

Homework 3, 2D Gaussian Quadrature

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1. 實際積分值計算

網路計算機：

[integral from 0 to 4 of integral from 0 to 4 of \(sin\(2x\)\)^2-\(cos\(3y\)\)^2+1 - Symbolab](#)

$$\int_0^4 \int_0^4 (\sin(2x))^2 - (\cos(3y))^2 + 1 dx dy = 16 - \frac{1}{3} \sin(24) - \frac{1}{2} \sin(16)$$

自己算：

$$\begin{aligned} \int_0^4 \int_0^4 [(\sin 2x)^2 - (\cos 3y)^2 + 1] dx dy &= \int_0^4 \int_0^4 \left[\frac{1}{2}(1 - \cos 4x) - \frac{1}{2}(1 + \cos 6y) + 1 \right] dx dy \\ &= \int_0^4 \left[4 - 2 \cos 6y - \frac{1}{8} \sin 16 \right] dy = 16 - \frac{1}{3} \sin 24 - \frac{1}{2} \sin 16 \quad (\text{計算結果相同}) \end{aligned}$$

➔ **$I = 16.445811112334741466156629030593$**

2. conformation mapping

$$I = \int_c^d \int_a^b f(x, y) dx dy = \int_{-1}^1 \int_{-1}^1 f\left(\frac{(b-a)x + (a+b)}{2}, \frac{(d-c)y + (c+d)}{2}\right) \cdot \frac{b-a}{2} \cdot \frac{d-c}{2} dx dy$$

3. 計算程式碼

```
ld func_mapping(ld x, ld y, double a, double b, double c, double d) {  
ld s = pow(sin(2.0 * ((b - a) * x + b + a) / 2.0), 2.0);  
ld t = pow(cos(3.0 * ((d - c) * y + d + c) / 2.0), 2.0);  
return (s - t + 1.0) * (b - a) * (d - c) / 4.0;  
}
```

4. ld count(int cell, int n) {

```
ld ans = 0;  
for (int i = 0; i < n; i++)  
    for (int j = 0; j < n; j++)  
        for (int x = 0; x < 4; x += 4 / cell)  
            for (int y = 0; y < 4; y += 4 / cell)  
                ans += coef[n][i] * coef[n][j] *  
                    func_mapping(root[n][i], root[n][j], x, x + 4 / cell, y, y + 4 / cell);  
return ans;  
}
```

5. 執行結果

actual value: 16.4458111123347415

cell = 1, N = 2: 10.496708585783 absolute error = 5.949102526551 relative error = 0.566758853781

cell = 1, N = 3: 17.880218530472 absolute error = 1.434407418137 relative error = 0.080223148039

cell = 1, N = 4: 19.686646209607 absolute error = 3.240835097272 relative error = 0.164620985350

cell = 4, N = 2: 15.097671982560 absolute error = 1.348139129775 relative error = 0.089294503903

cell = 4, N = 3: 16.558685402320 absolute error = 0.112874289985 relative error = 0.006816621443

cell = 4, N = 4: 16.440957281890 absolute error = 0.004853830445 relative error = 0.000295227970

cell = 16, N = 2: 16.445396092283 absolute error = 0.000415020051 relative error = 0.000025236245

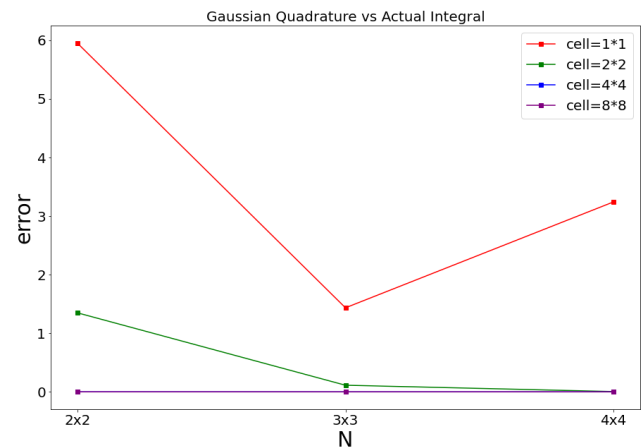
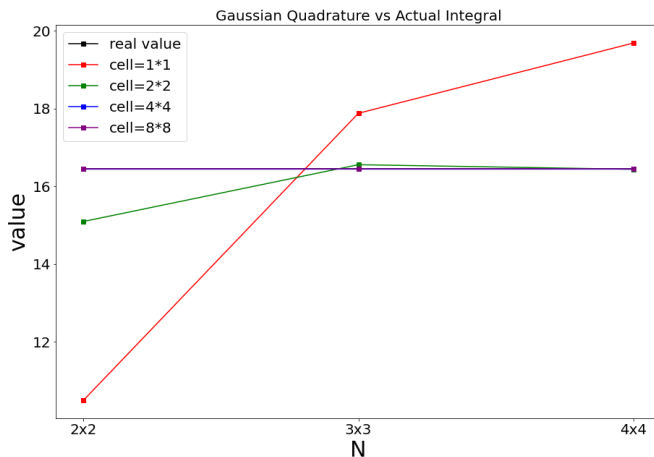
cell = 16, N = 3: 16.445813030815 absolute error = 0.000001918480 relative error = 0.000000116655

