

Try again once you are ready

Try again

Grade received 70% Latest Submission Grade 60% To pass 80% or higher

1. Suppose a MAC system  $(S, V)$  is used to protect files in a file system 0 / 1 point

by appending a MAC tag to each file. The MAC signing algorithm  $S$

is applied to the file contents and nothing else. What tampering attacks

are not prevented by this system?

- ☐ Changing the last modification time of a file.
- ☒ Replacing the contents of a file with the concatenation of two files on the file system.
- ☐ Changing the first byte of the file contents.
- ☐ Replacing the tag and contents of one file with the tag and contents of a file from another computer protected by the same MAC system, but a different key.

☒ Incorrect The MAC tag will fail to verify if any file data is changed.

2. Let  $(S, V)$  be a secure MAC defined over  $(K, M, T)$  where  $M = \{0, 1\}^n$  and  $T = \{0, 1\}^{128}$ . That is, the key space is  $K$ , message space is  $\{0, 1\}^n$ , and tag space is  $\{0, 1\}^{128}$ . 0 / 1 point

Which of the following is a secure MAC: (as usual, we use  $\parallel$  to denote string concatenation)

- ☒  $S'(k, m) = S(k, m \oplus 1^n)$  and  $V'(k, m, t) = V(k, m \oplus 1^n, t)$ .

☒ Correct a forger for  $(S', V')$  gives a forger for  $(S, V)$ .

- ☐  $S'(k, m) = S(k, m)$  and  $V'(k, m, t) = [V(k, m, t) \text{ or } V(k, m \oplus 1^n, t)]$  (i.e.,  $V'(k, m, t)$  outputs  $1$  if  $t$  is a valid tag for either  $m$  or  $m \oplus 1^n$ )

- ☐  $S'(k, m) = S(k, m \oplus m)$  and  $V'(k, m, t) = V(k, m \oplus m, t)$
- ☒  $S'((k_1, k_2), m) = (S(k_1, m), S(k_2, m))$  and  $V'((k_1, k_2), m, (t_1, t_2)) = [V(k_1, m, t_1) \text{ and } V(k_2, m, t_2)]$  (i.e.,  $V'((k_1, k_2), m, (t_1, t_2))$  outputs  $1$  if both  $t_1$  and  $t_2$  are valid tags)

☒ Correct a forger for  $(S', V')$  gives a forger for  $(S, V)$ .

- ☐  $S'(k, m) = [t \leftarrow S(k, m), \text{ output } (t, t)]$  and  $V'(k, m, (t_1, t_2)) = \begin{cases} V(k, m, t_1) & \text{if } t_1 = t_2 \\ 0 & \text{otherwise} \end{cases}$  (i.e.,  $V'(k, m, (t_1, t_2))$  only outputs  $1$  if  $t_1$  and  $t_2$  are equal and valid)

- ☐  $S'(k, m) = \begin{cases} S(k, 1^n) & \text{if } m = 0^n \\ S(k, m) & \text{otherwise} \end{cases}$  and  $V'(k, m) = \begin{cases} V(k, 1^n, t) & \text{if } m = 0^n \\ V(k, m, t) & \text{otherwise} \end{cases}$

You didn't select all the correct answers

3. Recall that the ECBC-MAC uses a fixed IV (in the lecture we simply set the IV to 0). Suppose instead we 0 / 1 point

chose a random IV for every message being signed and include the IV in the tag.

In other words,  $S(k, m) := (r, \text{ECBC}_r(k, m))$

where  $\text{ECBC}_r(k, m)$  refers to the ECBC function using  $r$  as the IV. The verification algorithm  $V$  given key  $k$ , message  $m$ ,

and tag  $(r, t)$  outputs  $1$  if  $t = \text{ECBC}_r(k, m)$  and outputs  $0$  otherwise.

The resulting MAC system is insecure.

An attacker can query for the tag of the 1-block message  $m$

and obtain the tag  $(r, t)$ . He can then generate the following existential forgery: (we assume that the underlying block cipher

operates on  $n$ -bit blocks)

- ☐ The tag  $(r \oplus 1^n, t)$  is a valid tag for the 1-block message  $m \oplus 1^n$ .
- ☐ The tag  $(r, t \oplus r)$  is a valid tag for the 1-block message  $0^n$ .
- ☐ The tag  $(r \oplus t, m)$  is a valid tag for the 1-block message  $0^n$ .
- ☒ The tag  $(m \oplus t, r)$  is a valid tag for the 1-block message  $0^n$ .

☒ Incorrect The right half of the tag,  $r$ , is not likely to be the result of the CBC MAC.

4. Suppose Alice is broadcasting packets to 6 recipients 1 / 1 point

$B_1, \dots, B_6$ . Privacy is not important but integrity is.

In other words, each of  $B_1, \dots, B_6$  should be assured that the packets he is receiving were sent by Alice.

Alice decides to use a MAC. Suppose Alice and  $B_1, \dots, B_6$  all share a secret key  $k$ . Alice computes a tag for every packet she

sends using key  $k$ . Each user  $B_i$  verifies the tag when receiving the packet and drops the packet if the tag is invalid.

