Homework 6, Jacobi method for computing eigenvalues

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1. 執行結果(for 作業 I 到 III 小題)

```
A:
  3.000000  17.000000   8.000000   6.000000
 17.000000   4.000000   5.000000  12.000000
  8.000000   5.000000   0.000000   7.000000
  6.000000  12.000000   7.000000   2.000000
k = 1, theta = -0.770697
A:
-13.507351  -0.000000   2.256229  -4.055068
 -0.000000  20.507351   9.160209  12.788918
  2.256229   9.160209   0.000000   7.000000
 -4.055068  12.788918   7.000000   2.000000
-----------------------------------------
k = 15, theta = -0.000002
A:
-15.701484   0.000000   0.000001   0.000000
  0.000000  31.204741  -0.000000  -0.000009
  0.000001  -0.000000  -2.362431   0.000000
  0.000000  -0.000009   0.000000  -4.140826
k = 16, theta = -0.000000
A:
-15.701484   0.000000   0.000001   0.000000
  0.000000  31.204741  -0.000000   0.000000
  0.000001  -0.000000  -2.362431   0.000000
  0.000000   0.000000   0.000000  -4.140826
Need 16 times iterations

Orthogonality between eigen-vectors:
orMtx:(驗證相互垂直)
  1.000000   0.000000   0.000000   0.000000
  0.000000   1.000000   0.000000   0.000000
  0.000000   0.000000   1.000000  -0.000000
  0.000000   0.000000  -0.000000   1.000000
EigenValues = -15.701484 31.204741 -2.362431 -4.140826
The eigen-vectors are:
x[0] =   0.604922  -0.670858  -0.249994   0.348599
x[1] =   0.559090   0.609250   0.341226   0.446987
x[2] =  -0.426080  -0.354842   0.648479   0.521552
x[3] =  -0.374090   0.229892  -0.632884   0.637703
```

Residual of ||A*v - u*v|| = 31.899210 32.688567 13.230114 9.273424

2-norm of ||A*v - u*v|| = 48.447232

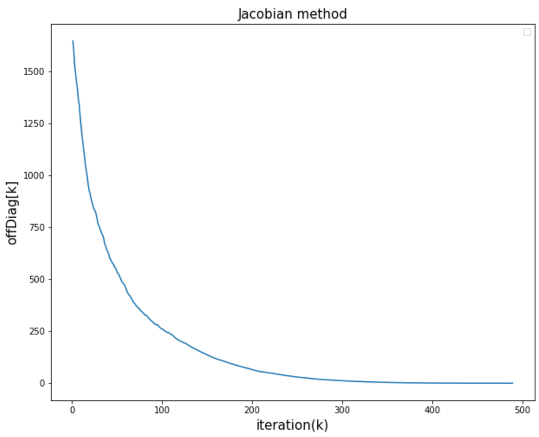
max eigenvalue = 31.204741

min eigenvalue = -2.362431

Condition number = 13.208740

2. 執行結果(for 作業Ⅳ到Ⅴ小題)

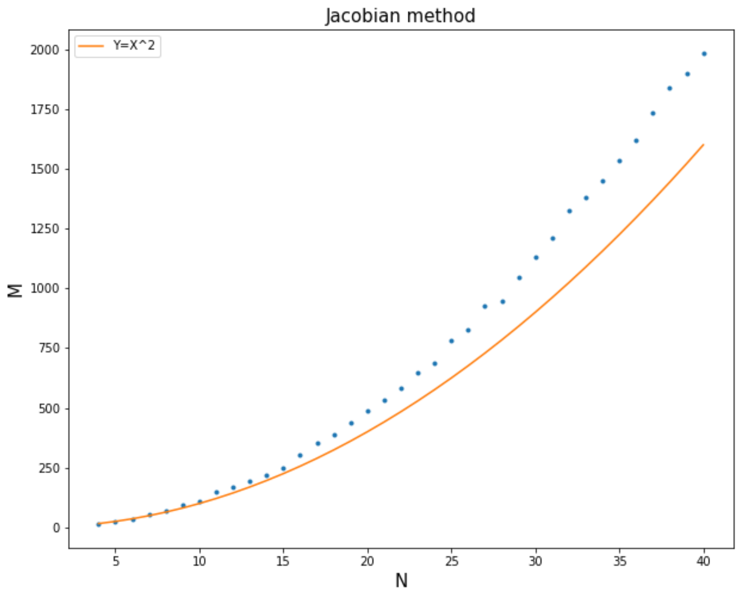
   N=20 offDiag[k]=$\sum_{i<j}|a_{ij}|$的結果圖



   EigenValues = 0.979783 33.8702 -44.6381 -36.9023 34.6742 17.0156 6.43481 -5.48172 -16.0842 -31.3934 -1.32521 -14.6742 47.4565 25.9205 23.246 -7.39834 -25.196 178.474 8.53992 -27.5175

