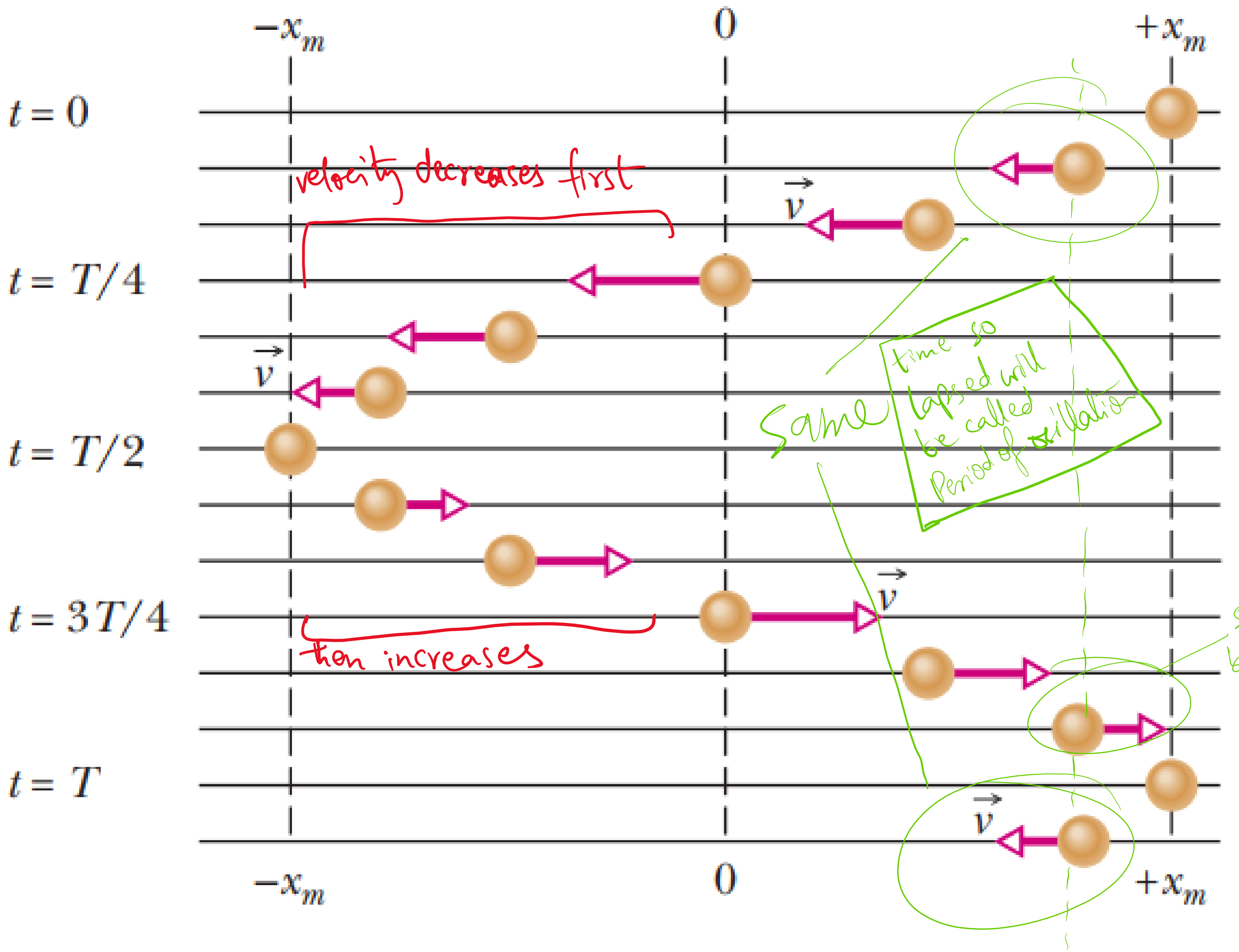
Oscillatory Motion

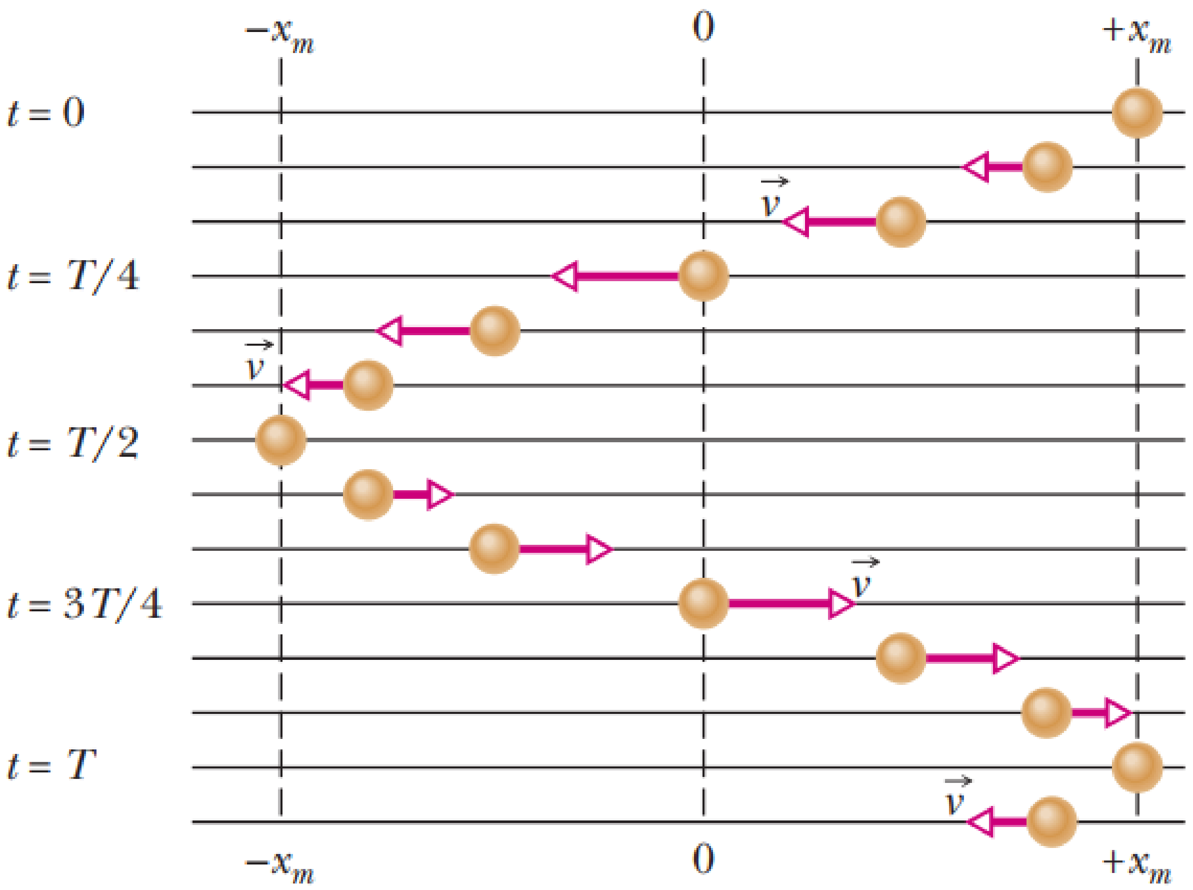
Universe also contains Oscillating objects



