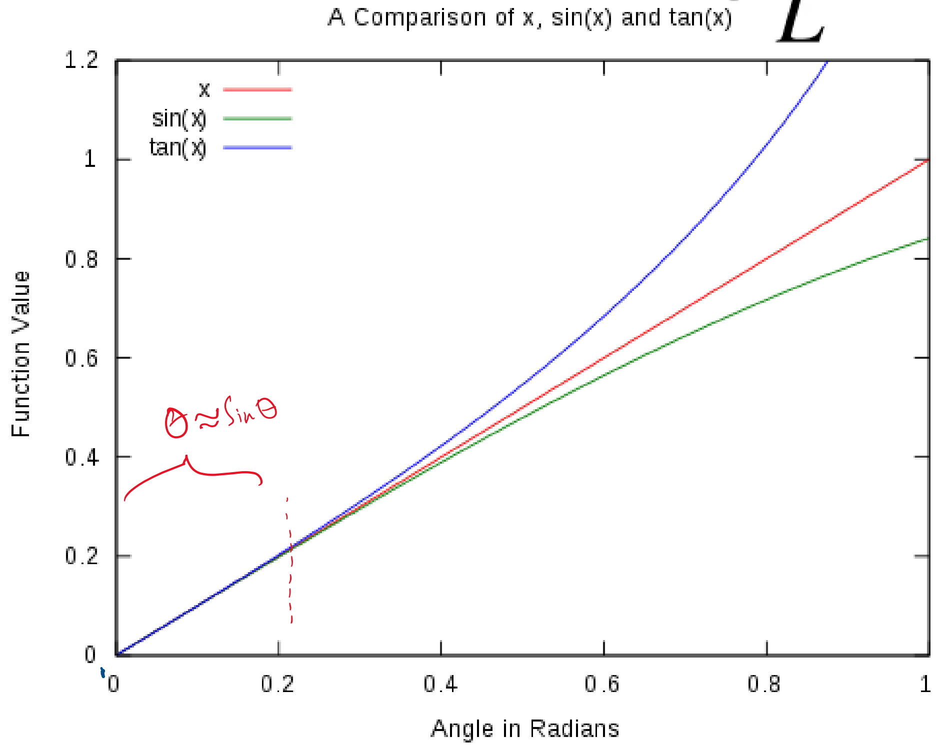
Force is opposite to displacement

$$F_\theta = -mg\sin\theta$$

if angle $\theta$ is *small*,

$$F_\theta = -mg\theta$$

$$= -mg\frac{x}{L} = -\frac{mg}{L}x$$



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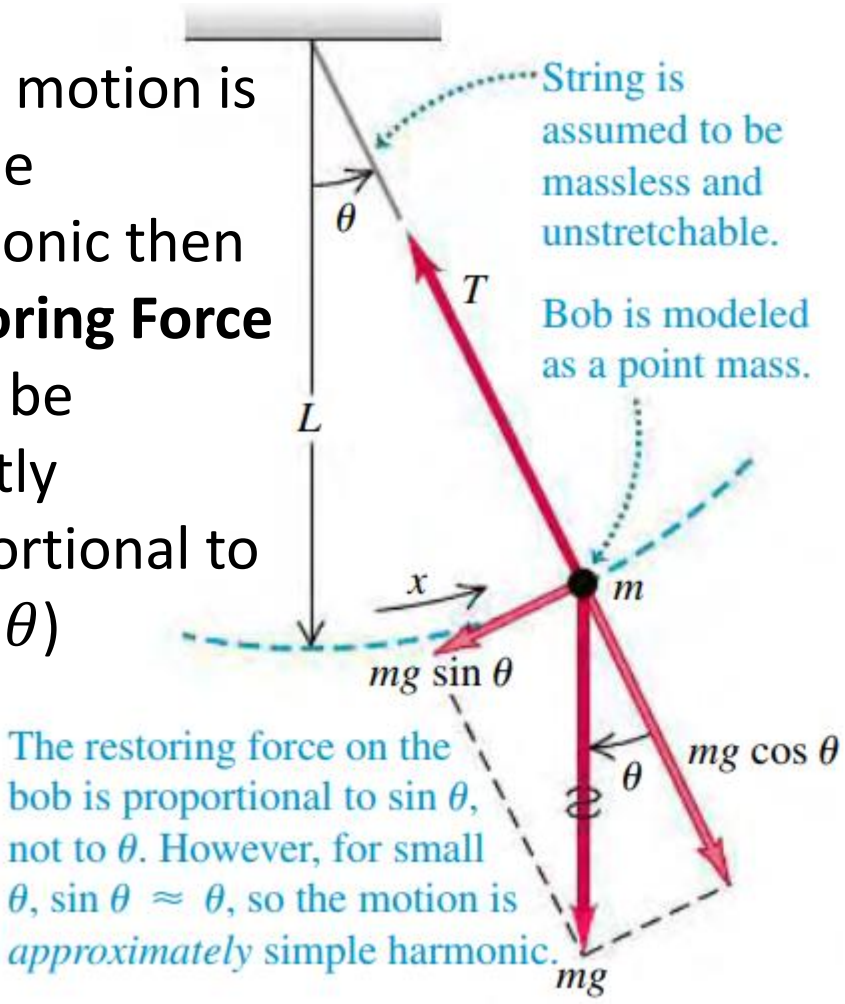



$$F_\theta = -mg\sin\theta$$

if angle $\theta$ is *small*,

$$F_\theta = -mg\theta$$

$$= -mg\frac{x}{L} = -\frac{mg}{L}x = -m\omega^2(x)$$

**Angular frequency of simple pendulum, small amplitude**

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{mg/L}{m}} = \sqrt{\frac{g}{L}}$$

