Code No: 123AH **R15**

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD
B.Tech II Year I Semester Examinations, April/May - 2018
MATHEMATICS – III
(Common to EEE, ECE, EIE, ETM)

Time: 3 Hours **Max. Marks: 75**

Note: This question paper contains two parts A and B.
Part A is compulsory which carries 25 marks. Answer all questions in Part A.
Part B consists of 5 Units. Answer any one full question from each unit. Each question carries 10 marks and may have a, b, c as sub questions.

PART- A

(25 Marks)

1.a) Find the particular integral of $x^2\frac{d^2y}{dx^2} - 6x\frac{dy}{dx} + 10y = x^2$. [2]

b) Find the singular points of the differential equation
$x^3(x-1)\frac{d^2y}{dx^2} + 2(x-1)\frac{dy}{dx} + y = 0$. [3]

c) Prove that $P_n'(1) = \frac{1}{2}n(n+1)$. [2]

d) Express $J_3(x)$ in terms of J_0 and J_1. [3]

e) Find the analytic function whose real part is xy. [2]

f) Evaluate $\int_0^{1+i}(x^2 - iy)dz$ along the path $y = x^2$. [3]

g) Find the zeros of the function $\sin\left(\frac{1}{z}\right)$ [2]

h) Show that the function e^z has an essential singularity at $z = \infty$. [3]

i) Find the fixed points of the transformation $w = \frac{z-1+i}{z+2}$. [2]

j) Find the points at which $w = \cosh z$ is not conformal. [3]

PART-B

(50 Marks)

2. Solve the equation in series $x^2y'' + xy' + (x^2 - 4)y = 0$. [10]

OR

3. Solve $(x+a)^2\frac{d^2y}{dx^2} - 4(x+a)\frac{dy}{dx} + 6y = x$ [10]

4. State and prove the generating function for $P_n(x)$. [10]

OR

5.a) Prove that $(n+1)P_{n+1}(x) = (2n+1)xP_n(x) - nP_{n-1}(x)$.

b) Prove that $\frac{d}{dx}\left(J_0(x)\right) = -J_1(x)$. [5+5]

6. State and prove Cauchy's intergral formula. [10]

OR

7. Verify Cauchy's theorem for the integral of z^3 taken over the boundary of the rectangle with vertices -1, 1, 1+ *i*, -1+ *i*. [10]

8. State and prove Laurent series for the function $f(z)$. [10]

OR

9. Evaluate $\int_0^{2\pi} \frac{sin^2\theta}{a+bcos\theta} d\theta$; (a>b>0). [10]

10. Find the bilinear transform which maps the points $z = 0, -i, -1$ into the points $w = i, 1, 0$. Find the image of the line $y = mx$ under this transformation. [10]

OR

11. Determine the region of the $w-$plane into which the region bounded by $\frac{1}{2} \leq x \leq 1$ and $\frac{1}{2} \leq y \leq 1$ is mapped under the transformation $w = z^2$. [10]