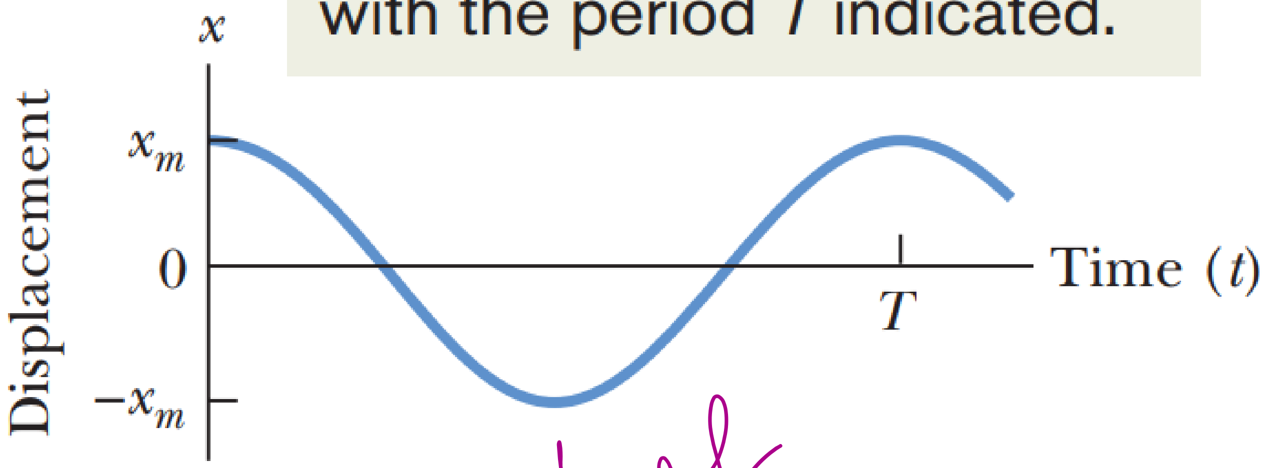
sudden change of velocity

Reflection ≠ Oscillation

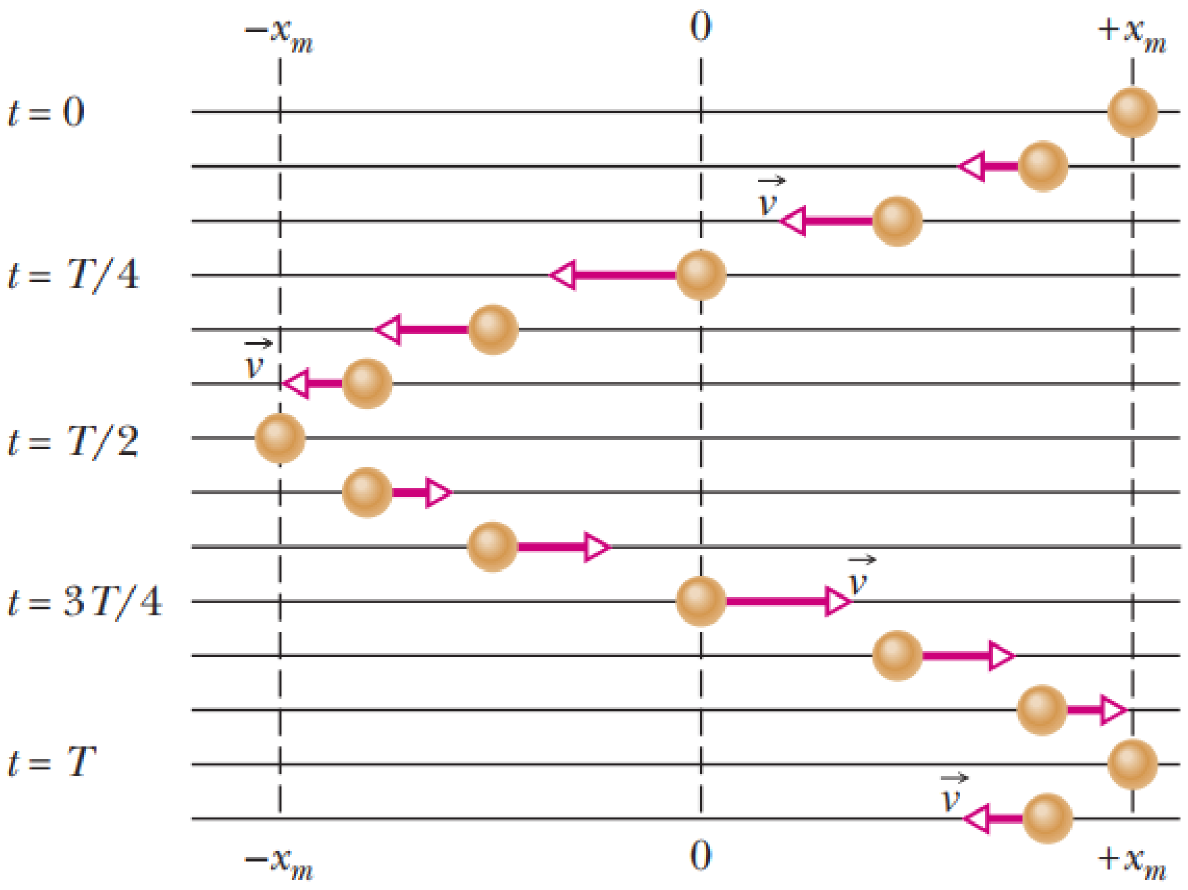
periodic in motion



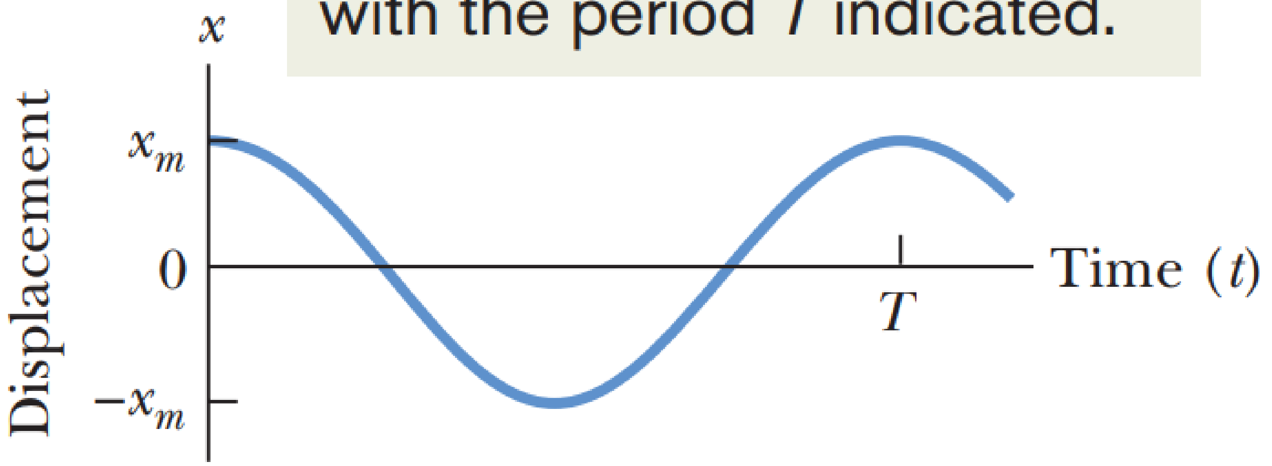
This is a graph of the motion, with the period T indicated.



graph of this motion



This is a graph of the motion, with the period *T* indicated.



$$T = \frac{1}{f}.$$

$$1 \text{ hertz} = 1 \text{ Hz} = 1 \text{ oscillation per second} = 1 \text{ s}^{-1}.$$

Assuming a suitable solution

Displacement at time t

Phase

$$\underbrace{x(t)}_{\text{Displacement at time } t} = \underbrace{x_m}_{\text{Amplitude}} \cos(\overbrace{\omega t + \phi}^{\text{Phase}})$$