Acceleration due to gravity

Pendulum length

Pendulum mass (cancels)

**Period of simple pendulum, small amplitude**

$$T = \frac{2\pi}{\omega} = \frac{1}{f} = 2\pi\sqrt{\frac{L}{g}}$$

Pendulum length

Acceleration due to gravity

Angular frequency

Frequency

# Please note the approximation

**Period of simple pendulum, small amplitude**

$$T = \frac{2\pi}{\omega} = \frac{1}{f} = 2\pi\sqrt{\frac{L}{g}}$$

Angular frequency ($\omega$); Frequency ($f$); Pendulum length ($L$); Acceleration due to gravity ($g$)

Without Approximation

$$T = 2\pi\sqrt{\frac{L}{g}}\left(1 + \frac{1^2}{2^2}\sin^2\frac{\Theta}{2} + \frac{1^2 \cdot 3^2}{2^2 \cdot 4^2}\sin^4\frac{\Theta}{2} + \cdots\right)$$

the value of T increases by $\sin^2$ factors

# Add a Force agent


Condition for maximum resonance

$$\boxed{\omega_d = \omega \qquad \text{(resonance)},}$$

Mass and Damping constant is also involved in amplitude variation

Maximum value of driving force

Amplitude of a forced oscillator

$$A = \frac{F_{\text{max}}}{\sqrt{(k - m\omega_d^2)^2 + b^2\omega_d^2}}$$

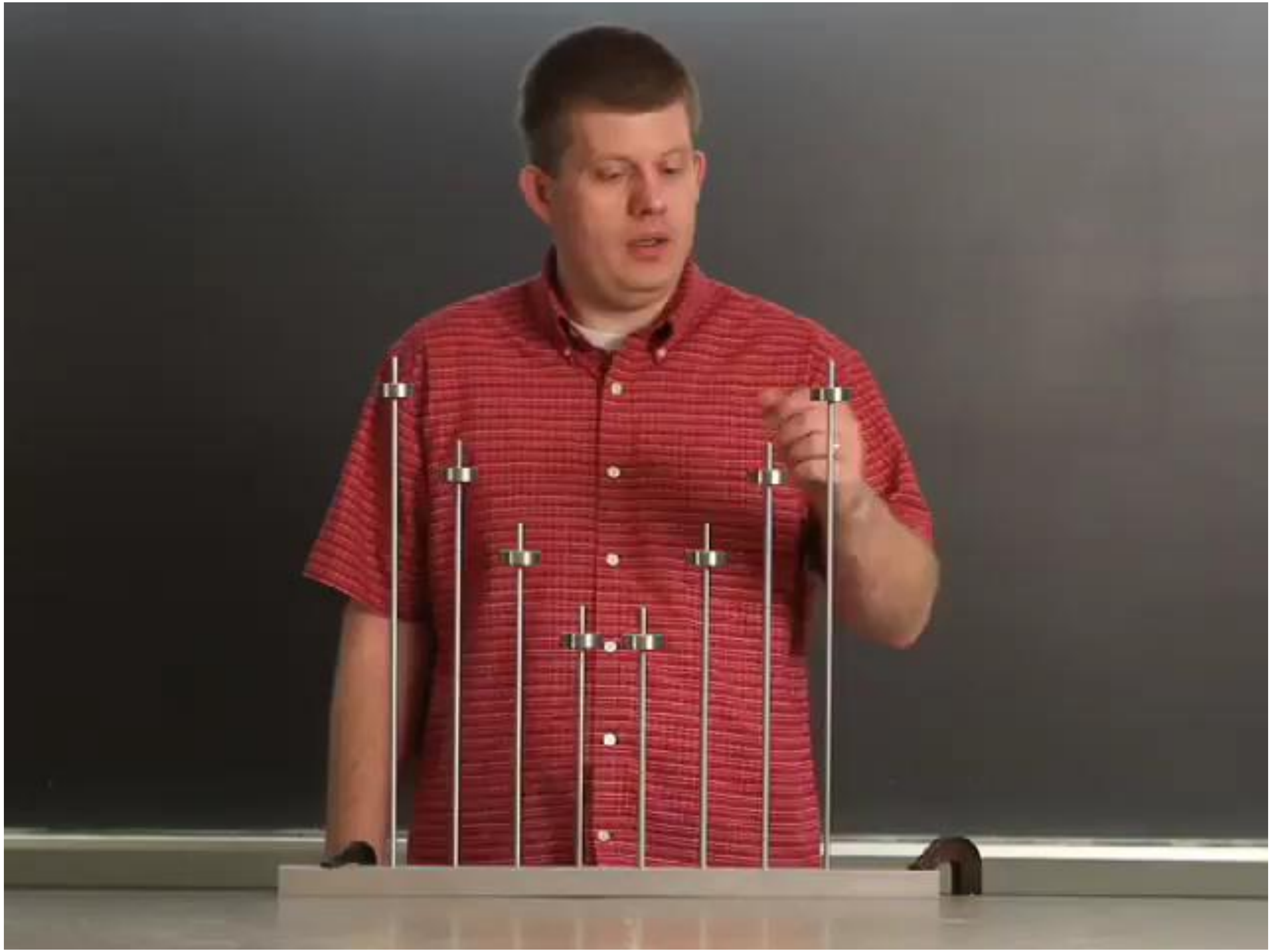
Damping constant

Force constant of restoring force

Mass

Driving angular frequency

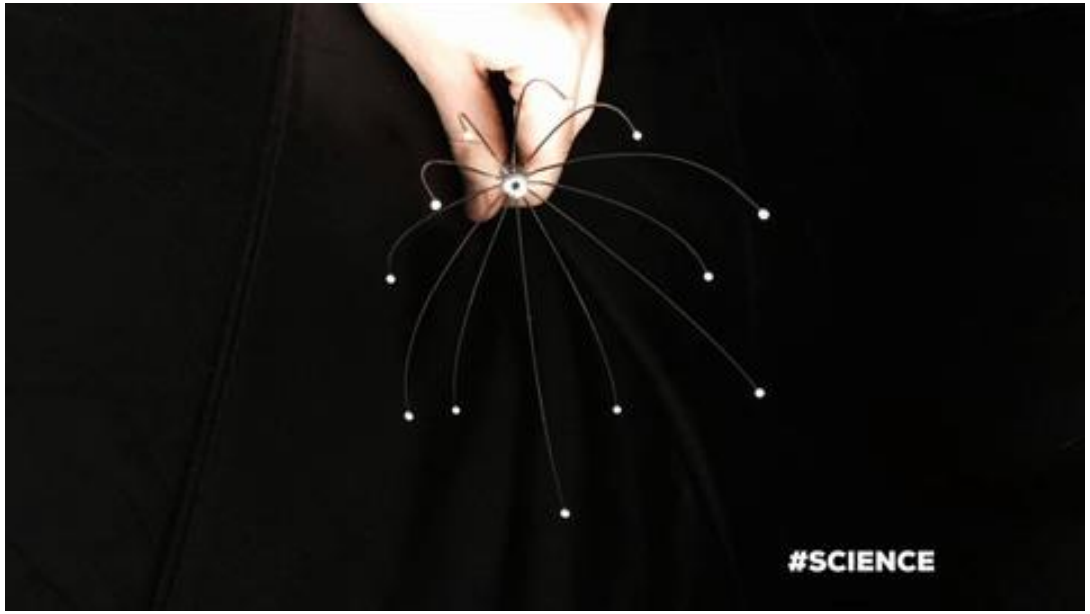
Lecture 14



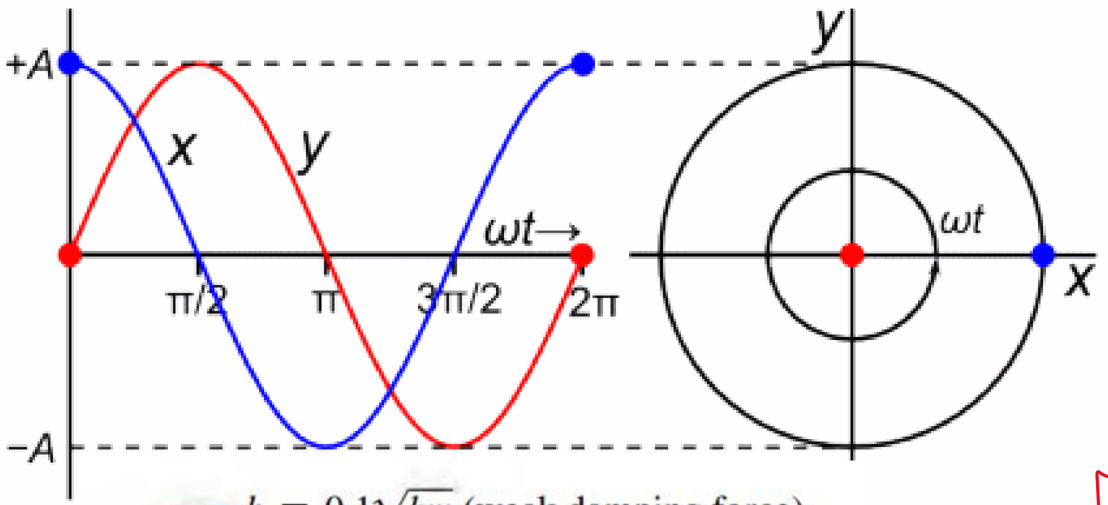
Demonstration QR

# Resonance

Lecture 14

# Recap



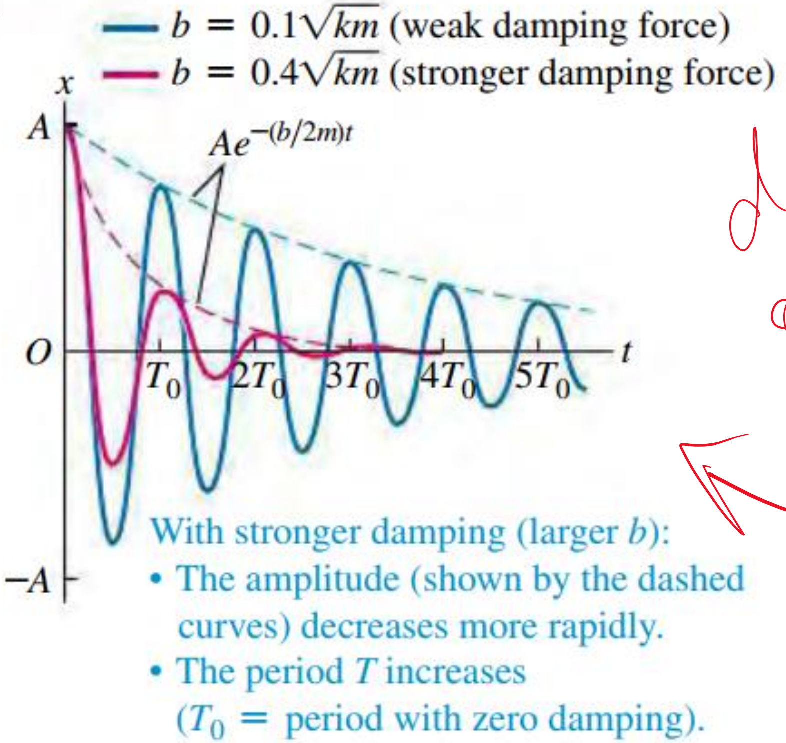

free oscillations

$$x(t) = x_m \cos(\omega t + \phi)$$

$$a(t) = -\omega^2 x(t)$$

damped oscillation

$$x(t) = x_m e^{-bt/2m} \cos(\omega' t + \phi),$$

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**Displacement of oscillator, little damping**

