An estimation of a DSGE model with effective lower bound in the Euro Area

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Abstract

This paper investigates the dynamics of the Euro Area business cycles under the ELB by estimating a non-linear DSGE model, and contrasting it with a model estimated with pre-crisis data and another without the ELB constraint. The analysis examines the responses of key macro variables to significant shocks, such as the NIRP, risk premiums, Government consumption, and interest rate shocks. The results from the non-linear model show that the Phillips Curve has flattened, with increased wage and price stickiness, that the ECB has shifted its policy focus toward output stabilization and strong response to inflation under the ELB. The non-linear model captures more persistent and volatile effects of the NIRP, risk premium, Government consumption, and interest rate shocks. These findings suggest that in the ELB environment, accommodative monetary policy, enhanced fiscal interventions, and structural reforms aimed at improving wage flexibility and financial stability are crucial for fighting economic downturns.

Keywords: Non-linear DSGE, Bayesian estimation, Effective lower bound, Euro Area

JEL classification: C11, E32, E52

1. Introduction

Since the unfolding of the Global Financial Crisis (GFC), many macro-models have been developed in an attempt to provide explanations to why previous generations of models have failed to predict one of the greatest crisis to date. For instance, Gertler and Karadi (2011) introduced financial frictions in a quantitative monetary DSGE model to capture the role of the financial sector, Christiano et al. (2015) used a medium-scale DSGE model to endogenize the labor force participation rate. However, these models were systematically estimated on pre-2008 data, hence

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failing to account for the crisis in the data. Subsequent researches accounted for the post-2008 data, but faced with the issue of occasionally binding constraints (OBCs) posed by the nominal interest rates at their effective lower bound (ELB). Handling the non-linearity induced by the ELB proved to be computationally demanding. For instance, Gust et al. (2017) estimated a fully non-linear DSGE model featuring an OBC, by relying on the Particle Filter (PF).[1] Guerrieri and Iacoviello (2015) provided a tool that helps to circumvent the curse of dimensionality issues of the PF, by developing a piecewise linear perturbation method to handle OBCs, such as the ELB on nominal interest rates. Not only their first-order perturbation approach enables the analysis of models with large number of state variables, but also is computationally fast. More recently, Boehl (2022a) proposed a faster solution method (by a factor of 1,500 with respect to Guerrieri and Iacoviello (2015)'s algorithm). Boehl (2022a)'s algorithm solves for the perfect foresight path of piecewise linear dynamic models by providing a closed-form solution for the complete expected trajectory of the endogenous variables as a function of the expected duration at the ELB (the 'ELB spell duration').

This paper contributes to the literature by (i) completing a review of existing frameworks that deal with OBCs in general equilibrium models and (ii) evaluating the performance of the recent algorithm in Boehl (2022a) to investigate the business dynamics of the Euro Area. Of a particular interest is negative interest rate policy (NIRP) implemented by the ECB. The Euro Area was severely affected by the GFC and in response, the ECB rapidly cut its main rate from 3.75% on 15 October 2008 to 1% by 13 May 2009. However, economic activity remained weak. The Sovereign debt crisis that started in 2010 put more pressure on the ECB to engage in additional rate cuts. In June 2014, the ECB made the unprecedented decision to introduce negative interest rates, becoming the first major central bank to do so. It lowered the Deposit Facility Rate to -0.10%. This marked the start of NIRP in the Euro Area. Given the challenge of modeling the economy in this negative rate environment, the main objective of the paper is to compare the performance of an estimated DSGE model that ignores the ELB with a model that features the ELB constraint. In this comparison exercise between the non-linear model and the linear one, I show that the latter systematically predicts low or absent responses of

[1] The PF is particularly appealing (for virtually infinite particles,1,500,000 particles in the case of Gust et al. (2017), the filter yields unbiased parameter estimates irrespective of the model's degree of non-linearity). But it comes at an expensive computational cost because of the curse of dimensionality that is introduced by a large-scale model. Such a model requires *de facto* more particles relatively to a small- or medium-scale model

macro variables to economic shocks. Using the estimates from the non-linear model, the results point to a lower risk aversion with respect to the linear model in the Euro Area, a flattened Phillips curve, and a strong response of the ECB to the output gap and to the inflation. The Government consumption, NIRP, and risk premium shocks are prominent shocks explaining the business cycle in the area.

The rest of the paper proceeds as follows. Section 2 depicts the literature on some available solution methods, the ZLB in the Euro Area, and draws the contribution of the paper. Section 3 presents the set of methodology the paper relies on. Section 4 discusses the data, the calibration of the parameters and the priors, and ends on a presentation of the results of the estimation. Section 5 concludes.

2. Related literature

As aforementioned, estimating DSGE models with OBCs, such as irreversible investment, borrowing constraint, or the zero lower bound (ZLB) is a highly difficult task because of the multiple challenges entailed, particularly for medium- and large-scale models. For instance, one of the challenges is the computational complexity, which implies that the solution time in non-linear models increases with the number of state variables (Fernández-Villaverde and Rubio-Ramírez, 2007; Richter and Throckmorton, 2016).

A snippet of existing algorithms dealing with OBCs

There are numerous works that employ various and diverse methods to solve and estimate non-linear models with OBCs. Early works focused essentially on small-scale models. Christiano and Fisher (2000) used parameterized expectations algorithms (PEAs) for approximating the solution of a dynamic model with an occasionally binding inequality constraint. By characterizing the solution of the model as either a policy and Lagrange multiplier function, or a conditional expectation function, they were able to transform the non-linear regression into a linear one with orthogonal explanatory variables. The PEAs are the class of algorithms which parameterize the conditional expectation of future variables rather than solving directly for policy functions. For better numerical stability and estimation accuracy, the authors used Chebyshev polynomials for interpolating and approximating functions within the PEAs, which yield a smoother approximation over the fixed interval grid.

Adjemian and Juillard (2010) relied on the Simulated Method of Moments to estimate a medium-scale DSGE model and used the Extended Path method to account for the full non-linearities of the deterministic part of the model. According to the authors, it is possible to handle the ZLB with the Extended Path approach since this approach does not rely on a strong smoothness assumption, thus can deal with the non differentiabilities caused by the max function in the Taylor rule (no OBC would be able to affect the expectations of the agents). The Extended Path approach uses a perfect foresight solver to deal with the non-linearities introduced by the OBCs. This method extends the traditional simulation of stochastic forward–looking models by accounting for the full non-linearities of the deterministic part of the model and solving only approximately the effect of future uncertainty. The proposed solution uses the expected value of future shocks which are set to zero by construction, instead of computing the expected non-linear effects of future shocks.

Guerrieri and Iacoviello (2015) developed a piecewise linear perturbation approach, by adapting a first-order perturbation method to solve dynamics models with OBCs. Standard perturbation methods are unable to capture OBCs because they provide only a local approximation, hence the need to adapt them. Their approach is to treat the OBCs as different regimes of the same model. As such, the OBC is slack under one regime and binding in the other. The piecewise linear solution ensures a link of the first-order approximation around the same point under each regime. Compared to high-quality numerical solutions, their method can accurately capture the essential dynamics of the models with OBCs and allows for efficient solving of models that would otherwise be complex due to the OBCs.

Gust et al. (2017) used Bayesian techniques via a MCMC algorithm to estimate a non-linear medium-scale model in which the interest-rate lower bound is occasionally binding. Instead of solving the model with an algorithm that keeps the non-linearity in the Taylor rule but log-linearizes the other equilibrium conditions like in Guerrieri and Iacoviello (2015), the authors rely on an algorithm that retains the full non-linearity of the model. Their solution method accounts for the effect of uncertainty about the ZLB on households and firms decisions, by not imposing perfect foresight. The model is solved for a minimum state variable solution via a projection method in the same vein as Christiano and Fisher (2000) and the Particle Filter is used to evaluate the likelihood.

In a recent paper, Boehl and Strobel (2024a) combined multiple previous contributions to provide an efficient and robust estimation of medium- and large-scale DSGE models with OBCs using a non-linear Bayesian likelihood approach. Employing the Ensemble Kalman Filter, a hybrid filtering system between the Kalman Filter and the Particle Filter, Boehl and Strobel (2024a) estimated a non-linear DSGE model and found that this hybrid filter delivered a good approximation of the likelihood function. The non-linear DSGE model is solved with the piecewise linear method.

I consider the Boehl and Strobel (2024a)'s set of methods to solve and estimate a DSGE model for the Euro Area, which takes into account the binding ELB on nominal interest rates in the estimation procedure. While most of the models focused predominantly on the case of the US economy, the mutation of the financial crisis into a Sovereign debt crisis later on in the Euro Area called for a different treatment. Until the beginning of the financial crisis, the overnight rate in the Euro Area interbank market was closely replicating the European Central Bank (ECB)'s Marginal Refinancing Operations rate. With the crisis being exacerbated by the Sovereign debt crisis, the overnight rate switched to track the ECB's Deposit Facility Rate. Eventually, the latter dropped to negative rates, leading the interest rate on reserves to turn negative too. The challenge of modeling the economy in this new state is overcome by employing several empirical techniques from Boehl and Strobel (2024a). I rely on their non-linear Bayesian likelihood estimation approach that embeds the Ensemble Kalman Filter (EnKF). The EnKF is helpful to estimate not only small or medium but also large non-linear DSGE models and to obtain fast likelihood approximations. In addition to the EnKF, Boehl (2022a)'s solution method is used to handle the ELB. Lastly, the Differential-Independence Mixture Ensemble Monte Carlo Markov Chain (DIME MCMC) method developed in Boehl (2022b) is used to sample from high-dimensional posterior distributions.

On the ZLB in the Euro Area

Some of the existing literature on the implications of the ZLB for the Euro Area highlight the crucial role of the ZLB on the dynamics of the variables. For instance, Kollmann et al. (2016) used an estimated three-region DSGE model to analyze the post-crisis slump in the Euro Area and the US, controlling for the Rest of the World. They found that the persistent Euro Area slump was driven by a combination of adverse supply and demand shocks, and particularly low productivity growth, high risk premiums on investment, and low aggregate demand. In their not-for-publication appendix, the authors used Guerrieri and Iacoviello (2015) algorithm to solve a version of their model in which the ZLB is imposed as an OBC. They confirmed that the ZLB was binding in the Euro Area in 2013 and 2014 but was not enough to explain that productivity and investment risk premium shocks were the most important factor of the Euro Area growth during and after the crisis. The authors additionally performed IRFs with fiscal shock with a linear model and a non-linear model. They found that the linear model systematically underestimates the responses. Gomes et al. (2015) relied on a multi-country DSGE model with features including the ZLB to simulate the effects of various policy measures. A sequence of unexpected shocks to the global economy ensures that the ZLB is hit. The agents are assumed to correctly forecast the results of each shock as they hit the economy, but are unaware of the future shocks about to hit. They used the non-linear perfect foresight algorithm to solve the model. For the Euro Area, Gomes et al. (2015) found a gradual decline in real activity and inflation until the point where agents realize the ZLB will be hit. However, inflation, output, and consumption dynamics show similar paths, with or without ZLB in the case of the Euro Area when compared with the US. They traced back the difference to the higher Calvo parameters for the Euro Area. Using the second generation ECB's New Area-Wide Model for the Euro Area, Coenen et al. (2021) conducted stochastic simulations of the model including an occasionally binding ELB constraint. They showed that the constraint can have a negative impact on inflation and the output gap, and amplify the volatility in these variables. They estimated the ELB to bind for an average duration of roughly 12 quarters. The authors simulated the non-linear model with the ELB constraint using the Extended Path algorithm. Overall, Hirose and Inoue (2015) demonstrated that missing to account for ZLB in the estimation leads to biased estimates of the structural shocks.

This paper revisits the importance of considering estimates from the DSGE model that incorporates the ELB to analyse the business cycles dynamics in the Euro Area. Closely following the work by Boehl and Strobel (2024a), it seeks to highlight the performance of a DSGE model that features the ELB relative to a model that ignores such a constraint.[2]

[2]It is worth noting that Boehl and Strobel (2024b) performed a similar exercise on Euro Area data, but with a focus on the role of financial frictions.

3. Methodology

This section is based on findings in Boehl (2022a) for the solution method, Boehl and Strobel (2024a) for the filtering method, and Boehl (2022b) for the posterior sampling. Additional details can be found in those papers.

3.1. The solution method

To solve for the non-linear DSGE model, I consider the piecewise linear solution method of Boehl (2022a). I provide hereafter a brief overview of the method. The complete solution, as it appears in the original paper is offered in the appendix B.

Assuming a detrended model, that is, linearized around its steady-state balanced growth, one can represent the original model (with variable vector $y_t \in \mathbb{R}^{n_y}$ and shock vector $\varepsilon_t \in \mathbb{R}^{n_z}$), subject to the ELB as a piecewise linear model of the form

$$A \begin{bmatrix} c_t \\ s_{t-1} \end{bmatrix} + b \max \left\{ p \begin{bmatrix} E_t c_{t+1} \\ s_t \end{bmatrix} + m \begin{bmatrix} c_t \\ s_{t-1} \end{bmatrix}, \bar{r} \right\} = E_t \begin{bmatrix} c_{t+1} \\ s_t \end{bmatrix} \tag{1}$$

$\begin{bmatrix} c_t \\ s_{t-1} \end{bmatrix}$ is a re-ordering of $\begin{bmatrix} y_t \\ \varepsilon_t \end{bmatrix}$, latent state variables and all current shocks are included in s_{t-1}, and all forward looking variables in c_t, A the system matrix, $\bar{r}$ the minimum value of the constrained variable r_t ($r_t = \max \left\{ p \begin{bmatrix} E_t c_{t+1} \\ s_t \end{bmatrix} + m \begin{bmatrix} c_t \\ s_{t-1} \end{bmatrix}, \bar{r} \right\}$), b the vector of the effects of r_t on all other variables. p and m recover the effects of those variables on r_t. Let's denote by $(k, l) \in \mathbb{N}_0^+$ the expected duration of the ELB spell and the expected number of periods before the ELB binds, respectively.

