$TVar$, α, β	Type variables
$IVar$, ϑ, θ	RGN index variables
$VVar$, f, x	Value variables
i, n	

$$\tau \quad ::= \qquad\qquad\qquad\qquad\qquad \text{Types}$$

	Int	
	$\tau_1 \rightarrow \tau_2$	
	$\tau_1 \times .. \times \tau_n$	
	unit	S
	α	
	$\forall\,\alpha.\tau$	
	$\mathbf{RGN}\,\theta\,\tau$	
	$\mathbf{RGNRef}\,\theta\,\tau$	
	$\mathbf{RGNHnd}\,\theta$	
	$\mathbf{RGNPf}\,(\theta_1 \preceq \theta_2)$	
	$\forall\,\vartheta.\tau$	
	$\tau[\tau_1/\alpha]$	M
	$\tau[\theta/\vartheta]$	M
	(τ)	M

$$e \quad ::= \qquad\qquad\qquad\qquad\qquad \text{Terms}$$

	i
	$e_1 + e_2$
	x
	$\lambda x : \tau.e$
	$e_1\,e_2$
	$\langle e_1, .., e_n \rangle$
	$\mathbf{sel}\,i\,e$
	$\Lambda\alpha.e$
	$e[\tau]$
	$\mathbf{let}\,x = e_1\,\mathbf{in}\,e_2$
	$\mathbf{runRGN}\,[\tau]v$
	κ
	$\Lambda\vartheta.e$
	$e[\theta]$

$$\kappa \quad ::= \qquad\qquad\qquad\qquad\qquad \text{RGN commands}$$

	$\mathbf{returnRGN}\,[\theta][\tau]v$
	$\mathbf{thenRGN}\,[\theta][\tau_1][\tau_2]v_1\,v_2$
	$\mathbf{letRGN}\,[\theta][\tau]v$
	$\mathbf{newRGNRef}\,[\theta][\tau]v\,v'$
	$\mathbf{readRGNRef}\,[\theta][\tau]v$
	$\mathbf{writeRGNRef}\,[\theta][\tau]v\,v'$

$$v \quad ::= \qquad\qquad\qquad\qquad\qquad \text{Values}$$

	i
	x
	$\lambda x : \tau.e$
	$\langle v_1, .., v_n \rangle$
	$\Lambda\alpha.e$
	κ
	$\Lambda\vartheta.e$

$$\Delta \quad ::= \qquad\qquad\qquad\qquad\qquad \text{Type and index contexts}$$

| .
| Δ, α
| Δ, ϑ

Γ ::=   Value contexts
| .
| $\Gamma, x : \tau$

terminals   ::=
| λ
| Λ
| $\vdash_{type}$
| $\vdash_{exp}$
| $\vdash_{index}$
| $\vdash_{vctxt}$
| $\vdash_{ctxt}$
| $\rightarrow$
| $\forall$
| $\preceq$
| $\langle$
| $\rangle$
| $.$
| $\times$
| $\in$

formula   ::=
| *judgement*
| $formula_1 \quad .. \quad formula_i$
| $x \in \mathbf{dom}\,(\Gamma)$
| $\Gamma(x) = \tau$

Jtype   ::=
| $\vdash_{ctxt} \Delta ; \Gamma$
| $\Delta \vdash_{type} \tau$
| $\Delta \vdash_{index} \theta$
| $\Delta ; \Gamma \vdash_{exp} e : \tau$

judgement   ::=
| *Jtype*

user_syntax   ::=
| *TVar*
| *IVar*
| *VVar*
| i
| τ
| e
| κ
| v

$$
\begin{array}{l}
| \quad \Delta \\
| \quad \Gamma \\
| \quad terminals \\
| \quad formula
\end{array}
$$

$$\boxed{\vdash_{ctxt} \Delta; \Gamma}$$

$$\boxed{\Delta \vdash_{type} \tau}$$

$$\boxed{\Delta \vdash_{index} \theta}$$

$$\boxed{\Delta; \Gamma \vdash_{exp} e : \tau}$$

$$
\frac{\vdash_{ctxt} \Delta; \Gamma}{\Delta; \Gamma \vdash_{exp} i : \mathbf{Int}} \quad \text{E\_INT}
$$

$$
\frac{\Delta; \Gamma \vdash_{exp} e_1 : \mathbf{Int} \quad \Delta; \Gamma \vdash_{exp} e_2 : \mathbf{Int}}{\Delta; \Gamma \vdash_{exp} e_1 + e_2 : \mathbf{Int}} \quad \text{E\_SUM}
$$

$$
\frac{\vdash_{ctxt} \Delta; \Gamma \quad x \in \mathbf{dom}\,(\Gamma) \quad \Gamma(x) = \tau}{\Delta; \Gamma \vdash_{exp} x : \tau} \quad \text{E\_VAR}
$$

$$
\frac{\Delta; \Gamma, x : \tau_1 \vdash_{exp} e : \tau}{\Delta; \Gamma \vdash_{exp} \lambda x : \tau_1.e : \tau_1 \to \tau} \quad \text{E\_ABS}
$$

$$
\frac{\Delta; \Gamma \vdash_{exp} e_1 : \tau_1 \to \tau \quad \Delta; \Gamma \vdash_{exp} e_2 : \tau_1}{\Delta; \Gamma \vdash_{exp} e_1\, e_2 : \tau} \quad \text{E\_APP}
$$