$$x = Ae^{-(b/2m)t}\cos(\omega' t + \phi)$$

Initial amplitude · Damping constant · Mass · Time · Angular frequency of damped oscillations · Phase angle

**Angular frequency of oscillator, little damping**

$$\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$$

Force constant of restoring force · Damping constant · Mass

# Waves

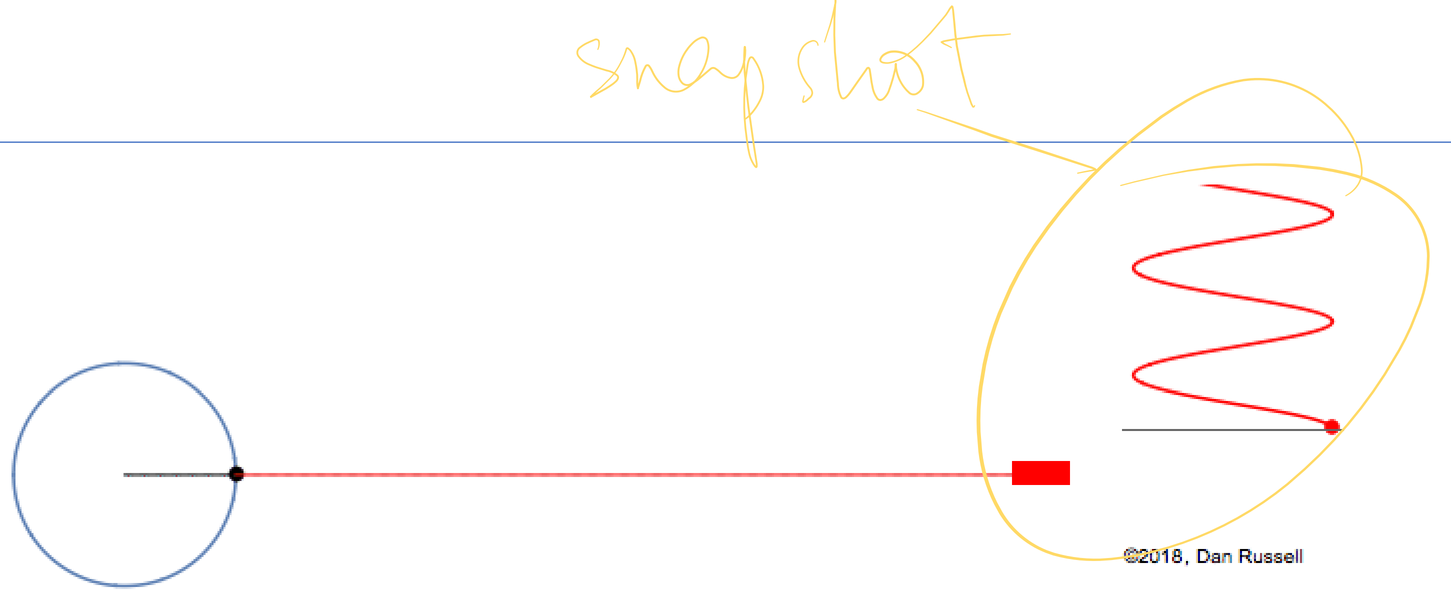

Spatial propagation of oscillations form wave pattern

***Mechanical waves.*** Those which can be explained via Newtonian Mechanics

***Electromagnetic waves.