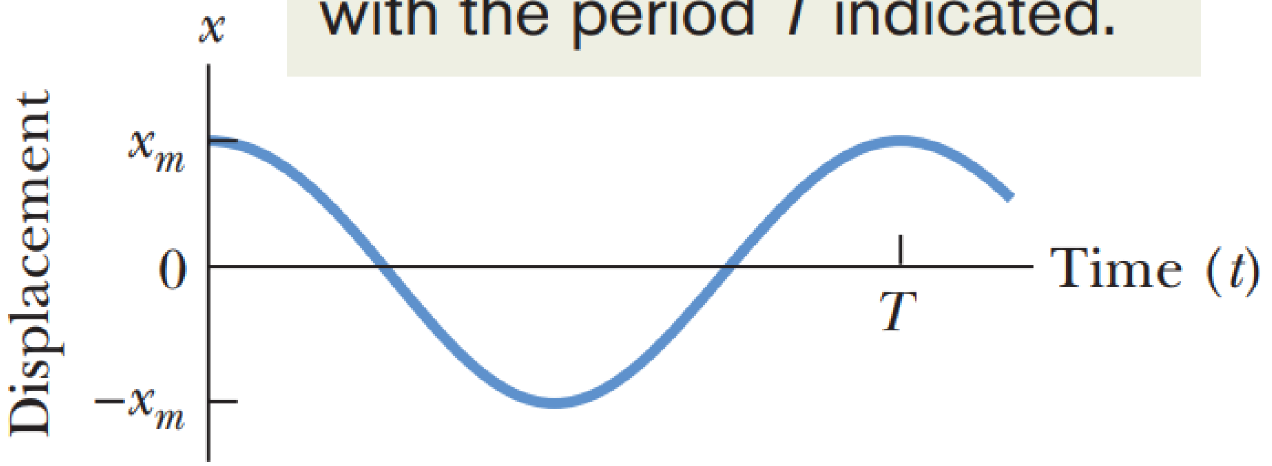
Amplitude

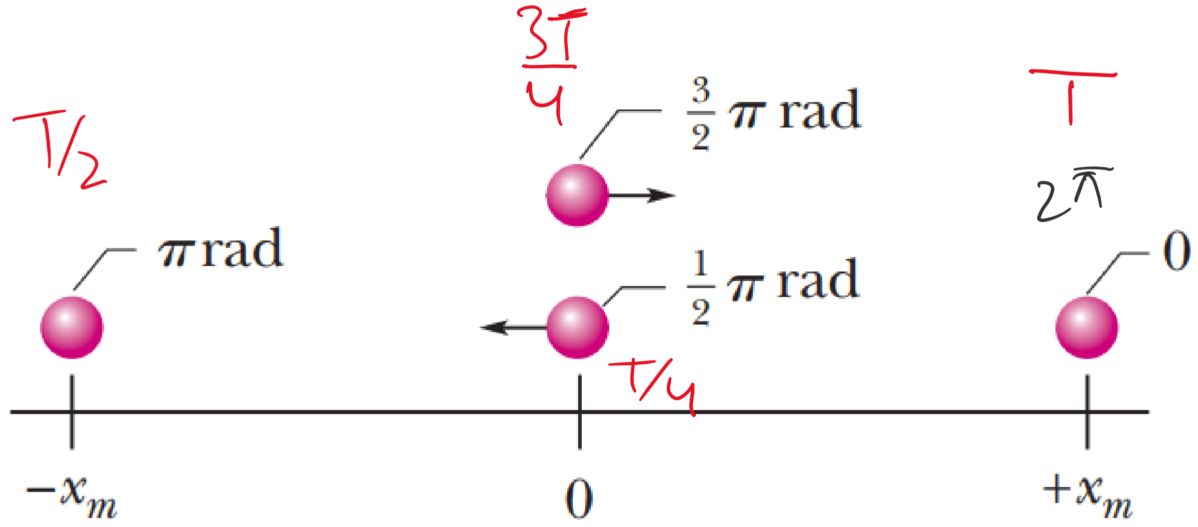
Time

Angular frequency

Phase constant or phase angle

This is a graph of the motion, with the period T indicated.





at $t=0$, $\theta=0$

$\cos(0) = \cos(t=0)$

$f = \frac{1}{T}$

$fT = 1$

at $t=T$, $\theta = 2\pi$

$\cos(2\pi) = \cos(\boxed{\quad} T)$

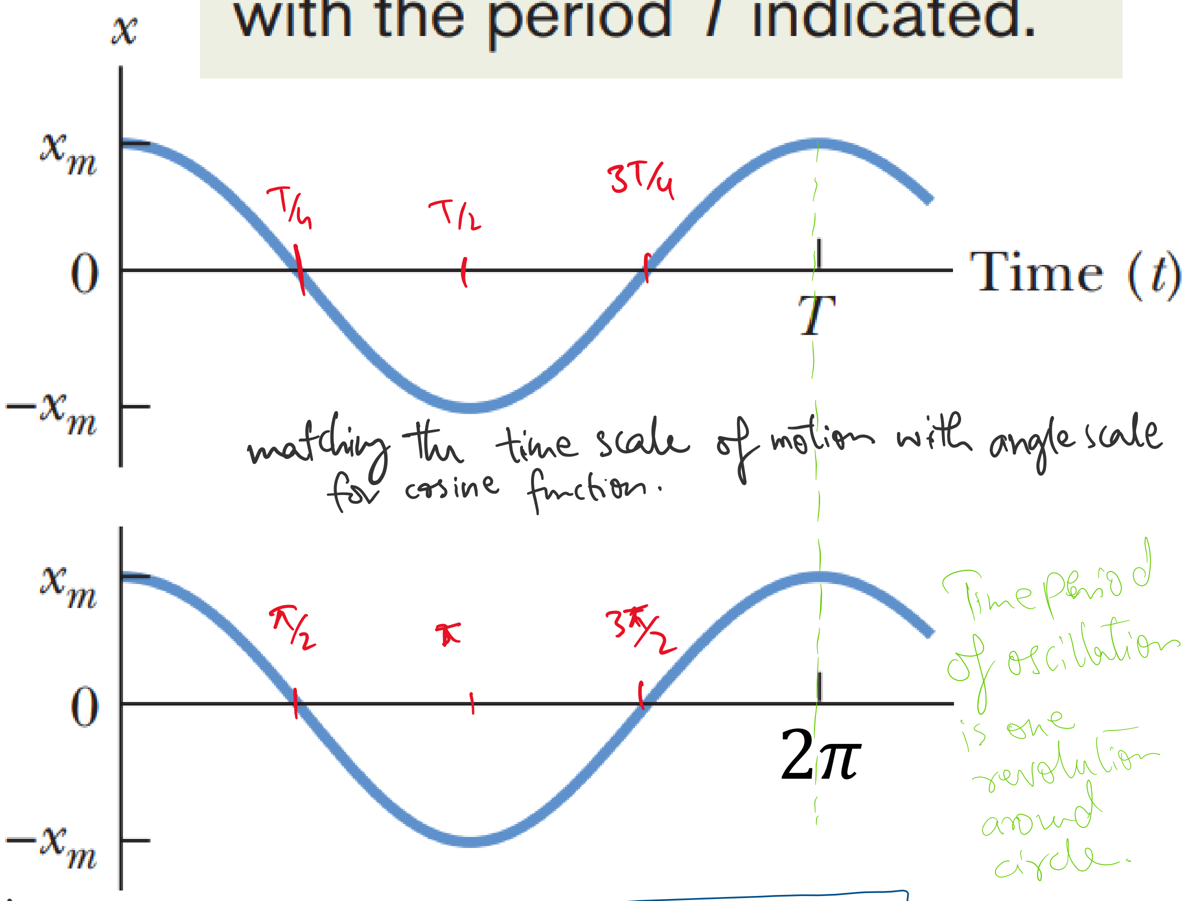
use adjustment constant

$\cos(2\pi) = \cos(\underbrace{2\pi f}_{\omega} T)$

$$\boxed{x_m \cos \omega t}$$

here, small t represents any point in time

This is a graph of the motion, with the period T indicated.



matching the time scale of motion with angle scale for cosine function.

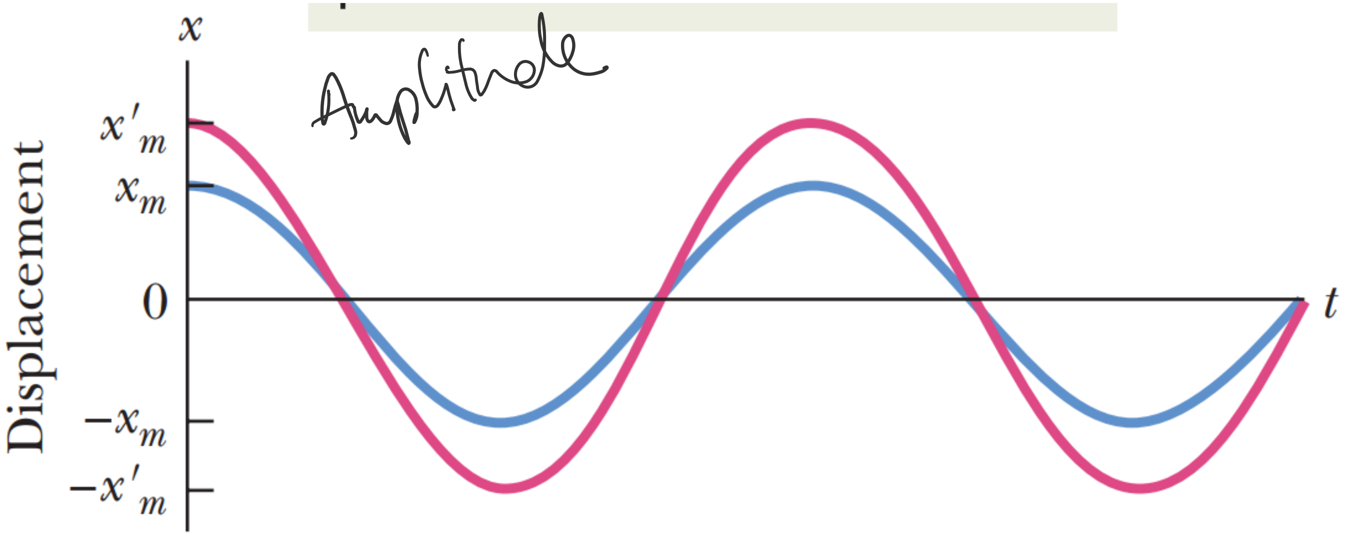
Time period of oscillation is one revolution around circle.

