213L5: Continuous Probability Distribution

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Cont. Rand Var.

$\mu = E(X) = \int_{-\infty}^{\infty} x f(x)\, dx$

$\sigma^2 = V(X) = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)\, dx$

$= \int_{-\infty}^{\infty} x^2 f(x)\, dx - \mu^2$

$\sigma = \sqrt{\sigma^2}$

PDF

1) $f(x) \geq 0$
2) $\int_{-\infty}^{\infty} f(x)\, dx = 1$
3) $P(a \leq X \leq b) = \int_a^b f(x)\, dx$

for any x_1, x_2:

$P(x_1 \leq X \leq x_2)$

$= P(x_1 < X \leq x_2)$

$= P(x_1 \leq X < x_2)$

$= P(x_1 < X < x_2)$

CDF

$F(x) = P(X \leq x) = \int_{\infty}^{x} f(u)\, du$

Normal Approximation

Binomial

$Z = \frac{X - np}{\sqrt{np(1-p)}}$ $\quad np > 5$, $n(1-p) > 5$

hypergeometric distribution $\approx$ binomial distribution $\approx$ normal distribution

$\frac{n}{N} < 0.1$ $\qquad np > 5$, $n(1-p) > 5$

Poisson

$Z = \frac{X - \lambda}{\sqrt{\lambda}}$ $\quad \lambda > 5$

$E(X) = V(X) = \lambda$

Cont. Uni. Rand Var.

$\mu = E(X) = \frac{a+b}{2}$

$\sigma^2 = V(X) = \frac{(b-a)^2}{12}$

PDF:

$f(x) = \frac{1}{b-a}, \quad a \leq x \leq b$

CDF

$F(x) = \int_a^x \frac{1}{b-a}\, du$

$= \frac{x-a}{b-a}$

$$F(x) = \begin{cases} 0 & , x < a \\ \frac{x-a}{b-a} & , a \leq x < b \\ 1 & , b \leq x \end{cases}$$

Norm. Rand. Var

PDF $\quad N(\mu, \sigma^2)$

$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{\frac{-(x-\mu)^2}{2\sigma^2}}$

$P(\mu - \sigma < X < \mu + \sigma) = 0.6827$

$P(\mu - 2\sigma < X < \mu + 2\sigma) = 0.9545$

$P(\mu - 3\sigma < X < \mu + 3\sigma) = 0.9973$

Std. Norm. Rand. Var.

$Z = \frac{X - \mu}{\sigma}$ "z-value obtained by standardizing X"

$\mu = 0 \quad \sigma^2 = 1$

PDF

$z = \frac{(x-\mu)}{\sigma}$ @ Appendix II

CDF

$\Phi(z) = P(Z \leq z)$

Exp. Rand. Var.

$\mu = E(X) = \frac{1}{\lambda} \quad \sigma^2 = V(X) = \frac{1}{\lambda^2} \quad \lambda > 0$

PDF

$f(x) = \lambda e^{-\lambda x} \quad 0 \leq x < \infty$

// Distance between successive counts of Poisson process

CDF

$P(X > x) = 1 - F(x) = e^{-\lambda x}$

Lack of Memory Property

$P(X < t_1 + t_2 \mid X > t_1) = P(X < t_2)$

Erlang Rand. Var

PDF

$f(x) = \frac{\lambda^r x^{r-1} e^{-\lambda x}}{(r-1)!}$ $\quad x > 0,\ r = 1, 2, 3 \ldots$

// interval length until r counts occur in Poisson

Gamma Rand. Var.

PDF

$f(x) = \frac{\lambda^r x^{r-1} e^{-\lambda x}}{\Gamma(r)}$ $\quad x > 0,\ r > 0$

// if $r = int(r)$, $X \to$ Erlang

Weibull Rand Var

PDF

$f(x) = \frac{\beta}{\delta}\left(\frac{x}{\delta}\right)^{\beta-1} e^{-\left(\frac{x}{\delta}\right)^{\beta}}$

$x > 0$

$\delta > 0$ // scale

$\beta > 0$ // shape

Lognormal Rand. Var

PDF

$f(x) = \frac{1}{x\omega\sqrt{2\pi}} e^{-\frac{(\ln x - \theta)^2}{2\omega^2}}$ $\quad 0 < x < \infty$

// $X = \exp(W)$ where W has normal distribution $\mu = \theta$, $\sigma^2 = \omega^2$

$E(X) = e^{\theta + \frac{\omega^2}{2}} \quad V(X) = e^{2\theta + \omega^2}(e^{\omega^2} - 1)$

1. Continuous Random Variables
2. Probability Distribution & PDF
3. Cumulative Distribution Functions
4. Mean & Variance of Continuous Random Variable
5. Continuous Uniform Distribution
6. Normal Distribution
7. Normal Approximation to Binomial & Poisson Distributions
8. Exponential Distribution
9. Erlang & Gamma Distribution
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11. Lognormal Distribution