A Computation and Communication Efficient Method for Distributed Nonconvex Problems in the Partial Participation Setting

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1. Distributed Stochastic Optimization Problem

We are solving the nonconvex distributed optimization problem:

$$\min_{x\in\mathbb{R}^d} f(x) \stackrel{\text{def}}{=} \frac{1}{n}\sum_{i=1}^{n} f_i(x)$$

devices / machines

model parametes / features

Loss on local data $\mathcal{D}_i$ stored on device i

$$f_i(x) = \mathrm{E}_{\xi\sim\mathcal{D}_i}\left[f_i(x;\xi)\right]$$

The datasets $\mathcal{D}_1,\ldots,\mathcal{D}_n$ can be arbitrarily heterogeneous

2. Goal

We want to find a stationary point of the optimization problem:

Find a (possibly random) vector $x \in \mathbb{R}^d$ such that

$$\mathbb{E}\left[\|\nabla f(x)\|^2\right] \leq \varepsilon$$

3. Unbiased Compressor

$$\mathcal{C}_i \in \mathbb{U}(\omega_i) \qquad \begin{array}{c}\mathbb{E}\left[\mathcal{C}_i(v)\right] = v \quad \forall v\in\mathbb{R}^d \\ \mathbb{E}\left[\|\mathcal{C}_i(v)-v\|^2\right] \leq \omega_i\|v\|^2 \quad \forall v\in\mathbb{R}^d\end{array}$$

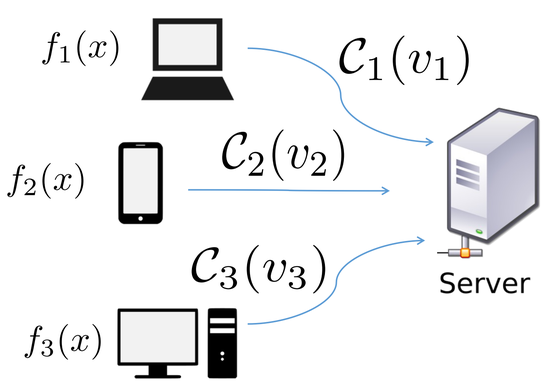
Example: *RandK* compressor

$$d=5; K=1 \qquad \begin{pmatrix}4\\-7\\2\\1\\-3\end{pmatrix} \xrightarrow{\mathcal{C}_i} 5\times\begin{pmatrix}0\\0\\2\\0\\0\end{pmatrix}$$

Random entry

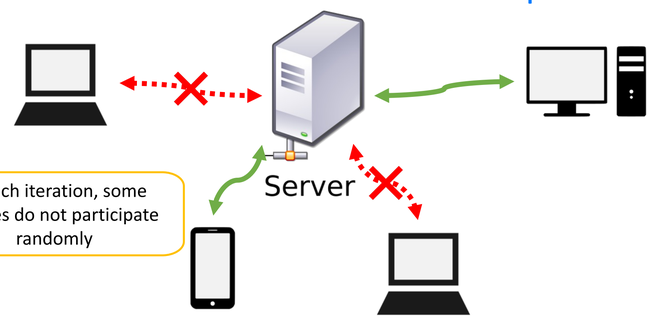
$$\omega_i = \frac{d}{K}-1 = \frac{5}{1}-1 = 4$$

4. Compressed Communication Approach



Clients send compressed vectors $\mathcal{C}_i(v_i)$ to the Server

5. Main Problem: Partial Participation



In each iteration, some devices do not participate randomly

Goal: Design a method that would have *the optimal oracle complexity* and *the state-of-the-art communication complexity* in the partial participation setting

6. New Method: DASHA-PP-MVR

$$x^{t+1} = x^t - \gamma g^t \qquad g^t = \frac{1}{n}\sum_{i=1}^{n} g_i^t$$

The server aggregates information and does the step

if i^{th} node is participating, then

$$k_i^{t+1} = \nabla f_i(x^{t+1},\xi_i^{t+1}) - \nabla f_i(x^t,\xi_i^{t+1}) - b(h_i^t - \nabla f_i(x^t,\xi_i^{t+1})),$$

$$h_i^{t+1} = h_i^t + \frac{1}{p_a}k_i^{t+1},$$

$$g_i^{t+1} = g_i^t + \mathcal{C}_i\left(\frac{1}{p_a}k_i^{t+1} - \frac{a}{p_a}(g_i^t - h_i^t)\right),$$

The nodes calculate new vectors and send the compressed vectors

else

$$h_i^{t+1} = h_i^t, \quad g_i^{t+1} = g_i^t,$$

where p_a is the probability that the node participates, and a and b are momentums.

7. Theoretical Guarantees

Corollary (Informal)**.** The communication complexity equals

$$\mathcal{O}\left(\frac{d\sigma}{\sqrt{p_a}\sqrt{n}\varepsilon} + \frac{d}{p_a\sqrt{n}\varepsilon}\right),$$

SOTA communication complexity

and the expected number of stochastic gradient calculations per node equals

$$\mathcal{O}\left(\frac{\sigma^2}{\sqrt{p_a}n\varepsilon} + \frac{\sigma}{p_a\varepsilon^{3/2}n}\right).$$

optimal oracle complexity

Our assumptions are standard:

Assumption 1. There exists $f^* \in \mathbb{R}$ such that $f(x) \geq f^*$ for all $x \in \mathbb{R}$.