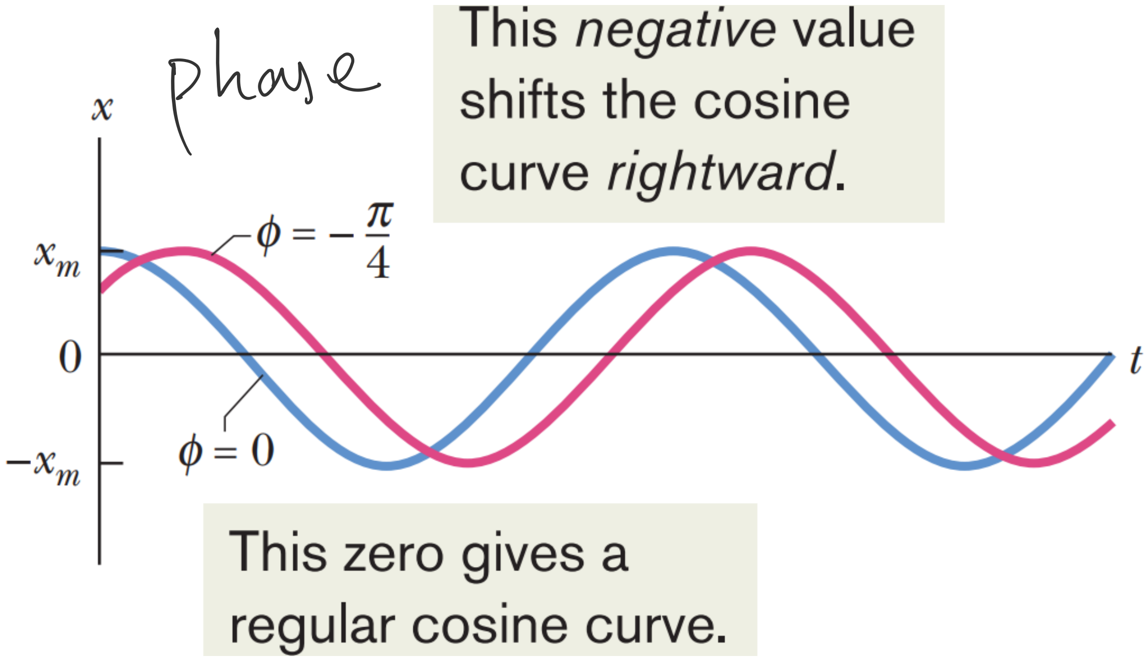
ω remains constant for oscillation.

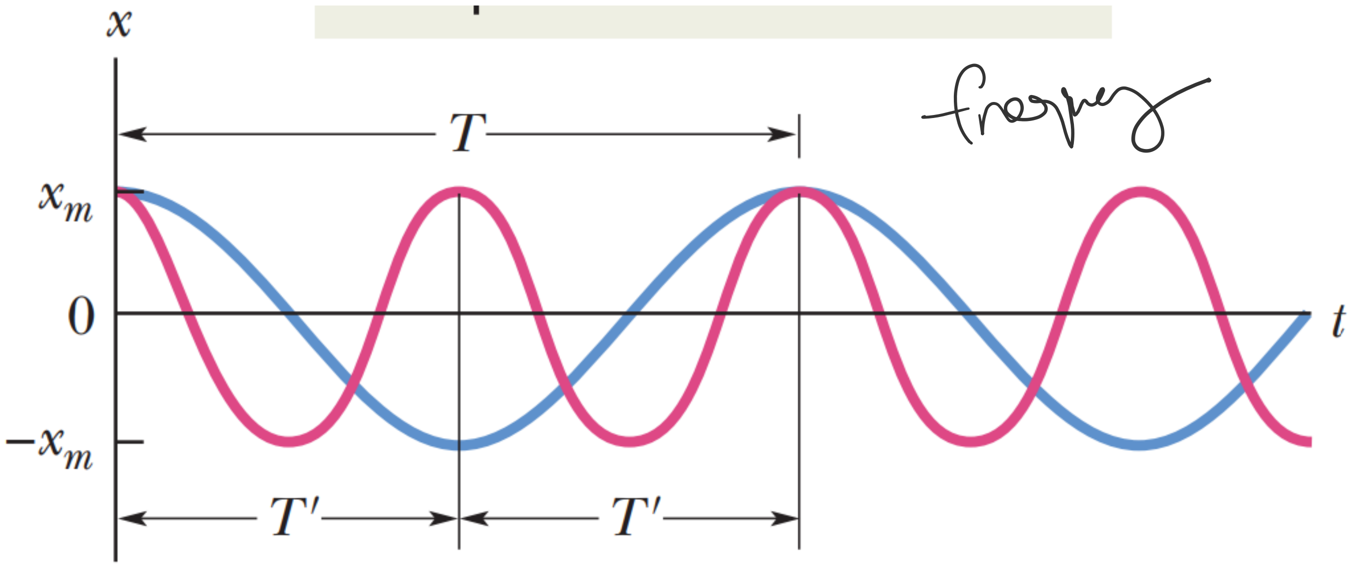
$\boxed{\omega = 2\pi f = \frac{2\pi}{T}}$