*** can be explained by Electromagnetic theory (Maxwell's equations)

***Matter waves.*** explained by Quantum Mechanics

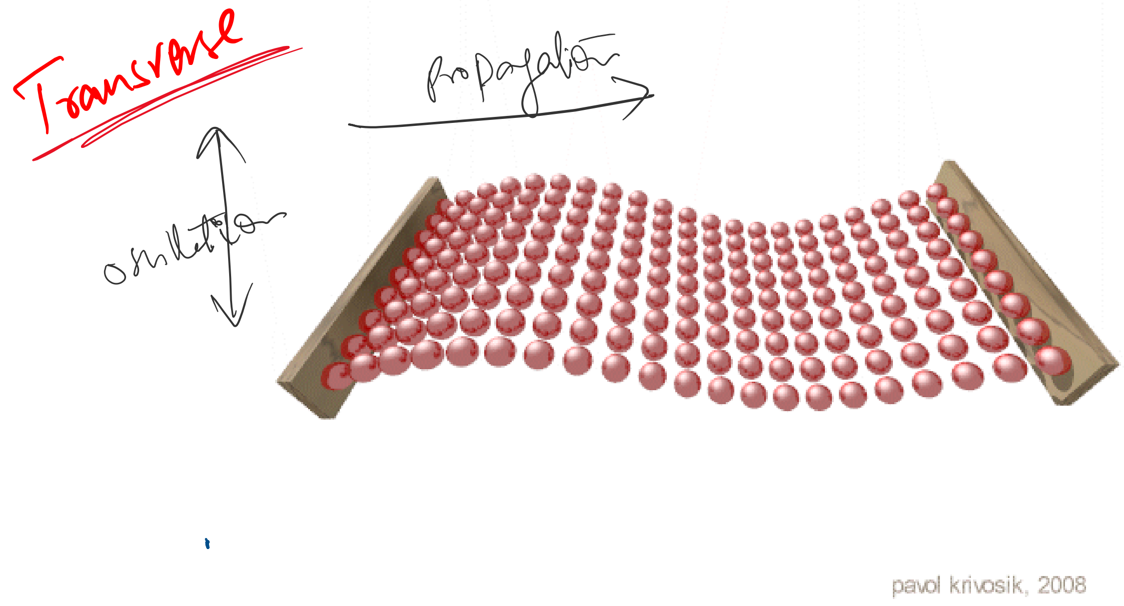
# Waves




More general classification

by Mechanical waves here
- Transverse
- Longitudinal

Spatial propagation of oscillations form wave pattern




***Mechanical waves.***

***~~Electromagnetic waves.~~***

***~~Matter waves.~~***

# Waves


Spatial propagation of oscillations form wave pattern

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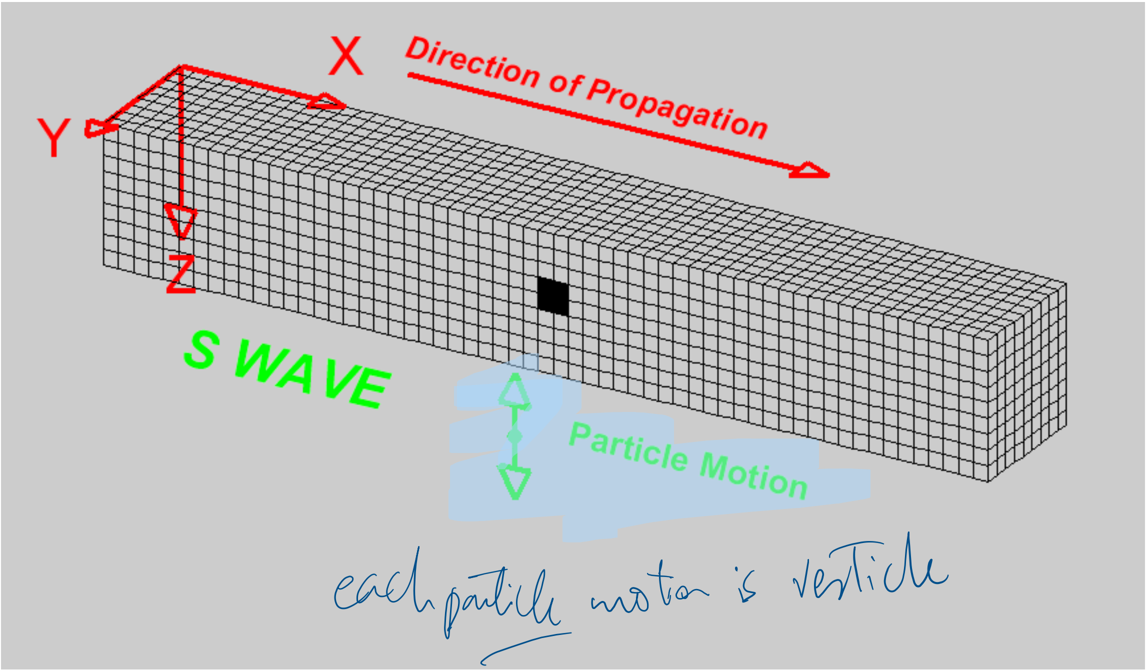


