Preamble

Mathematics is universally accepted as the queen of all sciences. This fact has been confirmed with the advances made in Science and Technology. Mathematics has become an imperative prerequisite for all the branches of science such as Physics, Statistics, and Computer Science etc. This proposed syllabus in Mathematics for Semester I and Semester II of the B.Sc. Program aims at catering to the needs of the students of all these branches.

The student learns two courses in each Semester wherein Course I deals with "Calculus and Analytic Geometry" and Course II deals with "Discrete Mathematics."

Course I: There are two kinds of sets one that is discreet and the other that is in continuum. Properties that can be studied in discreet sets are learnt in Discreet Mathematics which is our Course II and Properties on Sets which are in Continuum are studied in this Course. In this course we are not deleting any topic introduced in the earlier syllabus but few applications have been added. The way earlier syllabus was framed we found it difficult to complete to the extent we wanted mainly due to the placement of the topics, due to which lot of time was wasted in recapitulating the topic which had been dealt with in the First semester. Since all the topics were found to be important for any field it was necessary to keep these topics in the new syllabus. We have only placed them in such a fashion that there is no need to repeat the topic at a later stage. This gives us more time to do more applications which we have included.

Course II aims at development of logical reasoning and also studies various applications in science and technology. In order to have a continuous flow of topics, Set theory, Functions and Relations are dealt with in the first unit. Divisibility in integers followed by Congruence's are studied in the subsequent units. This makes it easier for a student to understand congruence's. As compared to the existing syllabus wherein counting principles were studied along with some units in the first and second term, the student will now learn counting principles as one full unit in unit I, Semester II. Elements of Graph theory are studied in the last unit of Semester II which adds a flavor of applications of Mathematics useful to Computer Science students in networking etc. The concepts of Sterling numbers, Recurrence equations and study of countable/ uncountable sets was removed as it was observed that these topics were better understood by the students of the third year B.Sc. class.

F. Y. B. Sc.

Course I (First Semester) **<u>Calculus and Analytic Geometry</u>**

Unit I Analytic Geometry (15 Lectures)

(a) Review of vectors in $\mathbf{R}^2$ and $\mathbf{R}^3$, Component form of vectors, basic notions such as addition and scalar multiplication of vectors, dot product of vectors, orthogonal vectors, length(norm) of a vector, Unit vector, distance between two vectors, Cross product of vectors in $\mathbf{R}^3$, Scalar triple product (box product), Vector projections.

(b) Lines and planes in space, equation of sphere, cylinders and quadric surfaces.

(c) Polar co-ordinates in $\mathbf{R}^2$, polar graphing with examples like $r = \sin\theta$, $r = \cos 2\theta$, $r = a(1 - \text{Cos}\,\theta)$.

(d) Relationship between Polar and Cartesian co-ordinates in $\mathbf{R}^2$, Cylindrical and spherical Co-ordinates in $\mathbf{R}^3$ and relationships of these co-ordinates with Cartesian co-ordinates and each other.

Unit II Properties of real number system and sequences (15 Lectures)

(a) Intervals in $\mathbf{R}$, neighbourhoods and deleted neighbourhoods of a real number. Absolute value function and its properties. Archimedean property, Hausdorff property, Density theorem (without proof), bounded subsets of $\mathbf{R}$, infimum and supremum of a set.

(b) Sequence of real numbers, $\varepsilon - n_0$ definition, limit of a sequence, algebra of convergent sequences, monotonic sequences.

Unit III Limits and Continuity of real valued functions of one and two variables. (15 Lectures)

(a) ε - δ definition of limit of a real valued function, ε - δ definition of one sided limit of a real valued function. Formal definition of infinite limits and limits as x approaches $\pm\infty$.

(i) Statement of rules for finding limits: sum rule, difference rule, product rule, constant multiple rule, quotient rule.

(ii) Sandwich theorem for limits(without proof)

(iii) Limit of composite function.(without proof)

(b) (i) Continuity of a real valued function at a point in terms of limits, and two sided limits.

(ii) Continuity of a real valued function at end points of domain. Types of discontinuity.

(iii) Algebra of continuous functions.

(iv) Constructing a function having finitely many prescribed points of discontinuity over an interval.

(v) Statements of properties of continuous function such as the following:

(i) Intermediate value property.

(ii) A continuous function on a closed and bounded interval is bounded and attains its bounds.

(c) Open disc in $\mathbf{R}^2$, boundary of open disc, closed disc in $\mathbf{R}^2$, and bounded regions, unbounded regions in $\mathbf{R}^2$.

ε - δ definition of limit of a real valued function of two variables (only brief statement).

(d) Statement of rules for finding limits in two variables, sum rule, difference rule, product rule, constant multiple rule, quotient rule and power rule.

Definition of continuity of function in two variables in terms of limits.

(e) Concept of path. Limit of a function along paths. Two path test for non-existence of a limit.

(f) Calculating $\lim_{(x,y)\to(0,0)} f(x,y)$ by changing to polar Coordinates (illustrating with examples)

References:

(1) Calculus and Analytic Geometry, G. B. Thomas and R. L. Finney, Ninth Edition, Addison – Wesley, 1998.

(2) Calculus by H. Anton

(3) Methods of real analysis by Richard R. Goldberg

Suggested topics for project that can be given to the students:

1) Graphs of standard function using any of the mathematical software such as winplot, maxima (freeware).

2) Properties of the function and the concept of limit and continuity.There applications such as Bisection method, Regula falsi method to find roots of a polynomial

Course I (second semester) Differential calculus

Unit 1.Differentiability of real valued functions of a single variable (15 Lectures)

(a) Derivative of a real valued function of one variable at a point. Geometrical interpretation as slope of the tangent. Derivative as rate of change of a quantity. Notion of derivative at a point as local linear approximation to the function.

(b) Differentiability of functions defined on an interval, Algebra of differentiable functions. Chain rule, derivative using parameterization, derivative of inverse function, implicit functions, derivative of one function with respect to another function.

(c) Relation between differentiability and continuity of a function. Examples of continuous function which is not differentiable, derivable function whose derivative is not continuous. Higher order derivatives. Functions which are differentiable twice but not thrice, etc. Leibnitz rule for nth order derivative of product of two functions.

(d) Mean value theorems: Rolle's, Lagrange's. L'Hospital rule (without proof)and Taylor's theorem (without proof)

Unit 2. Differentiability of a Scalar valued functions (15 Lectures)

(a) Differentiability of real valued functions of two: partial derivatives, derivatives, second order partial derivatives, continuity of partial derivatives, and definition of differentiability in terms of existence and continuity of partial derivatives. Increment theorem (without proof) Concept of linearization of a differentiable function at a point.

(b) Directional derivative, relationship between directional derivatives and partial derivatives in case of differentiable functions. Gradient of a scalar valued function at a point. Concept of tangent plane to a surface at a given point. Normal vector to a surface at a point.

Unit3. Applications of derivatives: (15 Lectures)

(a) Real valued functions of one variable: increasing and decreasing nature related to sign of derivative. Concavity of the graph and its relation to the second derivative. Local maximum, local minimum and points of inflection. Finding extreme values of function using second derivative test. Use of Taylor's theorem in deciding nature of critical points when second derivative test fails.

(c) Use of Taylor's theorem in finding approximate values. Taylor's polynomial as an approximation of given function. Concept of Taylor' series and examples of series for exponential, trigonometric and logarithmic functions.

(d) Extreme values of functions of two. Saddle points. Method of Lagrange multipliers to find extreme values.

Reference: Calculus and Analytic Geometry, G. B. Thomas and R. L. Finney, Ninth Edition, Addison – Wesley, 1998.

Tutorials and assignments should cover large number of computational as well as theoretical problems.

Suggested topics for project that can be given to the students:

1) Newton-Raphson method.
2) Velocity and acceleration of a particle moving in a curve.
3) Concept of radial and transverse components of velocity and acceleration.
4) Word problems involving measurements of area, volume, revenue functions.
5) Motion of a projectile.

Course II (First semester) Discrete Mathematics I

UNIT I: Relations and functions (15 Lectures)

(a) Review of set theory, Relations, Equivalence relations, partitions of sets, Equivalence classes.

(b) Definition of a function. Domain, co-domain and the range of a function with examples of special functions such as constant, identity, inclusion, projection, floor and ceiling functions. Injective, surjective and bijective functions. Composition of functions. Invertible functions and the inverse of a function. Direct and inverse image.

(c) Binary operations, simple examples.

(d) Finite and infinite sets, cardinality of sets.

UNIT II: Integers and divisibility (15 Lectures)

(a) Well Ordering Principle, Mathematical Induction (First and Second principle).Recursive definition of a sequence, Fibonacci sequence.

(b) Divisibility in Z: Definition and elementary properties. Division Algorithm.

(c) G.C.D.and L.C.M of two integers. Basic properties of G.C.D. including G.C.D.of two integers a and b can be expressed as ma+nb. Euclidean Algorithm, Euclid's Lemma.

(d)
- (i) Unique Factorization Theorem.
- (ii) The set of primes is infinite.
- (iii) The set of primes of the type 4n-1 and 4n+1 is infinite.

UNIT III: Congruences (15 Lectures)

(a) Congruences: Definition and elementary properties.
(b) Euler phi-function and examples.