Alice notices that this scheme is insecure because user  $B_1$  can use the key  $k$  to send packets with a valid tag to

users  $B_2, \dots, B_6$  and they will all be fooled into thinking that these packets are from Alice.

Instead, Alice sets up a set of 4 secret keys  $S = \{k_1, \dots, k_4\}$ .

She gives each user  $B_i$  some subset  $S_i \subseteq S$

of the keys. When Alice transmits a packet she appends 4 tags to it

by computing the tag with each of her 4 keys. When user  $B_i$  receives a packet he accepts it as valid only if all tags corresponding

to his keys in  $S_i$  are valid. For example, if user  $B_1$  is given keys  $\{k_1, k_2\}$  he will accept an incoming packet only if the first and second tags are valid. Note that  $B_1$  cannot validate the 3rd and 4th tags because he does not have  $k_3$  or  $k_4$ .

How should Alice assign keys to the 6 users so that no single user can forge packets on behalf of Alice and fool some other user?

- ☐  $S_1 = \{k_1, k_2\}, S_2 = \{k_1\}, S_3 = \{k_1, k_4\}, S_4 = \{k_2, k_3\}, S_5 = \{k_2, k_4\}, S_6 = \{k_3, k_4\}$
- ☒  $S_1 = \{k_2, k_4\}, S_2 = \{k_2, k_3\}, S_3 = \{k_3, k_4\}, S_4 = \{k_1, k_3\}, S_5 = \{k_1, k_2\}, S_6 = \{k_1, k_4\}$

☒ Correct Every user can only generate tags with the two keys he has. Since no set  $S_i$  is contained in another set  $S_j$ , no user  $i$  can fool a user  $j$  into accepting a message sent by  $i$ .

- ☐  $S_1 = \{k_1, k_2\}, S_2 = \{k_1, k_3\}, S_3 = \{k_1, k_4\}, S_4 = \{k_2, k_3\}, S_5 = \{k_2, k_4\}, S_6 = \{k_4\}$

- ☐  $S_1 = \{k_1, k_2\}, S_2 = \{k_2, k_3\}, S_3 = \{k_3, k_4\}, S_4 = \{k_1, k_3\}, S_5 = \{k_1, k_2\}, S_6 = \{k_1, k_4\}$

5. Consider the encrypted CBC MAC built from AES. Suppose we 1 / 1 point

compute the tag for a long message  $m$  comprising of  $n$  AES blocks.

Let  $m'$  be the  $n$ -block message obtained from  $m$  by flipping the last bit of  $m$  (i.e. if the last bit of  $m$  is  $b$  then the last bit

of  $m'$  is  $b \oplus 1$ ). How many calls to AES would it take to compute the tag for  $m'$  from the tag for  $m$  and the MAC key? (in this question please ignore message padding and simply assume that the message length is always a multiple of the AES block size)

- ☒ 4
- ☐ 5
- ☐  $n$
- ☐ 6

☒ Correct You would decrypt the final CBC MAC encryption step done using  $k_2$ , the decrypt the last CBC MAC encryption step done using  $k_1$ , flip the last bit of the result, and re-apply the two encryptions.

6. Let  $H : M \rightarrow T$  be a collision resistant hash function. 0 / 1 point

Which of the following is collision resistant:

(as usual, we use  $\parallel$  to denote string concatenation)

- ☐  $H'(m) = H(0)$
- ☐  $H'(m) = H(|m|)$  (i.e. hash the length of  $m$ )
- ☐  $H'(m) = H(m) \oplus H(m \oplus 1^{|m|})$  (where  $m \oplus 1^{|m|}$  is the complement of  $m$ )
- ☒  $H'(m) = H(H(m))$

☒ Correct a collision finder for  $H'$  gives a collision finder for  $H$ .

- ☐  $H'(m) = H(m) \oplus H(m)$

- ☐  $H'(m) = H(m) \parallel H(m)$

- ☒  $H'(m) = H(m \parallel m)$

☒ Correct a collision finder for  $H'$  gives a collision finder for  $H$ .

You didn't select all the correct answers

7. Suppose  $H_1$  and  $H_2$  are collision resistant 1 / 1 point

hash functions mapping inputs in a set  $M$  to  $\{0, 1\}^{256}$ .

Our goal is to show that the function  $H_2(H_1(m))$  is also collision resistant. We prove the contra-positive:

suppose  $H_2(H_1(\cdot))$  is not collision resistant, that is, we are given  $x \neq y$  such that  $H_2(H_1(x)) = H_2(H_1(y))$ .

We build a collision for either  $H_1$  or for  $H_2$ .

