Boolean Algebra

Introduction

- Logic is a formal method of reasoning
- Propositions are the elementary atomic statements that can be either true or false.
- Propositional Logic is the way of representing logic symbolically through propositions and logical connectives.

Types of Propositions

1. Simple - Does not contain any other proposition as a part.
2. Compound - When two or more propositions are connected using a connective.

Connectors / Operators

1)and (Conjunction) - Both arguments are true . (&,.,A)
2)or (Disjunction) - One of the two arguments or both are true. (+,V)
3)not (Negation) - Operator. It converts the argument to opposite. (~,')
4)Conditional(Implication) - If one argument is true, then the other argument must also be true. (→, ⇒)
5)Biconditional(Equivalence) - Either both arguments are true or both are false. (↔, ⇔)

Truth Table

- A table of all possible truth values is called a truth table.
- A truth value is the truth or falsity of a proposition.

P	Q	P+Q	P.Q	P→Q	P⇔Q	~P	~Q
0	0	0	0	1	1	1	1
0	1	1	0	1	0	1	0
1	0	1	0	0	0	0	1
1	1	1	1	1	1	0	0

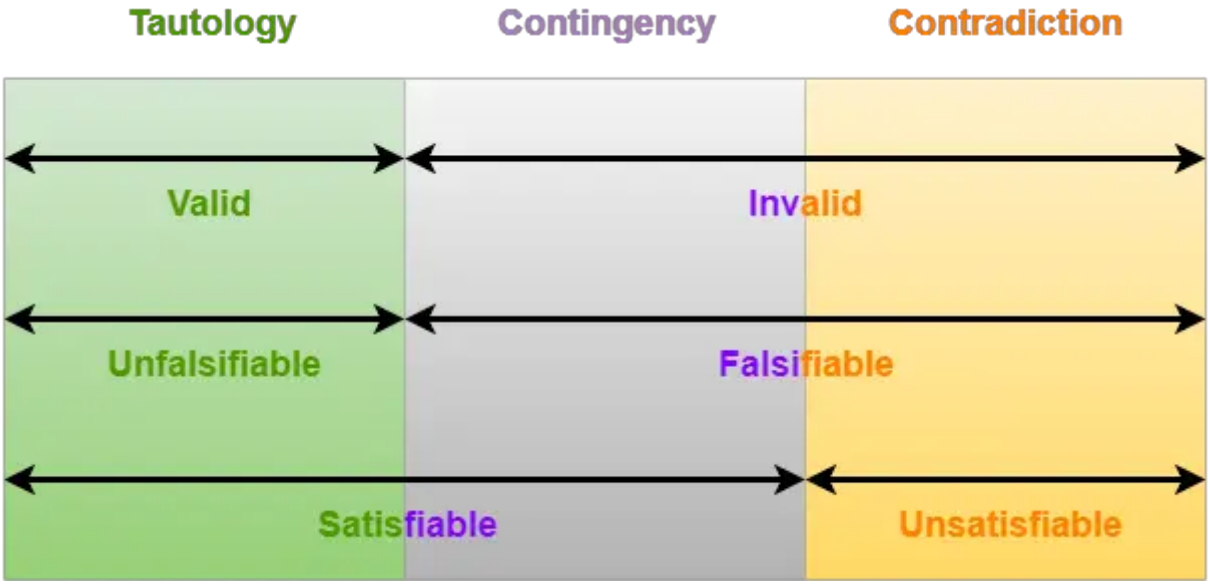
Syllogism

- An instance of a form of reasoning in which a conclusion is drawn (whether validly or not) from two given or assumed propositions (premises), each of which shares

a term with the conclusion, and shares a common or middle term not present in the conclusion.
- Two statements are consistent only if their conjunction is not a contradiction

Types

1. Contingencies (Satisfactory): Propositions that have some combination of 1's and 0's in their truth table
2. Tautologies (Valid): Propositions that have only 1's in their truth table
3. Contradictions (Invalid): Propositions that have only 0's in their truth table
   There are more (falsifiable, valid, and satisfiable and their compliments) and their expressions are shown in the graph below. Should clear up the meanings.



Laws of Boolean Algebra

1. Properties of 0
   p+0=p
   p.0=0

2. Properties of 1
   p+1=1
   p.1=p

3. Indentity or Idempotence Law
   p+p=p
   p.p=p

4. Commutative Law
   p+q=q+p

p.q=q.p

5. Associative Law
   p+(q+r) = (p+q)+r
   p.(q.r) = (p.q).r

6. Complimentary Law
   p+(~p) = 1
   p.(~p) = 0

7. Distributive Law
   p+(q.r) = (p+q).(p+r)
   p.(q+r) = (p.q)+(q.r)
   p+p'.q = p+q

8. Absorption Law
   p+(p.q) = p
   p.(p+q) = p

9. De Morgans Law
   (p+q)' = p'.q'
   (p.q)' = p'+q'

Forms of Boolean Expression

Category I - SOP/POS

- SOP (Sum of Products): When the boolean expression consists of entirely of minterms. Literal 1, Complement 0
- POS (Product of Sums): When the boolean expression consists of entirely of maxterms. Literal 0, Complement 1

Minterms are the products of all literals with or without bar within the logic system. Maxterms are the sum of all literals with or without bar within the logic system.

Category II - Cardinal/Canonical

- Canonical Form: Expression is in form of variables. (SOP ∑, POS π)
- Cardinal Form: Expression is in form of numbers. (∑, π)

Conversion to Canonical Form

If It is in SOP form, multiply 1 to each missing term. (P+P' = 1)
If it is in POS form, add 0 to each missing term (P.P' = 0), Then open brackets.

Principle of Duality

Principle of Duality: Using a boolean expression we can derive another boolean expression.
Interchange AND with OR, 1 with 0 to arrive at an expression's dual.