CS18200 Spring 2025 Homework 7

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Problem 1 (Mandatory)

1. *I, Maninder Kaur, affirm that I have not given or received any unauthorized help on this assignment and that this work is my own. What I have submitted is expressed and explained in my own words. I have not used any online websites that provide a solution. I will not post any parts of this problem set to any online platform and doing so is a violation of course policy.*

Problem 2

(a) $(464 \cdot 358) \mod 21$

First, we find each number modulo 21:

$$464 \mod 21 = 464 - 21 \times 22 = 464 - 462 = 2$$
$$358 \mod 21 = 358 - 21 \times 17 = 358 - 357 = 1$$

Now multiply the results:

$$(2 \cdot 1) \mod 21 = 2 \mod 21$$

Final answer: $\boxed{2}$

(b) $2^{77} \mod 11$

since 11 is prime:

$$2^{10} \equiv 1 \mod 11$$

We can write:

$$2^{77} = 2^{7\times 10+7} = (2^{10})^7 \cdot 2^7 \equiv 1^7 \cdot 128 \mod 11$$
$$128 \mod 11 = 128 - 11 \times 11 = 128 - 121 = 7$$

Final answer: $\boxed{7}$

(c) $(3^{25} + 5^{18}) \mod 17$

Compute each term separately:

For $3^{25} \mod 17$: $3^{16} \equiv 1 \mod 17$

$$3^{25} = 3^{16} \cdot 3^9 \equiv 1 \cdot 3^9 \mod 17$$
$$3^2 = 9$$
$$3^4 = 81 \equiv 81 - 4 \times 17 = 81 - 68 = 13 \mod 17$$
$$3^8 \equiv 13^2 = 169 \equiv 169 - 9 \times 17 = 169 - 153 = 16 \mod 17$$
$$3^9 \equiv 16 \times 3 = 48 \equiv 48 - 2 \times 17 = 14 \mod 17$$

For $5^{18} \mod 17$:

$$5^{16} \equiv 1 \mod 17$$
$$5^{18} = 5^{16} \cdot 5^2 \equiv 1 \cdot 25 \equiv 8 \mod 17$$

Now add them:

$$14 + 8 = 22 \equiv 22 - 17 = 5 \mod 17$$

Final answer: $\boxed{5}$

(d) $(7^{163}) \mod 19$

$7^{18} \equiv 1 \mod 19$

$$163 = 18 \times 9 + 1$$
$$7^{163} = (7^{18})^9 \cdot 7^1 \equiv 1^9 \cdot 7 \equiv 7 \mod 19$$

Final answer: $\boxed{7}$

Problem 3

Given:

- Prime $p = 4969$
- Primary function: $h(k) = k \mod p$
- Secondary function: $g(k) = (k+1) \mod (p-2)$
- Probing sequence: $h(k,i) = (h(k) + i \cdot g(k)) \mod p$ for $i = 0, 1, 2, \ldots$

We compute memory locations for all 10 keys:

1. **Key** $k_1 = 132489971$

$$\begin{aligned} h(k_1) &= 132489971 \mod 4969 \\ &= 132489971 - 4969 \times \left\lfloor \frac{132489971}{4969} \right\rfloor \\ &= 132489971 - 4969 \times 26663 = 1744 \\ g(k_1) &= (132489971 + 1) \mod 4967 = 132489972 \mod 4967 \\ &= 132489972 - 4967 \times 26673 = 1581 \\ \text{Location} &= h(k_1, 0) = 1744 \end{aligned}$$

2. **Key** $k_2 = 509496993$

$$\begin{aligned} h(k_2) &= 509496993 \mod 4969 \\ &= 509496993 - 4969 \times 102536 = 578 \\ g(k_2) &= 509496994 \mod 4967 \\ &= 509496994 - 4967 \times 102577 = 2169 \\ \text{Location} &= h(k_2, 0) = 578 \end{aligned}$$