This will prove that if  $H_1$  and  $H_2$  are collision resistant then so is  $H_2(H_1(\cdot))$ . Which of the following must be true:

- ☒ Either  $x, y$  are a collision for  $H_1$  or  $H_1(x), H_1(y)$  are a collision for  $H_2$ .
- ☐ Either  $x, y$  are a collision for  $H_2$  or  $H_1(x), H_1(y)$  are a collision for  $H_1$ .
- ☐ Either  $H_2(x), H_2(y)$  are a collision for  $H_1$  or  $x, y$  are a collision for  $H_2$ .
- ☐ Either  $x, y$  are a collision for  $H_1$  or  $x, y$  are a collision for  $H_2$ .

☒ Correct if  $H_2(H_1(x)) = H_2(H_1(y))$  then either  $H_1(x) = H_1(y)$  and  $x \neq y$ , thereby giving us a collision on  $H_1$ . Or  $H_1(x) \neq H_1(y)$  but  $H_2(H_1(x)) = H_2(H_1(y))$  giving us a collision on  $H_2$ . Either way we obtain a collision on  $H_1$  or  $H_2$  as required.

8. In this question you are asked to find a collision for the compression function: 1 / 1 point

$f_1(x, y) = \text{AES}(y, x) \oplus y$ ,

where  $\text{AES}(x, y)$  is the AES-128 encryption of  $y$  under key  $x$ .

Your goal is to find two distinct pairs  $(x_1, y_1)$  and  $(x_2, y_2)$  such that  $f_1(x_1, y_1) = f_1(x_2, y_2)$ .

Which of the following methods finds the required  $(x_1, y_1)$  and  $(x_2, y_2)$ ?

- ☒ Choose  $x_1, y_1, y_2$  arbitrarily (with  $y_1 \neq y_2$ ) and let  $v := \text{AES}(y_1, x_1)$ . Set  $x_2 = \text{AES}^{-1}(y_2, v \oplus y_1 \oplus y_2)$
- ☐ Choose  $x_1, y_1, y_2$  arbitrarily (with  $y_1 \neq y_2$ ) and let  $v := \text{AES}(y_1, x_1)$ . Set  $x_2 = \text{AES}^{-1}(y_2, v \oplus y_2)$
- ☐ Choose  $x_1, y_1, y_2$  arbitrarily (with  $y_1 \neq y_2$ ) and let  $v := \text{AES}(y_1, x_1)$ . Set  $x_2 = \text{AES}^{-1}(y_2, v \oplus y_1)$
- ☐ Choose  $x_1, y_1, x_2$  arbitrarily (with  $x_1 \neq x_2$ ) and let  $v := \text{AES}(y_1, x_1)$ . Set  $y_2 = \text{AES}^{-1}(x_2, v \oplus y_1 \oplus x_2)$

☒ Correct You got it!

9. Repeat the previous question, but now to find a collision for the compression function  $f_2(x, y) = \text{AES}(x, x) \oplus y$ . 1 / 1 point

Which of the following methods finds the required  $(x_1, y_1)$  and  $(x_2, y_2)$ ?

- ☒ Choose  $x_1, x_2, y_1$  arbitrarily (with  $x_1 \neq x_2$ ) and set  $y_2 = y_1 \oplus \text{AES}(x_1, x_1) \oplus \text{AES}(x_2, x_2)$
- ☐ Choose  $x_1, x_2, y_1$  arbitrarily (with  $x_1 \neq x_2$ ) and set  $y_2 = y_1 \oplus \text{AES}(x_1, x_1)$
- ☐ Choose  $x_1, x_2, y_1$  arbitrarily (with  $x_1 \neq x_2$ ) and set  $y_2 = \text{AES}(x_1, x_1) \oplus \text{AES}(x_2, x_2)$
- ☐ Choose  $x_1, x_2, y_1$  arbitrarily (with  $x_1 \neq x_2$ ) and set  $y_2 = y_1 \oplus x_1 \oplus \text{AES}(x_2, x_2)$

☒ Correct Awesome!

10. Let  $H : M \rightarrow T$  be a random hash function where  $|M| \gg |T|$  (i.e. the size of  $M$  is much larger than the size of  $T$ ). 1 / 1 point

In lecture we showed that finding a collision on  $H$  can be done with  $O(|T|^{1/2})$  random samples of  $H$ . How many random samples would it take until we obtain a three way collision, namely distinct strings  $x, y, z$  in  $M$  such that  $H(x) = H(y) = H(z)$ ?

- ☒  $O(|T|^{2/3})$
- ☐  $O(|T|^{1/2})$
- ☐  $O(|T|)$
- ☐  $O(|T|^{1/3})$

☒ Correct An informal argument for this is as follows: suppose we collect  $n$  random samples. The number of triples among the  $n$  samples is  $n$  choose 3 which is  $O(n^3)$ . For a particular triple  $x, y, z$  to be a 3-way collision we need  $H(x) = H(y)$  and  $H(x) = H(z)$ . Since each one of these two events happens with probability  $1/|T|$  (assuming  $H$  behaves like a random function) the probability that a particular triple is a 3-way collision is  $O(1/|T|^2)$ . Using the union bound, the probability that some triple is a 3-way collision is  $O(n^3/|T|^2)$  and since we want this probability to be close to 1, the bound on  $n$  follows.