2.1 Logical Equivalence and Truth Tables

Statement (propositional) forms

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- A statement form (or propositional form) is an expression made up of statement variables (such as p,q, and r) and logical connectives (such as $\sim$, $\wedge$, and $\vee$) that becomes a statement when actual statements are substituted for the component statement variables.

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- The truth table for a given statement form displays the truth values that correspond to all possible combinations of truth values for its component statement variables.

Truth Tables for statement forms

Examples

Find the truth tables for the following statement forms:

1. $p \vee \sim q$

Truth Tables for statement forms

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Find the truth tables for the following statement forms:

1. $p \vee \sim q$
2. $p \vee (q \wedge r)$

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Find the truth tables for the following statement forms:

1. $p \lor \sim q$
2. $p \lor (q \land r)$
3. $(p \lor q) \land (p \lor r)$

Logical Equivalence

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- Two statements are called logically equivalent if, and only if, they have logically equivalent forms when identical component statement variables are used to replace identical component statements.

Examples

Examples (de Morgan's Laws)

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1. We have seen that $\sim(p \wedge q)$ and $\sim p \vee \sim q$ are logically equivalent.
2. Show that $\sim(p \vee q) \equiv \sim p \wedge \sim q$.
3. Show that $\sim(p \wedge q)$ and $\sim p \wedge \sim q$ are **not** logically equivalent.

Tautologies and Contradictions

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- A tautology is a statement form that is always true regardless of the truth values of the individual statements substituted for its statement variables.
- A contradiction is a statement form that is always false regardless of the truth values of the individual statements substituted for its statement variables.

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- Example: $p \vee \sim p$.
- A contradiction is a statement form that is always false regardless of the truth values of the individual statements substituted for its statement variables.
- Example: $p \wedge \sim p$.

Logical Equivalence

Theorem 2.1.1 Logical Equivalences

Given any statement variables p, q, and r, a tautology $\mathbf{t}$ and a contradiction $\mathbf{c}$, the following logical equivalences hold.

1.	*Commutative laws:*	$p \wedge q \equiv$	$p \vee q \equiv$
2.	*Associative laws:*	$(p \wedge q) \wedge r \equiv$	$(p \vee q) \vee r \equiv$
3.	*Distributive laws:*	$p \wedge (q \vee r) \equiv$	$p \vee (q \wedge r) \equiv$
4.	*Identity laws:*	$p \wedge \mathbf{t} \equiv$	$p \vee \mathbf{c} \equiv$
5.	*Negation laws:*	$p \vee {\sim}p \equiv \mathbf{t}$	$p \wedge {\sim}p \equiv \mathbf{c}$
6.	*Double negative law:*	${\sim}({\sim}p) \equiv p$	
7.	*Idempotent laws:*	$p \wedge p \equiv p$	$p \vee p \equiv p$
8.	*Universal bound laws:*	$p \vee \mathbf{t} \equiv \mathbf{t}$	$p \wedge \mathbf{c} \equiv \mathbf{c}$
9.	*De Morgan's laws:*	${\sim}(p \wedge q) \equiv {\sim}p \vee {\sim}q$	${\sim}(p \vee q) \equiv {\sim}p \wedge {\sim}q$
10.	*Absorption laws:*	$p \vee (p \wedge q) \equiv p$	$p \wedge (p \vee q) \equiv p$
11.	*Negations of* $\mathbf{t}$ *and* $\mathbf{c}$*:*	${\sim}\mathbf{t} \equiv \mathbf{c}$	${\sim}\mathbf{c} \equiv \mathbf{t}$

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Theorem 2.1.1 Logical Equivalences

Given any statement variables p, q, and r, a tautology $\mathbf{t}$ and a contradiction $\mathbf{c}$, the following logical equivalences hold.

1. *Commutative laws:*	$p \wedge q \equiv q \wedge p$	$p \vee q \equiv q \vee p$
2. *Associative laws:*	$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	$(p \vee q) \vee r \equiv p \vee (q \vee r)$
3. *Distributive laws:*	$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
4. *Identity laws:*	$p \wedge \mathbf{t} \equiv p$	$p \vee \mathbf{c} \equiv p$
5. *Negation laws:*	$p \vee \sim p \equiv \mathbf{t}$	$p \wedge \sim p \equiv \mathbf{c}$
6. *Double negative law:*	$\sim(\sim p) \equiv p$	
7. *Idempotent laws:*	$p \wedge p \equiv p$	$p \vee p \equiv p$
8. *Universal bound laws:*	$p \vee \mathbf{t} \equiv \mathbf{t}$	$p \wedge \mathbf{c} \equiv \mathbf{c}$
9. *De Morgan's laws:*	$\sim(p \wedge q) \equiv \sim p \vee \sim q$	$\sim(p \vee q) \equiv \sim p \wedge \sim q$
10. *Absorption laws:*	$p \vee (p \wedge q) \equiv p$	$p \wedge (p \vee q) \equiv p$
11. *Negations of* $\mathbf{t}$ *and* $\mathbf{c}$*:*	$\sim \mathbf{t} \equiv \mathbf{c}$	$\sim \mathbf{c} \equiv \mathbf{t}$

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Use the logical equivalence of Theorem 2.1.1 to prove that the following are logical equivalences:

1. $(\sim p \vee q) \wedge (\sim q) \equiv \sim (p \vee q)$.
2. $(p \wedge \sim q) \vee (\sim p \vee q) \equiv \mathbf{t}$.