CS182 Homework # 1

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June 21, 2025

Question 0

I, Maninder (Kaurman) Kaur, affirm that I have not given or received any unauthorized help on this assignment and that this work is my own. What I have submitted is expressed and explained in my own words. I have not used any online websites that provide a solution. I will not post any parts of this problem set to any online platform and doing so is a violation of course policy.

Question 1

Let P(x) be the statement "$x^2 \geq x$." Suppose the domain consists of real numbers (i.e., $x \in R$). Determine the truth values of the following:

(a) $P(2)$

(b) $P(0.5)$

(c) $P(-3)$

(d) $\exists x \neg P(x)$

(e) $\exists x P(x)$

(f) $\forall x P(x)$

Question 2

For each of the following logical statements, determine whether it is a tautology (always true), a contradiction (always false), or neither. For tautologies/contradictions, use logical equivalences to show it. If the statement is neither (a contingency), provide two counterexamples: true and false cases. Use logical equivalences to simplify the statement first. In both cases, state the names of the equivalences used in your proof or simplification. DO NOT use truth tables in this question.

(a) $(\neg(p \wedge q) \implies (\neg q \wedge p)) \vee (q \wedge \neg p)$

(b) $((\neg p \implies q) \wedge (q \vee (p \implies \neg q))) \vee (\neg q \implies \neg(p \wedge \neg q))$

Question 3

Determine whether the following propositional statements are satisfiable. If a propositional statement is not satisfiable, use equivalences to show it is equivalent to F. (State the names of the equivalences used in your proof). If a propositional statement is satisfiable, provide an assignment of truth values to the variables that makes the statement true.

(a) $(p \land (q \lor \neg p)) \land \neg q$

(b) $((\neg p \lor q) \land \neg r) \lor (p \land \neg r)$

(c) $(p \land (\neg q \lor r)) \implies (\neg r \land p)$

Question 4

Let the universe be all animals. Let $R(x)$ be the predicate "x is a raccoon", $D(x)$ be "x is a deer", $V(x)$ be "x is violent", and $F(x, y)$ be "x and y are friends". Express the following sentences using predicate logic.
Note: $F(x, y)$ and $F(y, x)$ are logically equivalent for any x and y. Both are acceptable.

(a) There exists a violent deer that is not friends with any raccoon.

(b) Every raccoon that is not violent is friends with at least one deer.

Question 5

Let the universe be the set of all integers. Let $S(x)$ represent "x is a square number" and $N(x)$ represent "x is negative" We want to say that "every square number is non-negative". Which one of these is a correct way to state this using formal logic, and why are other options incorrect? Explain.

(a) $\neg\exists(S(x) \implies N(x))$

(b) $\forall x(S(x) \implies \neg N(x))$

(c) $\forall x(S(x) \wedge \neg N(x))$

(d) $\exists x(S(x) \wedge \neg N(x))$

Question 6

Which of the two expressions correctly describes $\exists! x P(x)$, that is, "there exists a unique x such that P(x) is true"? Explain your answer.

(a) $\exists x(P(x) \wedge \forall y(P(y) \implies x = y))$

(b) $\exists x \forall y(P(y) \implies x = y)$

Question 7

Write the negation of $\forall x \exists y \forall z (P(x,y) \implies Q(y) \vee R(x,z))$ **such that the negation symbols immediately precede the predicates. Show your steps to receive full credit.**

Question 8

Alice is a hardworking CS 182 student who has decided to achieve an A+ in the course. Consider the following premises:

1. **If the alarm rings in the morning, Alice wakes up early.**
2. **If Alice wakes up early, Alice studies in the morning.**
3. **If Alice did not wake up early, Alice does not study in the morning.**
4. **If the alarm did not ring, Alice does not wake up early.**

Using propositional logic and logical equivalence laws, prove or disprove the claim: The alarm rings if and only if Alice studies.