3. **Key** $k_3 = 546332190$

$$\begin{aligned} h(k_3) &= 546332190 \mod 4969 = 546332190 - 4969 \times 109938 = 1908 \\ g(k_3) &= 546332191 \mod 4967 = 546332191 - 4967 \times 109971 = 3934 \\ \text{Location} &= h(k_3, 0) = 1908 \end{aligned}$$

4. **Key** $k_4 = 034367980$

$$\begin{aligned} h(k_4) &= 34367980 \mod 4969 = 34367980 - 4969 \times 6916 = 1536 \\ g(k_4) &= 34367981 \mod 4967 = 34367981 - 4967 \times 6917 = 922 \\ \text{Location} &= h(k_4, 0) = 1536 \end{aligned}$$

5. **Key** $k_5 = 047900151$

$$\begin{aligned} h(k_5) &= 47900151 \mod 4969 = 47900151 - 4969 \times 9639 = 0 \\ g(k_5) &= 47900152 \mod 4967 = 47900152 - 4967 \times 9641 = 2145 \\ \text{Location} &= h(k_5, 0) = 0 \end{aligned}$$

6. **Key** $k_6 = 329938157$

$$\begin{aligned} h(k_6) &= 329938157 \mod 4969 = 329938157 - 4969 \times 66393 = 980 \\ g(k_6) &= 329938158 \mod 4967 = 329938158 - 4967 \times 66399 = 2925 \\ \text{Location} &= h(k_6, 0) = 980 \end{aligned}$$

7. **Key** $k_7 = 212228844$

$$\begin{aligned} h(k_7) &= 212228844 \mod 4969 = 212228844 - 4969 \times 42709 = 23 \\ g(k_7) &= 212228845 \mod 4967 = 212228845 - 4967 \times 42713 = 1134 \\ \text{Location} &= h(k_7, 0) = 23 \end{aligned}$$

8. **Key** $k_8 = 325510778$

$$\begin{aligned} h(k_8) &= 325510778 \mod 4969 = 325510778 - 4969 \times 65503 = 3151 \\ g(k_8) &= 325510779 \mod 4967 = 325510779 - 4967 \times 65509 = 1096 \\ \text{Location} &= h(k_8, 0) = 3151 \end{aligned}$$

9. **Key** $k_9 = 353354519$

$$\begin{aligned} h(k_9) &= 353354519 \mod 4969 = 353354519 - 4969 \times 71111 = 0 \\ g(k_9) &= 353354520 \mod 4967 = 353354520 - 4967 \times 71117 = 2141 \\ \text{Location} &= h(k_9, 0) = 0 \quad \text{(Collision with } k_5\text{)} \\ \text{Next try} &= h(k_9, 1) = (0 + 1 \times 2141) \mod 4969 = 2141 \end{aligned}$$

10. **Key** $k_{10} = 053708912$

$$\begin{aligned} h(k_{10}) &= 53708912 \mod 4969 = 53708912 - 4969 \times 10809 = 431 \\ g(k_{10}) &= 53708913 \mod 4967 = 53708913 - 4967 \times 10811 = 1136 \\ \text{Location} &= h(k_{10}, 0) = 431 \end{aligned}$$

Final assignments:

Key	Memory Location
k_1	1744
k_2	578
k_3	1908
k_4	1536
k_5	0
k_6	980
k_7	23
k_8	3151
k_9	2141
k_{10}	431

Problem 4

Let a_i be the total minutes watched through day i:

$$a_i = \sum_{k=1}^{i} t_k$$

Consider the sequence $\{a_i \mod 24\}$ for $i = 0, 1, ..., 140$ (21 weeks). By constraint:

$$a_{i+7} - a_i \leq 10 \quad \text{(weekly limit)}$$

$$a_i \geq i \quad \text{(at least 1 minute/day)}$$

There are 24 possible residue classes modulo 24. By the pigeonhole principle, among 141 partial sums, at least $\lceil 141/24 \rceil = 6$ must share the same residue.