Assumption 2. For all $i \in [n]$, the function f_i is L_i–smooth.

The two assumptions below are provided for the stochastic setting.

Assumption 4. For all $i \in [n]$ and for all $x \in \mathbb{R}^d$, the stochastic gradient $\nabla f_i(x;\xi)$ is unbiased and has bounded variance, i.e.,

$$\mathrm{E}_\xi\left[\nabla f_i(x;\xi)\right] = \nabla f_i(x), \qquad \text{and} \qquad \mathrm{E}_\xi\left[\|\nabla f_i(x;\xi) - \nabla f_i(x)\|^2\right] \leq \sigma^2,$$

where $\sigma^2 \geq 0$.

Assumption 5. For all $i \in [n]$ and for all $x, y \in \mathbb{R}$, the stochastic gradient $\nabla f_i(x;\xi)$ satisfies the mean-squared smoothness property, i.e.,

$$\mathrm{E}_\xi\left[\|\nabla f_i(x;\xi) - \nabla f_i(y;\xi) - (\nabla f_i(x) - \nabla f_i(y))\|^2\right] \leq L_\sigma^2\|x-y\|^2.$$

8. Comparison with Previous Methods

Table 1: Abbr.: *VR* (Variance Reduction) = Does a method have the optimal oracle complexity $\mathcal{O}\left(\frac{\sigma^2}{\varepsilon}+\frac{\sigma}{\varepsilon^{3/2}}\right)$? *PP* (Partial Participation) = Does a method support partial participation? *CC* = Does a method have the communication complexity equals to $\mathcal{O}\left(\frac{\omega}{\sqrt{n}\varepsilon}\right)$?

Method	*VR*	*PP*	*CC*	Limitations
SPIDER, SARAH, PAGE, STORM (Fang et al., 2018; Nguyen et al., 2017) (Li et al., 2021a; Cutkosky and Orabona, 2019)	✓	✗	✗	—
MARINA (Gorbunov et al., 2021)	✓	✗	✓	Suboptimal convergence rate.
FedPAGE (Zhao et al., 2021b)	✗	✗	✗	Suboptimal oracle complexity $\mathcal{O}\left(\frac{\sigma^2}{\varepsilon^2}\right)$.
FRECON (Zhao et al., 2021a)	✗	✓	✓	—
FedAvg (McMahan et al., 2017; Karimireddy et al., 2020b)	✗	✓	✗	Bounded gradients (dissimilarity) assumption of f_i.
SCAFFOLD (Karimireddy et al., 2020b)	✗	✓	✗	Suboptimal convergence rate.
MIME (Karimireddy et al., 2020a)	✗	✓	✗	Calculates full gradient. Bounded gradients (dissimilarity) assumption of f_i. Suboptimal oracle compl. $\mathcal{O}\left(1/\varepsilon^{3/2}\right)$ in the finite-sum setting.
CE-LSGD (for Partial Participation) (Patel et al., 2022) (concurrent work)	✓	✓	✗	Bounded gradients (dissimilarity) assumption of f_i. Suboptimal oracle compl. $\mathcal{O}\left(1/\varepsilon^{3/2}\right)$ in the finite-sum setting.
DASHA (Tyurin and Richtárik, 2023)	✓ or ✗	✗ or ✓	✓	—
DASHA-PP (new)	✓	✓	✓	—

9. New Methods DASHA-PP-PAGE and DASHA-PP-FINITE-MVR in the Finite-Sum Setting

In the finite-sum setting, we assume that $f_i(x) = \frac{1}{m}\sum_{j=1}^{m} f_{ij}(x)$.

Corollary (Informal)**.** The communication complexity is

$$\mathcal{O}\left(d + \frac{d}{p_a\varepsilon\sqrt{n}}\right),$$

SOTA communication complexity

and the expected number of gradient calculations per node equals

$$\mathcal{O}\left(m + \frac{\sqrt{m}}{p_a\varepsilon\sqrt{n}}\right).$$

optimal oracle complexity

Our assumptions are standard:

Assumption 1. There exists $f^* \in \mathbb{R}$ such that $f(x) \geq f^*$ for all $x \in \mathbb{R}$.

Assumption 2. For all $i \in [n]$, the function f_i is L_i–smooth.

The next assumption is used in the finite-sum setting.

Assumption 3. For all $i \in [n], j \in [m]$, the function f_{ij} is L_{ij}-smooth.

References

[1] Alexander Tyurin, Peter Richtárik
DASHA: Distributed Nonconvex Optimization with Communication Compression and Optimal Oracle Complexity
ICLR 2023

[2] Cutkosky, Ashok, and Francesco Orabona
Momentum-based variance reduction in non-convex SGD
NeurIPS 2019