(a) There is no injection from $\mathbb{N}_n$ to $\mathbb{N}_m$ if n>m, = {1,2,3,……..n}.
Pigeonhole principle and its applications.
Number of functions from a finite set X to a finite set Y.

(c) (i) Euler's theorem (without proof).
(ii) Fermat's little Theorem
(iii)Wilson's theorem.
(d) Introduction to $\mathbf{Z_n}$, addition and multiplication modulo n with their properties. Solutions of linear congruences.

References

Elementary Number theory, David M. Burton, UBS, New delhi.

Discrete Mathematics,Norman L. Biggs, Clarendon press, Oxford 1989

Course II (Second semester) Discrete Mathematics II

UNIT I: Counting Principles and Permutations (15 Lectures)

Number of injective functions from a finite set X to a finite set Y where $|X| \leq |Y|$.

Number of surjective functions from a finite set X to a finite set Y where $|X| \geq |Y|$.

(b) Inclusion and exclusion principle, derangements on n symbols, the number d_n of derangements of $\{1,2,3,\ldots\ldots\ldots n\}$. Binomial theorem, Pascal's triangle, Power set of a set with n elements has 2^n elements, Multinomial theorem.

(c) Permutations on n symbols. The set S_n and the number of permutations in S_n is n!

Compositions of two permutations, as a binary operation in S_n compositions of permutations is non commutative if $n \geq 3$.

Cycles and transpositions, representations of a permutation as a product of disjoint cycles, Listing permutations in S_3 , S_4 etc.

Sign of a permutation, sign of transposition is -1, multiplicative property of sign, odd and even permutations as product of disjoint cycles, number of even permutations in S_n is n!/2.

(d) Partition of a positive integer, its relation to decomposition of a permutation as product of disjoint cycles, conjugate of a permutation.

Unit II Complex numbers and Polynomial(15 Lectures)

(a) Review of a complex number, De-Moivre's Theorem. Roots of unity, Roots of a complex number.

(b) Polynomials **:** Polynomials in one variable with real coefficients. Division Algorithm (without proof). G.C.D of two polynomials(without proof).

(c) Root of a polynomial, multiplicity of a root, Remainder Theorem, Factor Theorem Relation between the roots and the coefficients of a polynomial. Examples.

(d) Complex roots are in conjugate pairs, factorization of a real polynomial as a product of linear and quadratic polynomials over IR. Fundamental theorem of algebra, Eisenstein's criterion(without proof).

Unit III Elements of Graph Theory and applications(15 Lectures)

(a) Definition of a graph, vertex, edge, degree of a vertex, complete graphs, regular graphs complement of a graph

(b) (i) Walks, trails, paths, circuit, cycle, connected graph.
(ii) components of a graph, Bridge, cut vertex.

(c) Trees, characterization of trees, spanning trees,

(d) Eulerian Graphs, Necessary and sufficient condition for a graph to be Eulerian, Chinese Postman problem.

(e) Hamiltonian graphs, Travelling Postman Problem

(f) Incidence matrix of a graph, finding number of walks and triangles using incidence matrix.

References

Discrete mathematics, Norman Biggs, Clarendon press, Oxford 1989

Discrete Mathematics and its applications, Kenneth Rosen, McGraw Hill International edition, Mathematics Series.

Preamble

Mathematics is universally accepted as the queen of all sciences. This fact has been confirmed with the advances made in Science and Technology. Mathematics has become an imperative prerequisite for all the branches of science such as Physics, Statistics, Computer Science etc. The syllabus in Mathematics for Semester III and Semester IV of the B.Sc. Program aims at catering to the needs of the students of all these branches and also learns core areas of Mathematics.

The student learns three courses in each Semester wherein Course I deals with "Analysis and Differential Equations" and Course II deals with "Linear Algebra" and in Course III the students learns "Computational Mathematics"

Course I "Analysis and Differential Equations" course code USMT31/41

Corse II "Linear Algebra" course code USMT32/42

Course III "Computational Mathematics" Course code USMT33/43

In this course the students are exposed to various areas of the real world where mathematics are applied. In this course they apply the topics learnt in Analysis and Linear Algebra to areas of Management studies, financial mathematics, computer Science and various other disciplines. In both the semester we have made great many changes in the syllabus and this is more so in course III. In unit 1 of this course we take off "Graphs" from where we left off in semester II course II. In Unit 2 and 3 we learn "Trees" and "Colouring of Graphs". Colouring of Graphs is newly introduced. In semester IV unit 1 and 2 deals with "Numerical methods" which is same as that is learnt in the present syllabus as unit 2 and 3 with substantial addition and minor deletions. Unit 3 deals with "Introduction to Financial Mathematics". Changes that have been made in these courses puts the students at a better advantage compared to the syllabus offered by other colleges as they learn practically everything the other students learn but much more without compromising on the quality.

Course I Sem III (Calculus and Analysis) Course code USMT31

Unit I Differential Equations of order 1 [15 lectures]

First Order Differential equations. Homogeneous differential equations, Exact differential equations, integrating factors, First order linear equations, Bernouli's equation, Riccati's equation.

Applications in Geometry, Physics, Life Sciences, such as finding family of curves orthogonal to the given family, inductance in a circuit, exponential growth of bacteria.

Unit II properties of real numbers [15 lectures]

Review of properties of real numbers . Bounded subsets of real numbers, supremum and infimum. Archemedian property of real numbers, Hausdorff Property, A,M.-G.M. inequality and Cauchy Scwartz Inequality.

Review of convergent sequences, Cauchy Sequences, completeness of $\mathbb{R}$ using lub property.

Continuity and Sequential continuity of functions. Uniform continuity of a function. Function continuous on a closed and bounded interval is uniformly continuous.

Unit III Series of real numbers and power series. [15 lectures]

Convergence of a series of real numbers. The Geometric series and the p-series. Cauchy condensation test, Leibnitz' test for alternating series.

Limit superior and limit inferior of a sequence of real numbers. Comparison test, Ratio test and Root test.

Power series with real coefficients. Radius of convergence and interval of convergence of a power series. Assuming convergence, a function can be represented by a unique power series. The series representation of exponential function, sine and cosine functions, Logarithm as an inverse of the exponential function. Definition of hyperbolic functions in terms of power series.

References:

(1) Introduction to Real Analysis *by* Robert Bartle and Donald Sherbat, *Springer verlag.*

(2) Methods of Real analysis *by* R. R. Goldberg, *Oxford IBH Publishing Company, New Delhi.*

(3) Basic elements of Real Analysis *by* M.H. Protter, *Springer Verlag.*

(4) Differential Equations with Applications and Historical notes, *by* G. F. Simmons, *McGraw Hill*

Course II Sem III (Linear Algebra) Course code USMT32

Unit I Matrices and system of equation(15 lectures)

Matrices over **R**. Properties of matrices

System of homogeneous and non- homogeneous linear equations.

Solutions of m homogeneous equations in n unknowns by elimination and their geometric interpretation. Existence of non trivial solution of such a system for m<n.

Matrix representation of homogeneous and non- homogeneous linear equations. Elementary row operations on matrices, Row Echelon form of a matrix and solving equations using Gauss elimination method.

Unit II Vector Spaces

Definition of vector space and examples. Subspaces, sum and intersection of subspaces, direct sum of vector space, linear combination of vectors, linear span of a subset of a vector space.

Linear dependence and independence, Basis and Dimensions of a Vector Space.

Row and Column Spaces of a matrix, Row rank and Column rank and their equivalence. Computing rank of a matrix by row reduction.

Unit III Inner Product spaces

Dot product in Euclidean space , norm of a vector. General inner product spaces. Cauchy Schwarz inequality.

Orthogonality,Pythagoras Theorem and geometric applications Orthogonal projection onto a line,
Orthogonal and orthonormal sets. Orthonormal. basis, Gram-Schmidt Orthogonalization, Orthogonal complement.

References:

(1) **Linear Algebra. A Geometric Approach**,S.Kumaresan,*Prentice-Hall of India Private Limited, New Delhi.*

(2) **Introduction to Linear Algebra,** Serge Lang, *Springer Verlag.*

(3) **Linear Algebra**, *Schaum Series.*

Course III (Computational Mathematics) Course code USMT33

Unit I Graphs and (15 Lectures)

Review: Simple graphs, Complete graphs, Regular graphs, Complement of a graph Walks, trails, paths, circuit, cycle, connected graph. Components of a graph, Bridge, cut vertex. Eulerian Graphs,Trees.

Special Graphs such as Wheel, Multipartite graphs, Directed graphs.

Representation of graphs and Graph Isomorphism's:
i) Adjacency matrix; Incident Matrix; Adjacency list
ii) Isomorphisms of simple graphs.

Number of walks in graph and its relationship with its adjacency matrix, Triangles in a graph.
Connectivity:
i) Strongly connected and Weakly connected graphs
ii) Shortest path problem: Dijkstra's algorithm

Simple properties of graphs
Such as Hand shaking lemma.
G or its complement is always connected.
A self complementary graph should have 4n or 4n+1 number of vertices.
An undirected graph has even number of odd vertices,
A simple graph with at least 2 vertices has atleast two vertices with the same degree.
In a simple graph on p vertices and p-1 number of edges has either a pendent or an isolated vertex.