Find two sums $a_j > a_i$ with $a_j \equiv a_i \mod 24$:

$$a_j - a_i \equiv 0 \mod 24$$

$$\Rightarrow \sum_{k=i+1}^{j} t_k = 24$$

Since $a_{i+7} - a_i \leq 10$, the consecutive days cannot span more than 3 weeks (otherwise the difference would be ≥ 24). Therefore, there must exist some window of ≤ 21 days where the total is exactly 24 minutes.

Problem 5

(a)

We must select:

- 4 Purdue from 14: $\binom{14}{4}$
- 2 IU from 8: $\binom{8}{2}$
- 1 Ivy Tech from 6: $\binom{6}{1}$

Total combinations:

$$\binom{14}{4} \times \binom{8}{2} \times \binom{6}{1} = 1001 \times 28 \times 6 = \boxed{168168}$$

(b)

Constraints:

- At least 5 Purdue (≥ 5 from 14)
- No more than 2 IU (≤ 2 from 8)

We enumerate all valid cases:

$$\begin{aligned} &\binom{14}{5}\left[\binom{8}{0}\binom{6}{3} + \binom{8}{1}\binom{6}{2} + \binom{8}{2}\binom{6}{1}\right] \\ +&\binom{14}{6}\left[\binom{8}{0}\binom{6}{2} + \binom{8}{1}\binom{6}{1} + \binom{8}{2}\binom{6}{0}\right] \\ +&\binom{14}{7}\left[\binom{8}{0}\binom{6}{1} + \binom{8}{1}\binom{6}{0}\right] \\ +&\binom{14}{8}\binom{8}{0}\binom{6}{0} \end{aligned}$$

Calculating each term:

$$\begin{aligned} &2002 \times [1 \times 20 + 8 \times 15 + 28 \times 6] \\ +&3003 \times [1 \times 15 + 8 \times 6 + 28 \times 1] \\ +&3432 \times [1 \times 6 + 8 \times 1] \\ +&3003 \times 1 \\ &= 2002 \times 248 + 3003 \times 91 + 3432 \times 14 + 3003 \\ &= 496,496 + 273,273 + 48,048 + 3,003 = \boxed{820,820} \end{aligned}$$

(c)

Constraints:

- 14 Purdue (P), 8 IU (I), 6 Ivy Tech (T)
- Each IU must stand immediately behind a Purdue

- No two Ivy Tech students adjacent

Solution approach:

1. First arrange the 14 Purdue students, creating 15 possible gaps (including ends):

$$\_P\_P\_\cdots\_P\_$$

2. Place IU students in gaps immediately after Purdue students:

$$\text{Choose 8 gaps from 15: } \binom{15}{8} \text{ ways}$$

3. Place Ivy Tech students in remaining 7 gaps (15 total - 8 used) with no two in same gap:

$$\text{Choose 6 gaps from 7: } \binom{7}{6} \text{ and arrange: } 6!$$

4. Arrange groups:

$$14! \text{ (Purdue)} \times 8! \text{ (IU)} \times 6! \text{ (Ivy Tech)}$$

Total arrangements:

$$14! \times 8! \times 6! \times \binom{15}{8} \times \binom{7}{6} = \boxed{14! \times 8! \times 6! \times 6435 \times 7}$$

Problem 6

(a)

First character must be a letter (A,B,C) - 3 choices. Remaining 4 characters from remaining 5 (no repeats):

$$3 \times P(5,4) = 3 \times 120 = 360$$

Final answer: $\boxed{360}$

(b)

Exactly 2 letters (must be in alphabetical order) and 3 digits:

- Choose 2 letters from 3: $C(3,2) = 3$ (order fixed)
- Choose 3 digits from 3: $C(3,3) = 1$
- Choose positions for letters: $C(5,2) = 10$

Total: $3 \times 1 \times 10 = 30$ Final answer: $\boxed{30}$

(c)

All letters appear before all digits. Possible patterns: LLLDD, LLDLD, etc. But we want all L before all D, so only LLLDD is valid.

Choose 3 letters from 3 and 2 digits from 3:

$$P(3,3) \times P(3,2) = 6 \times 6 = 36$$

Final answer: $\boxed{36}$