Angular frequency

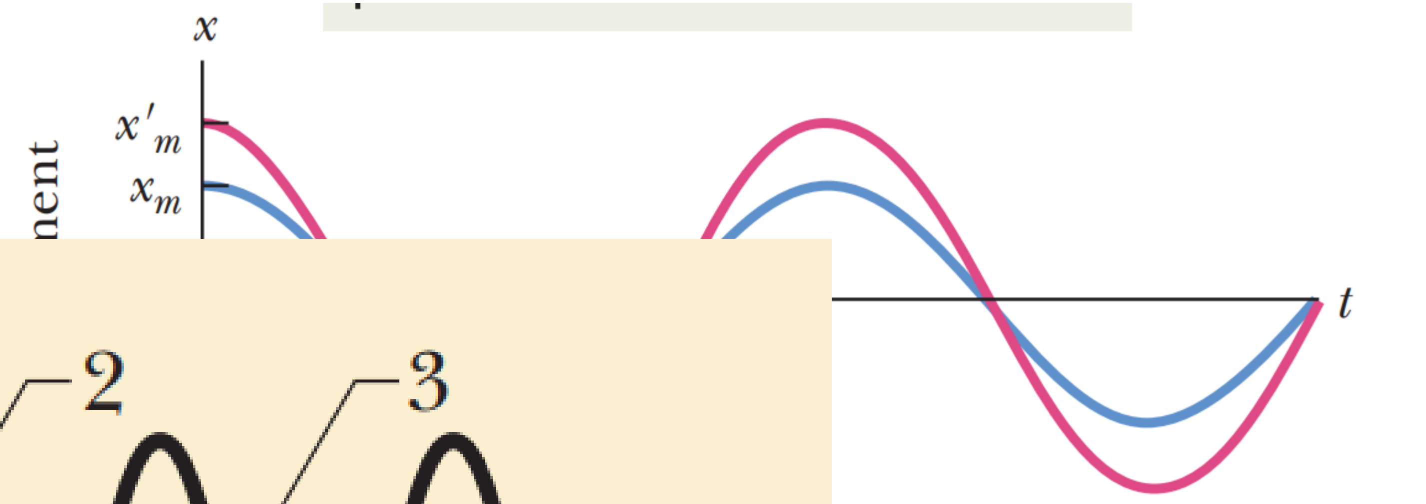
$$x(t) = x_m \cos(\omega t + \phi)$$

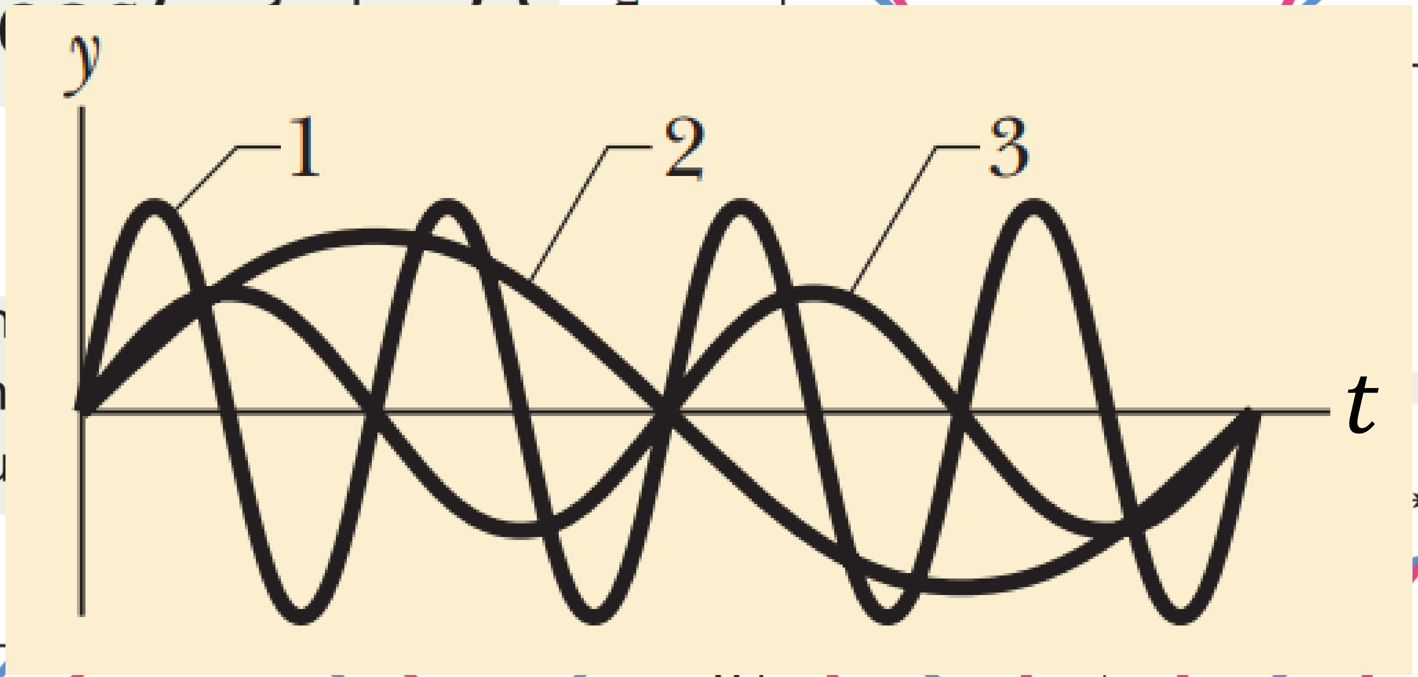


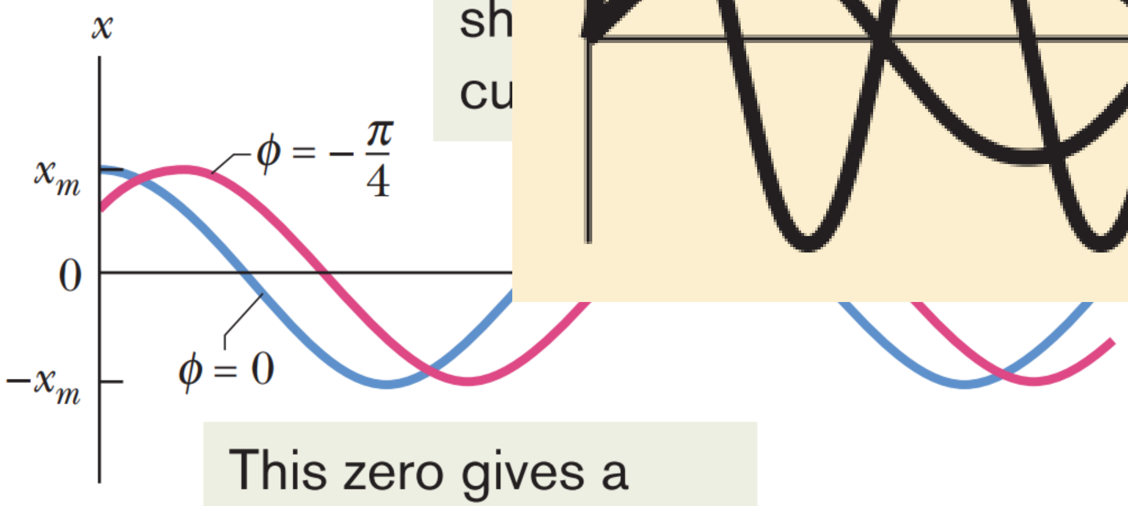




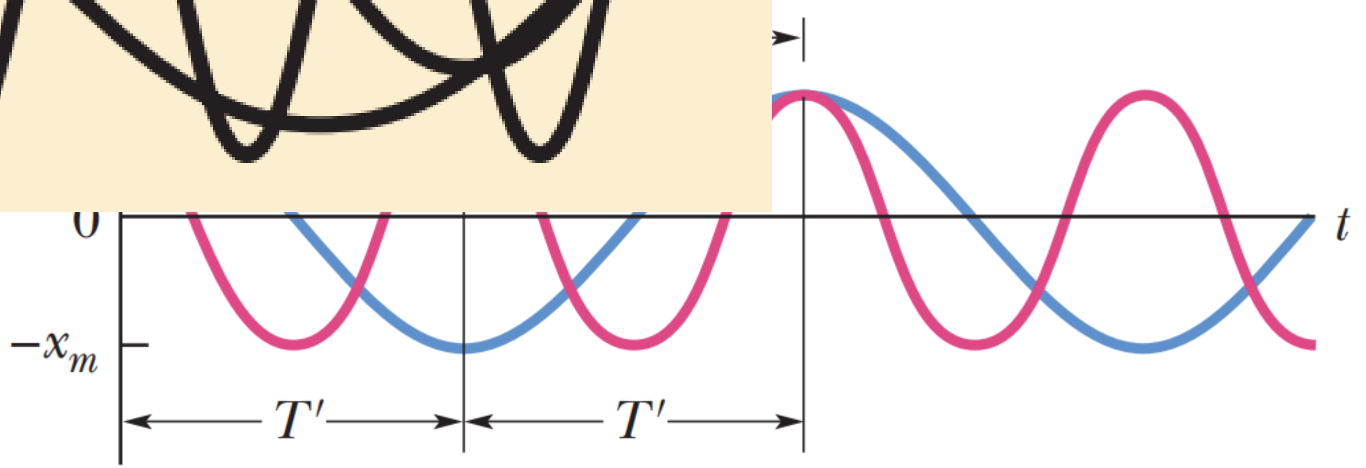
$$x(t) = x_m$$







This zero gives a regular cosine curve.



