R16

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD
B.Tech I Year I Semester Examinations, December - 2017
MATHEMATICS-II
(Common to CE, ME, MCT, MMT, AE, MIE, PTM, CEE, MSNT)
Time: 3 hours   **Max. Marks: 75**

Note: This question paper contains two parts A and B.
Part A is compulsory which carries 25 marks. Answer all questions in Part A.
Part B consists of 5 Units. Answer any one full question from each unit. Each
question carries 10 marks and may have a, b as sub questions.

PART- A

(25 Marks)

1.a) Find the Laplace transform of $\cosh^3 2t$. [2]

b) Find the Laplace transform of $e^{-3t}(2\cos 5t - 3\sin 5t)$. [3]

c) Evaluate the improper integral $\int_0^\infty \sqrt{x}e^{-x^2}dx$ using Gamma function. [2]

d) Evaluate the improper integral $\int_0^1 \frac{dx}{\sqrt{-\ln x}}$ using Beta and Gamma functions. [3]

e) Find the area bounded by the curves $x^2 = y^3, x = y$ using double integration. [2]

f) Change the order of the integration $\int_{y=0}^1 \int_{x=0}^{y+4} \frac{2y+1}{x+1}dxdy$ and evaluate the integral. [3]

g) Find $\nabla\phi$, when $\phi = 3x^2 y - y^3 z^2$ at the point (1, -2, -1). [2]

h) Find the directional derivative of the function $f(x, y, z) = 2xy + z^2$ at the point (1, -1, 3) in the direction of the vector $i + 2j + 2k$. [3]

i) If $R = t\bar{i} - t^2\bar{j} + (t-1)\bar{k}$ and $S = 2t^2\bar{i} + 6t\bar{k}$, evaluate $\int_0^2 R. S\ dt$. [2]

j) Evaluate the line integral $\oint_C (3x^2 - 8y^2)dx + (4y - 6xy)dy$, where C is the boundary of the region $y = \sqrt{x}, y = x$. [3]

PART-B

(50 Marks)

2.a) Find the Laplace transform of $\sin\sqrt{t}$. Hence find $L\left(\frac{\cos\sqrt{t}}{\sqrt{t}}\right)$.

b) Prove that $\int_{t=0}^\infty \int_{u=0}^t e^{-t}\left(\frac{\sin u}{u}\right)dudt = \frac{\pi}{4}$. [5+5]

OR

3.a)   Find the inverse Laplace transform of $\ln\left(\dfrac{s+1}{s-1}\right)$.

  b)   Find the inverse Laplace transform of $\dfrac{1}{s^3(s^2+a^2)}$ using the convolution theorem.

[5+5]

4.a)   Prove that $\displaystyle\int_0^a \dfrac{dx}{(a^n-x^n)^{1/n}} = \dfrac{\pi}{n}\cos ec\left(\dfrac{\pi}{n}\right)$.

  b)   Evaluate $\displaystyle\int_0^\pi x\sin^7 x\cos^4 x\,dx$ using Beta and Gamma functions.   [5+5]

OR

5.   Prove that $\displaystyle\int_0^{\pi/2} \dfrac{d\theta}{\sqrt{1-\dfrac{1}{2}\sin^2\theta}} = \dfrac{\left[\Gamma(1/4)\right]^2}{4\sqrt{\pi}}$.   [10]

6.a)   Evaluate $\displaystyle\int_0^1\int_0^{\sqrt{1-x^2}}\int_0^{\sqrt{1-x^2-y^2}} \dfrac{dz\,dy\,dx}{\sqrt{1-x^2-y^2-z^2}}$, by changing to spherical polar coordinates.

  b)   Evaluate the integral $\displaystyle\int_{-1}^1\int_0^z\int_{x-z}^{x+z} (x+y+z)dz\,dy\,dx$.   [5+5]

OR

7.   Find by triple integration, the volume of the paraboloid of revolution $x^2+y^2=4z$ cut off by the plane $z=4$.   [10]

8.   Prove the following vector identities.

  a) $\nabla(\phi_1\phi_2) = \phi_1\nabla(\phi_2) + \phi_2\nabla(\phi_1)$ b) $\nabla\left(\dfrac{\phi_1}{\phi_2}\right) = \dfrac{\phi_2\nabla\phi_1 - \phi_1\nabla\phi_2}{\phi_2^2}, \phi_2 \neq 0$.   [5+5]

OR

9.   If $R = xi + yj + zk$, show that: a) $\nabla r = \dfrac{R}{r}$ b) $\nabla\left(\dfrac{1}{r}\right) = -\dfrac{R}{r^3}$ c) $\nabla r^n = nr^{n-2}R$

  d) $\nabla(a.R) = a$, where a is a constant vector and $r = |R|$.   [10]

10.   State the Stokes' theorem. Verify it for the vector field $F = (2x-y)i - yz^2j - y^2zk$ over the upper half surface of $x^2+y^2+z^2 = 1$, bounded by its projection on the xy - plane.   [10]

OR

11.   State the Green's theorem in a plane. Verify it for $\displaystyle\oint_c e^{-x}(\sin y\,dx + \cos y\,dy)$ where c is the rectangle with the vertices $(0,0)$, $(\pi,0)$, $\left(\pi, \dfrac{\pi}{2}\right)$ and $\left(0, \dfrac{\pi}{2}\right)$.   [10]

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