Propn: $PSL_2(\mathbb{R})$ is triply transitive on $\partial\mathbb{H}$.

Pf: ~~Look at~~ For any distinct $a_1, a_2, a_3 \in \partial\mathbb{H}$, look at
(In clockwise orientation)

$$z \longrightarrow \frac{z-a_1}{z-a_2} \cdot \frac{a_3-a_2}{a_3-a_1}$$

This takes (a_1, a_2, a_3) to $(0, \infty, 1)$

Lenghts and Distances in $\mathbb{H}$.

We have a Riemannian metric $ds^2_{hyp} = \dfrac{dx^2+dy^2}{y^2}$
on $\mathbb{H}$ called the hyperbolic metric.

Let γ be a smooth curve in $\mathbb{H}$. i.e. $\gamma : [0,1] \to \mathbb{H}$, sm

then length of $\ell = \ell(\gamma) := \displaystyle\int_0^1 \|\gamma'(t)\|_{hyp}\, dt$.

$$= \int_0^1 \frac{\|\gamma'(t)\|_{euc}}{\operatorname{Im}(\gamma(t))}\, dt$$

If $\gamma(t) = (x(t), y(t))$

$$\ell(\gamma) = \int_0^1 \frac{\sqrt{x'(t)^2 + y'(t)^2}}{y(t)}\, dt.$$