Position

$$x(t) = x_m \cos(\omega t + \phi)$$

$v(t) = \frac{dx(t)}{dt}$

velocity

$$v(t) = -\omega x_m \sin(\omega t + \phi)$$

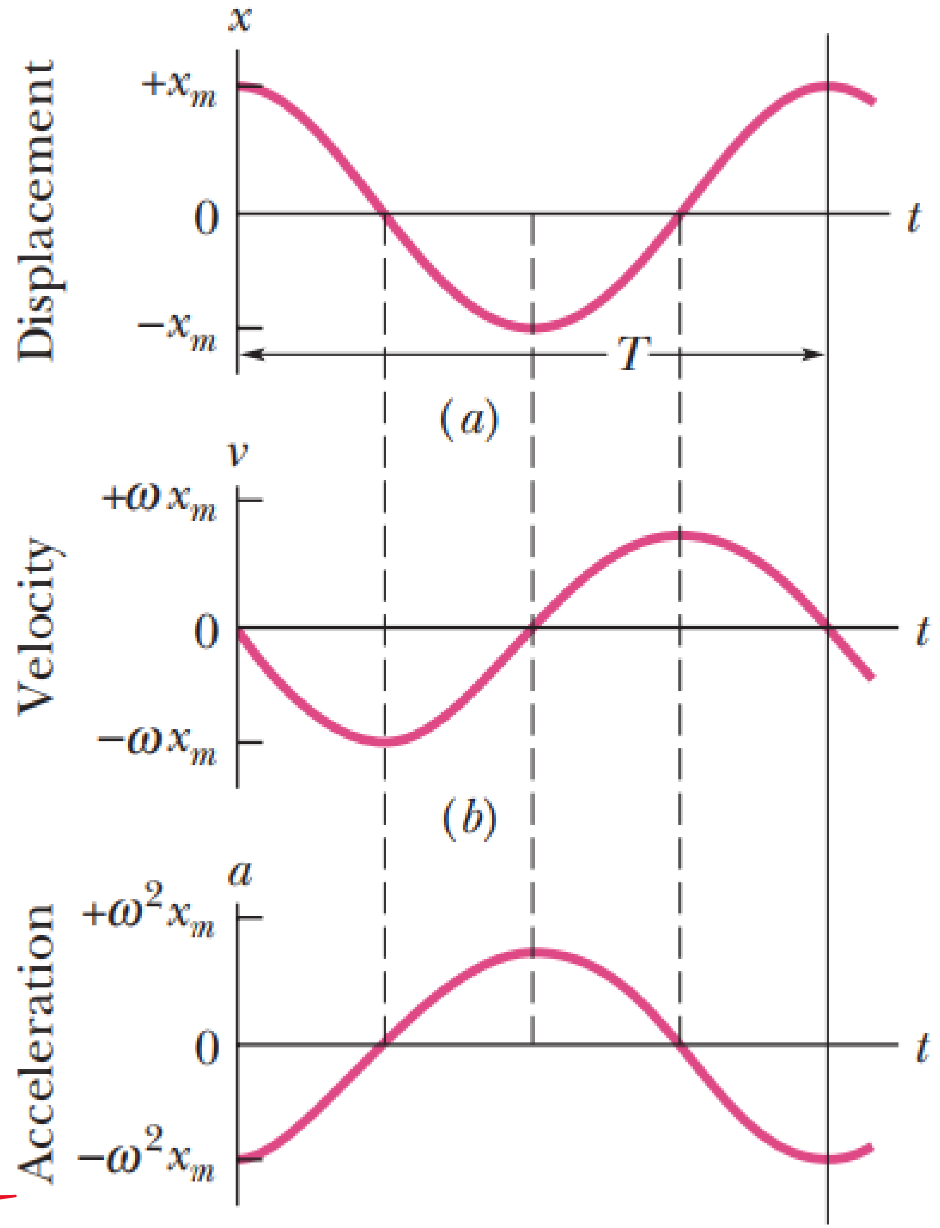
acceleration

$a(t) = \frac{dv(t)}{dt} = \frac{d^2x(t)}{dt}$

$$a(t) = -\omega^2 x_m \cos(\omega t + \phi)$$

Oscillation signature

$$\boxed{a(t) = -\omega^2 x(t)}$$

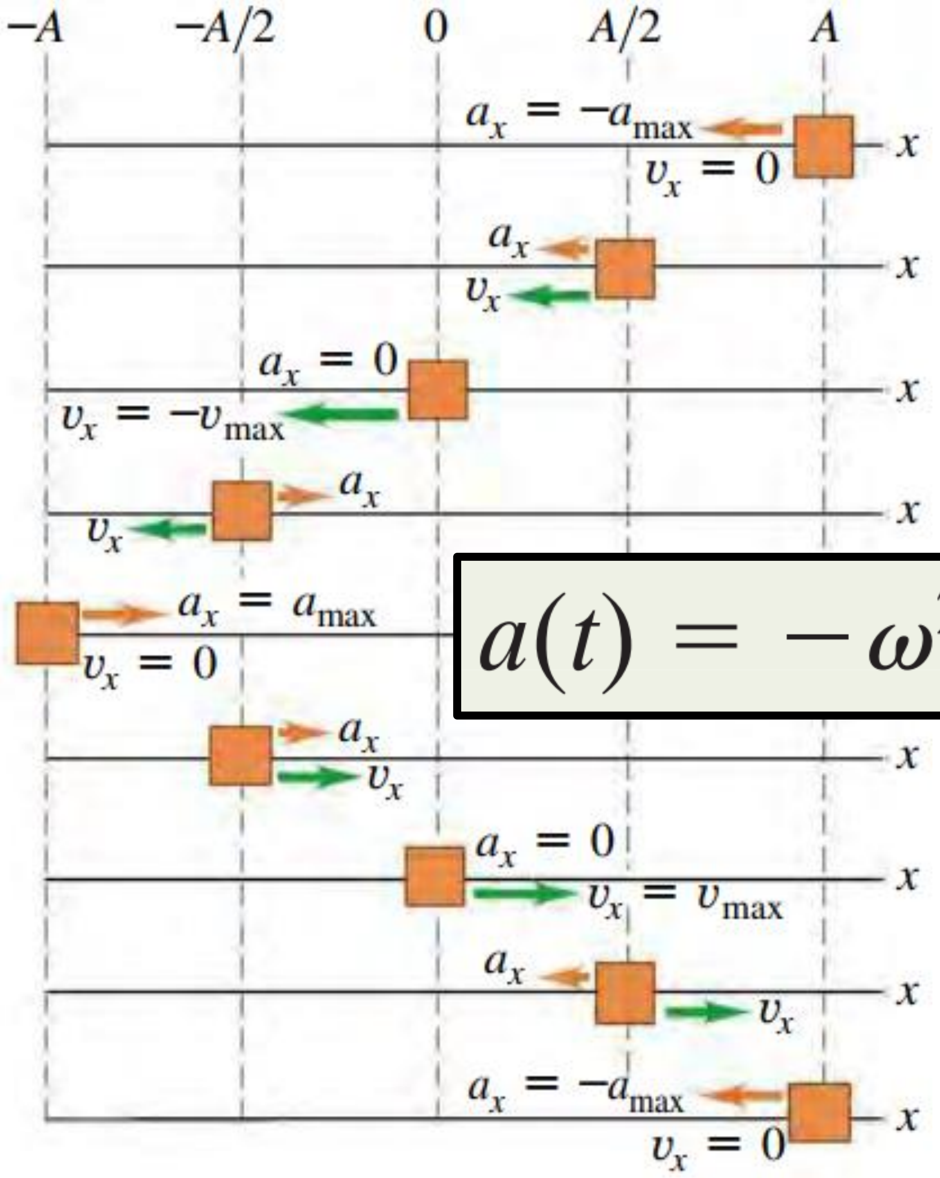


Extreme values here mean ...

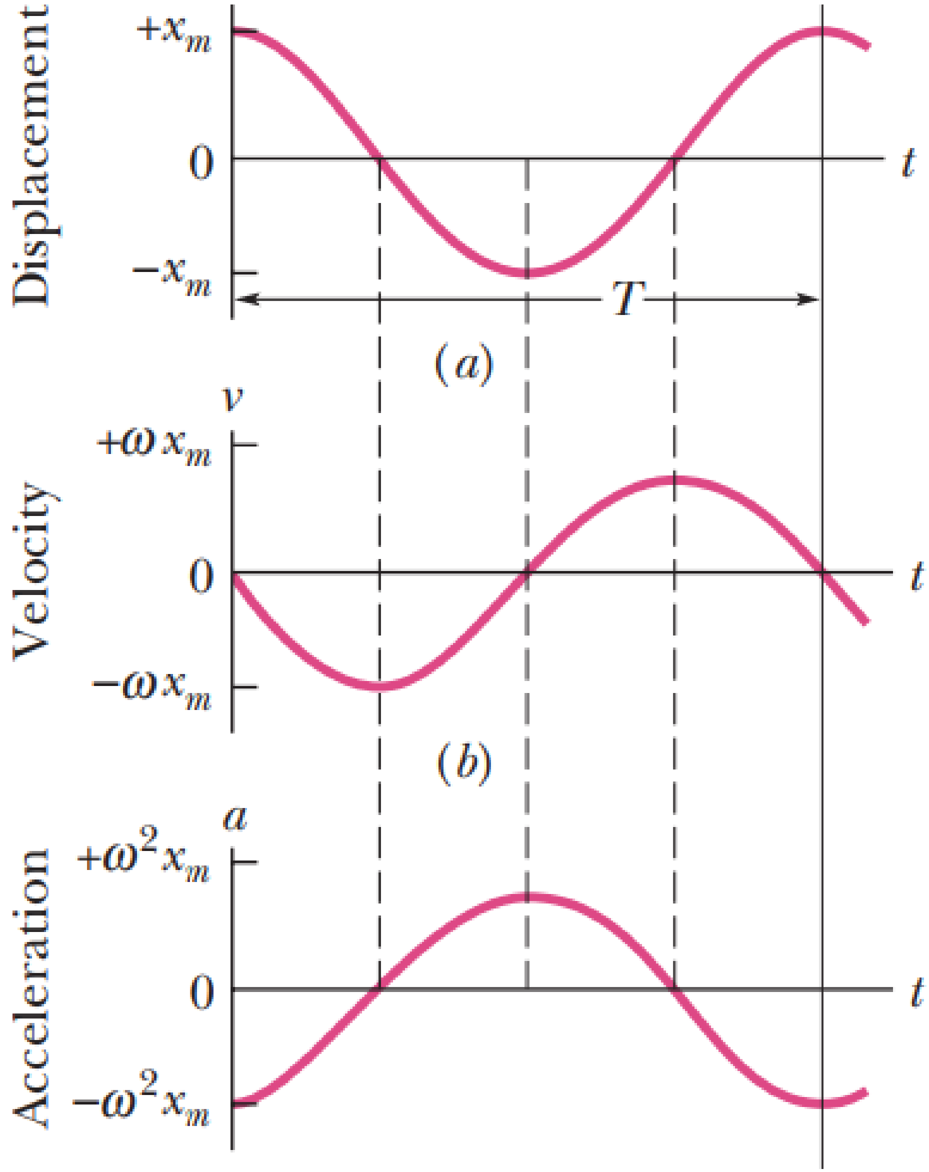
zero values here and ...