Unit II Trees (15 lectures)

Trees, Forests, subtrees,
Trees as models
Rooted trees, m-ary trees.
Tree traversal (preorder, inorder, post order)
Application of Trees:
Binary Search Trees, Locating and adding items to a Binary Search Tree. Decision Trees (simple examples). Game Trees, Minimax strategy and the value of a vertex in a Game Tree. Examples of games such as Nim and Tic-tac-toe.

Spanning trees: Breadth first search trees
Minimum Spanning trees: Prim's Algorithm, Kruskal's algorithm
Properties such as
A simple graph is connected iff it has a spanning tree
A connected graph T is a tree iff every edge is a bridge.
A tree on n vertices has n- 1 number of edges,
T be a tree then T U {e} (e any edge not in T) contains a circuit.
Number of pendant in a binary tree, extending it to m-ary tree,

Unit III Eulerian, Hamilton, Planar graphs and Colouring in a graph (15 Lectures)

Euler paths and circuits, Hamilton Paths and circuits.
A graph in which every vertex has degree at least 2 has a circuit.
A graph is Eulerian iff degree of every vertex is even.
Konisberg 7 bridge problem.
Introduction to edge colouring and vertex colouring in a simple graph. Vertex and edge chromatic number of a graph, Computation of the vertex and edge chromatic number. Brook's theorem (without proof), Vizing theorem (without proof)
Planar graph and Euler formula, 5 colour theorem (without proof) four colour theorem(without proof). In any simple connected planar graphs having f regions, n vertices and e edges the following inequalities hold:
$e \geq \frac{3}{2}f$ and $e \leq 3n - 6$. $K_{3,3}$ is not a planar graph.

Chromatic polynomial of some simple graph such as trees, cycles, complete graph wheel etc.

References:

1) Graph theory with application by J. A. Bondy and U. S. R. Murty

(Freely downloadable)

2) Graph Theory by Reinhard Diestel Electronic edition *Springer Verlag. (Freely downloadable)*

3) Graph theory with application, Narsingh DeoPrentice Hall publication

Practical:

1) A) Finding the adjacency and incident matrix of a given graph and drawing the graph for a given adjacency matrix / incident matrix.
   B) Walks and triangles in a graph.
   C) Finding the Adjacency list of a given Di-graph and drawing the di-graph given its adjacency list
2) A) Strongly and weakly connected graph
   B) Dijkstra's algorithm
3) A) Tree traversal
   B) Game tree
4) A) BFS tree
   B) Prim algorithm
   C) Kruskal's algorithm
5) A) Eulerian graphs
   B) Hamiltonian graph
   C) Planar graph
6) A) Chromatic number
   B) Chromatic polynomial of simple graphs

Course I Sem IV (Calculus and Analysis) Course code USMT41

Unit I Metric Properties of real numbers [15 lectures]

Open sets and closed sets in IR. Limit points of a set. Characterization of closed set as set containing all its limit points.

Nested Interval Property, Bolzano Wierestrass theorem, Compact subsets of real numbers. Charecterisation of compact sets in IR as closed and bounded sets . Image of a continuous function defined on closed and bounded sets is closed and bounded. Proof of intermediate value property of continuous functions. A continuous function defined on a closed and bounded interval attains its maximum and minimum values.

Unit II Riemann Integration [15 lectures]

Partition of an interval, definitions of upper and lower Riemann sums of a function with respect to a partition, Upper and lower Riemann sums over an integral [a,b], Riemann Integrability. Criterion for Riemann integrability in terms of $U(P, f) - L(P, f) < \varepsilon$. Simple Properties of Riemann integrable functions

Integrability of monotonic functions, continuous functions, pieces wise continuous functions, a function with infinitely many discontinuities for which the set of discontinuities can be covered by open intervals whose total length can be made as small as possible.

Fundamental Theorem of Calculus.

Unit III Applications of Integration [15 lectures]

Applications of Definite integral. Its interpretation as area under a curve. Volume of region obtained by rotating a curve in the plane about X axis or Y Axis.

Double and triple Integrals. Calculation of area and Volume using integrals.

Statement of Fubini's theorem. Simple problems involving change in order of integration, use of polar coordinates, spherical and cylindrical coordinates.

Calculation of center of mass, moment of inertia for plane laminae and solid bodies. (Simple examples only)

References:

(1) Introduction to Real Analysis *by* Robert Bartle and Donald Sherbat, *Springer verlag.*

(2) Methods of Real analysis *by* R. R. Goldberg, *Oxford IBH Publishing Company, New Delhi.*

(3) Basic elements of Real Analysis *by* M.H. Protter, *Springer Verlag.*

(4) Calculus vol. 2 *by* T. Apostol, *John Wiley*.

(5) Calculus and Analytic geometry *by* G. B. Thomas and Finney, *Addison-Wesley*.

Course II Sem IV (Linear Algebra) Course code USMT42

Unit I Linear Transformations [15 lectures]

Definition of Linear Transformation and examples. Determining a linear transformation by its values on a basis. Kernel and Image of a Linear transformation, Rank-Nullity theorem, composite of a linear transformation, inverse of a linear transformation, Linear Isomorphism, linear functionals, dual of a vector space.

Representation of a linear transformation by a matrix, matrix of sum, scalar multiple, inverse and composite of linear transformation.

Equivalence of rank of a matrix and a linear transformation associated with it. The solutions of non homogeneous system of linear equations represented by AX=B.

Unit II Eigen values and Eigen vectors [15 lectures]

Eigen values and Eigen vectors of a linear transformation $T:V \rightarrow V$ for a finite dimensional real vector space V.

Eigen values and Eigen vectors of n x n matrices and eigenspaces.

Linear independence of eigen vectors corresponding to distinct eigen values of a matrix(linear transformation).

characteristic polynomial of an n x n real matrix and properties, characteristic roots.
characteristic polynomial of a linear transformation $T:V \rightarrow V$ for a finite dimensional real vector space V.

Similar matrices. Cayley Hamilton Theorem(statement only) and examples.

Unit III Determinants [15 lectures]
Definition of a determinant using multilinear alternating functions..Properties of determinants.

Existence and uniqueness of determinant via permutations. Computation of 2 x 2, 3 x 3 matrices ,diagonal matrices. $\det(A^t) = \det(A)$ and $\det(AB)= \det(A)\det(B)$.

Laplace expansion of a determinant. Vandermonde determinant. determinant.of upper triangular and lower triangular matrices.

Linear dependence and independence of vectors using determinants. Exixtence and uniqueness of the solutions of the system AX=B with det(A) ≠ 0. Cofactors, minors, adjoint of a matrix and related results.

Cramer’s rule, Determinant as area and volume.

References:

(1) **Linear Algebra. A Geometric Approach**,S.Kumaresan,Prentice-*Hall of India Private Limited, New Delhi.*

(2) **Introduction to Linear Algebra,** Serge Lang, *Springer Verlag.*

(3) **Linear Algebra**, *Schaum Series.*

Semester IV (Computational Mathematics) Course code USMT43

Unit I Numerical methods I (15 lectures)

Errors in Numerical calculations:
i) Inherent errors: Round off errors, Truncating errors.
ii) Absolute errors; relative errors and percentage errors.
iii) Control of Numerical Errors.

Numerical methods to solve an equation:
i) Secant method
ii) Newton Raphson method
iii) Muller's method

Solving system of equations:
i) Gauss Seidel method
ii) LU factorization: Do-Little method
iii) Eigen Value problems

Interpolation:
i) Newton Gregory forward interpolation
ii) Lagrange's interpolation

Unit II Numerical methods II (15 lectures)

Numerical differentiation:
i) High-Accuracy Differentiation formulas
ii) Derivatives of unequally spaced Data.

Numerical Integration:
i) Simpson $\frac{1}{3}$rd rule and $\frac{3}{8}$rule
ii) Boole's rule
iii) Weddles rule

Numerical Solution to Ordinary Differential Equations:
One Step methods:

i) Taylors method
ii) Euler's method
iii) Runga Kutta's fourth order method

Multi step method:

Adam – Bashforth – Moulton method.
Milne – Simpson Method.

Unit III Introduction to Financial Mathematics(15lectures)

Simple market model
(i) Basic Notions & Assumptions
(ii) No Arbitrage Principle
(iii) One step Binomial Model
(iv) Risk & Return
(v)Idea of Forward Contracts, Call & Put options
(vi) Managing Risk with options

Risk- Free Assets
(i) Time value of Money
(ii) Money Market

Risky Assets
(i) Dynamics of stock prices
(ii) Binomial Tree Model & Risk Neutral Probability

Applications of No-Arbitrage Principle
(i) To the Binomial Tree Model
(ii) To Pricing Forward Contracts
(iii) To Put-Call Parity in Options

Practical:

1	**a)**	**Solving an equation**
	b)	**Solving a system of equations**
2	**a)**	**Gauss-Seidel method**
	b)	**DO-Little LU decomposition method**
	c)	**Eigen value problems**
3	**a)**	**Numerical integration**

	b)	**Ranga-Kutta's 4^{th} order method**
4		**Multi step method**
5	**a)**	**Risk free asset problems**
	b)	**Risky asset problems**
6		**No Arbitrage problems**

References:

<u>Numerical Methods:</u>

1) Numerical methods by E. Balaguruswamy*, Tata McGraw Hill*

2) Introductory methods of Numerical Analysis. By S. S. Sastry.

3) Numerical Methods for engineers. By Steven C. Chapra and Raymond Canale; Fifth Edition; *Tata McGraw hill education private ltd*.

