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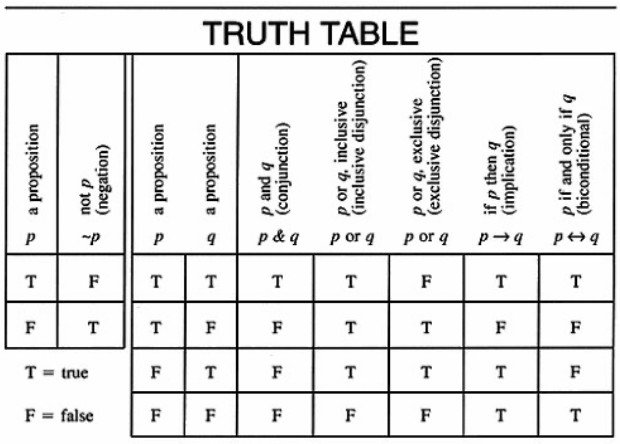
CS 161 – Assignment 5

1a)

Valid: α is true for all worlds
Unsatisfiable: There is no world where α is true

(Smoke ⇒ Fire) ⇒ (¬Smoke ⇒ ¬Fire)
A ⇒B ≡ ¬A ∨ B
¬(¬Smoke ∨ Fire) ∨ (Smoke ∨ ¬Fire)



Smoke	Fire	¬(¬Smoke ∨ Fire) ∨ (Smoke ∨ ¬Fire)	α
F	F	¬(¬F ∨F) ∨ (F∨ ¬F)	T
F	T	¬(¬ F∨T) ∨ (F∨ ¬T)	F
T	F	¬(¬ T∨F) ∨ (T∨ ¬F)	T
T	T	¬(¬ T∨T) ∨ (T∨ ¬T)	T

a) Neither

1b)

(Smoke ⇒ Fire) ⇒ ((Smoke ∨ Heat) ⇒ Fire)
(¬Smoke ∨ Fire) ⇒ ((Smoke ∨ Heat) ⇒ Fire)
(¬Smoke ∨ Fire) ⇒ (¬ (Smoke ∨ Heat) ∨ Fire)
¬(¬Smoke ∨ Fire) ∨ (¬ (Smoke ∨ Heat) ∨ Fire)
¬(¬Smoke ∨ Fire) ∨ ((¬Smoke ∧¬ Heat) ∨ Fire)

Smoke	Fire	Heat	¬(¬Smoke ∨ Fire) ∨ ((¬Smoke ∧¬ Heat) ∨ Fire)	α
F	F	F	¬(¬ F∨ F) ∨ ((¬F∧¬F) ∨ F)	T
F	F	T	¬(¬ F∨ F) ∨ ((¬F∧¬T) ∨ F)	F
F	T	F	¬(¬F ∨ T) ∨ ((¬F∧¬F) ∨ T)	T
F	T	T	¬(¬ F∨ T) ∨ ((¬F∧¬T) ∨ T)	T
T	F	F	¬(¬T ∨ F) ∨ ((¬T∧¬F) ∨ F)	T
T	F	T	¬(¬ T∨ F) ∨ ((¬T∧¬T) ∨ F)	T
T	T	F	¬(¬ T∨ T) ∨ ((¬T∧¬F) ∨ T)	T
T	T	T	¬(¬ T∨ T) ∨ ((¬T∧¬T) ∨ T)	T

b) Neither

1c)

((Smoke ∧ Heat) ⇒ Fire) ⇔ ((Smoke ⇒ Fire) ∨ (Heat ⇒ Fire))
(¬(Smoke ∧ Heat) ∨ Fire) ⇔ ((¬Smoke ∨ Fire) ∨ (¬Heat ∨ Fire))
((¬Smoke ∨ ¬Heat) ∨ Fire) ⇔ ((¬Smoke ∨ Fire) ∨ (¬Heat ∨ Fire))

Smoke	Fire	Heat	((¬Smoke ∨ ¬Heat) ∨ Fire) ⇔ ((¬Smoke ∨ Fire) ∨ (¬Heat ∨ Fire))	α
F	F	F	((¬F∨ ¬F) ∨F) ⇔ ((¬F∨F) ∨ (¬F∨F)) T⇔T	T
F	F	T	((¬F∨ ¬T) ∨F) ⇔ ((¬F∨F) ∨ (¬T∨F)) T⇔T	T