Boehl (2022a) showed that the rational expectations solution to 1 for the state s periods ahead, c_{t+s}, s_{t+s-1}, as a function of s_{t-1}, k, and l can be written as

$$F_s(l,k,s_{t-1}) = A^{\max\{s-l,0\}}\widehat{A}^{\min\{l,s\}}\begin{bmatrix} f(l,k,s_{t-1}) \\ s_{t-1} \end{bmatrix} \tag{2}$$

$$+ (I-A)^{-1}\left(I - A^{\max\{s-l,0\}}\right)b\overline{r}$$

$$= E_t\begin{bmatrix} c_{t+s} \\ s_{t+s-1} \end{bmatrix} \tag{3}$$

with $\widehat{A} = (I-bp)^{-1}(A+bm)$ and

$$f(l,k,s_{t-1}) = \left\{ c_t : \Psi A^k \widehat{A}\begin{bmatrix} c_t \\ s_{t-1} \end{bmatrix} = -\Psi(I-A)^{-1}(I-A^k)b\overline{r} \right\} \tag{4}$$

$\Psi = [I \quad -\Omega]$, and $\Omega : c_t = \Omega s_{t-1}$ is the linear rational expectations solution of the unconstrained system.

3.2. The Ensemble Kalman Filter

The EnKF is a recursive Bayesian filter originally developed in meteorology and engineering, adapted to macroeconomic state-space models by Boehl and Strobel (2024a). Unlike the standard Kalman Filter which is limited to linear-Gaussian settings, or the PF which suffers from degeneracy and computational inefficiency in high-dimensional applications, the EnKF approximates the filtering distribution through a Monte Carlo ensemble of state trajectories. Each ensemble member is propagated through the non-linear model, and the ensemble is then updated using observed data through a Kalman-like linear shift, rather than re-weighting as in the PF. This avoids the curse of dimensionality associated with particle degeneracy and enables practical estimation of large DSGE models with non-linear policy rules, such as the ELB.

The EnKF computes an approximate likelihood at each time step by fitting a multivariate normal distribution to the forecast ensemble and updating it using the Kalman gain. This approach preserves the key advantage of Kalman filters (recursive, real-time updating) while allowing the model to retain its non-linear features.

3.3. The DIME MCMC sampler

The Differential-Independence Mixture Ensemble (DIME) MCMC sampler proposed in Boehl (2022b) allows to efficiently estimate DSGE models with complex, potentially multimodal posteriors and expensive likelihood functions. By combining the strengths of differential evolution and adaptive independence sampling within a parallel ensemble framework, its offers rapid convergence, robustness to posterior shape, and ease of implementation. The sampler operates an ensemble of Markov chains, each considered as a separate "walker", that evolves in parallel. Each chain is updated by drawing proposals from a mixture of two transition kernels: a local transition kernel (inspired by Differential Evolution (DE)), and a global kernel based on an adaptive Independence Metropolis-Hastings proposal. This dual strategy allows for rapid convergence ("burn-in") and accurate posterior exploration, addressing the limitations of Random Walk Metropolis-Hastings (RWMH) and standard DE-MCMC, especially under irregular or multimodal posteriors.

Each iteration of the DIME sampler proceeds as follows:

1. Update the global proposal distribution based on current ensemble.
2. For each chain, randomly choose either the local or global kernel with probability χ (by default: 10% global and 90% local).
3. Propose a candidate draw.
4. Compute the factor weight adjustment to ensure detailed balance.
5. Evaluate the posterior density.
6. Accept or reject the candidate using a Metropolis criterion.

This mixture approach addresses the limitations of both kernels: the global kernel prevents chains from getting stuck in local maxima, and the local kernel provides high-quality, adaptive exploration during burn-in.

4. Model analysis

4.1. Data and priors

To estimate the model, data for various key macroeconomic variables of the Euro Area are gathered, for the period 1998Q1:2019Q4. They are used to construct seven observables: the real

GDP growth, real consumption growth, real investment growth, real wage growth, labor hours, log change of the GDP deflator, and the ECB interest rate on reserves.

The observables are constructed as follows.

$$
\begin{aligned}
\text{GDP} &= 100 * \Delta ln\,(\text{GDP/WAP}) \\
\text{CONS} &= 100 * \Delta ln\,(\text{CONS/WAP}) \\
\text{INV} &= 100 * \Delta ln\,((\text{GFCF-IP})/\text{WAP}) \\
\text{LAB} &= \text{demeaned}\,(100 * ln\,(100^*\text{Hours/WAP})) \\
\text{WAGE} &= 100 * \Delta ln\,(\text{Wage/Hours/GDPDEF}) \\
\text{IOR} &= \max\,(\text{Euribor}/4,\ 0) \\
\text{NIR} &= \min\,(\text{Euribor}/4,\ 0)
\end{aligned}
$$

GDP is the Gross Domestic Product of the Euro Area 19 (constant prices, seasonally adjusted), *CONS* is their Private Final Consumption Expenditure (constant prices, seasonally adjusted), *GFCF* their total Gross Fixed Capital Formation (constant prices, seasonally adjusted), *IP* the Intellectual Property Products (constant prices, seasonally adjusted), *Wage* the total Wages and Salaries (current prices, seasonally adjusted), *Hours* the total Hours worked (seasonally adjusted), *GDPDEF* the GDP Deflator (seasonally adjusted), *WAP* the Working-Age Population (Age 15-74), and *Euribor* the quarterly averages of the EURIBOR 3-Month. They are extracted from the OECD's Quarterly National Accounts database, except *WAP* (OECD Economic Outlook), and *Euribor* (ECB). For the investment series, starting from 2015, the data has exhibited several outlier observations, characterized by sharp spikes followed by offsetting movements in adjacent quarters. This heightened volatility largely stems from changes in R&D capitalization reporting, notably the transfer of intellectual property assets to affiliated entities and the relocation of Public Limited Companies, especially in Ireland. To mitigate the resulting distortions for the estimation process, I apply linear interpolation to smooth the investment data.

These observables are matched with the model variables through the following measurement equations:

$$\text{Real GDP growth} = \overline{\gamma} + (y_t - y_{t-1}) + z_t \tag{5}$$

$$\text{Real consumption growth} = \overline{\gamma} + (c_t - c_{t-1}) + z_t \tag{6}$$

$$\text{Real investment growth} = \overline{\gamma} + (i_t - i_{t-1}) + z_t \tag{7}$$

$$\text{Real wage growth} = \overline{\gamma} + (w_t - w_{t-1}) + z_t \tag{8}$$

$$\text{Labor hours} = \overline{l} + l_t \tag{9}$$

$$\text{Inflation} = \overline{\pi} + \pi_t \tag{10}$$

$$\text{Interest rate on reserves} = \left(\frac{\overline{\pi}}{\beta \gamma^{-\sigma_c}} \right) * 100 + r_t \tag{11}$$

$$\text{Negative interest rate} = 0 + v_{nir,t} \tag{12}$$

The variable

$$Z_t = e^{\gamma t + \dfrac{1}{1-\alpha} \tilde{z}_t} \tag{13}$$

is used to de-trend all non-stationary variables. $\tilde{z}_t$ is the log productivity process and is linearly de-trended. It is defined as an AR(1) process $\tilde{z}_t = \rho_z \tilde{z}_{t-1} + \sigma_z \epsilon_z$. Expressing z_t (the growth rate of technology) in deviations from γ, $z_t = \dfrac{1}{1-\alpha} (\rho_z - 1) \tilde{z}_t + \dfrac{1}{1-\alpha} \sigma_z \varepsilon_z$.

To keep the notion of the effective lower bound valid in the model, Boehl and Strobel (2024b) proposed to split the interest rate on reserves into two rates: the IOR+ $= \max\{IOR, 0\}$ and NIR $= \min\{IOR, 0\}$. The lower bound for the quarterly nominal rate is $\overline{r} = 0$, $\overline{\pi}$ is the gross inflation.

Figure 1 shows the data for the observables and the implied observables when the EnKF is applied. It illustrates that the EnKF applied to the non-linear DSGE model effectively tracks the dynamics of the actual data. The model fit is tight across all the series. The high-quality fit supports the validity of the non-linear model specification and the estimation methodology using EnKF.

Calibration of most parameters and priors are set to match the ones found in Smets and Wouters

(2007). However, in the light of the evidence of a stronger and more persistence deviance of the economy from its steady-state path during the Global Financial Crisis, Kulish et al. (2017) proposed to use a tighter prior for $\overline{\gamma}$. This helps to soften the pull down of $\overline{\gamma}$ to the sample mean (Boehl and Strobel, 2024a).

As shown in Table 1, the standard errors of the disturbances follow an inverse-gamma distribution with mean 0.10 and 0.25 degree of freedom. The persistence of the disturbances and of the mark-ups has a beta distribution with mean 0.5 and standard deviation 0.2. The growth rate trend and the steady-state labor growth rate are normally distributed with means 0.44 and 0 and standard deviations 0.05 and 2, respectively. The steady-state inflation rate follows a gamma distribution with mean 0.625 and standard deviation 0.1.

For the monetary policy rule parameters, a Normal distribution characterizes the long-run reaction on inflation (mean 1.5 and standard deviation 0.25), on the output gap (mean 0.125 and standard deviation 0.05), and the short-run reaction coefficient to the change in the output gap (mean 0.125 and standard deviation 0.05). The persistence of the monetary policy rule is beta distributed with mean 0.75 and standard deviation 0.1.

The rest of the priors are also the same as in Smets and Wouters (2007).

Figure 1: Model observables

4.2. Posterior estimates and model results

The posteriors are obtained using the DIME MCMC algorithm of Boehl (2022b). As in Boehl and Strobel (2024a), 500 iterations out of 2,500 are kept to represent the posterior distribution. The estimations are run on a Linux machine powered by 20 CPUs Intel Xeon E5, with 2.50GHz and 64 GB RAM. They are performed with the Pydsge package.[3]

Before discussing the main results of the estimation, I showcase the performance of the model and the convergence of the parameters. Figures 2 and 3 show the traceplots of selected parameters following the use of the DIME MCMC method for posterior sampling. They show the 200 chains used for estimating the model, and for each model the last 500 iterations are kept. Thus, the posterior contains $500 \times 200 = 10{,}000$ parameter draws. The traceplots display the sampled values of the parameter across iterations. As observed in the right panels of the two figures, the traceplots fluctuate around a stable mean without any trends or drift.[4] This suggests that the chain has converged to the target distribution. Also, it is noted that the first initial iterations show instability as expected, but the convergence is quickly achieved for all the parameters from the 1,500th iteration. Hence, the 2,500 iterations are sufficiently enough to achieve convergence. For comparison, the trace of the chains using the typical 5,000 iterations is shown in Figure 3 for selected parameters. The plots reveal that the convergence is achieved very early.[5] Given those results and the estimation times of roughly 7,5 hours with 2,500 iterations and slightly less than 15,5 hours with 5,000 iterations, the DIME MCMC method can be regarded as efficient for fast convergence.

[3] Available at https://pydsge.readthedocs.io/en/latest/
[4] All the figures are available in the appendix C.
[5] All the figures are available in the same appendix.

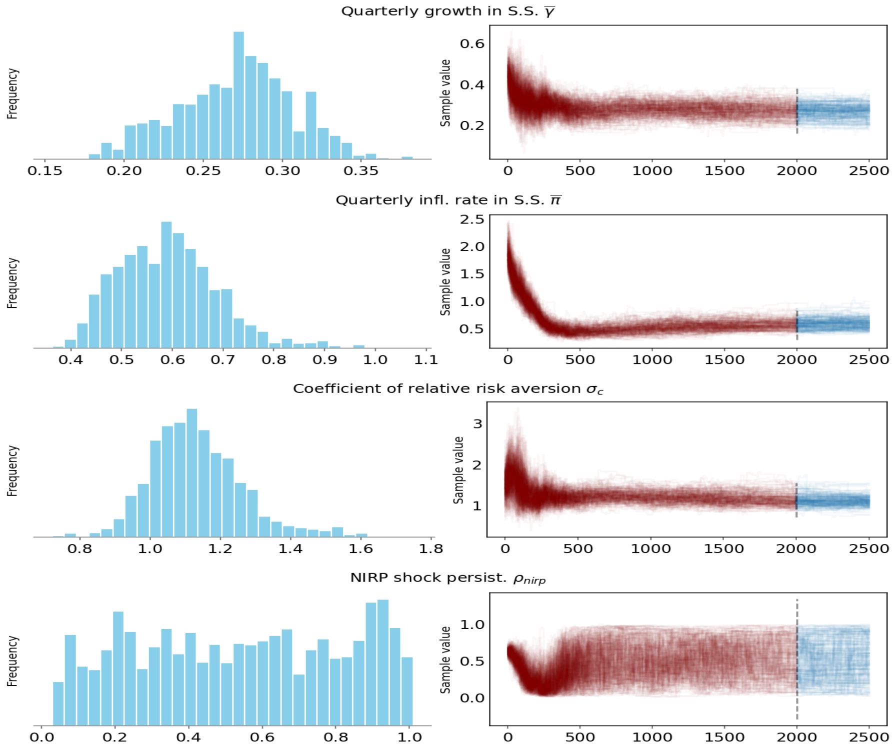

Figure 2: Traceplots of the 200 DIME chains for selected parameters, with 2,500 iterations.

