213L4: Discrete Probability Distributions

June 12, 2020 1:28 PM

1. Discrete Random Variables (DRV)
2. Probability Distributions & Probability Mass Function
3. Cumulative Distribution Functions
4. Mean & Variance of Discrete Random Variable
5. Discrete Uniform Distribution
6. Binomial Distribution
7. Geometric & Negative Binomial Distributions
8. Hypergeometric Distribution
9. Poisson Distribution

②③⑧

Ⓒ Probability Mass Function (PMF)

for DRV $X = x_1, x_2, \ldots, x_n$

1) $f(x_i) \geq 0$
2) $\sum_{i=1}^{n} f(x_i) = 1$
3) $f(x_i) = P(X = x_i)$

Ⓒ Cumulative Distribution Function

for DRV X, $F(x)$ satisfies:

1) $F(x) = P(X \leq x) = \sum_{x_i \leq x} f(x_i)$
2) $0 \leq F(x) \leq 1$
3) If $x \leq y$, then $F(x) \leq F(y)$

Ⓒ Hypergeometric Distribution

Hypergeometric random variable X w/ K successes, $N-K$ failures, $p = K/N$

$$f(x) = \frac{\binom{K}{x}\binom{N-K}{n-x}}{\binom{N}{n}}$$

$x = \max\{0, n+K-N\}$ to $\min\{K, n\}$

μ: $E(X) = np$

σ^2: $V(X) = np(1-p)\left(\frac{N-n}{N-1}\right)$ finite population correction factor

④⑤⑨

DRVs:

$\mu = E(X) = \sum_x x f(x)$

$\sigma^2 = V(X)$

$= E(X-\mu)^2 = \sum_x (x-\mu)^2 f(x)$

$= \sum_x x^2 f(x) - \mu^2$

$\sigma = \sqrt{\sigma^2}$

$E[h(X)] = \sum_x h(x) f(x)$

Ⓒ Discrete Uniform Distribution

iff each of n values have equal probability

$f(x_i) = 1/n$

$\mu = E(X) = \frac{b+a}{2}$

$\sigma^2 = \frac{(b-a+1)^2 - 1}{12}$

for consecutive ints $a, a+1, a+2, \ldots, b$ $\forall a \leq b$.

Ⓒ Poisson Distribution

Partition interval of real #s into subintervals of small enough length s.t. if counts are random,

1) $P(>1 \text{ count}) = 0$
2) $P(\text{count}_i)$ is equal $\forall i \in$ subinterval
3) Count is independent

Poisson random variable X w/ $0 < \lambda$

$$f(x) = \frac{e^{-\lambda}\lambda^x}{x!} \qquad x = 0, 1, \ldots$$

μ: $E(X) = \lambda$

σ^2: $V(X) = \lambda$

PMF of Binomial Random Variable X

$f(x) = \binom{n}{x} p^x (1-p)^{n-x}$

$x = 0, 1, \ldots, n$ $\quad 0 < p < 1$ $\quad n = 1, 2, \ldots$

$\mu = E(X) = np$

$\sigma^2 = V(X) = np(1-p)$

⑥⑦

Ⓒ Binomial Distribution

Random experiment consisting of n Bernouilli trials s.t.

1) trials are independent
2) only 2 possible outcomes per trial (SUCCESS & FAILURE)
3) constant probability of success, p, in each trial.

Ⓒ Geometric Binomial Distributions

Geometric random variable X w/

$0 < p < 1$ $\quad x = 1, 2, \ldots$

x: # of trials until first success

$f(x) = (1-p)^{x-1} p$

$\mu = E(X) = \frac{1}{p}$

$\sigma^2 = V(X) = (1-p)/p^2$

Ⓒ Negative Binomial Distribution

Negative binomial random variable w/

$0 < p < 1$ $\quad r = 1, 2, 3, \ldots$

$f(x) = \binom{x-1}{r-1}(1-p)^{x-r} p^r$

$x = r, r+1, r+2$

*Can be represented as sum of Geometric random variables

μ: $E(X) = r/p$

σ^2: $V(X) = r(1-p)/p^2$