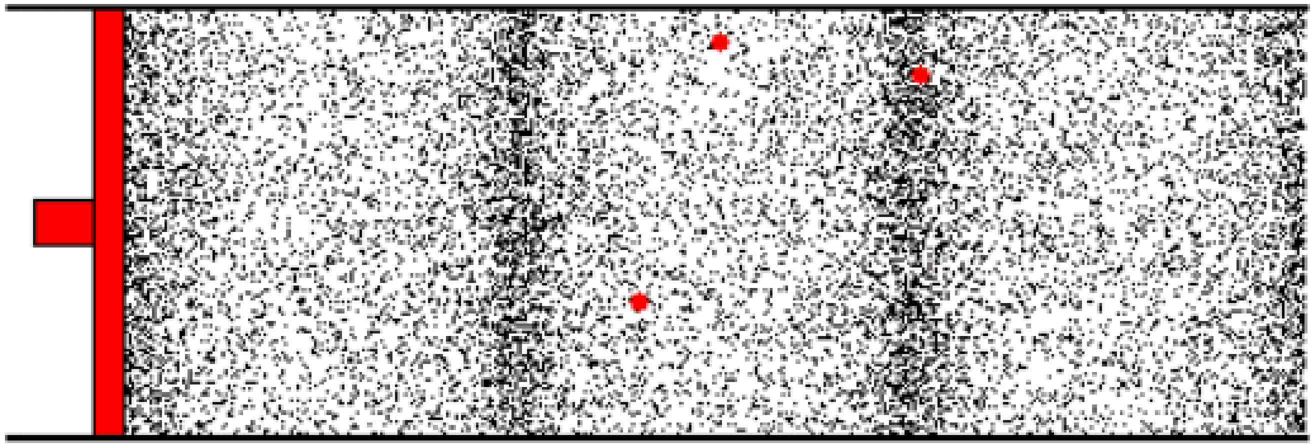
# Waves


Spatial propagation of oscillations form wave pattern

oscillations

Propagation

Longitudinal Wave



***Mechanical waves.***

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# Waves


Spatial propagation of oscillations form wave pattern

*Mechanical waves.*

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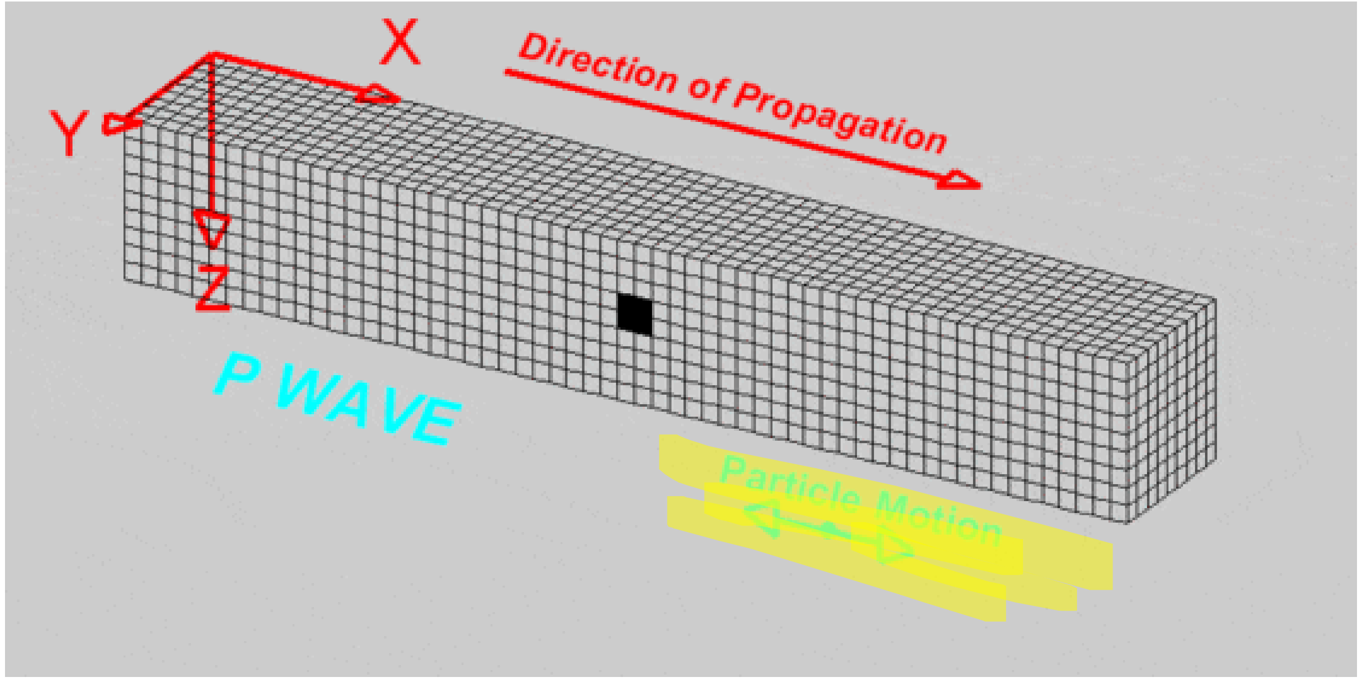


# Waves


Spatial propagation of oscillations form wave pattern

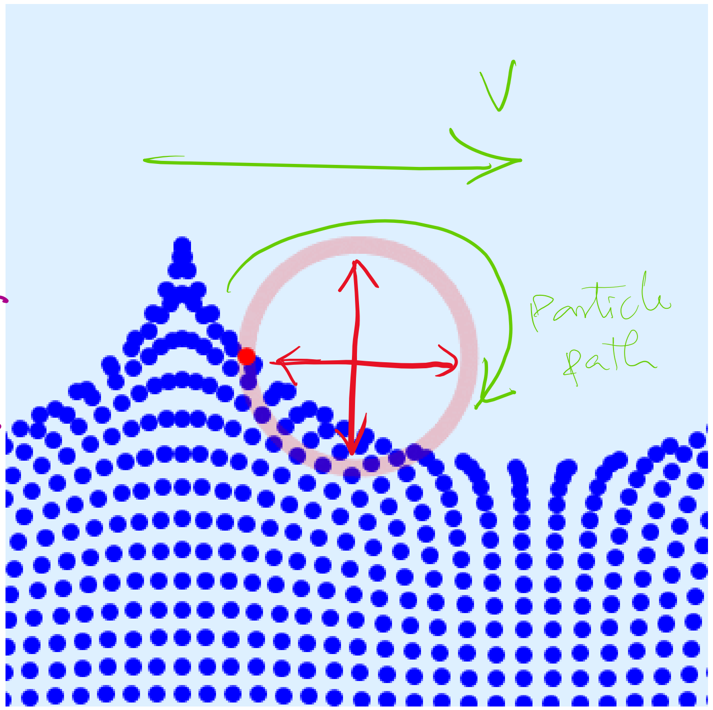
The vertical and horizontal oscillations can combine to give circular or elliptical oscillation.