F	T	F	$((\neg F \vee \neg F) \vee T) \Leftrightarrow ((\neg F \vee T) \vee (\neg F \vee T))$ $T \Leftrightarrow T$	T
F	T	T	$((\neg F \vee \neg T) \vee T) \Leftrightarrow ((\neg F \vee T) \vee (\neg T \vee T))$ $T \Leftrightarrow T$	T
T	F	F	$((\neg T \vee \neg F) \vee F) \Leftrightarrow ((\neg T \vee F) \vee (\neg F \vee F))$ $T \Leftrightarrow T$	T
T	F	T	$((\neg T \vee \neg T) \vee F) \Leftrightarrow ((\neg T \vee F) \vee (\neg T \vee F))$ $F \Leftrightarrow F$	T
T	T	F	$((\neg T \vee \neg F) \vee T) \Leftrightarrow ((\neg T \vee T) \vee (\neg F \vee T))$ $T \Leftrightarrow T$	T
T	T	T	$((\neg T \vee \neg T) \vee T) \Leftrightarrow ((\neg T \vee T) \vee (\neg T \vee T))$ $T \Leftrightarrow T$	T

c) Valid

2) *If the unicorn is mythical, then it is immortal, but if it is not mythical, then it is mortal and it is a mammal. If the unicorn is either immortal or a mammal, then it is horned. The unicorn is magical if it is horned.*

a)

P: mythical
Q: mortal
R: mammal
S: horned
U: magical

(If the unicorn is mythical, then it is immortal,)
P1: $(P \Rightarrow \neg Q)$
but (if it is not mythical, then it is (mortal and it is a mammal.))
P2: $\neg P \Rightarrow (Q \wedge R)$
If (the unicorn is either immortal or a mammal), then it is horned.
P3: $\neg Q \vee R \Rightarrow S$
The unicorn is magical if it is horned.
P4: $S \Rightarrow U$

b) Convert to CNF

$(P \Rightarrow \neg Q)$
$(\neg P \vee \neg Q)$
$(\neg P \Rightarrow (Q \wedge R))$
$(P \vee \neg(Q \wedge R))$
$(P \vee \neg Q) \wedge (P \vee R)$
$(\neg Q \vee R) \Rightarrow S$
$\neg(\neg Q \vee R) \vee S$
$((Q \wedge \neg R) \vee S)$
$(Q \vee S) \wedge (\neg R \vee S)$

$S \Rightarrow U$
$(\neg S \vee U)$

KB = P1: $(\neg P \vee \neg Q)$
P2: $(P \vee \neg Q) \wedge (P \vee R)$
P3: $(Q \vee S) \wedge (\neg R \vee S)$
P4: $(\neg S \vee U)$

c)

KB ⊨ P iff KB ∧¬P is UNSAT

P	Q	R	S	U	$(\neg P \vee \neg Q) \wedge (P \vee \neg Q) \wedge (P \vee R) \wedge (Q \vee S) \wedge (\neg R \vee S) \wedge (\neg S \vee U)$	α
F	T	T	T	T	$(\neg F \vee \neg T) \wedge (F \vee \neg T) \wedge (F \vee T) \wedge (T \vee T) \wedge (\neg T \vee T) \wedge (\neg T \vee T)$ T ∧ F ∧ T ∧ T ∧ T ∧ T	F
F	F	T	T	T	$(\neg F \vee \neg F) \wedge (F \vee \neg F) \wedge (F \vee T) \wedge (F \vee T) \wedge (\neg T \vee T) \wedge (\neg T \vee T)$ T ∧ T ∧ T ∧ T ∧ T ∧ T	T

- When P = F, Q = F, R = T, S = T, U = T, KB = True
    - Since KB ∧¬P is SAT then KB does not entail P
- KB ∧¬P is not refutation complete since we were unable to derive false through resolution. ¬P is possible to be derived from using modus ponens on the contrapositive of P1. This means that KB ∧¬P is not UNSAT. Therefore, KB does not entail P and thus KB does not entail mythical.

i)	KB does not entail Mythical

Using Deduction:

1	Q => ¬P	Contrapositive P1
2	¬P	Modus ponens 1
	¬P ⇒ (Q ∧ R)	P2
3	Q ⇒ (Q ∧ R)	1 & P2
4	¬Q ∨ (Q ∧ R)	Implication of 3
5	(¬Q ∧ Q) ∧ (¬Q ∧ R)	Convert 4 to CNF
6	(¬Q ∧ R)	And elimination 5
7	¬Q	And elimination 6
	¬Q ∨ R ⇒ S	P3
8	S	7 & P3
	S⇒U	P4
9	U	8 & P4

Using deduction from the KB, it is possible to deduce S and U,
KB ⊨ S and KB ⊨ U
Since S = Horned and U = Magical

ii)	KB ⊨ Horned
iii)	KB ⊨ Magical

3)

$P(oil) = .5$	$P(Gas) = .2$	$P(N) = .3$
$P(positive \mid oil) = .9$	$P(positive \mid Gas) = .3$	$P(positive \mid N) = .1$

Given Test = positive, what is probability that oil is present?

Bayes Rule:

$$P(oil \mid positive) = \frac{P(positive \mid oil)\, P(oil)}{P(positive)}$$

$$P(positive) = P(positive, oil, Gas, Neither)$$

$$P(positive, oil) = .9 \cdot .5 = .45$$
$$P(positive, Gas) = .3 \cdot .2 = .06$$
$$P(positive, N) = .1 \cdot .3 = .03$$
$$P(positive) = .54$$

$$P(oil \mid positive) = \frac{.9 \cdot .5}{.54} \approx .8\overline{33}$$