cell = 16, N = 4: 16.445811107530 absolute error = 0.000000004805 relative error = 0.000000000292
 cell = 64, N = 2: 16.445809593917 absolute error = 0.000001518418 relative error = 0.000000092329
 cell = 64, N = 3: 16.445811112770 absolute error = 0.000000000436 relative error = 0.000000000026
 cell = 64, N = 4: 16.445811112335 absolute error = 0.000000000000 relative error = 0.000000000000
 cell = 256, N = 2: 16.445811106427 absolute error = 0.000000005908 relative error = 0.000000000359
 cell = 256, N = 3: 16.445811112335 absolute error = 0.000000000000 relative error = 0.000000000000
 cell = 256, N = 4: 16.445811112335 absolute error = 0.000000000000 relative error = 0.000000000000

這個函數的結果跟預想中的相同，N 越大，cell 越多，誤差就會越少。

6. 分析結果

N 的選擇：

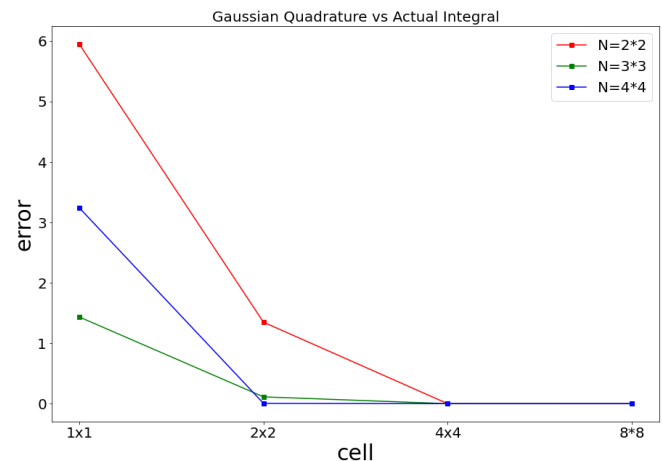
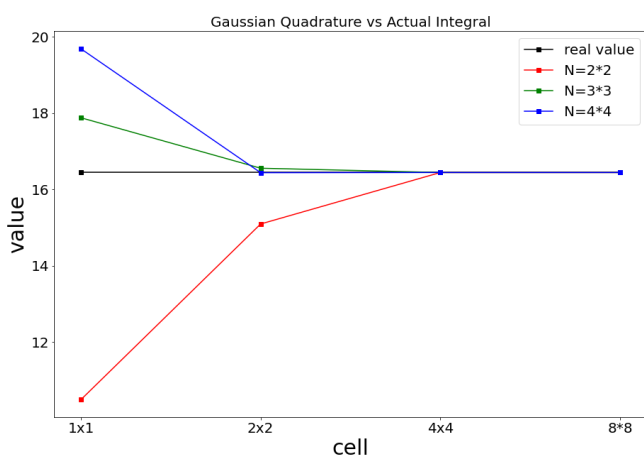


高斯積分值和實際積分值比較

高斯積分值和實際積分值的誤差(絕對值)

※cell=4*4 和 8*8 的數據十分接近，在圖中分離不明顯(跟實際積分值的線重疊)

cell 的選擇：



高斯積分值和實際積分值比較

高斯積分值和實際積分值的誤差(絕對值)

由圖可看出，大部分的數據顯示 N 越大，cell 越多，高斯積分就會越接近實際積分值。

此表顯示 N 變大或 cell 變多對誤差的影響：

	(N=2) / (N=3)	(N=3) / (N=4)
cell 1	4.147428723059	0.442604259422
cell 4	11.943721904742	23.254683339800
cell 16	216.327542526034	399.259229657342
cell 64	3485.708948407197	2786.681818181818
cell 256	118780.928571428565	0.482758620690

	cell 1/cell 4	cell 4/cell 16	cell 16/cell 64	cell 64/cell 256
N=2	4.412825349521	3248.371074452727	273.323950820146	257.013792497954
N=3	12.708008336790	58835.272401144393	4404.098202489112	8758.142857142857
N=4	667.68609533364	1010141.7074708967	30738.97727272727	1.517241379310

比起固定 cell 來比較 N，固定 N 比較 cell 時，cell 增加使得誤差減少的比率似乎較明顯，因此推論誤差受 cell 的影響較大，但兩個因素影響的差距並不大。