<u>Financial Mathematics:</u>

1) Mathematics for Finance An Introduction to Financial Engineering

Marek Capinski and Tomasz Zastawniak *Springer Undergraduate edition - 2003(Freely downloadable)*

2) Hull and Basu- Options, Futures and other derivatives

3) Paul Wilmott – Quantitative Finance

PREAMBLE

It gives us great pleasure in introducing the new syllabus for TYBSC.

In the first year, the students were exposed to basics of multivariable calculus and integers. They also dealt with various combinatorial problems.

In the second year, they learnt about properties of real numbers with an introduction to metric spaces, sequences and series of real numbers, ordinary differential equations, continuity, Riemann integration, double and triple integration along with its applications. In course II of SYBSC students learnt about linear algebra. In this course they dealt with finite dimensional vector spaces over the real numbers. They also learnt about linear transformations and eigen values and eigen vectors in detail. In course III they learnt Graph theory, Numerical methods and financial mathematics was introduced.

At TYBSC we will deal with four courses in mathematics.

Course I will deal with Multivariable calculus in greater detail along with some part of analysis.

Course II will continue with linear algebra, Group theory and rings in detail.

In course III they will learn topology of metric spaces.

In course IV students will learn number theory.

Apart from the mathematics course the students will also deal with the applied component "Computer programming" In this paper we are planning to deal with basics of C programming, Java, SQL, P/SQL and also introduce mathematical software namely 'Maxima

Revised Syllabus in Mathematics

Credit Based Semester and Grading System

Third Year B. Sc. 2015-16

Semester V

Course: Real Analysis and Multivariate Calculus
Course Code: USMT501

Unit I Differential calculus

(a) Review of functions from R^n to R (scalar _fields), Iterated limits.

(b) Limits and continuity of functions from R^n to R^m (Vector fields)

(c) Basic results on limits and continuity of sum, difference, scalar multiples of vector fields.

(d) Continuity and components of vector fields.

(e) Derivative of a scalar field with respect to a vector.

(f) Direction derivatives and partial derivatives of scalar fields.

(g) Mean value theorem for derivatives of scalar fields.

Reference for Unit I:

(1) Calculus, Vol. 2, T. Apostol, John Wiley.

(2) Calculus. J. Stewart. Brooke/Cole Publishing Co.

Unit II Differentiability

(a) Differentiability of a scalar field at a point (in terms of linear transformation). Total derivative. Uniqueness of total derivative of a differentiable function at a point. (Simple examples of finding total derivative of functions such as f(x; y) = x2+y2, f(x; y; z) = x+y+z, may be taken).

Differentiability at a point implies continuity, and existence of direction derivative at the point. The existence of continuous partial derivatives in a neighborhood of a point implies differentiability at the point.

(b) Gradient of a scalar field. Geometric properties of gradient, level sets and tangent planes.

(c) Chain rule for scalar fields.

(d) Higher order partial derivatives, mixed partial derivatives.

Sufficient condition for equality of mixed partial derivative.

Second order Taylor formula for scalar fields.

(e) Differentiability of vector fields. Representation of derivative using Jacobian matrix.

(i) Definition of differentiability of a vector field at a point.
Differentiability of a vector field at a point implies continuity.

(ii) The chain rule for derivative of vector fields (statement only). Computations using Jacobian matrices.

Reference for Unit II:

(1) Calculus, Vol. 2, T. Apostol, John Wiley.

(2) Calculus. J. Stewart. Brooke/Cole Publishing Co.

Unit III Differential Equations of second and higher orders.

(a) Review of Differential equations of order 1

(b) Second order Linear Differential Equations:

(i) The general second order linear differential equation. The linear differential equations with constant coefficients. Existence and Uniqueness Theorem for the solutions of a second order initial value problem (statement only).
The linear differential equations with constant coefficients and its solutions using auxilliary equation.

(ii) Homogeneous and non-homogeneous second order linear differential equations:
The space of solutions of the homogeneous equations as a vector space.
Wronskian and linear independence of the solutions.

The general solution of homogeneous differential equation. The use of known solutions to find the general solution of a homogeneous equations.

The general solution of a non-homogeneous second order equation, Complementary functions and particular integrals.

(iii) The homogeneous equation with constant coefficients, auxiliary equation, the general solution corresponding to real and distinct roots, real and equal roots and complex roots of the auxiliary equation.
(iv) Non-homogeneous equations: The method of undetermined coefficients. The method of variation of parameters.

(v) Euler's equation and its solution by converting it to a linear differential equation with constant coefficients,

(c) Extension of these ideas to linear differential equations of order n. Operator method.

Reference for Unit III:

Differential Equations with Applications and Historical Notes, G.F. Simmons, McGraw Hill.

Course: Algebra
Course Code: USMT502

Unit I. Quotient Space and Orthogonal Transformation (15 Lectures)

Review of vector spaces over R:

(a) Quotient spaces:

(i) For a real vector space V and a subspace W, the cosets $v + W$ and the quotient space V/W. First Isomorphism theorem of real vector spaces (Fundamental theorem of homomorphism of vector spaces.)

(ii) Dimension and basis of the quotient space V/W, when V is finite dimensional.

(b) (i) Orthogonal transformations and isometries of a real finite dimensional inner product space. Translations and reflections with respect to a hyperplane. Orthogonal matrices over R.

(ii) Equivalence of orthogonal transformations and isometries fixing origin on a finite dimensional inner product space. Characterization of isometries as composites of orthogonal transformations and isometries.

(iii) Orthogonal transformation of R^2. Any orthogonal transformation in R^2 is a reflection or a rotation.

(c) Characteristic polynomial of an $n \times n$ real matrix and a linear transformation of a finite dimensional real vector space to itself. Cayley Hamilton Theorem (Proof assuming the result A adj$(A) = I_n$ for an $n \times n$ matrix over the polynomial ring $R[t]$.)

Reference for Unit I:

(1) S. Kumaresan, Linear Algebra: A Geometric Approach.

(2) M. Artin. Algebra. Prentice Hall.

(3) T. Banchoff and J. Wermer, Linear Algebra through Geometry, Springer.

(4) L. Smith, Linear Algebra, Springer.

Unit II. Diagonalization and Orthogonal diagonalization (15 Lectures)

(a) Diagonalizability.

(i) Diagonalizability of an $n \times n$ real matrix and a linear transformation of a finite dimensional real vector space to itself.

Definition: Geometric multiplicity and Algebraic multiplicity of eigenvalues of an $n \times n$ real matrix and of a linear transformation.

(ii) An $n \times n$ matrix A is diagonalisable if and only if R^n has a basis of eigenvectors of A if and only if the sum of dimension of eigenspaces of A is n if and only if the algebraic and geometric multiplicities of eigenvalues of A coincide.

(b) Orthogonal diagonalization

(i) Orthogonal diagonalization of $n \times n$ real symmetric matrices.

(ii) Application to real quadratic forms. Positive definite, semidefinite matrices. Classification in terms of principal minors. Classification of conics in R^2 and quadric surfaces in R3.

Reference for Unit II:

(1) S. Kumaresan, Linear Algebra: A Geometric Approach.

(2) M. Artin. Algebra. Prentice Hall.

(3) T. Banchoff and J. Wermer, Linear Algebra through Geometry, Springer.

(4) L. Smith, Linear Algebra, Springer.

Unit III. Groups and subgroups (15 Lectures)

(a) Definition and properties of a group. Abelian group. Order of a group, finite and infinite groups. Examples of groups including

(i) Z, Q, R, C under addition.

(ii) $Q^*(= Q\backslash\{0\})$, $R^*(= R\backslash\{0\})$, $C^*(= C\backslash\{0\})$, Q^+(= positive rational numbers) under multiplication.

(iii) Zn, the set of residue classes modulo n under addition.

(iv) $U(n)$, the group of prime residue classes modulo n under multiplication.

(v) The symmetric group S_n.

(vi) The group of symmetries of a plane figure. The Dihedral group D_n as the group of symmetrices of a regular polygon of n sides (for n = 3,4).

(vii) Klein 4- group.

(viii) Matrix groups $M_{m\times n}(R)$ under addition of matrices, $GL_n(R)$, the set of invertible real matrices, under multiplication of matrices.

(b) Subgroups and Cyclic groups.

(i) S^1 as subgroup of C, μ_n the subgroup of n-th roots of unity.

(ii) Cyclic groups (examples of Z, Zn, and μ_n) and cyclic subgroups.

(iii) The Center $Z(G)$ of a group G as a subgroup of G.

(iv) Cosets, Lagrange's theorem.

(c) Group homomorphisms and isomorphisms. Examples and properties. Automorphisms of a group, inner automorphisms.

Reference for Unit III:

(1) I.N. Herstein, Algebra.

(2) P.B. Bhattacharya, S.K. Jain, S. Nagpaul. Abstract Algebra.

