State Transition representation of Soft Component Automaton

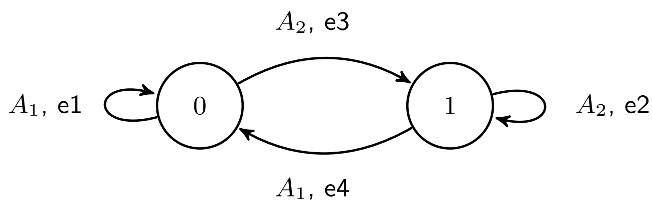

Figure 1: Soft Component Automaton A

Logical representation of Soft Component Automaton

Synthax. The synthax of a formula $\phi \in L$ is as follows :

$$\phi := \quad e \quad | \quad t_1 = t_2 \quad | \quad R(t_1, .., t_n) \quad | \quad \phi_1 \wedge \phi_2 \quad | \quad \neg\phi \quad | \quad \exists x \phi$$

where $e \in \mathbb{E}$.

Given that A'_i is the action A_i in conjunction with the state constraints, the logical expression of A is:

$$\begin{aligned} \phi(A) = \quad & (s = 0 \wedge A_1 \wedge s' = 0 \wedge e_1) \vee (s = 0 \wedge A_2 \wedge s' = 1 \wedge e_3) \vee (s = 1 \wedge A_2 \wedge s' = 1 \wedge e_2) \vee (s = 1 \wedge A_1 \wedge s' = 3 \wedge e_4) \\ = \quad & (A'_1 \wedge e_1) \vee (A'_2 \wedge e_3) \vee (A'_2 \wedge e_2) \vee (A'_1 \wedge e_4) \end{aligned}$$

Semantics. The semantic is defined by $[\![.]\!]$ as follow :

$$\begin{aligned} [\![e]\!] = \quad & e \in \mathbb{E} \\ [\![t_1 = t_2]\!] = \quad & 1_{\mathbb{B}} \quad | \quad 0_{\mathbb{B}} \quad \in \mathbb{B} \\ [\![\phi \wedge \psi]\!] = \quad & [\![\phi]\!] \otimes [\![\psi]\!] \\ [\![\phi \vee \psi]\!] = \quad & [\![\phi]\!] \oplus [\![\psi]\!] \\ [\![\neg\phi]\!] = \quad & 1_{\mathbb{B}} - [\![\phi]\!] \quad \text{if } \phi \text{ has a binary semiring valuation} \end{aligned}$$

where $\otimes$ and $\oplus$ are two polymorphic operators. By polymorphic, this operator is interpreted regarding the type of the two operands semiring values. At this point, we assume that the evaluation of this operator is given for any possible composition of semiring values. The formula describing A can be interpreted as follow:

$$\begin{aligned} [\![\phi(A)]\!] = \quad & [\![(A'_1 \wedge e_1)]\!] \oplus [\![(A'_2 \wedge e_3)]\!] \oplus [\![(A'_2 \wedge e_2)]\!] \oplus [\![(A'_1 \wedge e_4)]\!] \\ = \quad & ([\![A'_1]\!] \otimes [\![e_1]\!]) \oplus ([\![A'_2]\!] \otimes [\![e_3]\!]) \oplus ([\![A'_2]\!] \otimes [\![e_2]\!]) \oplus ([\![A'_1]\!] \otimes [\![e_4]\!]) \end{aligned}$$

In this example, A_1 is a binary constraint, and is evaluated to $1_{\mathbb{B}}$ or to $0_{\mathbb{B}}$. The composition operator is define over $\mathbb{E} \times \mathbb{E}$ with any semiring $\mathbb{E}$. If it operates over $\mathbb{B} \times \mathbb{E}$, we can still interpret this operator over $\mathbb{E} \times \mathbb{E}$ by applying homomorphique transformation $h_{\mathbb{E}} : \mathbb{B} \to \mathbb{E}$ with $h_{\mathbb{E}}(0_{\mathbb{B}}) = 0_{\mathbb{E}}$ and $h_{\mathbb{E}}(1_{\mathbb{B}}) = 1_{\mathbb{E}}$. Assuming that $e_i \in \mathbb{E}$,

$$[\![\phi(A)]\!] = \quad (h_{\mathbb{E}}([\![A'_1]\!]) \otimes [\![e_1]\!]) \oplus (h_{\mathbb{E}}([\![A'_2]\!]) \otimes [\![e_3]\!]) \oplus (h_{\mathbb{E}}([\![A'_2]\!]) \otimes [\![e_2]\!]) \oplus (h_{\mathbb{E}}([\![A'_1]\!]) \otimes [\![e_4]\!])$$

Polymorphic operators are interpreted regarding the type of the semiring involved in the product. $T_{\mathbb{E}}$ is the type of the semiring $\mathbb{E}$ and the polymorphic operators are :

$$\otimes_{T_{\mathbb{E}_1} \times T_{\mathbb{E}_2}} : \mathbb{E}_1 \times \mathbb{E}_2 \to \mathbb{E}_3$$

where $\mathbb{E}_i$ are either atomic semirings (weighted, probabilistic, ..) or a product of atomic semirings. As an example, $T_{\mathbb{E}_1}$ could be the Energy semiring and $T_{\mathbb{E}_2}$ the Time semiring and interpret $\otimes$ as

Composition. Let A and B be two automata with only one transition ϕ_i and one semiring value e_i. We want to get the product of those two automata, according to their semiring preference. The composition operator is polymorphic at the level of automata composition, and is interpreted with the right and left hand side type of semiring value.

$$\phi(A) = \phi_1 \wedge e_1 \qquad\qquad \phi(B) = \phi_2 \wedge e_2$$

$$A \odot B \equiv \quad \phi(A) \wedge \phi(B) = \phi_1 \wedge e_1 \wedge \phi_2 \wedge e_2$$

$$\begin{aligned} \oplus(A \odot B) \equiv \quad & [\![\phi(A) \wedge \phi(B)]\!] = [\![\phi_1 \wedge e_1 \wedge \phi_2 \wedge e_2]\!] \\ & = [\![\phi_1]\!] \otimes e_1 \otimes [\![\phi_2]\!] \otimes e_2 \end{aligned}$$

Domain of a composed semiring value :
Semiring elements are maps from semiring type to value. The composition operator is the composition of the semiring in the intersection of the two maps, and the union of both fields otherwise. Let's take an example :

$$e_1 = \{\text{energy} : 5 \; ; \text{space} : 4\} \qquad\qquad e_2 = \{\text{energy} : 3 \; ; \text{time} : 0.1\}$$

$$\text{energy} \otimes \text{energy} = \text{energy} \odot \text{energy}$$

$$\text{space} \otimes \text{time} = \text{space} \triangleright \text{time}$$

The jion composition of e_1 and e_2 is :

$$\begin{aligned} e_1 \otimes e_2 = \quad & \{\text{energy} : 5 \otimes 3 \; ; \text{space} \triangleright \text{time} \; ; : 4 \triangleright 0.1\} \\ = \quad & \{\text{energy} : 15 \; ; \text{space} \triangleright \text{time} \; ; : 4 \triangleright 0.1\} \end{aligned}$$

Coproduct Semiring
Define the Semiring where the product belongs as a coproduct.