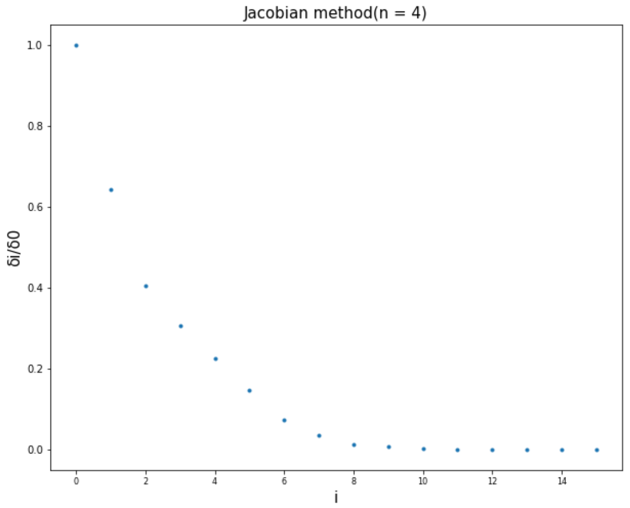
3. 執行結果(for 作業Ⅵ小題)

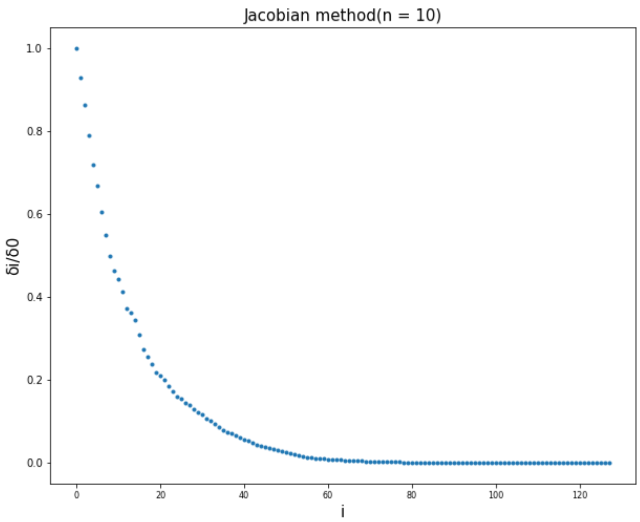
   N=4~40 的 iteration 次數

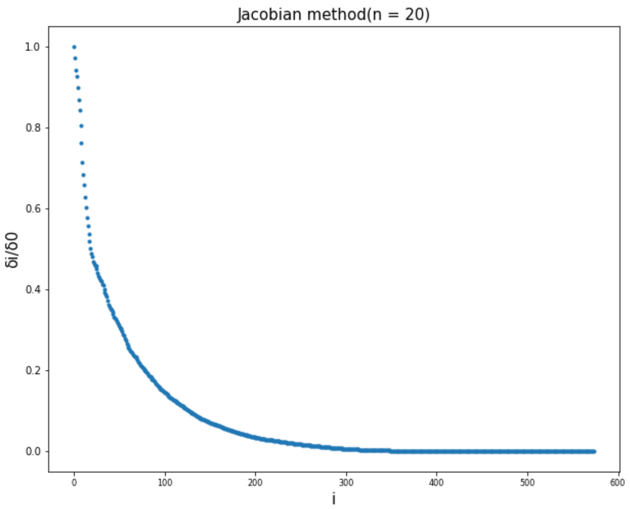


結論：iteration 的次數和 N 很接近二次的關係。

4. 執行結果(for 作業Ⅶ小題)







結果顯示是以 quadratically 的速率下降。

5. 若 A 是 symmetric 且 diagonal dominant，A 會較快收斂嗎？

特徵值和特徵向量會更準確嗎？

執行結果：

```
A:
 39.000000  12.000000  12.000000  10.000000
 12.000000  36.000000   6.000000  13.000000
 12.000000   6.000000  33.000000  10.000000
 10.000000  13.000000  10.000000  38.000000
-----------------------------------------------
k = 15, theta = 0.000000
A:
 29.975699   0.000000   0.000000   0.000000
  0.000000  26.907997  -0.000000   0.000000
  0.000000  -0.000000  20.656694  -0.000000
  0.000000   0.000000  -0.000000  68.459610

Need 15 times iterations
```

```
EigenValues = 29.975699 26.907997 20.656694 68.459610
The eigen-vectors are:
x[0] =   0.525475  -0.484862   0.498326  -0.490362
x[1] =   0.453239   0.499186  -0.514018  -0.530258
x[2] =  -0.464129   0.522656   0.558755  -0.446327
x[3] =   0.550486   0.492497   0.418628   0.528359

Residual of ||A*v - u*v|| = 19.490901 22.309540 22.013548 35.609555
2-norm of ||A*v - u*v|| = 51.285939

max eigenvalue = 68.459610
min eigenvalue = 20.656694
Condition number = 3.314161
```

結論：並無影響。

6. 心得

這個作業許多實驗結果都和預期的相同。

我覺得 Jacobi method 不是很容易收斂，有可能要上千次才能收斂。