Recommended Books

1. S.Kumaresan. Linear Algebra: A Geometric Approach, Prentice Hall of India Pvt Ltd, New Delhi.

2. I.N. Herstein. Topics in Algebra, Wiley Eastern Limited, Second edition.

3. N.S. Gopalakrishnan, University Algebra, Wiley Eastern Limited.

4. M. Artin, Algebra, Prentice Hall of India, New Delhi.

5. T. Banchoff and J. Wermer, Linear Algebra through Geometry, Springer.

6. L. Smith, Linear Algebra, Springer.

7. Tom M. Apostol, Calculus Volume 2, Second edition, John Wiley, New York, 1969.

8. P.B. Bhattacharya, S.K. Jain, and S.R. Nagpaul, Abstract Algebra, Second edition, Foundation Books, New Delhi, 1995.

9. J.B. Fraleigh, A first course in Abstract Algebra, Third edition, Narosa, New Delhi.

10. J. Gallian. Contemporary Abstract Algebra. Narosa, New Delhi.

Additional Reference Books

1. S. Lang, Introduction to Linear Algebra, Second edition, Springer Verlag, New York.

2. K. Hoffman and S. Kunze, Linear Algebra, Prentice Hall of India, New Delhi.

3. S. Adhikari. An Introduction to Commutative Algebra and Number theory. Narosa Publishing House.

4. T.W. Hungerford. Algebra. Springer.

5. D. Dummit, R. Foote. Abstract Algebra. John Wiley & Sons, Inc.

6. I.S. Luthar, I.B.S. Passi. Algebra, Vol. I and II.

Course: Topology of Metric Spaces

Course Code: USMT503

Unit I. Metric spaces (15 Lectures)

(a) (i) Metrics spaces: Definition, Examples, including R with usual distance, discrete metric space.

(ii) Normed linear spaces: Definition, the distance (metric) induced by the norm, translation invariance of the metric induced by the norm. Examples including

(1) R^n with sum norm $|| \ ||_1$, the Euclidean norm $|| \ ||_2$, and the sup norm $|| \ ||_\infty$.

(2) $C[a,b]$, the space of continuous real valued functions on $[a,b]$ with norms $||\,||_1$, $||\,||_2$, $|| \ ||_\infty$, where $||f||_1 = \int_a^b |f(t)|dt$, $||f||_2 = \left(\int_a^b |f(t)|^2 dt\right)^{\frac{1}{2}}$, $||f||_\infty = \sup\{|f(t)|, t \in [a,b]\}$.

(iii) Subspaces, product of two metric spaces.

(b) (i) Open ball and open set in a metric space (normed linear space) and subspace Hausdorff property. Interior of a set.

(ii) Structure of an open set in R, namely any open set is a union of a family of pairwise disjoint intervals.

(iii) Equivalent metrics, equivalent norms.

(c) (i) Closed set in a metric space (as complement of an open set), limit point of a set (A point which has a non-empty intersection with each deleted neighbourhood of the point), isolated point. A closed set contains all its limit points.

(ii) Closed balls, closure of a set, boundary of a set in a metric space.

(iii) Distance of a point from a set, distance between two sets, diameter of a set in a metric space.

Reference for Unit I:

1. S. Kumaresan, Topology of Metric spaces.
2. W. Rudin, Principles of Mathematical Analysis.

Unit II. Sequences (15 Lectures)

(a) (i) Sequences in a metric space.

(ii) The characterization of limit points and closure points in terms of sequences.

(iii) Dense subsets in a metric space.

(iv) Cauchy sequences and complete metric spaces. R^n with Euclidean metric is a complete metric space.

(b) Cantor's Intersection Theorem.

Reference for Unit II:

1. S. Kumaresan, Topology of Metric spaces.
2. W. Rudin, Principles of Mathematical Analysis.

Unit III. Continuity (15 Lectures)

$\varepsilon - \delta$ definition of continuity at a point of a function from one metric space to another.

(a) Characterization of continuity at a point in terms of sequences, open sets.

(b) Continuity of a function on a metric space. Characterization in terms of inverse image of open sets and closed sets.

(c) Algebra of continuous real valued functions.

(d) Uniform continuity in a metric space, definition and examples (emphasis on R).

(e) Urysohn's Lemma

Reference for Unit III:

S. Kumaresan, Topology of Metric spaces.

Course: Number Theory and its applications
Course Code: USMT5B4

Unit I. Prime numbers and congruences (15 Lectures)

(a) (i) Review of divisibility.

(ii) Primes: Definition, The fundamental theorem of Arithmetic Distribution of primes (There are arbitrarily large gaps between consecutive primes).

(b) Congruences

(i) Definition and elementary properties, complete residue system modulo *m*. Reduced residue system modulo *m*, Euler's function φ.

(ii) Euler's generalization of Fermat's little Theorem, Fermat's little Theorem, Wilson's Theorem. The Chinese remainder Theorem.

(iii) With Congruences of degree 2 with prime modulii.

Unit II. Diophantine equations and their solutions (15 Lectures)

Diophantine equations and their solutions

(a) The linear equations $ax + by = c$.

(b) Representation of prime as a sum of two squares.

(c) The equation $x^2 + y^2 = z^2$, Pythagorean triples, primitive solutions.

(d) The equations $x^4 + y^4 = z^2$ and $x^4 + y^4 = z^4$ have no solutions (x,y,z) with xyz 6= 0.

(e) Every positive integer n can be expressed as sum of squares of four integers, Universal quadratic forms $x^2 + y^2 + z^2 + t^2$.

Unit III. Quadratic Reciprocity (15 Lectures)

(a) Quadratic residues and Legendre Symbol. The Gaussian quadratic reciprocity law.

(b) The Jacobi Symbol and law of reciprocity for Jacobi Symbol.

(c) Special numbers; Fermat numbers; Mersene numbers; Perfect numbers, Amicable numbers.

Reference Books

1. I. Niven, H. Zuckerman and H. Montogomery. *Elementary number theory.* John Wiley & Sons. Inc.

2. David M. Burton. *An Introduction to the Theory of Numbers.* Tata McGraw Hill Edition

3. G. H. Hardy, and E.M. Wright, *An Introduction to the Theory of Numbers*. Low priced edition. The English Language Book Society and Oxford University Press, 1981.

4. Neville Robins. *Beginning Number Theory*, Narosa Publications.

5. S.D. Adhikari. *An introduction to Commutative Algebra and Number Theory*. Narosa Publishing House.
6. S. B. Malik. *Basic Number theory*. Vikas Publishing house.

7. N. Koblitz. *A course in Number theory and Crytopgraphy*. Springer.

8. M. Artin.*Algebra*. Prentice Hall.

9. K. Ireland, M. Rosen. *A classical introduction to Modern Number Theory*. Second edition, Springer Verlag.

Applied Component

SEMESTER V	
Course code	**Title**
USMT5A	**Computer Programming –I**

Unit I

Introduction to C Programming

(a) **Structure of C program**: Header and body, Concept of header files, Use of comments, Compilation of a program.

(b) **Data Concepts:** Variables, Constants, data types like: int, float char, double and void. Qualifiers: short and long size qualifiers, signed and unsigned qualifiers. Declaring variables, Scope of the variables according to block, Hierarchy of data types.

(c) **Types of operators:** Arithmetic, Relational, Logical, Compound Assignment, Increment and decrement, Conditional or ternary operators. Precedence and order of evaluation. Statements and Expressions.

(d) **Type conversions:** Automatic and Explicit type conversion.

(e) **Data Input and Output functions:** Formatted I/O: printf(), scanf(). Character I/O format: getch(), getche(), getchar(), getc(), gets(), putchar(), putc(), puts().

(f) **Iterations:** Control statements for decision making: (a) Branching: if statement, if..else statement , else.. if statement, nested if statement, switch statement. (b) Looping: while loop, do.. while, for loop, nested loop. (c) Loop interruption statements: break, continue.

.

Unit II

Arrays and Functions in C

(a) **#define directive**

(b) **Arrays:** (One and two dimensional), declaring array variables, initialization of arrays, accessing array elements.

(c) **Strings:** Declaring and initializing String variables, Character and string handling functions (strcpy, strcat, strchr, strcmp, strlen, strstr).

(d) **Mathematical functions** : exp(), ceil(), floor(),pow(), sqrt().

(e) **Functions:** Global and local variables, Function definition, return statement, calling a function.

(f) **Recursion:** Definition, Recursion functions for factorial, Fibonacci sequence, exponential function, G.C.D.

Unit III

Introduction to JAVA

(a) **Object-Oriented approach:** Comparison between structured and object oriented approach.Features of object-orientations: Abstraction, Inheritance, Encapsulation and Polymorphism.

(b) **Introduction:** History of Java, Java features, different types of Java programs, Differentiate Java with C. Java Virtual Machine.

(c) **Java Basics:** Variables and data types, declaring variables, literals: numeric, Boolean, character and string literals, keywords, type conversion and casting. Standard default values. Java Operators, Loops and Controls .

(d) **Classes:** Defining a class, creating instance and class members: creating object of a class; accessing instance variables of a class; creating method; naming method of a class; accessing method of a class; overloading method; 'this' keyword, constructor and Finalizer: Basic Constructor; parameterized constructor; calling another constructor; finalize() method; overloading constructor.

(e) **Arrays:** one and two-dimensional array, declaring array variables, creating array objects, accessing array elements

(f) **Access control:** public access, friendly access, protected access, private access.