Note: *The left panel shows a KDE of the parameter distribution. The right displays the trace of each of the chains over time. The red area of the trace corresponds to the burn-in (high-density region of the posterior, discarded) and the blue is the part kept to represent the posterior.* $\overline{\gamma}$ *and* σ_c *have a normal distribution,* $\overline{\pi}$ *is gamma-distributed, and* ρ_{nirp} *beta-distributed.*

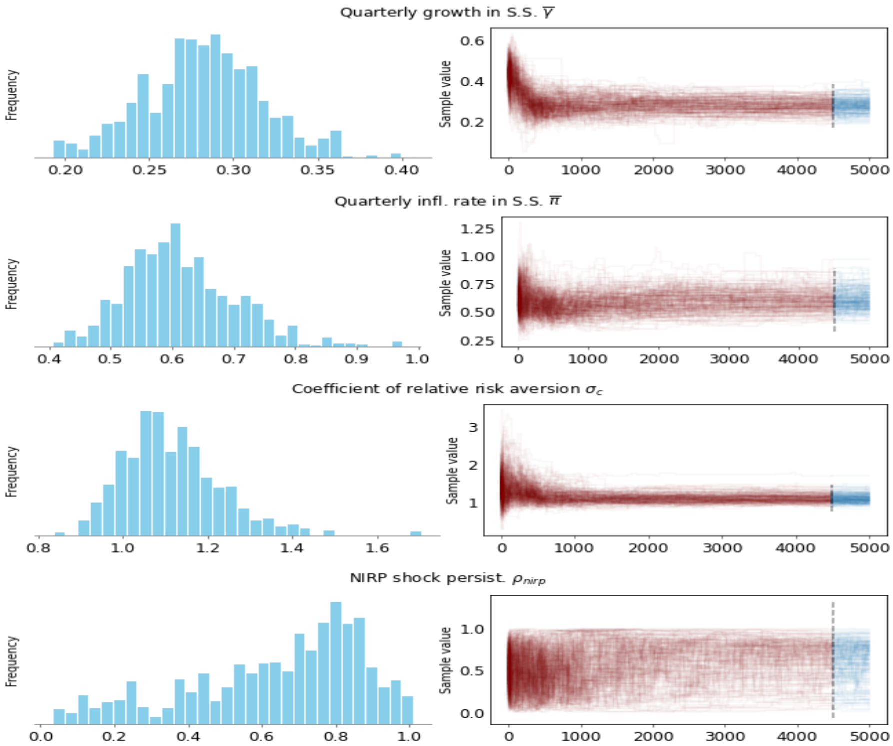

Figure 3: Traceplots of the 200 DIME chains for selected parameters, with 5,000 iterations.

Note: *The left panel shows a KDE of the parameter distribution. The right displays the trace of each of the chains over time. The red area of the trace corresponds to the burn-in (high-density region of the posterior, discarded) and the blue is the part kept to represent the posterior.* $\overline{\gamma}$ *and* σ_c *have a normal distribution,* $\overline{\pi}$ *is gamma-distributed, and* ρ_{nirp} *beta-distributed.*

Table 1: Prior and Posterior distributions of structural parameters and disturbances (non-linear vs linear model)

		Prior			**Posterior (Non-linear model)**					**Posterior (Linear model)**				
Parameter		Distrib.	Mean	sd/df	Mode	Mean	sd	5 perc.	95 perc.	Mode	Mean	sd	5 perc.	95 perc.
Coefficient of relative risk aversion	σ_c	normal	1.500	0.375	1.206	1.185	0.116	1.005	1.380	1.081	1.206	0.169	0.920	1.471
Elasticity of Labor Supply	σ_l	normal	2.000	0.750	-0.931	-0.939	0.043	-1.000	-0.878	1.346	0.593	1.212	-0.999	2.166
Discount factor	β_{tpr}	gamma	0.250	0.100	0.256	0.196	0.075	0.067	0.300	0.099	0.139	0.058	0.045	0.223
Habit formation	h	beta	0.700	0.100	0.673	0.705	0.047	0.621	0.777	0.808	0.729	0.071	0.613	0.842
Investment adjust. costs	S''	normal	4.000	1.500	5.049	5.507	1.076	3.907	7.258	2.405	3.416	1.291	1.238	5.460
Indexation prices	ι_p	beta	0.500	0.150	0.129	0.211	0.094	0.064	0.354	0.472	0.466	0.164	0.167	0.701
Indexation wages	ι_w	beta	0.500	0.150	0.289	0.356	0.110	0.179	0.532	0.662	0.484	0.164	0.211	0.756
Capital production share	α	normal	0.300	0.050	0.061	0.076	0.017	0.051	0.106	0.178	0.178	0.029	0.126	0.221
Degree of price stickiness	ζ_p	beta	0.500	0.100	0.707	0.719	0.073	0.595	0.831	0.446	0.515	0.105	0.340	0.688
Degree of wage stickiness	ζ_w	beta	0.500	0.100	0.775	0.788	0.041	0.724	0.856	0.706	0.616	0.110	0.431	0.788
Fixed costs	Φ_p	normal	1.250	0.125	1.448	1.323	0.114	1.144	1.507	1.099	1.129	0.100	0.958	1.286
Capital utilization cost	ψ	beta	0.500	0.150	0.704	0.555	0.139	0.359	0.827	0.652	0.565	0.181	0.280	0.876
Monetary Policy: Inflation response	ϕ_π	normal	1.500	0.250	1.361	1.329	0.255	0.926	1.739	1.400	1.753	0.219	1.420	2.130
Monetary Policy: Output gap response	ϕ_y	normal	0.125	0.050	0.274	0.262	0.034	0.207	0.319	0.015	0.073	0.060	-0.006	0.179
Monetary Policy: Diff. output gap response	ϕ_{dy}	normal	0.125	0.050	0.009	-0.001	0.028	-0.049	0.045	0.138	0.115	0.047	0.034	0.190
Monetary Policy: Interest rate smoothing	ρ	beta	0.750	0.100	0.896	0.904	0.018	0.875	0.933	0.697	0.730	0.100	0.572	0.895
Monetary Policy shock persist.	ρ_r	beta	0.500	0.200	0.303	0.305	0.080	0.183	0.441	0.604	0.462	0.231	0.092	0.828
NIRP shock persist.	ρ_{nirp}	beta	0.500	0.200	0.963	0.962	0.019	0.934	0.991	0.565	0.477	0.240	0.094	0.862
Govt consumption shock persist.	ρ_g	beta	0.500	0.200	0.990	0.972	0.017	0.953	0.996	0.354	0.526	0.250	0.155	0.950
Technology shock persist.	ρ_z	beta	0.500	0.200	0.985	0.946	0.038	0.886	0.993	0.580	0.530	0.248	0.189	0.995
Risk premium shock persist.	ρ_u	beta	0.500	0.200	0.949	0.940	0.012	0.919	0.959	0.759	0.763	0.270	0.321	0.998
Price markup shock persist.	ρ_p	beta	0.500	0.200	0.691	0.692	0.133	0.499	0.881	0.356	0.440	0.223	0.053	0.772
Wage markup shock persist.	ρ_w	beta	0.500	0.200	0.555	0.529	0.143	0.315	0.766	0.980	0.770	0.296	0.277	0.999
MEI shock persist.	ρ_i	beta	0.500	0.200	0.092	0.087	0.056	0.008	0.160	0.093	0.075	0.062	0.004	0.152
MA(1) price markup shock	μ_p	beta	0.500	0.200	0.540	0.478	0.133	0.263	0.695	0.568	0.522	0.239	0.129	0.899
MA(1) wage markup shock	μ_w	beta	0.500	0.200	0.480	0.429	0.151	0.162	0.657	0.505	0.425	0.225	0.042	0.767
Response of g_t to ε_t^z	ρ_{gz}	normal	0.500	0.250	1.030	1.148	0.197	0.864	1.471	0.682	0.520	0.245	0.118	0.917
Govt consumption shock	σ_g	inv.gamma	0.100	0.250	0.419	0.402	0.047	0.326	0.481	0.049	0.058	0.025	0.023	0.091
Risk premium shock	σ_u	inv.gamma	0.100	0.250	0.191	0.224	0.042	0.162	0.293	0.046	0.065	0.033	0.025	0.109
Technology shock	σ_z	inv.gamma	0.100	0.250	0.324	0.349	0.034	0.292	0.398	0.037	0.056	0.024	0.024	0.088
Monetary Policy shock	σ_r	inv.gamma	0.100	0.250	0.089	0.093	0.008	0.080	0.108	0.042	0.052	0.021	0.025	0.080
NIRP shock	σ_{nirp}	inv.gamma	0.100	0.250	0.010	0.010	0.001	0.009	0.012	0.040	0.053	0.022	0.025	0.085
Price markup shock	σ_p	inv.gamma	0.100	0.250	0.147	0.131	0.019	0.101	0.161	0.078	0.052	0.021	0.022	0.080
Wage markup shock	σ_w	inv.gamma	0.100	0.250	0.146	0.141	0.018	0.111	0.170	0.046	0.057	0.025	0.025	0.088
MEI shock	σ_i	inv.gamma	0.100	0.250	1.449	1.417	0.120	1.224	1.604	1.410	1.217	0.137	0.991	1.433
Quarterly growth in S.S.	$\overline{\gamma}$	normal	0.440	0.050	0.245	0.221	0.034	0.172	0.271	0.272	0.275	0.038	0.215	0.336
Log hours worked in S.S.	$\overline{l}$	normal	0.000	2.000	-0.401	-0.528	1.921	-3.567	2.653	-3.202	-0.722	1.850	-3.777	2.023
Quarterly infl. rate in S.S.	$\overline{\pi}$	gamma	0.625	0.100	0.570	0.566	0.068	0.449	0.666	0.489	0.554	0.096	0.400	0.702
Mean acceptance fraction			**NA**				**0.058**					**0.124**		

The mean values of the posterior of the persistence parameters in Table 1 (estimated with the non-linear solution method on the full sample 1998Q1-2019Q4) reveal that the data are remarkably informative on the stochastic processes of the shock disturbances, when compared with their prior counterpart. The most persistent shock for the Euro Area is the government spending shock (ρ_g), its estimated AR(1) coefficient is 0.97. The negative interest rate policy (NIRP) shock (ρ_{nirp}) is the second most persistent one with an AR(1) estimate of 0.96. The shocks on the technology and risk premium processes are also persistent ($\rho_z = 0.95$ and $\rho_u = 0.94$ respectively).

For the main parameters describing the behavior of households and firms in the model, the posteriors for the degree of price stickiness ζ_p (0.72) and wage stickiness ζ_w (0.79) are a little higher than their assumed mean prior. The elasticity of the cost of changing investment S'' displays a posterior mean (5.51) higher than the prior, hinting to the fact that the response of investment to changes in the value of capital will be slower. The fixed cost parameter Φ_p is also found to have a higher posterior estimate (1.32), while the share of the capital in production α has a very low posterior (0.08).

For the monetary policy reaction function (Eq. (A.2)), the results show that the central bank reacts strongly to the output (estimated mean of $\phi_y = 0.26$), as opposed to a change in the output for which no reaction is found ($\phi_{dy} = -0.00$). The estimated mean of the long-run reaction coefficient to inflation is $\phi_\pi = 1.33$, which is slightly smaller than the assumed prior of 1.5. The coefficient on the lagged interest rate has an estimated value of $\rho = 0.9$, thereby hinting at a substantial high degree of the interest rate smoothing.

The estimate of the quarterly growth rate of the trend $\overline{\gamma}$ is around 0.22, a value smaller than the average of the growth rate of the output over the sample (0.3). The steady-state inflation rate $\overline{\pi}$ shows a posterior mean of 0.57, or about 2.3% on an annual basis. The annual steady-state growth rate of the labor $\bar{l}$ is about -2.1%.

The posterior estimates of the linear model (that is, when ignoring the ELB) are displayed in Table 1. The posterior values for the shock persistence are around their prior values. In this model, the marginal efficiency of investment (MEI) shock however has a very low persistence ($\rho_i = 0.08$). This suggests that such a shock is short-lived and might not have a lasting impact on the other variables. The other shock processes still have posterior estimates of about half the prior values.

I now use the estimated posterior parameters from the full sample (which includes the ELB) to offer some interpretation of the Euro Area business cycles. The CRRA σ_c mean estimate is 1.19 in the full sample, and close to 1. Slightly above this value, variations in hours worked have no effect on consumption, through the Euler equation (Eq. A.3). This estimate is higher in the linear model (1.21), suggesting that the model attributes a stronger risk aversion to agents. The drop in the log hours worked in steady-state $\bar{l}$ (from the prior value of 0 to -0.53), is thus refrained from pulling excessively down the consumption level, thanks to the muted effect mentioned above. The estimated $\bar{l}$ is lower in the linear model (-0.72) than in the full sample. The difference with the non-linear model could be related to the opposite effect of the elasticity of labor supply σ_l (positive in the linear model, negative in the non-linear model). σ_l is -0.94 in the full sample with the ELB, its value is 0.60 in the linear model. This observation means that the labor supply did react positively to rising wages in the linear model, but the sharp drop in the labor supply following the GFC was not prevented by the better wages growth, as shown in Figure 1. The degree of price and wage stickiness are $\zeta_p = 0.72$ and $\zeta_w = 0.79$ in the full sample with ELB, respectively. Their respective indexation values are $\iota_p = 0.21$ and $\iota_w = 0.36$. In the full sample estimated linearly, they are $\zeta_p = 0.52$, $\zeta_w = 0.62$, $\iota_p = 0.47$ and $\iota_w = 0.48$. This suggests that the Phillips Curve in the Euro Area has flattened according to the full sample including the ELB. Indeed, from the linear model to the non-linear model, the degree of price and wage stickiness have increased, while there is a fall in the degree of their indexation (Smets and Wouters, 2007).