***Mechanical waves.***

***Electromagnetic waves.***

***Matter waves.***

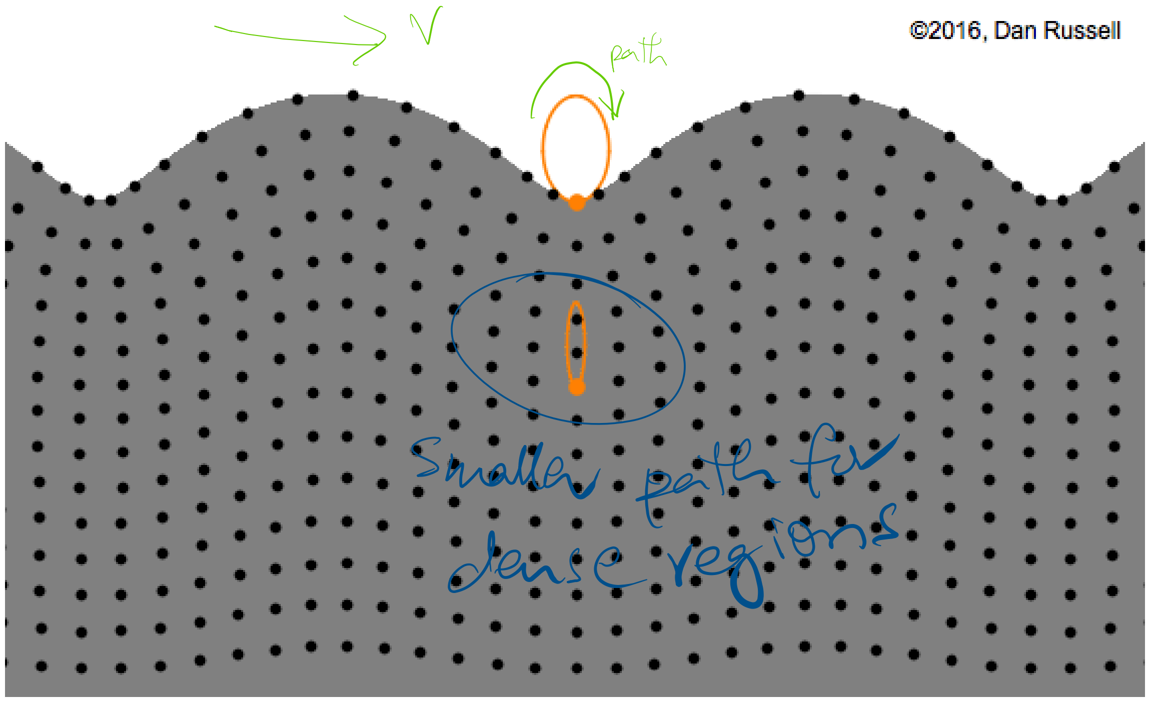
circular oscillation wave example



# Waves


Spatial propagation of oscillations form wave pattern

elliptical
oscillation wave
example




***Mechanical waves.***

***Electromagnetic waves.***

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# Wave function

(A mathematical function that can describe the path/trajectory of oscillations and waves)

Simplest wave model [in SHM (Transverse wave)]

$$y(x, t) = y_m \sin(kx - \omega t)$$

*Wave function*

$\lambda$ ——— $2\pi$

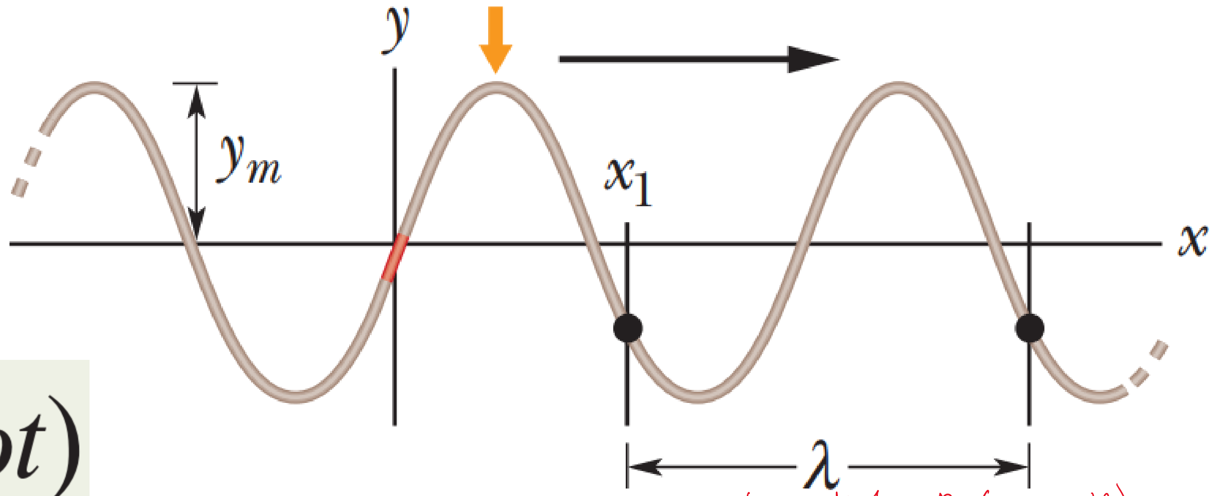
$1$ ——— $\frac{2\pi}{\lambda}$

$x$ ——— $\frac{2\pi}{\lambda}x$

$$k = \frac{2\pi}{\lambda} \quad \text{(angular wave number).}$$

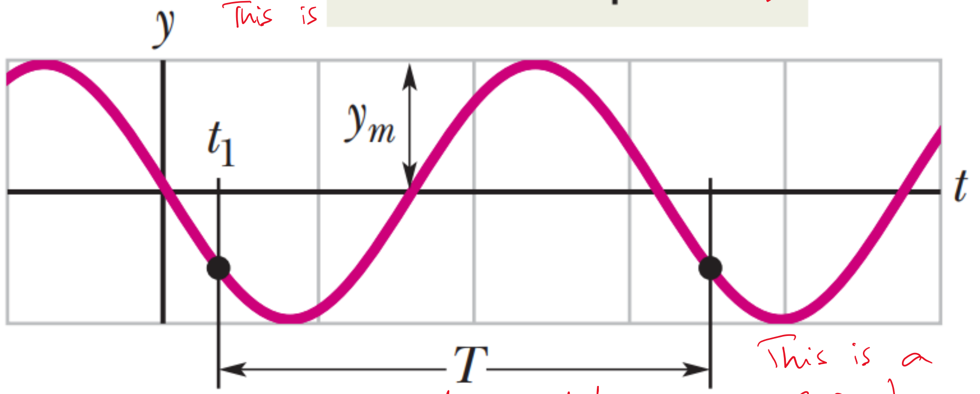
compare it with $\omega = \frac{2\pi}{T}$

This is snapshot, other picture is not.




