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Algorithms Design and Analysis
Assignment 1

1.

```
#include<iostream>
#include<vector>
using namespace std;

bool isPalindrome(string str)
{
   // str = clean_string(str);
    if(str.length() < 2)
        return false;
    for(int i=0; i<str.length()/2; i++)
    {
        if(str[i] != str[str.length()-1-i])
            return false;
    }
    return true;
}

int main()
{
    vector<string> test_cases = {"dad", "abc", "wasiteliotstoiletisaw", "dontnod",
    "s"};
    for(int i=0; i<test_cases.size(); ++i)
    {
        if(!isPalindrome(test_cases[i]))
            test_cases.erase(test_cases.begin() + i);
    }
    for(int i=0; i<test_cases.size();i++){
        cout << test_cases[i] << endl;
    }
    return 0;
}
```

2.Analysis

Let n represent the number of elements in the vector and m represent the number of characters each string has.

Best case for traversing n elements: O(n)
Worst case for traversing n elements: O(n)

Best case for isPalindrome fn(): O(1)
Worst case for isPalindrome fn(): O(m/2)

Overall time complexities:

Best case: O(n)
Worst case: O(n*m/2)

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1-15. $f(n) = n^2 + 3n^3$

For $n \geq 1$, $n^2 + 3n^3 \leq n^3 + 3n^3 = 4n^3$ if we take $c = 4$ and $N = 1$

$\therefore f(n) \in O(n^3)$

For $n \geq 0$, $n^2 + 3n^3 \geq 1 \times n^3$ if we take $c = 1$ and $N = 0$ to obtain that

$n^2 + 3n^3 \in \Omega(n^3)$

Since $f(n)$ is in both $O(n^3)$ and $\Omega(n^3)$, $1 \times n^3 \leq n^2 + 3n^3 \leq 4n^3$

therefore $f(n) \in O(n^3) \cap \Omega(n^3) = \Theta(n^3)$

$n^2 + 3n^3 \in \Theta(n^3)$, $c = 1$, $d = 4$

1-16. For $n \geq 1$, $6n^2 + 20n \leq n^3$

Take $c = 1$ and $N = 1$ to show that $6n^2 + 20n \in O(n^3)$

Proof by contradiction that $6n^2 + 20n \notin \Omega(n^3)$

Assuming $6n^2 + 20n \in \Omega(n^3)$ and some nonnegative integer N such that for $n \geq N$: $6n^2 + 20n \geq c * n^3$

If we divide both side of this inequality by $c * n^2$

for $n \geq N$, $\frac{6}{c} + \frac{20}{n} \geq n$

However, for any $n > \frac{6}{c} + \frac{20}{n}$, this inequality cannot hold, which means that it cannot hold for all $n \geq N$. This contradiction proves that n is not in $\Omega(n^3)$.

1-17. ~~We have~~ Repeatedly applying Properties 6 and 7, we have

$5n^5 \in \Theta(n^5)$

which means $4n^4 + 5n^5 \in \Theta(n^5)$

which means $6n^3 + 4n^3 + 5n^5 \in \Theta(n^5)$

which means $2n^2 + 6n^3 + 4n^3 + 5n^5 \in \Theta(n^5)$

which means $n + 2n^2 + 6n^3 + 4n^4 + 5n^5 \in \Theta(n^5)$

which means $7 + n + 2n^2 + 6n^3 + 4n^4 + 5n^5 \in \Theta(n^5)$

1-18. $p(n) = a_k n^k + a_{k-1} n^{k-1} + \dots + a_1 n + a_0$ where $a_k > 0$

Using property #2 which states that $g(n) \in \theta(f(n))$ iff $f(n) \in \theta(g(n))$

we can show that

$$1 \times a_k n^k \le a_k n^k + a_{k-1} n^{k-1} + \dots + a_1 n + a_0 \le n^k \cdot \sum_{i=0}^{i=k} a_i$$

$$\Rightarrow p(n) \in \theta(n^k)$$

Conversely, we can show that $n^k \in \theta(p(n))$

$$\frac{p(n)}{a_k n^k} \le n^k \le 1 \times p(n)$$

1-22. n^n, $n^n + \ln n$

$n!$

$10^n + n^{20}$

4^n

e^n

$(\lg n)!$

$n^{5/2}$

$5^{\lg n}$

$5n^2 + 7n$

$n \log n$, $\log(n!)$

$8 \cdot n + 12$

$n^{1/2}$

$(\log n)^2$