The central bank appeared to have increased its response to the output gap, with $\phi_y = 0.26$ (against 0.07 in the linear model). Its reaction to output growth is absent ($\phi_{dy} = -0.00$). While the estimates suggest that the central bank has a lower response to inflation in sample with the ELB ($\phi_\pi = 1.33$, against 1.75 in the linear cases), this reaction remains the strongest in the policy reaction function. These findings are consistent with the view that the ECB has fulfilled on its main mandate of price stability. The estimates confirmed that the ECB had gradually adjusted its policy interest rate, as shown by the very high interest rate smoothing parameter ρ. The persistence of some shocks (NIRP, Government consumption, technology, and risk premium) increased substantially in the full sample including the ELB. These shocks are likely to play a pivotal role in explaining the variations in the aggregate variables. Most of these results are in line with the previous literature that account for the ELB (Lindé et al., 2016; Kulish et al., 2017; Boehl and Strobel, 2024a).

Before using the parameters estimates for a policy analysis exercise, a comparative analysis with the closest papers on the ELB in the Euro Area is proposed in the Table 2.

Table 2: Comparison of of the mean values of the posterior distributions

			This paper	**BS24**	**KP3RV16** (mode)	**AGBM21**
Coefficient of relative risk aversion	σ_c	normal	1.185	1.435	1.47	1.24
Elasticity of Labor Supply	σ_l	normal	-0.939	0.627	2.32	0.84 (G)
Discount factor	β_{tpr}	gamma	0.196	0.207	NA	NA
Habit formation	h	beta	0.705	0.607	0.88	0.77
Investment adjust. costs	S''	normal	5.507	4.904	NA	NA
Indexation prices	ι_p	beta	0.211	0.325	NA	0.63
Indexation wages	ι_w	beta	0.356	0.313	NA	0.56
Capital production share	α	normal	0.076	0.272	NA	NA
Degree of price stickiness	ζ_p	beta	0.719	0.856	NA	0.13
Degree of wage stickiness	ζ_w	beta	0.788	0.777	0.97	0.32
Fixed costs	Φ_p	normal	1.323	1.730	NA	NA
Capital utilization cost	ψ	beta	0.555	0.307	NA	NA
Monetary Policy: Inflation response	ϕ_π	normal	1.329	1.653	2.23	NA
Monetary Policy: Output gap response	ϕ_y	normal	0.262	0.220	0.08	NA
Monetary Policy: Diff. output gap response	ϕ_{dy}	normal	-0.001	0.115	NA	NA
Monetary Policy: Interest rate smoothing	ρ	beta	0.904	0.919	NA	NA
Monetary Policy shock persist.	ρ_r	beta	0.305	0.375	0.85	0.40
NIRP shock persist.	ρ_{nirp}	beta	0.962	0.984	NA	NA
Govt consumption shock persist.	ρ_g	beta	0.972	0.966	0.95	NA
Technology shock persist.	ρ_z	beta	0.946	0.954	0.95	NA
Risk premium shock persist.	ρ_u	beta	0.940	0.962	0.94	0.80
Price markup shock persist.	ρ_p	beta	0.692	0.713	NA	NA
Wage markup shock persist.	ρ_w	beta	0.529	0.744	NA	NA
MEI shock persist.	ρ_i	beta	0.087	0.605	NA	NA
MA(1) price markup shock	μ_p	beta	0.478	0.733	0.24	NA
MA(1) wage markup shock	μ_w	beta	0.429	0.518	NA	NA
Response of g_t to ε_t^z	ρ_{gz}	normal	1.148	1.117	NA	NA
Govt consumption shock	σ_g	inv.gamma	0.402	0.234	0.06 (G)	NA
Risk premium shock	σ_u	inv.gamma	0.224	0.176	0.18 (G)	0.21
Technology shock	σ_z	inv.gamma	0.349	0.217	0.02 (G)	NA
Monetary Policy shock	σ_r	inv.gamma	0.093	0.080	0.09 (G)	0.11
NIRP shock	σ_{nirp}	inv.gamma	0.010	0.006	NA	NA
Price markup shock	σ_p	inv.gamma	0.131	0.147	NA	NA
Wage markup shock	σ_w	inv.gamma	0.141	0.108	NA	NA
MEI shock	σ_i	inv.gamma	1.417	0.328	NA	NA
Quarterly growth in S.S.	$\bar{\gamma}$	normal	0.221	0.297	NA	NA
Log hours worked in S.S.	$\bar{l}$	normal	-0.528	3.082	NA	NA
Quarterly infl. rate in S.S.	$\bar{\pi}$	gamma	0.566	0.576	NA	NA

Note: *BS24=Boehl and Strobel (2024b), KP3RV16=Kollmann et al. (2016), AGBM21=Andrade et al. (2021).* G: Gamma distribution

Since this paper follows the solution method and model used in Boehl and Strobel (2024a,b), the posterior estimates closely mimic the ones found in Boehl and Strobel (2024b). The noticeable differences concern the elasticity of labor supply σ_l and the investment shock σ_i. For the latter, the difference could be traced back to the behavior of the investment data. As explained in the data section, the investment data series has exhibited large unusual volatility which after smoothing the data, were still non-negligible for the estimation process. For the former, the negative σ_l could be counter-intuitive but given the post-crisis period and the negative interest rate environment, a potential explanation is that real wages rose while employment fell (Figure 1). Results from the paper are also generally in line with the findings of Kollmann et al. (2016) and Andrade et al. (2021).

4.3. The role of the NIRP and other shocks

In the following exercises, I focus on the implications of the negative interest rate policy, as well as the interest rate, risk premium, and government consumption shocks for the fluctuations of the macro variables. Particularly, the performance of the non-linear model as opposed to the linear model is assessed.

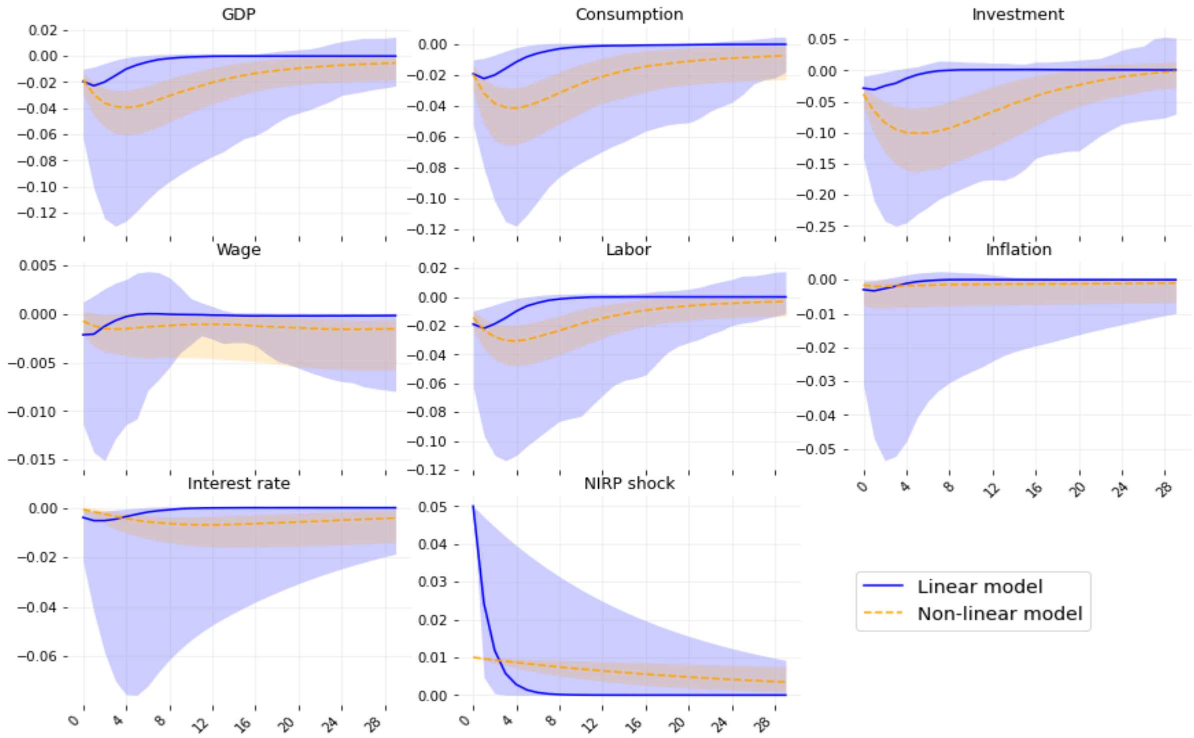

Figure 4: Impulse responses to a NIRP shock

Note: Medians over 250 simulations drawn from the posterior. The continuous blue line corresponds to the linear model, the dashed orange line corresponds to the non-linear model. Shaded area represents the 90% confidence band. The shock size in each model is the estimated posterior mean standard deviation of the shock.

Figure 4 shows the responses of the macro variables to the estimated posterior mean standard deviation impulse from the NIRP shock, for each model. The linear model has a bigger estimated size of the NIRP shock, which is fivefold the one in the non-linear model. However, the shock drops strongly in the linear model within the first 4 quarters, while the non-linear model shows a more persistent effect of the NIRP shock. As a result, most variables in the non-linear model do not come back to their steady-state level, even after 30 quarters. The shock induces a strong drop in the GDP and consumption as much as roughly 0.04%. The huge collapse in investment reflects the observed increase in the posterior estimate of the investment adjustment costs (from 3.42 to 5.51). Labor follows the pattern seen in GDP and consumption, and the drop in labor and consumption hints at strong non-separabilities between them (σ_c is not weak enough to prevent a drop in consumption following the drop in labor). Wage and inflation have a steadier dynamic. In the non-linear model, the NIRP has a more deflationary impact. The persistence of wage and inflation can be interpreted as a success in anchoring inflation expectations under the NIRP shock. The nominal interest rate falls initially and stay very low for the subsequent periods, even though there is a gradual increase. This is compatible with the lower-for-longer policy by the ECB, consistent with the findings of Bletzinger and Wieland (2017).

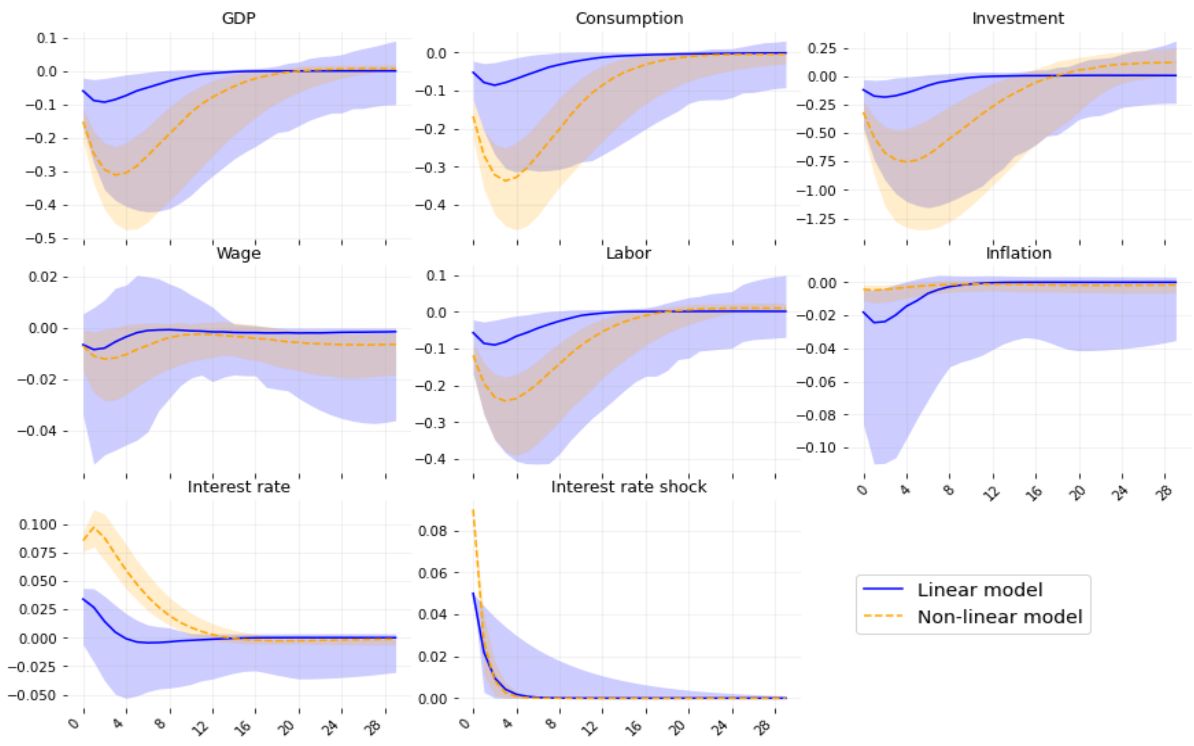

Figure 5: Impulse responses to an Interest rate shock

Note: Medians over 250 simulations drawn from the posterior. The continuous blue line corresponds to the linear model, the dashed orange line corresponds to the non-linear model. Shaded area represents the 90% confidence band. The shock size in each model is the estimated posterior mean standard deviation of the shock.

Following a shock to the nominal interest rate (Figure 5), the behavior of the shock process reveals a very similar pattern in both models (the shock size is 0.05 in the linear case and 0.09 in the non-linear one). The hike of the policy rate in the non-linear model is large, almost 50 b.p. more than in the linear model. Because of the short-lived shock, the policy rate returns to the steady-state level fast, after 3 years in the non-linear model (and only one in the linear setting). Similarly to the NIRP shock, the non-linear model captures a more volatile and stronger response to the interest rate shock, especially in terms of real activity (GDP, consumption, labor, and investment).

