Discrete Mathematics Structure

Notes

Rule of Inference:

Constructing more complicated valid statements using simple arguments, which are valid are known as rule of inference.

1. Modus Ponens

Suppose there are two premises, P and P → Q. Now, we will derive Q with the help of Modules Ponens like this:

P → Q

P

____

∴ Q

Example:

Suppose P → Q = "If we have a bank account, then we can take advantage of this new policy."

P = "We have a bank account."

Therefore, Q = "We can take advantage of this new policy."

2. Modus Tollens

Suppose there are two premises, P → Q and ¬Q. Now, we will derive ¬P with the help of Modules Tollens like this:

P → Q

¬Q

____

∴ ¬P

Example:

Suppose P → Q = "If we have a bank account, then we can take advantage of this new policy."

¬Q = "We cannot take advantage of this new policy."

Therefore, ¬P = "We don't have a bank account."

3. Hypothetical Syllogism

Suppose there are two premises, P → Q and Q → R. Now, we will derive P → R with the help of Hypothetical Syllogism like this:

$P \rightarrow Q$
$Q \rightarrow R$

$\therefore P \rightarrow R$

Example:

Suppose P → Q = "If my fiancé comes to meet me, I will not go to office."

Q → R = "If I will not go to office, I won't require to do office work."

Therefore, P → R = "If my fiancé come to meet me, I won't require to do office work."

4. Disjunction Syllogism

Suppose there are two premises ¬P and P ∨ Q. Now, we will derive Q with the help of Disjunction Syllogism like this:

$\neg P$
$P \vee Q$

$\therefore Q$

Example:

Suppose ¬P = "Harry birthday cake is not strawberry flavored."

P ∨ Q = "The birthday cake is either red velvet flavored or mixed fruit flavored."

Therefore, Q = "The birthday cake is mixed fruit flavored."

5. Addition

Suppose there is a premise P. Now, we will derive P ∨ Q with the help of Addition like this:

P

∴ $P \vee Q$

Example:

Suppose P be the proposition, "Harry is a hard working employee" is true

Here Q has the proposition, "Harry is a bad employee".

Therefore, "Either Harry is a hard working employee Or Harry is a bad employee".

6. Simplification:

Suppose there is a premise $P \wedge Q$. Now, we will derive P with the help of Simplification like this:

$P \wedge Q$

∴ P

Example:

Suppose $P \wedge Q$ = "Harry is a hard working employee, and he is the best employee in the office".

Therefore, "Harry is a hard working employee".

7. Conjunction

Suppose there are two premises P and Q. Now, we will derive $P \wedge Q$ with the help of conjunction like this:

P
Q

∴ $P \wedge Q$

Example:

Suppose P = "Harry is a hard working employee".

Suppose Q = "Harry is the best employee in the office".

Therefore, "Harry is a hard working employee and Harry is the best employee in the office".

8. Resolution

Suppose there are two premises $P \vee Q$ and $\neg P \vee R$. Now, we will derive $Q \vee R$ with the help of a resolution like this:

$P \vee Q$

$\neg P \vee R$

$\therefore Q \vee R$

Example:

Suppose $P \rightarrow Q$ = "If my fiancé comes to meet me, I will not go to office".

$\neg P \vee R$ = "If my fiancé did not come to met me, I won't require to do office work".

Therefore, $Q \vee R$ = "Either I will not go to office or I won't require to do office work".

9. Constructive Dilemma

Suppose there are two premises $(P \rightarrow Q) \wedge (R \rightarrow S)$ and $P \vee R$. Now, we will derive $Q \vee S$ with the help of a constructive dilemma like this:

$(P \rightarrow Q) \wedge (R \rightarrow S)$

$P \vee R$

$\therefore Q \vee S$

Example:

Suppose $P \rightarrow Q$ = "If my fiancé will come to meet me, I will not go to office".

$R \rightarrow S$ = "If my relatives will come, I will tell my employees that I will come".

$P \vee R$ = "Either my fiancé will comes to meet me or my relatives will come".

Therefore, $Q \vee S$ = "Either I will not go to office or I will tell my employees that I will come".

10. Destructive Dilemma

Suppose there are two premises $(P \rightarrow Q) \land (R \rightarrow S)$ and $\neg Q \lor \neg S$. Now, we will derive $\neg P \lor \neg R$ with the help of a Destructive dilemma like this:

$(P \rightarrow Q) \land (R \rightarrow S)$

$\neg Q \lor \neg S$

$\therefore \neg P \lor \neg R$

Example:

Suppose $P \rightarrow Q$ = "If my fiancé comes to meet me, I will not go to office".

$R \rightarrow S$ = "If my relatives come, I will tell my employees that I will come".

$\neg Q \lor \neg S$ = "Either I will go to office or I will tell my employees that I will not come".

Therefore, $\neg P \lor \neg R$ = "Either my fiancé will not come to meet me or my relatives will not come".