$$
\frac{\vdash_{ctxt} \Delta; \Gamma \quad \Delta, \alpha; \Gamma \vdash_{exp} e : \tau}{\Delta; \Gamma \vdash_{exp} \Lambda\alpha.e : \forall\,\alpha.\tau} \quad \text{E\_TABS}
$$

$$
\frac{\Delta; \Gamma \vdash_{exp} e : \forall\,\alpha.\tau \quad \Delta \vdash_{type} \tau_1}{\Delta; \Gamma \vdash_{exp} e[\tau_1] : \tau[\tau_1/\alpha]} \quad \text{E\_TAPP}
$$

$$
\frac{\Delta; \Gamma \vdash_{exp} e_1 : \tau_1 \quad \Delta; \Gamma, x : \tau_1 \vdash_{exp} e_2 : \tau}{\Delta; \Gamma \vdash_{exp} \mathbf{let}\, x = e_1 \, \mathbf{in}\, e_2 : \tau} \quad \text{E\_LET}
$$

$$
\frac{\Delta \vdash_{type} \tau \quad \Delta; \Gamma \vdash_{exp} v : \forall\,\vartheta.\mathbf{RGNHnd}\,\vartheta \to \mathbf{RGN}\,\vartheta\,\tau}{\Delta; \Gamma \vdash_{exp} \mathbf{runRGN}\,[\tau]v : \tau} \quad \text{E\_RUNRGN}
$$

$$
\frac{\vdash_{ctxt} \Delta; \Gamma \quad \Delta, \vartheta; \Gamma \vdash_{exp} e : \tau}{\Delta; \Gamma \vdash_{exp} \Lambda\vartheta.e : \forall\,\vartheta.\tau} \quad \text{E\_RABS}
$$

$$
\frac{\Delta; \Gamma \vdash_{exp} e : \forall\,\vartheta.\tau \quad \Delta \vdash_{index} \theta}{\Delta; \Gamma \vdash_{exp} e[\theta] : \tau[\theta/\vartheta]} \quad \text{E\_RAPP}
$$

$$\frac{\begin{array}{c}\Delta \vdash_{index} \theta \\ \Delta \vdash_{type} \tau \\ \Delta; \Gamma \vdash_{exp} v : \tau\end{array}}{\Delta; \Gamma \vdash_{exp} \textbf{returnRGN}\,[\theta][\tau]v : \textbf{RGN}\,\theta\,\tau} \quad \text{E\_RETURN}$$

$$\frac{\begin{array}{c}\Delta \vdash_{index} \theta \\ \Delta \vdash_{type} \tau_1 \\ \Delta \vdash_{type} \tau_2 \\ \Delta; \Gamma \vdash_{exp} v_1 : \textbf{RGN}\,\theta\,\tau_1 \\ \Delta; \Gamma \vdash_{exp} v_2 : \tau_1 \to \textbf{RGN}\,\theta\,\tau_2\end{array}}{\Delta; \Gamma \vdash_{exp} \textbf{thenRGN}\,[\theta][\tau_1][\tau_2]v_1\,v_2 : \textbf{RGN}\,\theta\,\tau_2} \quad \text{E\_THEN}$$

$$\frac{\begin{array}{l}\Delta \vdash_{index} \theta_1 \\ \Delta \vdash_{type} \tau \\ \Delta; \Gamma \vdash_{exp} v : \forall\vartheta_2.\textbf{RGNPf}\,(\theta_1 \preceq \vartheta_2) \to \textbf{RGNHnd}\,\vartheta_2 \to \textbf{RGN}\,\vartheta_2\,\tau\end{array}}{\Delta; \Gamma \vdash_{exp} \textbf{letRGN}\,[\theta_1][\tau]v : \textbf{RGN}\,\theta_1\,\tau} \quad \text{E\_LETRGN}$$

$$\frac{\begin{array}{c}\Delta \vdash_{index} \theta \\ \Delta \vdash_{type} \tau \\ \Delta; \Gamma \vdash_{exp} v : \textbf{RGNHnd}\,\theta \\ \Delta; \Gamma \vdash_{exp} v' : \tau\end{array}}{\Delta; \Gamma \vdash_{exp} \textbf{newRGNRef}\,[\theta][\tau]v\,v' : \textbf{RGN}\,\theta\,(\textbf{RGNRef}\,\theta\,\tau)} \quad \text{E\_NEW}$$

$$\frac{\begin{array}{c}\Delta \vdash_{index} \theta \\ \Delta \vdash_{type} \tau \\ \Delta; \Gamma \vdash_{exp} v : \textbf{RGNRef}\,\theta\,\tau\end{array}}{\Delta; \Gamma \vdash_{exp} \textbf{readRGNRef}\,[\theta][\tau]v : \textbf{RGN}\,\theta\,\tau} \quad \text{E\_READ}$$

$$\frac{\begin{array}{c}\Delta \vdash_{index} \theta \\ \Delta \vdash_{type} \tau \\ \Delta; \Gamma \vdash_{exp} v : \textbf{RGNRef}\,\theta\,\tau \\ \Delta; \Gamma \vdash_{exp} v' : \tau\end{array}}{\Delta; \Gamma \vdash_{exp} \textbf{writeRGNRef}\,[\theta][\tau]v\,v' : \textbf{unit}} \quad \text{E\_WRITE}$$

```
Definition rules:         17 good     0 bad
Definition rule clauses: 60 good     0 bad
```