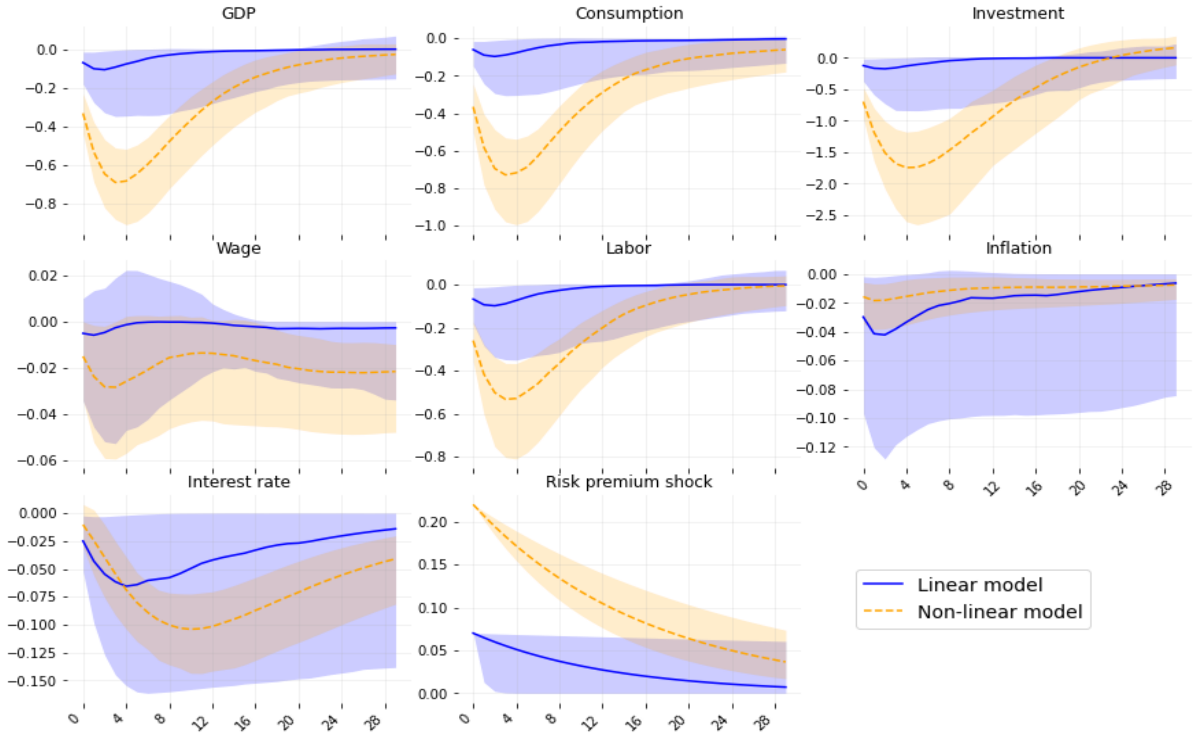

Figure 6: Impulse responses to a Risk premium shock

Note: Medians over 250 simulations drawn from the posterior. The continuous blue line corresponds to the linear model, the dashed orange line corresponds to the non-linear model. Shaded area represents the 90% confidence band. The shock size in each model is the estimated posterior mean standard deviation of the shock.

The risk premium shock has been prominently featured in the literature (Christiano et al., 2015; Smets and Wouters, 2007) as the key driver of the great recession. Its estimated size in the non-linear model is 0.22 and 0.07 in the linear one. Throughout the 30 quarters, the shock in the non-linear model is highly persistent and remains significantly different from zero (Figure 6). This reflects elevated risk premiums during and after the financial crisis, with investors demanding higher returns due to fears of defaults, sovereign debt crises, and general financial instability. The response of the variables to this shock is substantially large in the non-linear setting. The deep and prolonged contractions seen in GDP, consumption, investment, and labor reflect how the Euro Area's financial markets and economy were significantly impacted by elevated risk premiums. However, the dynamic of inflation stays very modest. A potential explanation is the somewhat estimated flat Phillips curve mentioned above.

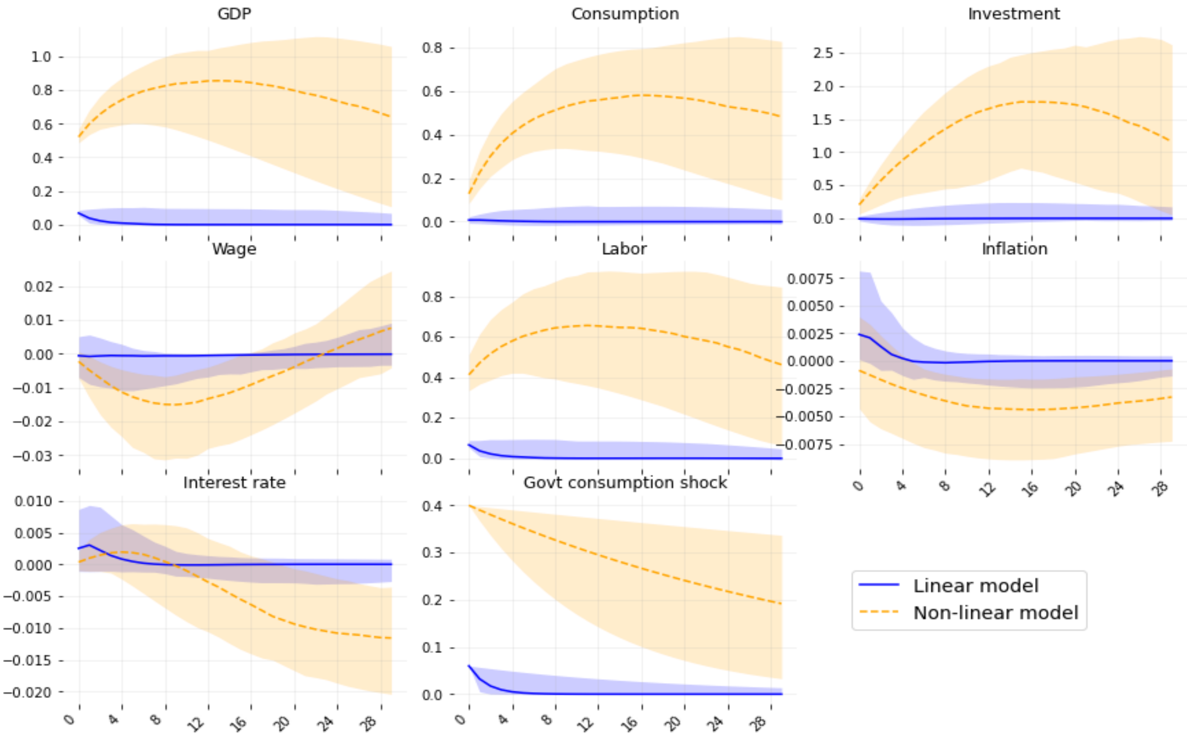

Figure 7: Impulse responses to a Government consumption shock

Note: Medians over 250 simulations drawn from the posterior. The continuous blue line corresponds to the linear model, the dashed orange line corresponds to the non-linear model. Shaded area represents the 90% confidence band. The shock size in each model is the estimated posterior mean standard deviation of the shock.

The Government consumption shock is the most persistent one. Its estimated size in the non-linear model (0.40) is also very large, relatively to the linear model (0.06), more than 6 times. The implications are depicted in Figure 7: the effects of the shock look non-existent in the linear setting, while they are very strong and volatile with the non-linear model. In particular, the shock induced a positive and persistent reaction from GDP, consumption, investment, and labor. This finding is in line with many empirical evidence of the amplified effect of the fiscal multiplier in the Euro Area at the ELB (Amendola et al., 2020; Flotho, 2014).

In summary, there is a clear evidence of better empirical performance of a DSGE model that is estimated with the ELB constraint, relatively to a model in which the constraint is ignored. Using the parameter estimates from both models, I assessed and compared how economic shocks, such as the NIRP, risk premiums, and government consumption shocks, have impacted key macroeconomic variables. The key findings show that accounting for the ELB implies a lower risk aversion, a flattened Phillips curve in the Euro Area, a strong response of the ECB to the output gap and to the inflation. Additionally, the IRFs show that the NIRP shock led to a drop in the GDP, consumption, investment, and labor. A risk premium shock has a similar effect on the variables, albeit stronger. Relatively to the linear model, the Government consumption shock has the largest and most persistent (positive) effect on GDP and other variables, consistent with the theory of a stronger fiscal multiplier during periods of low interest rates. The pattern of the nominal interest rate makes evident the ECB's policy of lower-for-longer under the ELB regime.

5. Conclusion

The GFC and the negative interest rate period it led to had challenged the conventional macroeconomic tools that policymakers and academics rely on. Among the multiple challenges, the one posed by the modeling of the occasionally binding constraint on the nominal interest rates requires a great deal of treatment. Using the fast solution method of Boehl (2022a) that is based on a piecewise linear perturbation framework, this paper has studied the business cycles in the Euro Area, with a focus on the economic implications of the ELB.

The findings of the paper are consistent with the previous literature that include the ELB in their analyses, hence reinforcing the importance of considering the ELB in the estimation. Namely, many parameter estimates from the full sample exhibit substantial differences with those from the pre-crisis sample or the linear model. This result highlights the added value of using the estimates from the full sample to perform economic analysis of the Euro Area. In this regard, the simulation exercise of various economic shocks confirms that a linear model systematically underestimates the responses of the macro variables. The results from the non-linear model show a stronger response of the variables.

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Appendices

A. The model

The canonical model of Smets and Wouters (2007) is considered as the baseline model to analyse the business cycles dynamics of the Euro Area. The model is adjusted to deal with an economy which is at the effective lower bound. The model is comprised of households, intermediate good producers, final good producers, the government, and the monetary authority.

Households maximize a nonseparable utility function, which depends on consumption of goods and on labor supply, over an infinite life horizon. Consumption enters the utility function relative to a time-varying external habit variable. Labor is differentiated by unions, with some monopoly power over wages. Intermediate good firms transform labor and capital from households into intermediate goods, which are sold to final good firms. Final good producers are monopolistically competitive.
Taking into account the ELB on the nominal interest rate, the interest rate policy set by the central bank is:

$$r_t = \max\left\{\overline{r}, r_t^n\right\} + v_{nir,t} \tag{A.1}$$

where $\overline{r}$ is the lower bound value. The stochastic negative shock $v_{nir,t}$ allows for policy disturbances to shift the central bank's rate on reserves into negative territory, while keeping at the same time agents' expectation at the classic lower bound on the nominal rate. The notional rate follows the feedback rule:

$$r_t^n = \rho r_{t-1}^n + (1-\rho)\left(\phi_\pi \pi_t + \phi_y \tilde{y}_t\right) + \phi_{dy}\Delta\tilde{y}_t + v_{r,t} \tag{A.2}$$

where $\tilde{y}_t$ is the output gap, $\Delta\tilde{y}_t = \tilde{y}_t - \tilde{y}_{t-1}$ its growth rate, ρ captures the degree of interest rate smoothing, ϕ_π, ϕ_y, and ϕ_{dy} are feedback coefficients. $v_{r,t}$ is the regular interest rate shock when the economy is away from the ELB. However, it is interpreted as a forward guidance whenever the nominal interest rate is zero, because it may not directly affect the level of the nominal rate, but it affects the expected path of the notional rate and hence alter the expected duration of the lower bound spell.

The linearized equilibrium conditions are presented below. The variables of the model are the log-deviation from their steady-state values.

The consumption Euler equation gives the dynamics of consumption:

$$\begin{aligned} c_t = {} & c_1\left(c_{t-1}-z_t\right)+(1-c_1)E_t\left[c_{t+1}-z_{t+1}\right]+c_2\left(l_t-E_t\left[l_{t+1}\right]\right) \\ & -c_3\left(r_t-E_t\left[\pi_{t+1}\right]+u_t\right) \end{aligned} \tag{A.3}$$

where c_t and l_t are respectively the consumption and labor supply of the households, $c_1 = \dfrac{h/\gamma}{(1+h/\gamma)}$, $c_2 = \dfrac{(\sigma_c-1)(W^hL/C)}{\sigma_c(1+h/\gamma)}$, $c_3 = \dfrac{(1-h/\gamma)}{(1+h/\gamma)\sigma_c}$. z_t is the exogenous process of total factor productivity. h, σ_c, γ, r_t, π_t, and u_t represent the degree of external habit formation in consumption, the CRRA (coefficient of relative risk aversion), the steady-state growth rate of the economy, the nominal interest rate, the inflation rate, and an exogenous risk premium shock, respectively. u_t drives a wedge between the central bank interest rate and the return on assets held by households.

The investment Euler equation gives the dynamics of investment:

$$i_t = i_1\left(i_{t-1}-z_t\right)+(1-i_1)E_t\left[i_{t+1}-z_{t+1}\right]+i_2q_t+v_{i,t} \tag{A.4}$$

where i_t and q_t denote respectively investment in physical capital and the real value of existing capital stock, $i_1 = \dfrac{1}{(1+\beta\gamma^{(1-\sigma_c)})}$, $i_2 = \dfrac{1}{(1+\beta\gamma^{(1-\sigma_c)}\gamma^2S'')}$. β is the discount factor applied by the households, S'' the steady-state value of the second derivative of the capital adjustment cost function, and $v_{i,t}$ a disturbance to the investment-specific technology process. Boehl and Strobel (2024a) refer to the latter as a shock on the MEI, which can be considered as a reduced-form way of capturing financial friction, because it drives a wedge between aggregate savings and aggregate investment (Justiniano et al., 2011).

