**Code No: 124DD** **R15**

**JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD**
**B.Tech II Year II Semester Examinations, May - 2017**
**MATHEMATICS – II**
**(Common to ME, MCT, MIE, MSNT)**

**Time: 3 Hours** **Max. Marks: 75**

**Note:** This question paper contains two parts A and B.
Part A is compulsory which carries 25 marks. Answer all questions in Part A. Part B consists of 5 Units. Answer any one full question from each unit. Each question carries 10 marks and may have a, b, c as sub questions.

## PART- A

**(25 Marks)**

1.a) Show that $\nabla r^n = nr^{n-2}\vec{r}$, where $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and $r = |\vec{r}|$. [2]

b) Find the values of $a, b, c$ so that
$\vec{F} = (x + 2y + az)\hat{i} + (bx - 3y - z)\hat{j} + (4x + cy + 2z)\hat{k}$ is irrotational. [3]

c) What are Dirichlet's conditions for the existence of Fourier series? [2]

d) Find the Fourier transform of $f(x) = e^{-|x|}$. [3]

e) Construct the forward difference table from the following data: [2]

| x: | 0 | 10 | 20 | 30 |
|---|---|---|---|---|
| y: | 0 | 0.174 | 0.347 | 0.518 |

f) Obtain the normal equations for fitting a straight line $y = ax + b$ to the data $(x_i, y_i)$, $i = 1, 2, \ldots\ldots\ldots n$. [3]

g) If the first two approximations $x_3$ and $x_4$ for the root of $x^3 - 3x - 4 = 0$ are 2.125 and -3 respectively, find $x_5$ by the method of false position. [2]

h) Find the LU decomposition for the matrix $A = \begin{pmatrix} 2 & 3 \\ 1 & 2 \end{pmatrix}$. [3]

i) Approximate $\int_0^{\pi} \sin x\, dx$ using the 2-point Gauss-Legendre formula. [2]

j) Evaluate $\int_0^1 \frac{dx}{x}$ using Simpson's $\frac{1}{3}$ rule with $h = \frac{1}{4}$. [3]

## PART-B

**(50 Marks)**

2.a) Find the values of $a$ and $b$ so that the surfaces $ax^2 - byz = (a+2)x$ and $4x^2y + z^3 = 4$ intersect orthogonally at the point (1,-1,2).

b) Prove that $\nabla \times (\nabla \times \vec{F}) = \nabla(\nabla \cdot \vec{F}) - \nabla^2 \vec{F}$. [5+5]

OR

3.a) Find the work done by the force $\vec{F}=\left(3x^2-6yz\right)\hat{i}+\left(2y+3xz\right)\hat{j}+\left(1-4xyz^2\right)\hat{k}$ in moving a particle from ( 0, 0, 0) to ( 1, 1, 1) along the curve $C: x=t, y=t^2, z=t^3$.

b) Use Green's theorem to evaluate $\oint_c \left(2xy-x^2\right)dx+\left(x^2+y^2\right)dy$, where $c$ is the boundary of the region enclosed by $y=x^2$ and $y^2=x$. [5+5]

4. Find the Fourier series expansion of the function $f(x)=\begin{cases}2+x, & -2\le x\le 0\\ 2-x, & 0<x\le 2\end{cases}$, $f(x+4)=f(x)$. Hence show that $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\ldots\ldots=\frac{\pi^2}{8}$. [10]

**OR**

5.a) Find the Fourier integral representation of $f(x)=\begin{cases}x, & |x|<1\\ 0, & |x|>1\end{cases}$.

b) Find the inverse Fourier sine transform of $\frac{e^{-as}}{s}$. [5+5]

6.a) If $y_{20}=24,\ y_{24}=32,\ y_{28}=35,\ y_{32}=40$, find $y_{25}$ using Gauss forward difference formula.

b) Use Lagrange's interpolation formula to find a polynomial of least degree which suits the following data: [5+5]

| x: | 0 | 1 | 3 | 4 |
|---|---|---|---|---|
| y: | 5 | 6 | 50 | 105 |

**OR**

7.a) Fit a polynomial of second degree to the following data by the method of least squares:

| x: | 0 | 1 | 2 |
|---|---|---|---|
| y: | 1 | 6 | 17 |

b) Fit a curve of the form $y=ae^{bx}$ for the following data: [5+5]

| x: | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| y: | 1.65 | 2.70 | 4.50 | 7.35 |

8.a) Find a root of the equation $e^x-x=2$ using bisection method correct to 3 decimal Places.

b) Compute $\sqrt{10}$ using Newton-Raphson method correct to 3 decimal places. [5+5]

**OR**

9. Solve the system of equations $10x+y+z=12, 2x+10y+z=13, 2x+2y+10z=14$ by Jacobi's iteration method and Gauss-Seidel iteration method. [10]

10.a) Evaluate $\int_0^1 \frac{dx}{1+x^2}$ using Trapezoidal rule with $h=\frac{1}{6}$.

b) Apply shooting method to solve the boundary value problem

$y''-6y^2=0,\ y(0)=1,\ y(0.5)=0.44$. [5+5]

**OR**

11.a) Given that $\frac{dy}{dx}=2+\sqrt{xy},\ y(1)=1$. Find approximate value of $y$ at $x=2$ using Euler's modified method.

b) Find the largest eigen vector and the corresponding Eigen value of the matrix $A=\begin{pmatrix}1 & 3 & -1\\ 3 & 2 & 4\\ -1 & 4 & 10\end{pmatrix}$ by power method. [5+5]

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