Unit IV

Use of Mathematical software: ONLY FOR PRACTICAL EVALUATION

(a) **Maxima:**

References:

- **(a)** Programming in ANSI C (Third Edition) : E Balagurusamy, TMH
- **(b)** Let us C by Yashwant Kanetkar, BPB.
- **(c)** Programming with Java: A Primer 4th Edition by E. Balagurusamy, Tata McGraw Hill.
- **(d)** Java The Complete Reference, 8th Edition, Herbert Schildt, Tata McGraw Hill

Additional References:

- **(a)** Mastering Algorithms with C, Kyle Loudon, Shroff Publishers.
- **(b)** Algorithms in C (Third Edition): Robert Sedgewick , Pearson Education Asia.
- **(c)** Programming in ANSI C by Ram Kumar, Rakesh Agrawal, TMH.
- **(d)** Programming with C (Second Edition): Byron S Gottfried (Adapted by Jitender Kumar Chhabra) Schaum's Outlines (TMH)
- **(e)** Programming with C : K R Venugopal, Sudeep R Prasad TMH Outline Series.
- **(f)** Unix and C : M.P. Bhave and S.A. Pateker, Nandu printers and publishers private limited.

- **(1)** Eric Jendrock, Jennifer Ball, D Carson and others, The Java EE 5 Tutorial, Pearson Education, Third Edition, 2003.
- **(2)** Ivan Bayross, Web Enabled Commercial Applications Development Using Java 2, BPB Publications, Revised Edition, 2006
- **(3)** Joe Wigglesworth and Paula McMillan, Java Programming: Advanced Topics, Thomson Course Technology (SPD), Third Edition, 2004

The Java Tutorials of Sun Microsystems Inc. http://docs.oracle.com/javase/tutorial

Suggested Practicals based on USMT501

1. Continuity of scalar valued and vector valued functions on R^n

2. Differentiability of scalar valued functions, partial derivatives and directional derivatives.

3. Gradient, level surfaces, tangent plane, Use of Taylor polynomials to find appraoximate values of functions with more than one variable.

4. Differentiability of vector valued functions. Chain rule. Jacobian matrices.

5. Differential equations of order 2.

6. Differential equations of higher orders and operator method.

7. Miscellaneous theoretical questions.

Suggested Practicals based on USMT502

1. Quotient spaces.
2. Orthogonal transformations,Isometries.
3. Diagonalization.
4. Orthogonal diagonalization.
5. Groups, Subgroups, Lagrange's Theorem, Cyclic groups and Groups of Symmetry.
6. Group homomorphisms, isomorphisms.
7. Miscellaneous Theoretical questions based on full USMT502

Suggested Practicals based on USMT503

(1). Metric spaces and normed linear spaces. Examples.

(2) Open balls, open sets in metric spaces, subspaces and normed linear spaces.

(3) Limit points: (Limit points and closure points, closed balls, closed sets, closure of a set, boundary of a set, distance between two sets). (4) Cauchy Sequences, completeness

(5) Continuity.

(6) Uniform continuity in a metric space.

(7) Miscellaneous Theoretical Questions based on full paper.

Suggested Practicals based on USMT5B4

(1) Primes, Fundamental theorem of Airthmetic.

(2) Congruences.

(3) Linear Diophantine equation.

(4) Pythagorean triples and sum of squares.

(5) The Gaussian quadratic reciprocity law.

(6) Jacobi symbols and law of reciprocity for Jacobi symbols.

(7) Miscellaneous Theoretical questions based on full paper.

Course code USMT5AP	Topics for Practical

(1) Practical related to Mathematical software Maxima

(2) Practical related to Mathematical software Maxima

(3) Practical related to Mathematical software Maxima

(4) Write a C program that illustrates the concepts of C operators, mathematical functions,

(5) Write a C program that illustrates the concepts of arrays.

(6) Write a C program that illustrates the concepts of string.

(7) Write a C program that illustrates the concepts of different iterations.

(8) Write a C program that illustrates the concepts of functions, recursion.

(9) Write a Java program to create a Java class: (a) without instance variables and methods, (b) with instance variables and without methods, (c) without instance variables and with methods.(d) with instance variables and methods.

(10)Write a Java program that illustrates the concepts of one, two dimension arrays.

(11)Write a Java program that illustrates the concepts of Java class that includes (a)constructor with and without parameters.

(12)Write a Java program that illustrates the concepts of Overloading methods.

Revised Syllabus in Mathematics

Credit Based Semester and Grading System Third Year B. Sc. 2015-16

Semester VI

Course: Real Analysis and Multivariate Calculus
Course Code: USMT601

Unit I Line Integrals.

(a) Review of Riemann integral

(b) Integration of vector fields

Line Integrals, Definition, Evaluation for smooth curves. Mass and moments for coils, springs, thin rods.
Vector fields, Gradient fields, Work done by a force over a curve in space, Evaluation of work integrals.

(d) Flow integrals and circulation around a curve.

(e) Flux across a plane curve.

(f) Path independence of the line integral in case of Conservative fields. Calculation of potential function.

(g) The Fundamental theorems of line integrals (without proof).

(h) Flux density (divergence), Circulation density (curl) at a point.

(i) Green's Theorem in plane (without proof), Evaluation of line integrals using Green's Theorem.

Reference for Unit I:

(1) Calculus. Vol. 2, T. Apostol, John Wiley.

(2) Calculus. J. Stewart. Brooke/Cole Publishing Co.

Unit II Surface integrals

(a) Review of double and triple integrals. Fubini's theorem over rectangles.

(b) Properties of Double and Triple Integrals:

(i) Integrability of sums, scalar multiples, products of integrable functions, and formulae for integrals of sums and scalar multiples of integrable functions.

(ii)Domain additivity of the integrals.

(iii) Integrability of continuous functions and functions having only finitely (countably) many discontinuities.

(v) Double and triple integrals over bounded domains.

(c) Change of variable formula for double integral and Change of variable formula for triple integrals

(d) (i) Parametric representation of a surface.

(ii) The fundamental vector product, definition and it being normal to the surface.

(iii) Area of a parameterized surface.

(e) (i) Surface integrals of scalar and vector fields (definition).

(ii) Independence of value of surface integral under change of parametric representation of the surface (statement only).

(iii) Stokes' theorem, (assuming general form of Green's theorem) Divergence theorem for a solid in 3-space bounded by an orientable closed surface for continuously differentiable vector fields.

Reference for Unit II:

(1) Calculus. Vol. 2, T. Apostol, John Wiley.

(2) Calculus. J. Stewart. Brooke/Cole Publishing Co.

Recommended books:

(1) Robert G. Bartle and Donald R. Sherbert. Introduction to Real Analysis, Second edition, John Wiley & Sons, INC.

(2) Richard G. Goldberg, Methods of Real Analysis, Oxford & IBH Publishing Co. Pvt. Ltd., New Delhi.

(3)Tom M. Apostol, Calculus Volume II, Second edition, John Wiley & Sons, New York.

(4)Berberian. Introduction to Real Analysis. Springer.

Additional Reference Books:

(1) J.E. Marsden and A.J. Tromba, Vector Calculus. Fifth Edition, http://bcs.whfreeman.com/marsdenvc5e/

(2) R. Courant and F. John, Introduction to Calculus and Analysis, Volume 2, Springer Verlag, New York.

(3) M.H. Protter and C.B. Morrey, Jr., Intermediate Calculus, Second edition, Springer Verlag, New York, 1996.

(4) D.V. Widder, Advanced Calculus, Second edition, Dover Pub., New York.
(5) Tom M. Apostol, Mathematical Analysis, Second edition, Narosa, New Delhi, 1974.

Unit III Uniform Convergence of sequences and series of functions.

(a) Point wise and uniform convergence of sequences and series of real-valued functions. Cauchy's criterion for uniform convergence. Weierstrass M-test. Examples.

(b) Continuity of the uniform limit (resp: uniform sum) of a sequence (resp: series) of real-valued functions. The integral and the derivative of the uniform limit (resp: uniform sum) of a sequence (resp: series) of real-valued functions on a closed and bounded interval. Examples.

(c) Power series in R. Radius of convergence. Interval of convergence. Uniform convergence. Validity of Term-by-term differentiation and integration of power series. Examples. Uniqueness of power series representation for a function in terms of its Taylor's series. Analytic and Non analytic functions.

(d) Classical functions defined by power series such as exponential, cosine and sine functions, and the basic properties of these functions.

Reference for Unit III:

Methods of Real Analysis, R.R. Goldberg. Oxford and International Book House (IBH) Publishers, New Delhi.

Recommended books:

(1) Robert G. Bartle and Donald R. Sherbert, Introduction to Real Analysis, Second edition, John Wiley & Sons, INC.

(2) Richard G. Goldberg, Methods of Real Analysis, Oxford & IBH Publishing Co. Pvt. Ltd., New Delhi.

(3) Tom M. Apostol, Calculus Volume II, Second edition, John Wiley & Sons, New York.

(4) J. Stewart. Calculus. Third edition. Brooks/Cole Publishing Co.

(5) Berberian. Introduction to Real Analysis. Springer.

Additional unit on differential Equations These topics may be given for projects.

(a) Linear systems of first order differential equations. Solutions using computation of eigen values. Relationship between a differential equation of order n and a system of n linear differential equations of order 1.

(b) Sturm Liouville Oscillation theory of differential equations for order 2

(c) Solution of linear diofferential equations using power series.

References Differential Equations with Applications and Historical Notes, G.F. Simmons, McGraw Hill.

Course: Algebra

Course Code: USMT602

Unit I. Normal subgroups (15 Lectures)

(a) (i) Normal subgroups of a group. Definition and examples including center of a group.