The accumulation of physical capital is given by:

$$\overline{k}_t = k_1\left(\overline{k}_{t-1} - z_t\right) + (1-k_1)i_t + k_2 v_{i,t} \tag{A.5}$$

where $\overline{k}_t$ denotes physical capital. $k_1 = \dfrac{(1-\delta)}{\gamma}$, $k_2 = (1-k_1)(1+\beta\gamma^{(1-\sigma_c)}\gamma^2 S'')$. δ is the depreciation rate of the capital.

The no-arbitrage condition between the rental rate of capital r_t^k and the risk free interest rate r_t is given by:

$$q_t = q_1 E_t\left[q_{t+1}\right] + (1-q_1)E_t\left[r_{t+1}^k\right] - \left(r_t - E_t\left[\pi_{t+1}\right] + u_t\right) \tag{A.6}$$

where $q_1 = \beta\gamma^{-\sigma_c}(1-\delta) = \dfrac{(1-\delta)}{r^k + (1-\delta)}$.

Since using effectively the physical capital happens only after a quarter after being installed, and it requires some utilization costs that are function of the rental rate of capital r_t^k, the effectively used capital k_t and the physical capital stock are linked through the following relationship:

$$k_t = \frac{(1-\psi)}{\psi} r_t^k + \overline{k}_{t-1} \tag{A.7}$$

where $\psi \in (0,1)$ is the parameter controlling for the costs of capital utilization.
On the supply side, the aggregate production function is the following:

$$y_t = \Phi\left(\alpha k_t + (1-\alpha)l_t\right) + (\Phi - 1)\frac{1}{1-\alpha}\tilde{z}_t \tag{A.8}$$

where α is the share of capital in the production function, $\tilde{z}_t$ is the linearly de-trended log productivity process. Φ is introduced because of the assumption of fixed cost in production. Real marginal costs for producing firms is given by:

$$mc_t = w_t + \alpha \left(l_t - k_t\right) \tag{A.9}$$

where w_t is the real wage set by unions. Cost minimization for intermediate good firms yields:

$$k_t = w_t - r_t^k + l_t \tag{A.10}$$

The aggregate resource constraint depends on an exogenous demand shifter g_t (comprised of exogenous variations in government spending and net exports), and the resource costs of capital utilization:

$$y_t = g_y g_t + c_y c_t + i_y i_t + r_y r_t^k - \frac{1}{1-\alpha}\tilde{z}_t \tag{A.11}$$

where $g_y = \dfrac{G}{Y}$ is the steady-state exogenous spending-output ratio, $c_y = \dfrac{C}{Y}$ the steady-state share of consumption in output, $i_y = \dfrac{I}{Y}$ the steady-state exogenous investment-output ratio, and $r_y = \dfrac{R^k K}{Y}\dfrac{1-\psi}{\psi}$. R^k is the steady-state rental rate of capital.

The assumption of monopoly power of final good producers and the presence of price stickiness as in Calvo (1983) yield the following New Keynesian Phillips Curve (NKPC):

$$\pi_t = \pi_1 \pi_{t-1} + \pi_2 E_t\left[\pi_{t+1}\right] + \pi_3 mc_t + v_{p,t} \tag{A.12}$$

where $\pi_1 = \dfrac{\iota_p}{\left(1+\beta\gamma^{(1-\sigma_c)}\iota_p\right)}$, $\pi_2 = \dfrac{\beta\gamma^{(1-\sigma_c)}}{\left(1+\beta\gamma^{(1-\sigma_c)}\iota_p\right)}$,
$\pi_3 = \dfrac{\left(1-\zeta_p\beta\gamma^{(1-\sigma_c)}\right)\left(1-\zeta_p\right)}{\left(1+\beta\gamma^{(1-\sigma_c)}\iota_p\right)\zeta_p\left((\Phi-1)\varepsilon_p+1\right)}$. ι_p is the degree of indexation to past inflation, ζ_p characterizes the degree of price stickiness, ε_p is the curvature of the Kimball goods market aggregator, $\Phi-1$ is the share of fixed costs in production which relates to the steady-state price mark-up through a zero-profit condition in equilibrium. A higher ε_p slows down the speed of adjustment of prices given that it increases the strategic complementarity between price setters. $v_{p,t}$ is the exogenous fluctuations in the price mark-up.

Similarly to the good market, in the monopolistic competitive labor markets, unions set the real wages. They bundle households' labor services and sell them to firms with a mark-up over the frictionless wage w_t^h:

$$w_t^h = \frac{1}{1-h}\left(c_t - \frac{y}{\gamma}c_{t-1} + \frac{y}{\gamma}z_t\right) + \sigma_l l_t \tag{A.13}$$

Assuming the Calvo (1983) type nominal rigidities and partial wage indexation, the wage Phillips Curve is written as follows.

$$\begin{aligned} w_t = w_1(w_{t-1} - z_t + \iota_w \pi_{t-1}) + (1-w_1)\left(E_t\left[w_{t+1} + z_{t+1} + \pi_{t+1}\right]\right) \\ - w_2\pi_t + w_3\left(w_t^h - w_t\right) + \upsilon_{w,t} \end{aligned} \tag{A.14}$$

where $w_1 = \frac{1}{1+\beta\gamma^{(1-\sigma_c)}}$, $w_2 = \frac{1+\beta\gamma^{(1-\sigma_c)}\iota_w}{1+\beta\gamma^{(1-\sigma_c)}}$, $w_3 = \frac{\left(1-\zeta_w\beta\gamma^{(1-\sigma_c)}\right)(1-\zeta_w)}{\left(1+\beta\gamma^{(1-\sigma_c)}\right)\zeta_w\left((\lambda_w-1)\varepsilon_w+1\right)}$. The expression $\left(w_t^h - w_t\right)$ is the inverse of wage mark-up. λ_w is the steady-state wage mark-up, ι_w is the wage indexation, ε_w is the curvature of the Kimball aggregator for labor services. $\upsilon_{w,t}$ is the exogenous fluctuations in the wage mark-up.

The interest rate policy equation is:

$$r_t = \max\left\{\bar{r}, r_t^n\right\} + \upsilon_{nir,t} \tag{A.15}$$

and for the notional rate:

$$r_t^n = \rho r_{t-1}^n + (1-\rho)\left(\phi_\pi\pi_t + \phi_y\tilde{y}_t\right) + \phi_{dy}\Delta\tilde{y}_t + \upsilon_{r,t} \tag{A.16}$$

The equations (A.3) to (A.16) determine 14 endogenous variables: c_t, i_t, $\bar{k}_t$, q_t, k_t, y_t, mc_t, r_t^k, l_t, π_t, w_t^h, w_t, r_t, and r_t^n. The stochastic behavior of the model is driven by eight exogenous processes:

$$u_t = \rho_u u_{t-1} + \varepsilon_t^u, \tag{A.17}$$

$$z_t = \rho_z z_{t-1} + \varepsilon_t^z, \tag{A.18}$$

$$g_t = \rho_g g_{t-1} + \varepsilon_t^g + \rho_{gz}\varepsilon_t^z, \tag{A.19}$$

$$v_{r,t} = \rho_r v_{r,t-1} + \varepsilon_t^r, \tag{A.20}$$

$$v_{nir,t} = \rho_{nir} v_{nir,t-1} + \varepsilon_t^{nir}, \tag{A.21}$$

$$v_{i,t} = \rho_i v_{i,t-1} + \varepsilon_t^i, \tag{A.22}$$

$$v_{p,t} = \rho_p v_{p,t-1} + \varepsilon_t^p - \mu_p \varepsilon_{t-1}^p, \tag{A.23}$$

$$v_{w,t} = \rho_w v_{w,t-1} + \varepsilon_t^w - \mu_w \varepsilon_{t-1}^w \tag{A.24}$$

where $\varepsilon_t^k \overset{i.i.d}{\sim} \mathcal{N}(0, \sigma_k^2)\ \forall\ k = \{r, i, p, w\}$ and similarly for $\{u_t, z_t, g_t\}$.

B. The full solution method

First, I start with some preliminary setup. Consider the following linear rational expectations model, which is subject to $j = 1, 2, \ldots, n_c$ OBCs:

$$r_{j,t} = \max\left\{a_j y_{t+1} + b_j y_t + c_j y_{t-1}, \overline{r}_j\right\}. \tag{B.1}$$

This constrained variable is subject to an OBC depicted by the set of vectors $\{a_j, b_j, c_j\}$ and a minimum value $\overline{r}_j$. Upon linearization through perturbation methods, the first order conditions can be written as

$$E_t\left[\mathfrak{A}z_{t+1} + \mathfrak{B}z_t + \mathfrak{C}z_{t-1} + \mathfrak{D}\epsilon_t + \sum_j^{n_c} \mathfrak{h}_j \max\left\{\mathfrak{a}_j z_{t+1} + \mathfrak{b}_j z_t + \mathfrak{c}_j z_{t-1} + \mathfrak{d}_j \epsilon_t, \overline{r}_j\right\}\right] = 0. \tag{B.2}$$

This equation can be reduced to

$$E_t\left[Ay_{t+1}+By_t+Cy_{t-1}+\sum_{j}^{n_c}h_j\max\left\{a_jy_{t+1}+b_jy_t+c_jy_{t-1},\bar{r}_j\right\}\right]=0. \tag{B.3}$$

when we define $y_t = (z_t, \epsilon_{t+1})$, $A = \begin{bmatrix} \mathfrak{A} & 0 \\ 0 & 0 \end{bmatrix}$, $B = \begin{bmatrix} \mathfrak{B} & 0 \\ 0 & I \end{bmatrix}$, $C = \begin{bmatrix} \mathfrak{C} & \mathfrak{D} \\ 0 & 0 \end{bmatrix}$, and $a_j = (\mathfrak{a}_j, 0)$, $b_j = (\mathfrak{b}_j, 0)$, $c_j = (\mathfrak{c}_j, \mathfrak{d}_j)$, $h_j = (\mathfrak{h}_j, 0)$ for each constraint j. z_t is the n-dimensional vector containing all the variables of the model, ϵ_t the n_ϵ-dimensional vector of i.i.d. exogenous shocks, $\mathfrak{A}, \mathfrak{B}, \mathfrak{C}$ generic $n \times n$ system matrices, and $\mathfrak{D}$ a $n \times n_\epsilon$ matrix. $h_j \in \mathbb{R}^n$ aggregates the coefficients of $r_{j,t}$ in each equation of the linear system.

Let's write the unconstrained system (where all constraints are slack) as

$$\widehat{A}y_{t+1}+\widehat{B}y_t+\widehat{C}y_{t-1}=0, \tag{B.4}$$

where

$$\widehat{A}=A+\sum_{j}^{n_c}h_ja_j, \tag{B.5}$$

$$\widehat{B}=B+\sum_{j}^{n_c}h_jb_j, \tag{B.6}$$

$$\widehat{C}=C+\sum_{j}^{n_c}h_jc_j, \tag{B.7}$$

$\widehat{A}$ is the unconstrained matrix for forward-looking variables y_{t+1} and consists of the matrix A (for the system in which all the variables bind) and a correction matrix for each $r_{j,t}$ (it contains the endogenous responses of $r_{j,t}$ if constraint j is slack).

Lastly, 3 Assumptions (see Boehl (2022a)) are further necessary: they ensure the existence and uniqueness of a solution for the unconstrained linear system and a solution for the model under perfect foresight (all uncertainty is resolved after t). The latter permits us to drop the expectations operator E_t in the next part.

B.1. Representing the model with a closed-form specification

Let's rewrite Eq. B.3 in the form:

$$Px_{t+1} = Mx_t + \sum_{j}^{n_c} h_j \max\left\{p_j x_{t+1} + m_j x_t, \overline{r}_j\right\} \tag{B.8}$$

$x_t = \begin{bmatrix} s_{t-1} \\ c_t \end{bmatrix}$, with c_t the controls or forward looking variables, s_{t-1} the states variables. The unconstrained system is denoted as

$$\widehat{P}x_{t+1} = \widehat{M}x_t, \tag{B.9}$$

with

$$\widehat{P} = \left(P + \sum_{j}^{n_c} h_j p_j\right) \text{ and } \widehat{M} = \left(M + \sum_{j}^{n_c} h_j m_j\right), \tag{B.10}$$

in an analogous way to Eqs. B.5-B.7.

Let's consider the simple case of $j = 1$, that is, when there is only a single constraint characterised by (h_0, p_0, m_0). Under the assumption of invertibility of P and $\widehat{P}$, Eq. B.8 becomes

$$x_{t+1} = \begin{cases} \widehat{G}x_t & \text{if} \quad p_0 x_{t+1} + m_0 x_t - \overline{r} \geq 0, \text{ i.e. } \quad \text{when the constraint is slack} \\ Gx_t + q_0\overline{r} & \text{if} \quad p_0 x_{t+1} + m_0 x_t - \overline{r} < 0, \text{ i.e. } \quad \text{when the constraint is binding,} \end{cases} \tag{B.11}$$

with $\widehat{G} = \widehat{P}^{-1}\widehat{M}$, $G = P^{-1}M$, and $q_0 = P^{-1}h_0$.

Let's consider the *no transition* to the constraint case (if the constraint is binding, it always does so already in the current period t). Denoting k_t the expected duration of the spell to the constraint in t, a rational expectations solution is the function λ such that

$$c_t = \lambda\left(k_t, s_{t-1}\right). \tag{B.12}$$

Using the QZ-decompostion method for instance, one can obtain the controls c_t for the unconstrained system $\widehat{G}$. If the matrix Ω is the linear solution for c_t, then:

$$c_t = \Omega s_{t-1} \text{ if } p_0 x_{t+1} + m_0 x_t \geq \overline{r} \tag{B.13}$$

Rearranging the terms and defining $\Psi = [-\Omega \quad I]$, Eq. B.13 can be rewritten

$$\Psi \begin{bmatrix} s_{t+k-1} \\ c_{t+k} \end{bmatrix} = 0 \quad \text{if} \quad p_0 x_{t+k+1} + m_0 x_{t+k} \geq \overline{r}, \tag{B.14}$$

meaning that whenever the system is unconstrained, Eq. B.14 must hold for every future period $t + k$.