Displacement of wave (wavelength)

This is not a snapshot, other picture is.




Period of oscillation (Time period)

This is a graph

# Wave function

$$\overbrace{y(x,t)}^{\text{Displacement}} = \overbrace{y_m}^{\text{Amplitude}} \overbrace{\sin\,(\overbrace{kx - \omega t}^{\text{Phase}})}^{\text{Oscillating term}}$$

Angular wave number

Position

Time

Angular frequency

# Wave function

To take snapshot,

$$y(x,0) = y_m \sin kx.$$

fix a point in time
t=0

To plot a graph,

fix a point in space
x=0

$$y(0,t) = y_m \sin(-\omega t)$$
$$= -y_m \sin \omega t$$

## The generalized wave function

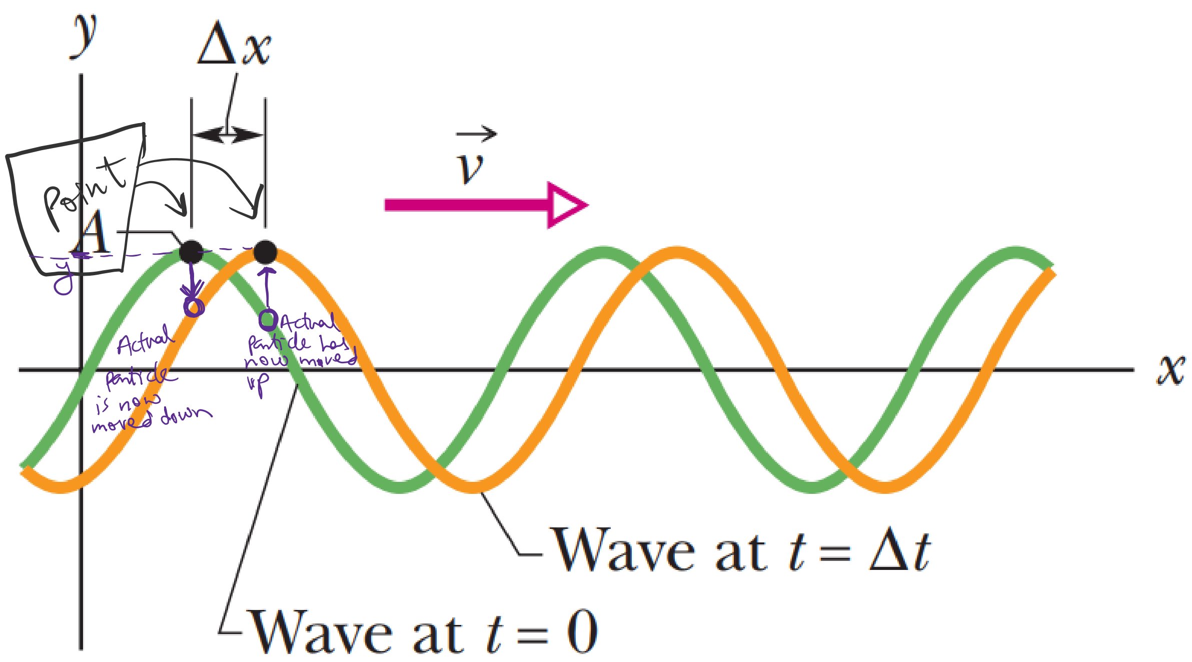
$$\boxed{y = y_m \sin(kx \pm \omega t + \phi).}$$

will add information for second wave

# Wave Velocity

point $A$ retains its displacement

$$kx - \omega t = \text{a constant.}$$




$$k\frac{dx}{dt} - \omega = 0$$

$$\frac{dx}{dt} = v = \frac{\omega}{k}.$$

$$\boxed{v = \frac{\omega}{k} = \frac{\lambda}{T} = \lambda f} \quad \text{(wave speed)}$$

The **frequency** *f* of a wave is defined as $1/T$ and is related to the angular frequency $\omega$ by; $\boldsymbol{\omega = 2\pi f}$

We define the **period** of oscillation *T* of a wave to be the time any string element takes to move through one full oscillation.

The **wavelength** of a wave is the distance (parallel to the direction of the wave's travel) between repetitions of the shape of the wave (or *wave shape*).

The **amplitude** $y_m$ of a wave is the magnitude of the maximum displacement of the elements from their equilibrium positions as the wave passes through them.

The **phase** of the wave is the *argument* $kx - \omega t$ of the sine. As the wave sweeps through a string element at a particular position *x*, the phase changes linearly with time *t*. This means that the sine also changes, oscillating between +1 and -1.

We call *k* the **angular wave number** of the wave; its SI unit is the radian per meter, or the inverse meter.

A phase constant φ in the wave function:

$\mathbf{y = y_m \sin(kx - \omega t + \varphi)}$.

The value of φ can be chosen so that the function gives some other displacement and slope at $x = 0$ when $t = 0$.