  (ii) Quotient group.

  (iii) Alternating group A_n, cycles. Listing normal subgroups of A_4, S_3.

(b) First Isomorphism theorem (or Fundamental Theorem of homomorphisms of groups).

(c) Cayley's theorem.

(d) External direct product of a group. Properties of external direct products. Order of an element in a direct product, criterion for direct product to be cyclic.

(e) Classification of groups of order ≤ 5.

Reference for Unit I:

(1) I.N. Herstein. Algebra.

(2) P.B. Bhattacharya, S.K. Jain, S. Nagpaul. Abstract Algebra.

Unit II. Ring theory (15 Lectures)

(a) (i) Definition of a ring. (The definition should include the existence of a unity element.)

  (ii) Properties and examples of rings, including $\mathbb{Z}$, $\mathbb{Q}$, $\mathbb{R}$, $\mathbb{C}$, $M_n(\mathbb{R})$, $\mathbb{Q}[X]$, $\mathbb{R}[X]$, $\mathbb{C}[X]$, $\mathbb{Z}[i]$, $\mathbb{Z}[\sqrt{2}]$, $\mathbb{Z}[\sqrt{-5}]$, $\overline{\mathbb{Z}n}$.

  (iii) Commutative rings.

  (iv) Units in a ring. The multiplicative group of units of a ring.

  (v) Characteristic of a ring.

  (vi) Ring homomorphisms. First Isomorphism theorem of rings.

  (vii) Ideals in a ring, sum and product of ideals in a commutative ring.

  (viii) Quotient rings.

(b) Integral domains and fields. Definition and examples.

  (i) A finite integral domain is a field.

  (ii) Characteristic of an integral domain, and of a finite field.

(c) (i) Construction of quotient field of an integral domain (Emphasis on Z, Q).

(ii) A field contains a subfield isomorphic to Zp or Q.

Reference for Unit II:

(1) M. Artin. Algebra.
(2) N.S. Gopalkrishnan. University Algebra.

(3) P.B. Bhattacharya, S.K. Jain, S. Nagpaul. Abstract Algebra.

Unit III. Factorization. (15 Lectures)

(a) Prime ideals and maximal ideals. Definition and examples. Characterization in terms of quotient rings.

(b) Polynomial rings. Irreducible polynomials over an integral domain. Unique Factorization Theorem for polynomials over a field.

(c) Divisibility in an integral domain, irreducible and prime elements, ideals generated by prime and irreducible elements.

(d) (i) Definition of a Euclidean domain (ED), Principal Ideal Domain (PID), Unique Factorization Domain (UFD). Examples of ED: Z, $F[X]$, where F is a field, and Z$[i]$.

(ii) An ED is a PID, a PID is a UFD.

(iii) Prime (irreducible) elements in R$[X]$, Q$[X]$, Z$p[X]$. Prime and maximal ideals in polynomial rings.

(iv) Z$[X]$ is not a PID. Z$[X]$ is a UFD (Statement only).

Reference for Unit III:

(1) M. Artin. Algebra.

(2) N.S. Gopalkrishnan. University Algebra.

(3) P.B. Bhattacharya, S.K. Jain, S. Nagpaul. Abstract Algebra.

Recommended Books

1. S.Kumaresan. Linear Algebra: A Geometric Approach, Prentice Hall of India Pvt Ltd, New Delhi.

2. I.N. Herstein. Topics in Algebra, Wiley Eastern Limited, Second edition.

3. N.S. Gopalakrishnan, University Algebra, Wiley Eastern Limited.

4. M. Artin, Algebra, Prentice Hall of India, New Delhi.

5. T. Banchoff and J. Wermer, Linear Algebra through Geometry, Springer.

6. L. Smith, Linear Algebra, Springer.

7. Tom M. Apostol, Calculus Volume 2, Second edition, John Wiley, New York, 1969.

8. P.B. Bhattacharya, S.K. Jain, and S.R. Nagpaul, Abstract Algebra, Second edition, Foundation Books, New Delhi, 1995.

9. J.B. Fraleigh, A first course in Abstract Algebra, Third edition, Narosa, New Delhi.

10. J. Gallian. Contemporary Abstract Algebra. Narosa, New Delhi.

Additional Reference Books

1. S. Lang, Introduction to Linear Algebra, Second edition, Springer Verlag, New York.

2. K. Hoffman and S. Kunze, Linear Algebra, Prentice Hall of India, New Delhi.

3. S. Adhikari. An Introduction to Commutative Algebra and Number theory. Narosa Publishing House.

4. T.W. Hungerford. Algebra. Springer.

5. D. Dummit, R. Foote. Abstract Algebra. John Wiley & Sons, Inc.

6. I.S. Luthar, I.B.S. Passi. Algebra, Vol. I and II.

Course: Topology of Metric Spaces
Course Code: USMT603

Unit I. Fourier Series (15 lectures)

(a) Fourier series of functions on $C[-\pi,\pi]$,

(b) Dirichlet kernel, Fejer kernel, Cesaro summability of Fourier series of functions on $C[-\pi,\pi]$

(c) Bessel's inequality and Pareseval's identity

(d) Convergence of the Fourier series in L^2 norm.

Reference for Unit I:

1. R. Goldberg. Methods of Real Analysis.
2. S. Kumaresan, Topology of Metric spaces.

Unit II. Compactness (15 lectures)

(a) Definition of a compact set in a metric space (as a set for which every open cover has a finite subcover). Examples, properties such as

  (i) continuous image of a compact set is compact.
  (ii) compact subsets are closed.
  (iii) a continuous function on a compact set is uniformly continuous.

(b) Characterization of compact sets in R^n: The equivalent statements for a subset of R^n to be compact:

  (i) Heine-Borel property.
  (ii) Closed and boundedness property.
  (iii) Bolzano-Weierstrass property.
  (iv) Sequentially compactness property.

Reference for Unit II:

1. S. Kumaresan, Topology of Metric spaces.
2. W. Rudin, Principles of Mathematical Analysis.

Unit III. Connectedness (15 lectures)

(a) (i) Connected metric spaces. Definition and examples.

  (ii) Characterization of a connected space, namely a metric space X is connected if and only if every continuous function from X to $\{1,-1\}$ is a constant function.

  (iii) Connected subsets of a metric space, connected subsets of R.

(iv) A continous image of a connected set is connected.

(b) (i) Path connectedness in R^n, definitions and examples.

(ii) A path connected subset of R^n is connected.

(iii) An example of a connected subset of R^n which is not path connected.

Reference for Unit III:

1. S. Kumaresan, Topology of Metric spaces.
2. W. Rudin, Principles of Mathematical Analysis.

Recommended Books

1. S. Kumaresan. Topology of Metric spaces.
2. R.G. Goldberg Methods of Real Analysis, Oxford and IBH Publishing House, New Delhi.
3. W. Rudin. Principles of Mathematical Analysis. McGraw Hill, Auckland, 1976.
4. P.K. Jain, K. Ahmed. Metric spaces. Narosa, New Delhi, 1996.
5. G.F. Simmons. Introduction to Topology and Modern Analysis. McGraw Hill, New York, 1963.

Additional Reference Books

1. T. Apostol. Mathematical Analysis, Second edition, Narosa, New Delhi, 1974.
2. E.T. Copson. Metric spaces. Universal Book Stall, New Delhi, 1996.
3. Sutherland. Topology.
4. D. Somasundaram, B. Choudhary. A first course in Mathematical Analysis. Narosa, New Delhi.
5. R. Bhatia. Fourier series. Texts and readings in Mathematics (TRIM series), HBA, India.

Course: Number Theory and its applications

Course Code: USMT6B4

Unit I. Continued Fractions (15 Lectures)

(a) Finite continued fractions

(b) (i) Infinite continued fractions and representatio of an irrational number by an infinite simple continued fraction.

(ii) Rational approximations to irrational numbers, Order of convergence, Best possible approximations.

(iii) Periodic continued fractions.

Unit II. Pell's equation, Arithmetic function and Special numbers (15 Lectures)

(a) Pell's equation $x^2 - dy^2 = n$, where d is not a square of an integer. Solutions of Pell's equation. (The proofs of convergence theorems to be omitted).

(b) Algebraic and transcendental numbers. The existence of transcendental numbers.

(c) Arithmetic functions of number theory: $d(n)$ (or $\tau(n)$), $\sigma(n)$ and their properties. $\mu(n)$ and the M$\ddot{o}$bius inversion formula.

(d) Special numbers: Fermats numbers, Perfect numbers, Amicable numbers. Pseudo primes, Carmichael numbers.

Unit III. Cryptography (15 Lectures)

(a) Basic notions such as encryption (enciphering) and decryption (deciphering).

Cryptosystems, symmetric key cryptography. Simple examples such as shift cipher, affine cipher, hill's cipher.

(b) Concept of Public Key Cryptosystem; RSA Algorithm.

Reference Books

1. I. Niven, H. Zuckerman and H. Montogomery. *Elementary number theory.* John Wiley & Sons. Inc.
2. David M. Burton. *An Introduction to the Theory of Numbers.* Tata McGraw Hill Edition
3. G. H. Hardy, and E.M. Wright, *An Introduction to the Theory of Numbers*. Low priced edition. The English Language Book Society and Oxford University Press, 1981.
4. Neville Robins. *Beginning Number Theory*, Narosa Publications.
5. S.D. Adhikari. *An introduction to Commutative Algebra and Number Theory*. Narosa Publishing House.
6. S. B. Malik. *Basic Number theory*. Vikas Publishing house.
7. N. Koblitz. *A course in Number theory and Crytopgraphy*. Springer.