extreme values here.

In SHM, the acceleration a is proportional to the displacement x but opposite in sign, and the two quantities are related by the square of the angular frequency ω.



$$a(t) = -\omega^2 x(t)$$



Extreme values here mean ...

zero values here and ...

extreme values here.

Checkpoint 2

Which of the following relationships between a particle's acceleration a and its position x indicates simple harmonic oscillation: (a) $a = 3x^2$, (b) $a = 5x$, (c) $a = -4x$, (d) $a = -2/x$? For the SHM, what is the angular frequency (assume the unit of rad/s)?

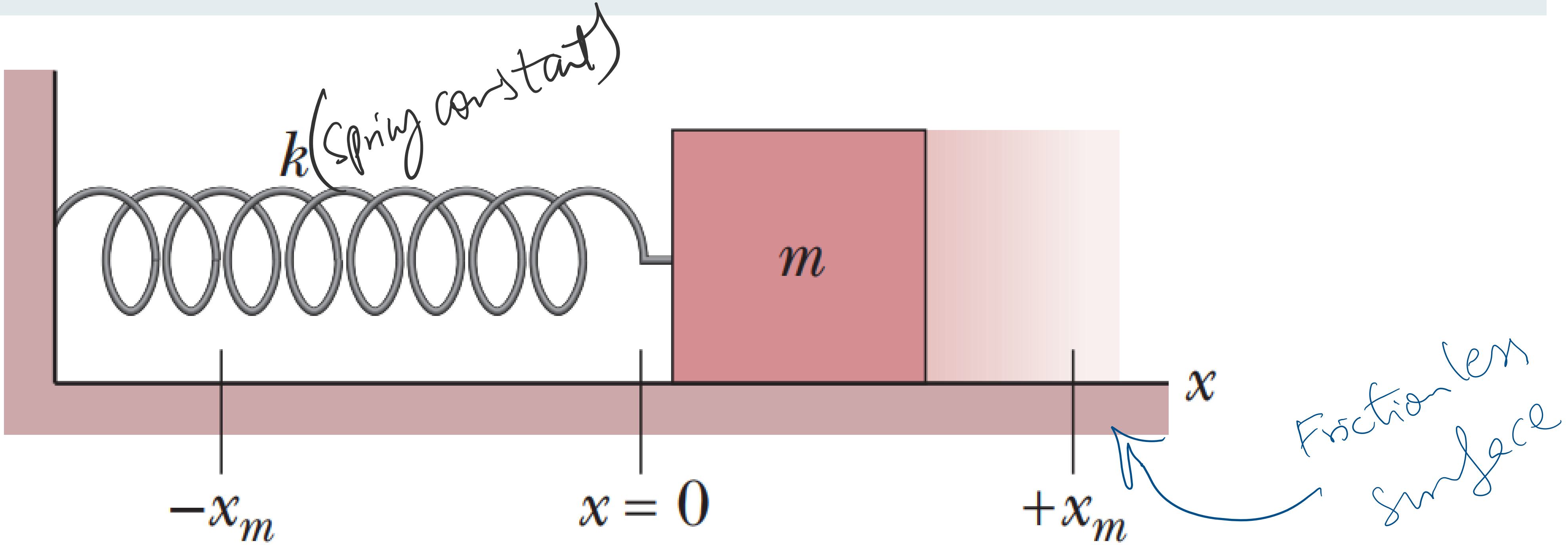
Lecture 12

No mention of source

In SHM, the acceleration a is proportional to the displacement x but opposite in sign, and the two quantities are related by the square of the angular frequency ω.

Simple harmonic motion is the motion of a particle when the force acting on it is proportional to the particle's displacement but in the opposite direction.