For the system in which the constraint is binding, if one assumes that it binds at time t and this will be the case until $t + k$, iterating k periods forwards gives

$$\Psi \begin{bmatrix} s_{t+k-1} \\ c_{t+k} \end{bmatrix} = G^k x_t + (I - G)^{-1}(I - G^k) q_0 \overline{r}, \tag{B.15}$$

G^k is the matrix power, $(I - G)^{-1}(I - G^k) = \sum_{i=0}^{k-1} G^i$ is the result of a geometric series of matrices. By combining Eqs. B.14 and B.15, Boehl (2022a) shows that

$$\lambda\left(k, s_{t-1}\right) = \left(c_t : \Psi G^k \begin{bmatrix} s_{t-1} \\ c_t \end{bmatrix} = -\Psi(I - G)^{-1}(I - G^k) q_0 \overline{r}\right). \tag{B.16}$$

For the case where we assume a transition to the constraint, by relaxing the assumption that the constraint holds immediately at time t after a shock occurs, Boehl (2022a) shows that the solution becomes

$$\lambda\left(l, k, s_{t-1}\right)=\left(c_t: \Psi G^k \widehat{G}^l\left[\begin{array}{c} s_{t-1} \\ c_t \end{array}\right]=-\Psi(I-G)^{-1}(I-G^k) q_0 \bar{r}\right). \tag{B.17}$$

where l is the number of periods in the unconstrained system $\widehat{G}$ until the system reaches the constraint.

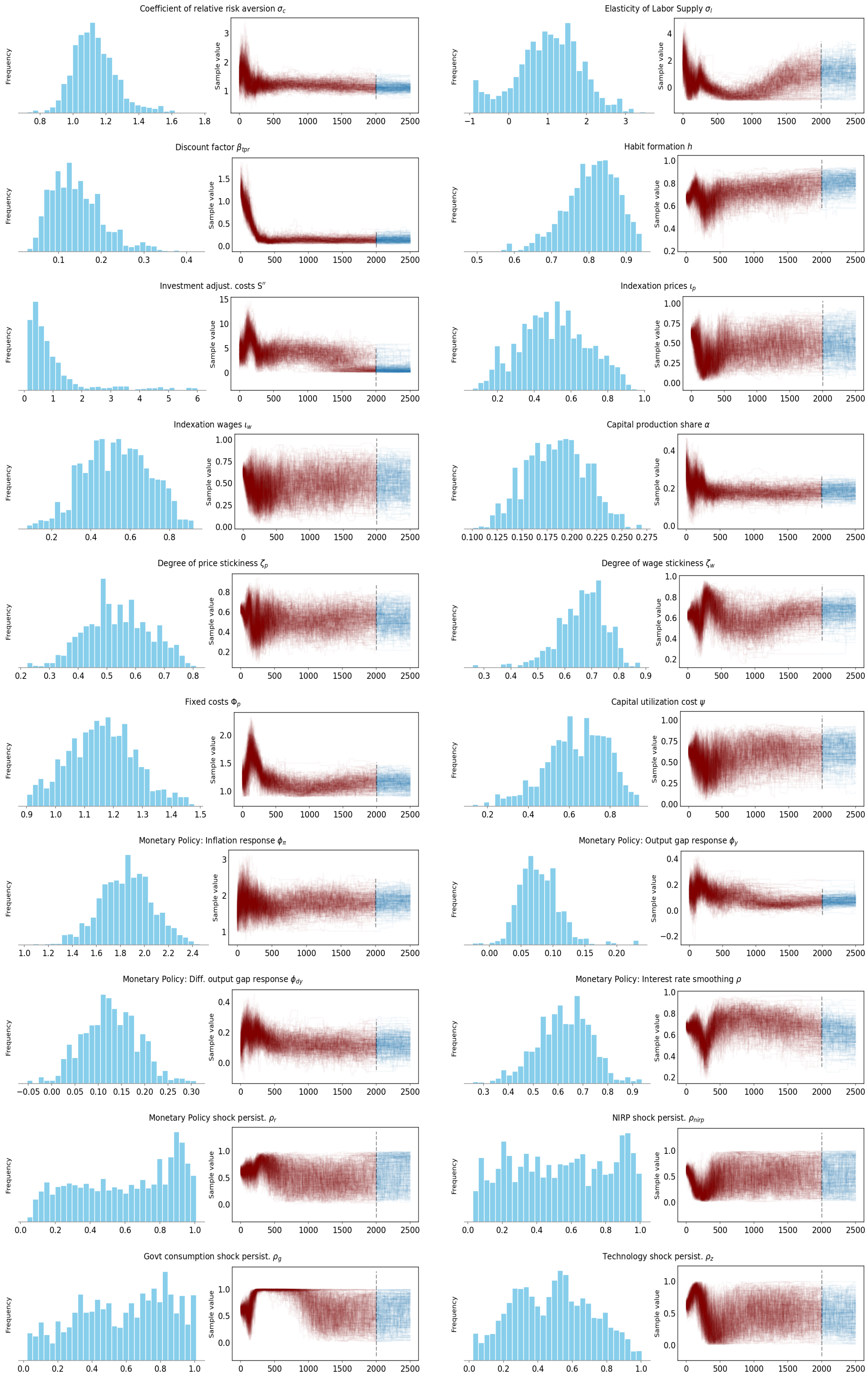
If we denote the perfect foresight solution of variable vector x_{t+j} in period $t+j$ by $[x_{t+j} \mid (l,k)]$, $[x_{t+j} \mid (l_t,k_t)]$ conditional on assuming spell durations of (l_t,k_t) can be expressed as a function of Λ, using B.15 and B.17

$$\begin{aligned} \left[x_{t+j} \mid (l_t,k_t)\right]=\Lambda_j\left(l_t, k_t, s_{t-1}\right)= & G^{\max\{j-l_t,0\}} \widehat{G}^{\min\{l_t,j\}}\left[\begin{array}{c} \lambda(l_t, k_t, s_{t-1}) \\ s_{t-1} \end{array}\right] \\ & +(I-G)^{-1}\left(I-G^{\max\{j-l_t,0\}}\right) q_0 \bar{r}. \end{aligned} \tag{B.18}$$

Additionally, Proposition 1 and Proposition 2 in Boehl (2022a) give the rational expectations equilibrium conditions, for the non-transition and the transition cases of the constraint.

To complete the set of methodologies for the estimation, Boehl and Strobel (2024a) and Boehl (2022b) contain details about the non-linear filtering system (the Ensemble Kalman Filter) and the posterior sampling (the Differential-Independence Mixture Ensemble Monte Carlo Markov).

C. The posterior distribution



Coefficient of relative risk aversion σ_c
Elasticity of Labor Supply σ_l
Discount factor β_{tpr}
Habit formation h
Investment adjust. costs S''
Indexation prices ι_p
Indexation wages ι_w
Capital production share α
Degree of price stickiness ζ_p
Degree of wage stickiness ζ_w
Fixed costs Φ_p
Capital utilization cost ψ
Monetary Policy: Inflation response ϕ_π
Monetary Policy: Output gap response ϕ_y
Monetary Policy: Diff. output gap response ϕ_{dy}
Monetary Policy: Interest rate smoothing ρ
Monetary Policy shock persist. ρ_r
NIRP shock persist. ρ_{nirp}
Govt consumption shock persist. ρ_g
Technology shock persist. ρ_z
Frequency
Sample value

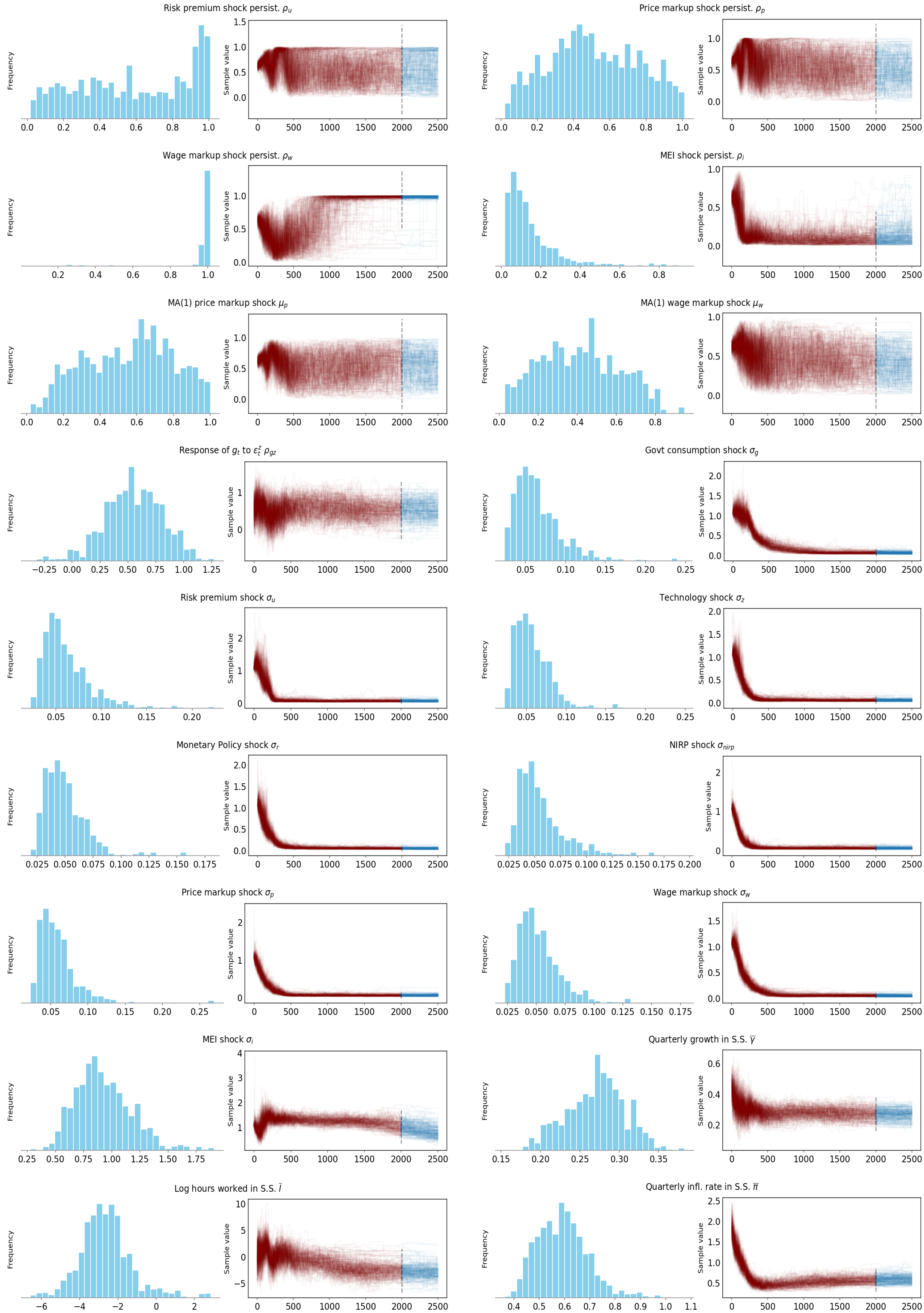
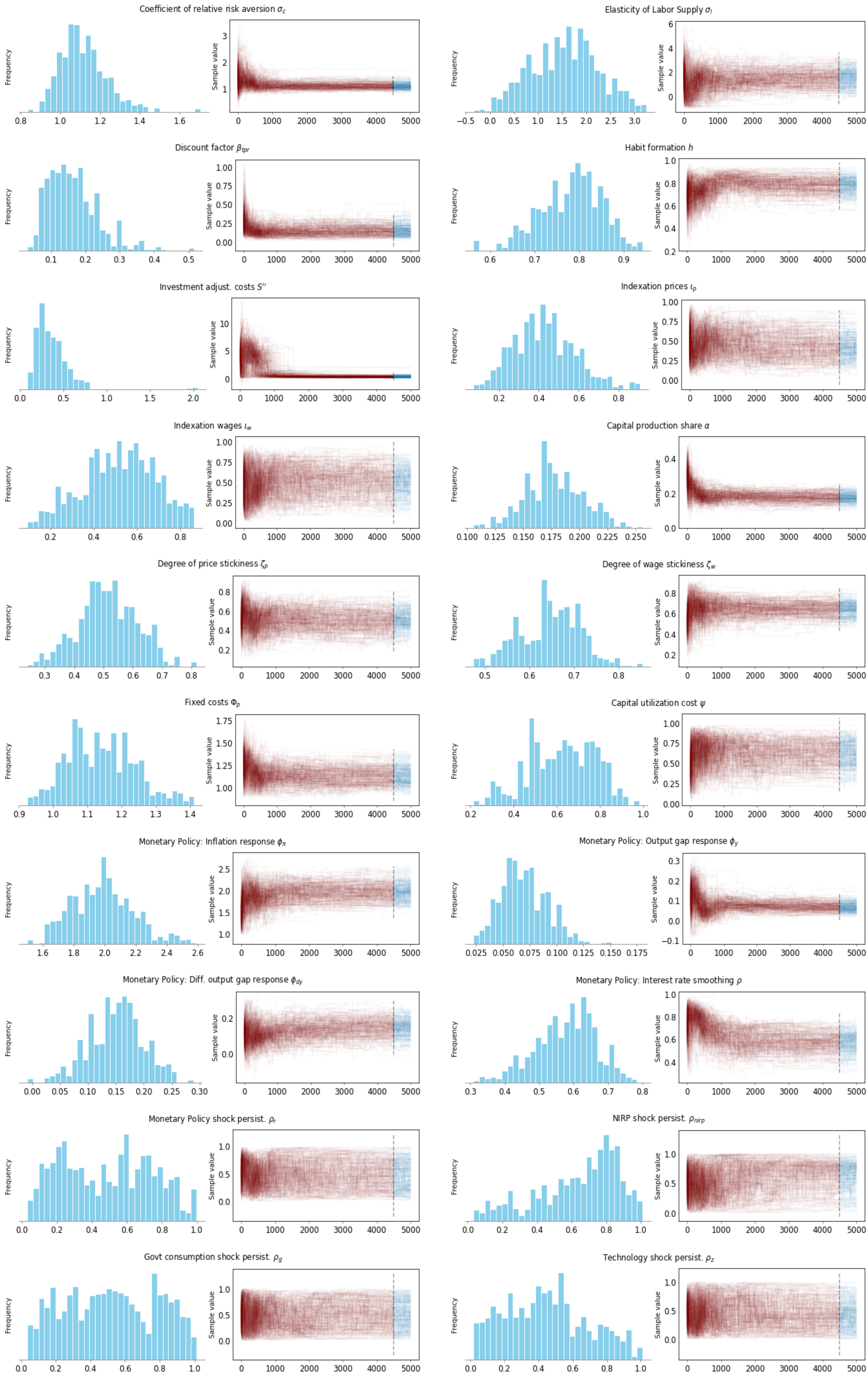


Figure C.8: Traceplots of the 200 DIME chains for all parameters, with 2,500 iterations.