8. M. Artin.*Algebra*. Prentice Hall.

9. K. Ireland, M. Rosen. *A classical introduction to Modern Number Theory*. Second edition, Springer Verlag.

10. William Stalling. *Cryptology and network security*.

SEMESTER VI

Course code	Title
USMT6A	**Computer Programming–II**

..

Unit I

JAVA: INHERITANCE AND APPLETS

(a) **Inheritance:** Various types of inheritance, super and subclasses, keywords- ‘extends’; ‘super’, overriding method, final and abstract class: final variables and methods; final classes, abstract methods and classes. Concept of interface.

(b) **Exception Handling and Packages:** Need for Exception Handling, Exception Handling techniques: try and catch; multiple catch statements; finally block; usage of throw and throws. Concept of package. Integer class method: parseInt()**.**

(c) **JAVA Applets:** Difference of applet and application, creating applets, applet life cycle, passing parameters to applets.

(d) **Graphics, Fonts and Color:** The graphics class, painting, repainting and updating an applet, sizing graphics. Font class, draw graphical figures - lines and rectangle, circle and ellipse, drawing arcs, drawing polygons. Working with Colors: Color methods, setting the paint mode.

(e) **AWT package:** Containers: Frame and Dialog classes, Components: Label; Button; Checkbox; TextField, TextArea. **ava Basics:** Variables and data types, declaring variables, literals: numeric, Boolean, character and string literals, keywords, type conversion and casting. Standard default values. Java Operators, Loops and Controls .

.

Unit II

Relational Database Management System

(a) **Introduction to Database Concepts:** Database, Overview of database management system. Database Languages- Data Definition Language (DDL) and Data Manipulation Languages (DML).

(b) **Entity Relation Model:** Entity, attributes, keys, relations, Designing ER diagram, integrity constraints over relations, Conversion of ER to relations with and without constraints.

(c) **SQL commands and Functions:**

(i) **Creating and altering tables:** CREATE statement with constraints like KEY, CHECK, DEFAULT, ALTER and DROP statement.

(ii) **Handling data using SQL:** selecting data using SELECT statement, FROM clause, WHERE clause, HAVING clause, ORDER BY, GROUP BY,DISTINCT and ALL predicates, Adding data with INSERT statement, changing data with UPDATE statement, removing data with DELETE statement.

(iii) **Functions:** Aggregate functions-AVG, SUM, MIN, MAX and COUNT, Date functions-ADD_MONTHS(), CURRENT_DATE(), LAST_DAY(), MONTHS_BETWEEN(), NEXT_DAY(). String functions- LOWER(), UPPER(), LTRIM(), RTRIM(), TRIM(), INSTR(), RIGHT(), LEFT(), LENGTH(), SUBSTR(). Numeric functions: ABS(), EXP(), LOG(), SQRT(), POWER(), SIGN(), ROUND(number).

(iv) **Joining tables:** Inner, outer and cross joins, union.

.

Unit III

Introduction to PL/SQL

(a) **Fundamentals of PL/SQL:** Defining variables and constants, PL/SQL expressions and comparisons: Logical Operators, Boolean Expressions, CASE ExpressionsHandling, Null Values in Comparisons and Conditional Statements,

(b) **PL/SQL Datatypes:**Number Types, Character Types, Boolean Type. Datetime and Interval Types.

(c) **Overview of PL/SQL Control Structures:**Conditional Control: IF and CASE Statements, IF-THEN Statement, IF-THEN-ELSE Statement, IF-THEN-ELSIF Statement, CASE Statement,

(d) **Iterative Control:** LOOP and EXIT Statements, WHILE-LOOP, FOR-LOOP, Sequential Control: GOTO and NULL Statements.

.

Unit IV

Use of Mathematical software: ONLY FOR PRACTICAL EVALUATION

(a) **Maxima:**

References:

(a) Programming with Java: A Primer 4th Edition by E. Balagurusamy, Tata McGraw Hill.
(b) Java The Complete Reference, 8th Edition, Herbert Schildt, Tata McGraw Hill
(c) Database Management Systems, Ramakrishnam, Gehrke, McGraw-Hill
(d) Ivan Bayross, "SQL,PL/SQL -The Programming language of Oracle", B.P.B. Publications, 3rd Revised Edition.
(e) George Koch and Kevin Loney , ORACLE "The Complete Reference", Tata McGraw Hill,New Delhi.

Additional References:

(1) Eric Jendrock, Jennifer Ball, D Carson and others, The Java EE 5 Tutorial, Pearson Education, Third Edition, 2003.
(2) Ivan Bayross, Web Enabled Commercial Applications Development Using Java 2, BPB Publications, Revised Edition, 2006
(3) Joe Wigglesworth and Paula McMillan, Java Programming: Advanced Topics, Thomson Course Technology (SPD), Third Edition, 2004
(4) The Java Tutorials of Sun Microsystems Inc. http://docs.oracle.com/javase/tutorial

a) Elsmasri and Navathe, "Fundamentals of Database Systems", Pearson Education.
b) Peter Rob and Coronel, "Database Systems, Design, Implementation and Management", Thomson Learning
c) C.J.Date, Longman, "Introduction to database Systems", Pearson Education.
d) Jeffrey D. Ullman, Jennifer Widom, "A First Course in Database Systems", Pearson Education.
e) Martin Gruber, "Understanding SQL",B.P.B. Publications.
f) Michael Abbey, Michael J. Corey, Ian Abramson, Oracle 8i – A Beginner's Guide, Tata McGraw-Hill.

Suggested Practicals based on USMT601

1. Line Integral
2. calculation of divergence and curl. Green's theorem.
3. Double and triple integrals.
4. Surface integrals, Stoke's theorem.
5. Uniform convergence of sequences and series of functions.
6. Continuity, integribility, differentiability under uniform convergence and power series.
7. Miscellaneous theoretical questions.

Suggested Practicals based on USMT602

1. Normal subgroups and quotient groups.
2. Cayley's Theorem and external direct product of groups.
3. Rings, Integral domains and fields.
4. Ideals, prime ideals and maximal ideals.
5. Ring homomorphism, isomorphism.
6. Euclidean Domain, Principal Ideal Domain and Unique Factorization Domain.
7. Miscellaneous Theoretical questions based on full paper.

Suggested Practicals based on USMT603

(1) Fourier series

(2) Parseval's identity.

(3) Compact sets in a metric space, Compactness in R^n (emphasis on R, R^2). Properties.

(4) Continuous image of a compact set.

(5) Connectedness, Path connectedness.

(6) Continuous image of a connected set.

(7) Miscellaneous Theoretical Questions based on full paper.

Suggested Practicals based on USMT6B4

(1) Finite continued fractions.

(2) Infinite continued fractions.

(3) Pell's equations.

(4) Arithmatic functions of number theory, Special numbers.

(5) Cryptosytems (Private key).

(6) Public Key Cryptosystems. RSA Algorithm.

(7) Miscellaneous Theoretical questions based on full paper.

Course code USMT6AP	Topics for Practical

(1) Practical related to Mathematical software Maxima

(2) Practical related to Mathematical software Maxima

(3) Practical related to Mathematical software Maxima

(4) Write a Java program to demonstrate inheritance by creating suitable classes.

(5) Write a Java program to demonstrate overriding and overloading in a sub class.

(6) Write a program that illustrates the error handling using exception handling.

(7) Write a Java applet to demonstrate graphics, Font and Color classes.

(8) Write a Java applet by using AWT package.

(9) Creating a single table with/ without constraints and executing queries. Queries containing aggregate, string and date functions fired on a single table.

(10) Updating tables, altering table structure and deleting table Creating and altering a single table and executing queries. Joining tables and processing queries.

(11) Writing PL/SQL Blocks with basic programming constructs.

(12) Writing PL/SQL Blocks with control structures.

Evaluation pattern:

Internal Assessment of Theory Core Courses Per Semester Per Course (Total 40 marks)

(a) One Assignments: .. 10 Marks.

(b) One Class Test: .. 30 Marks.

Semester End Theory Examination (Total 60 marks)

Theory: At the end of the semester, examination of two and half hours duration and 60 marks based on the three units shall be held for each course. Pattern of Theory question paper at the end of the semester for each course:

There shall be four compulsory Questions with internal option.

Question1 based on Unit I, Question 2 based on Unit II, Question 3 based on Unit III, Question 4 based on all Units combined.

Semester End Practical Examination (Total 100 marks)

Total evaluation is of 100 marks-

(a) Semester End practical examination on computer - 80 marks.

(One question from each unit)

(b) Journal - 10 Marks.

(c) Viva based on the journal / practical - 10 Marks.

1. The semester end practical examination on the machine will be of THREE hours.

2. Students should carry a certified journal with minimum of 06 practicals (mentioned in the practical topics) at the time of examination.

3. Number of students per batch for the regular practical should not exceed 20. Not more than two students are allowed to do practical experiment on one computer at a time.

Workload

Theory : 4 lectures per week .

Practical: 2 practical's each of 2 lecture periods per week per batch. Two lecture periods of the practicals shall be conducted in succession