Physical source mentioned



Newton's Second Law

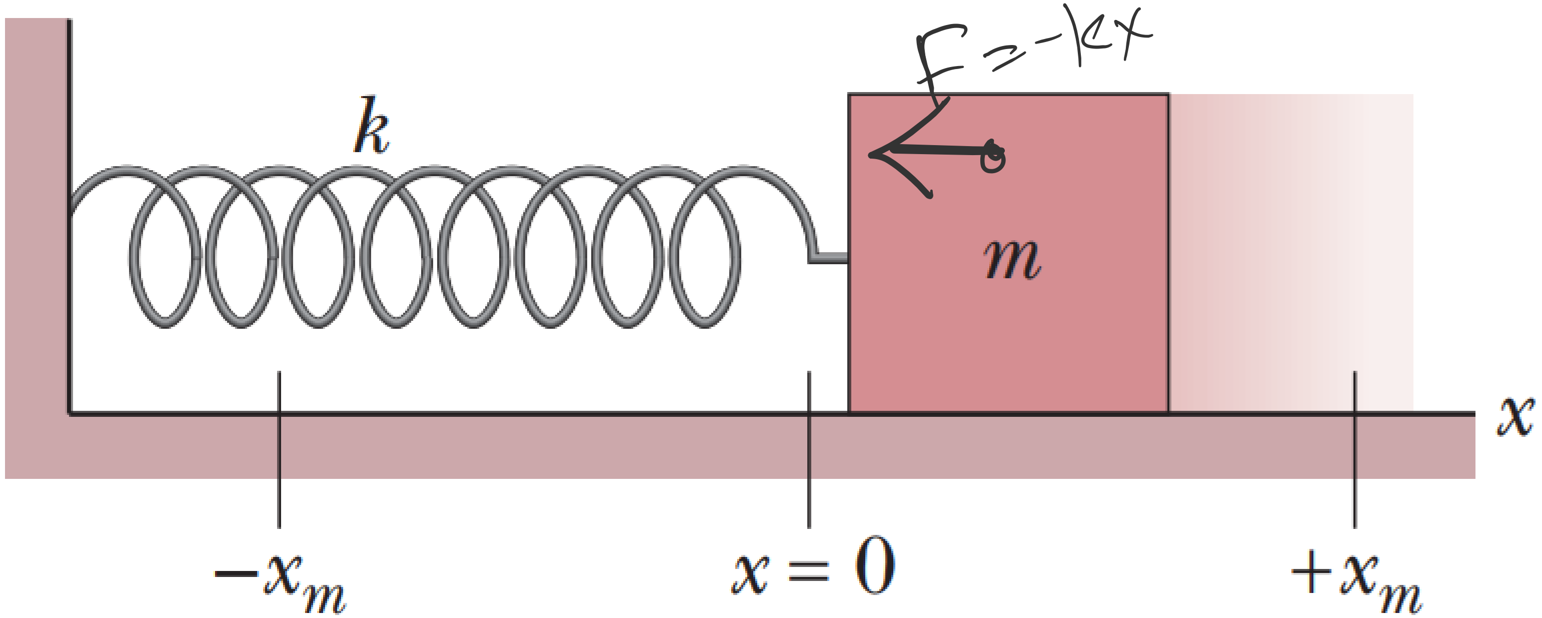
Same as

$$F = ma = m(-\omega^2 x) = -(m\omega^2)x. \qquad F = -kx,$$

general motion for every SHM

Restoring Force due to spring

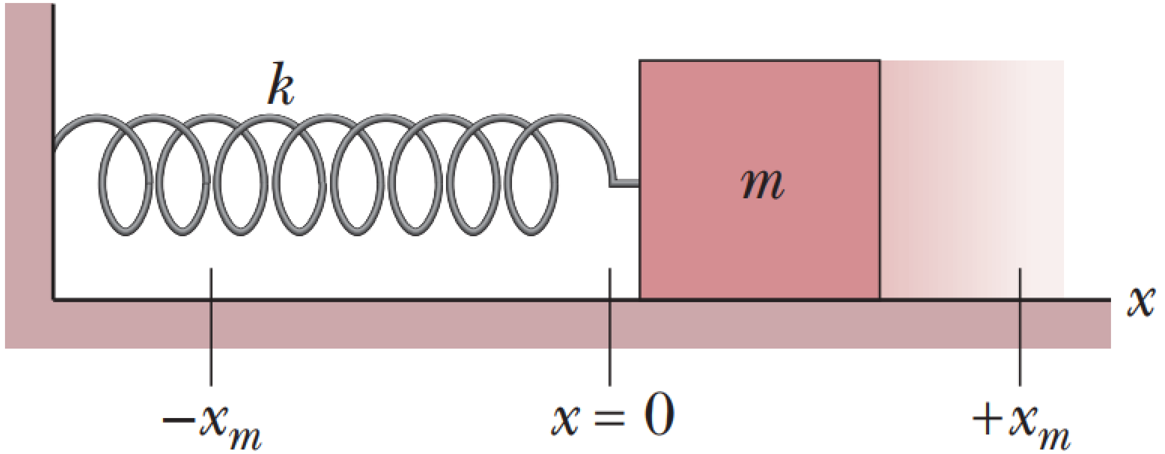
Simple harmonic motion is the motion of a particle when the force acting on it is proportional to the particle's displacement but in the opposite direction.



$$F = ma = m(-\omega^2 x) = -(m\omega^2)x. \quad F = -kx,$$

equating the forms

$$k = m\omega^2.$$



Natural oscillation frequency

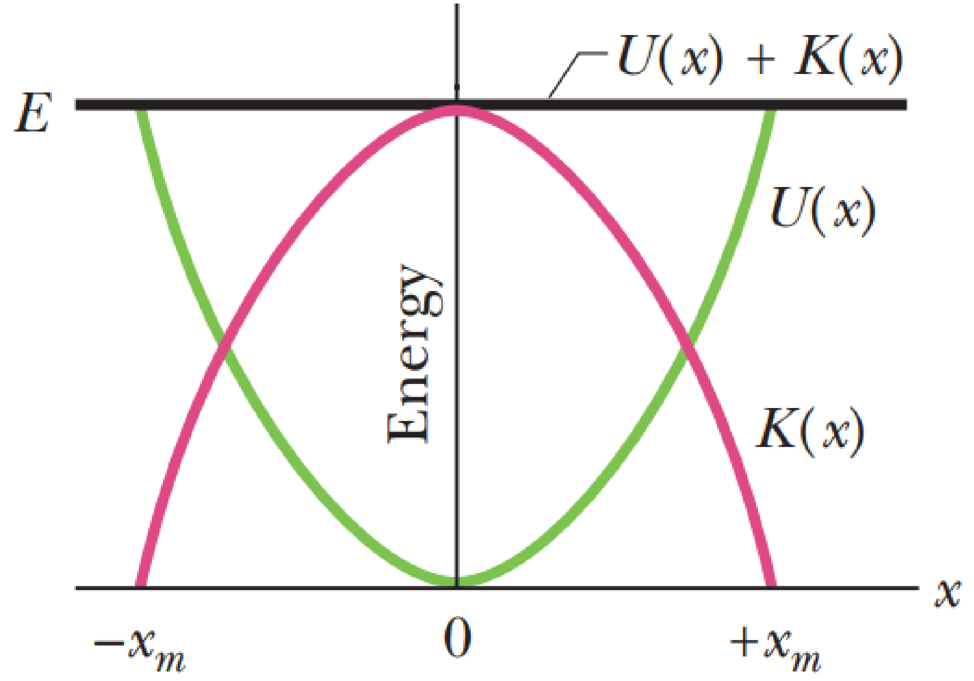
$$\omega = \sqrt{\frac{k}{m}} \quad \text{(angular frequency).}$$

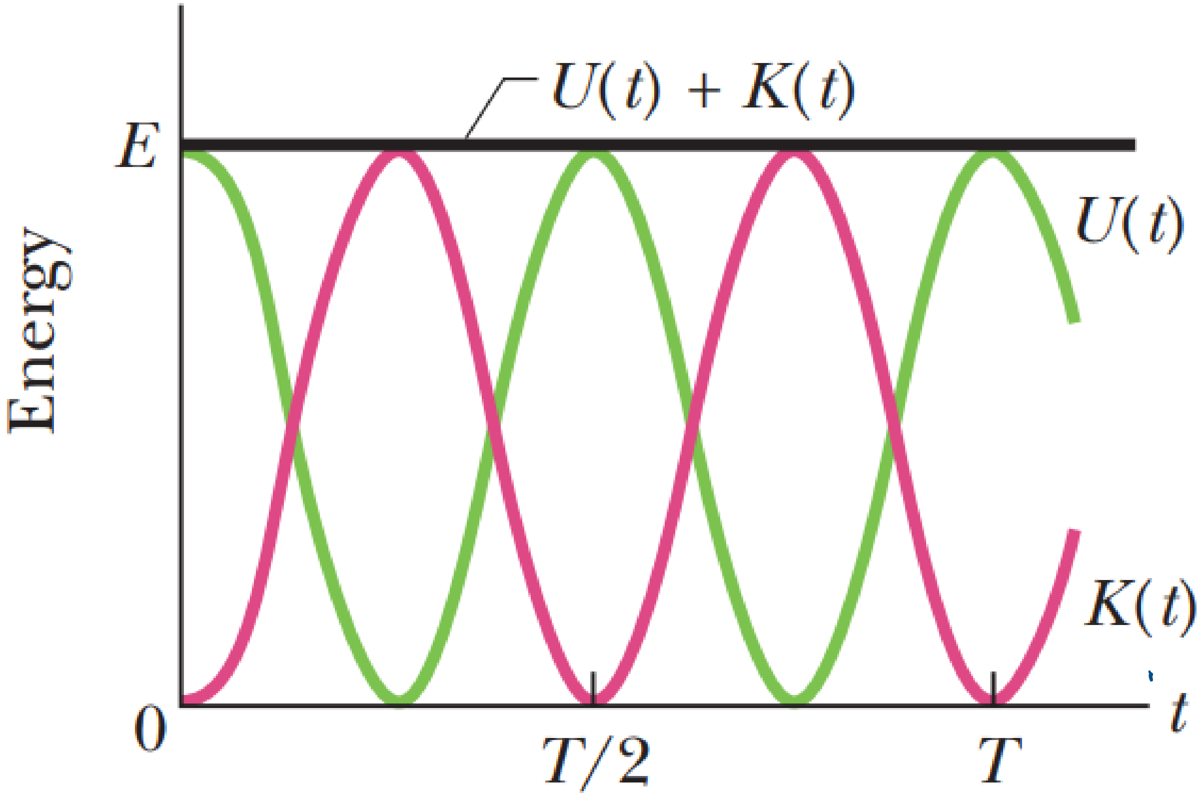
$$T = 2\pi\sqrt{\frac{m}{k}} \quad \text{(period).}$$

elastic Potential energy due to spring

$$U(t) = \tfrac{1}{2}kx^2 = \tfrac{1}{2}kx_m^2 \cos^2(\omega t + \phi).$$

$$K(t) = \tfrac{1}{2}mv^2 = \tfrac{1}{2}kx_m^2 \sin^2(\omega t + \phi).$$





$$E = U + K = \tfrac{1}{2}kx_m^2.$$

Total energy remains constant

HW

Checkpoint 4

In Fig. 15-7, the block has a kinetic energy of 3 J and the spring has an elastic potential energy of 2 J when the block is at $x = +2.0$ cm. (a) What is the kinetic energy when the block is at $x = 0$? What is the elastic potential energy when the block is at (b) $x = -2.0$ cm and (c) $x = -x_m$?