Coefficient of relative risk aversion σ_c
Elasticity of Labor Supply σ_l
Discount factor β_{tpr}
Habit formation h
Investment adjust. costs S''
Indexation prices ι_p
Indexation wages ι_w
Capital production share α
Degree of price stickiness ζ_p
Degree of wage stickiness ζ_w
Fixed costs Φ_p
Capital utilization cost ψ
Monetary Policy: Inflation response ϕ_π
Monetary Policy: Output gap response ϕ_y
Monetary Policy: Diff. output gap response ϕ_{dy}
Monetary Policy: Interest rate smoothing ρ
Monetary Policy shock persist. ρ_r
NIRP shock persist. ρ_{nirp}
Govt consumption shock persist. ρ_g
Technology shock persist. ρ_z
Frequency
Sample value

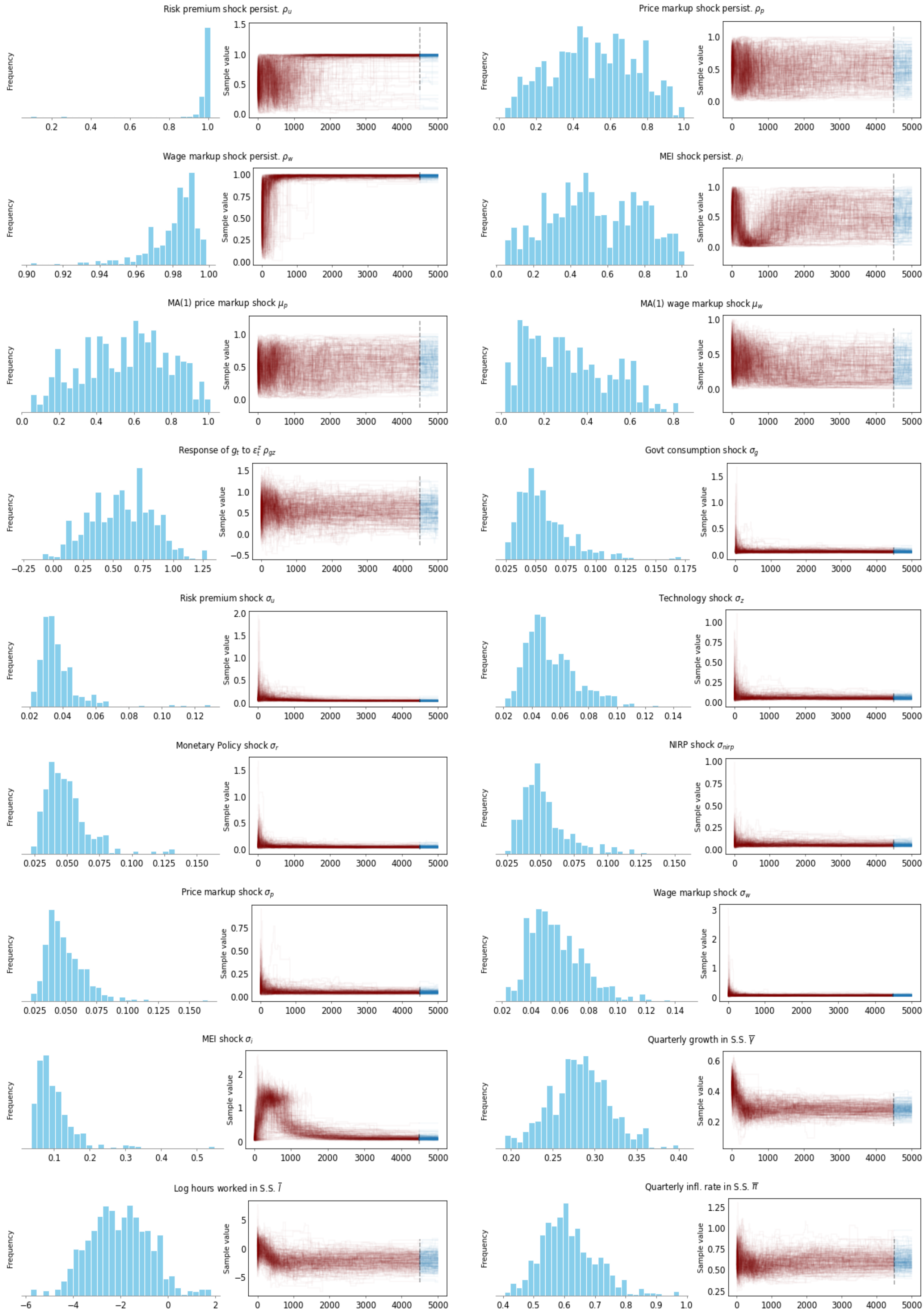


Figure C.9: Traceplots of the 200 DIME chains for all parameters, with 5,000 iterations.

D. The pre-crisis vs the full sample results

For the posterior estimates using the pre-crisis sample (1998Q1-2007Q4) (Table D.1), the mean value of most posteriors are closer to the priors than those of the full sample are, as expected. For the shock persistence, they are all fairly close to the 0.50 prior mean value but the one of wage markup. This high persistence of the wage markup ($\rho_w = 0.91$) indicates that the wage markup shock might explain most of the forecast error variance of the real variables in the long run, in the event the pre-crisis estimates are used for model applications. The shocks themselves have posterior mean values of about half the size of their assumed prior mean.

Table D.1: Prior and Posterior distributions of structural parameters and disturbances (full vs pre-crisis sample)

		Prior			Posterior (Full sample)					Posterior (Pre-crisis sample)				
Parameter		Distrib.	Mean	sd/df	Mode	Mean	sd	5 perc.	95 perc.	Mode	Mean	sd	5 perc.	95 perc.
Coefficient of relative risk aversion	σ_c	normal	1.500	0.375	1.206	1.185	0.116	1.005	1.380	1.346	1.231	0.199	0.908	1.550
Elasticity of Labor Supply	σ_l	normal	2.000	0.750	-0.931	-0.939	0.043	-1.000	-0.878	2.092	1.347	0.844	-0.144	2.675
Discount factor	β_{tpr}	gamma	0.250	0.100	0.256	0.196	0.075	0.067	0.300	0.126	0.153	0.062	0.051	0.244
Habit formation	h	beta	0.700	0.100	0.673	0.705	0.047	0.621	0.777	0.704	0.744	0.088	0.611	0.884
Investment adjust. costs	S''	normal	4.000	1.500	5.049	5.507	1.076	3.907	7.258	4.525	4.017	1.339	1.668	6.041
Indexation prices	ι_p	beta	0.500	0.150	0.129	0.211	0.094	0.064	0.354	0.460	0.489	0.164	0.205	0.750
Indexation wages	ι_w	beta	0.500	0.150	0.289	0.356	0.110	0.179	0.532	0.474	0.507	0.163	0.233	0.764
Capital production share	α	normal	0.300	0.050	0.061	0.076	0.017	0.051	0.106	0.290	0.312	0.045	0.244	0.395
Degree of price stickiness	ζ_p	beta	0.500	0.100	0.707	0.719	0.073	0.595	0.831	0.632	0.499	0.104	0.322	0.659
Degree of wage stickiness	ζ_w	beta	0.500	0.100	0.775	0.788	0.041	0.724	0.856	0.703	0.654	0.085	0.531	0.806
Fixed costs	Φ_p	normal	1.250	0.125	1.448	1.323	0.114	1.144	1.507	1.263	1.165	0.113	0.976	1.348
Capital utilization cost	ψ	beta	0.500	0.150	0.704	0.555	0.139	0.359	0.827	0.411	0.509	0.168	0.250	0.804
Monetary Policy: Inflation response	ϕ_π	normal	1.500	0.250	1.361	1.329	0.255	0.926	1.739	1.783	1.670	0.218	1.348	2.064
Monetary Policy: Output gap response	ϕ_y	normal	0.125	0.050	0.274	0.262	0.034	0.207	0.319	0.046	0.055	0.039	-0.004	0.124
Monetary Policy: Diff. output gap response	ϕ_{dy}	normal	0.125	0.050	0.009	-0.001	0.028	-0.049	0.045	0.092	0.128	0.048	0.042	0.199
Monetary Policy: Interest rate smoothing	ρ	beta	0.750	0.100	0.896	0.904	0.018	0.875	0.933	0.625	0.780	0.082	0.656	0.916
Monetary Policy shock persist.	ρ_r	beta	0.500	0.200	0.303	0.305	0.080	0.183	0.441	0.497	0.411	0.212	0.073	0.760
NIRP shock persist.	ρ_{nirp}	beta	0.500	0.200	0.963	0.962	0.019	0.934	0.991	0.511	0.470	0.231	0.086	0.822
Govt consumption shock persist.	ρ_g	beta	0.500	0.200	0.990	0.972	0.017	0.953	0.996	0.685	0.508	0.248	0.111	0.916
Technology shock persist.	ρ_z	beta	0.500	0.200	0.985	0.946	0.038	0.886	0.993	0.613	0.508	0.243	0.114	0.901
Risk premium shock persist.	ρ_u	beta	0.500	0.200	0.949	0.940	0.012	0.919	0.959	0.326	0.484	0.235	0.077	0.839
Price markup shock persist.	ρ_p	beta	0.500	0.200	0.691	0.692	0.133	0.499	0.881	0.252	0.484	0.239	0.080	0.858
Wage markup shock persist.	ρ_w	beta	0.500	0.200	0.555	0.529	0.143	0.315	0.766	0.913	0.914	0.073	0.842	0.993
MEI shock persist.	ρ_i	beta	0.500	0.200	0.092	0.087	0.056	0.008	0.160	0.575	0.500	0.253	0.117	0.918
MA(1) price markup shock	μ_p	beta	0.500	0.200	0.540	0.478	0.133	0.263	0.695	0.596	0.530	0.236	0.153	0.916
MA(1) wage markup shock	μ_w	beta	0.500	0.200	0.480	0.429	0.151	0.162	0.657	0.356	0.315	0.174	0.032	0.585
Response of g_t to ε_t^z	ρ_{gz}	normal	0.500	0.250	1.030	1.148	0.197	0.864	1.471	0.399	0.513	0.250	0.084	0.891
Govt consumption shock	σ_g	inv.gamma	0.100	0.250	0.419	0.402	0.047	0.326	0.481	0.040	0.057	0.025	0.025	0.092
Risk premium shock	σ_u	inv.gamma	0.100	0.250	0.191	0.224	0.042	0.162	0.293	0.049	0.060	0.030	0.025	0.097
Technology shock	σ_z	inv.gamma	0.100	0.250	0.324	0.349	0.034	0.292	0.398	0.060	0.053	0.022	0.025	0.082
Monetary Policy shock	σ_r	inv.gamma	0.100	0.250	0.089	0.093	0.008	0.080	0.108	0.051	0.050	0.019	0.024	0.076
NIRP shock	σ_{nirp}	inv.gamma	0.100	0.250	0.010	0.010	0.001	0.009	0.012	0.051	0.058	0.027	0.024	0.091
Price markup shock	σ_p	inv.gamma	0.100	0.250	0.147	0.131	0.019	0.101	0.161	0.056	0.056	0.028	0.024	0.090
Wage markup shock	σ_w	inv.gamma	0.100	0.250	0.146	0.141	0.018	0.111	0.170	0.080	0.067	0.027	0.028	0.103
MEI shock	σ_i	inv.gamma	0.100	0.250	1.449	1.417	0.120	1.224	1.604	0.052	0.057	0.024	0.027	0.090
Quarterly growth in S.S.	$\overline{\gamma}$	normal	0.440	0.050	0.245	0.221	0.034	0.172	0.271	0.403	0.367	0.042	0.298	0.435
Log hours worked in S.S.	$\overline{l}$	normal	0.000	2.000	-0.401	-0.528	1.921	-3.567	2.653	-1.786	-1.313	1.195	-3.277	0.650
Quarterly infl. rate in S.S.	$\overline{\pi}$	gamma	0.625	0.100	0.570	0.566	0.068	0.449	0.666	0.559	0.567	0.085	0.417	0.699
Mean acceptance fraction			**NA**				**0.058**					**0.142**		