213L6: Point Estimation of Parameters

June 16, 2020 10:22 AM

1. Introduction
2. General Concepts of Point Estimation
3. Methods of Point Estimation
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5. Sampling Distributions of Means

Statistical inference:
- Parameter estimation
- Hypothesis testing

© Point Estimate of population parameter θ

↳ single numerical value $\hat{\theta}$ of Statistic $\hat{\Theta}$

Point Estimator

μ: $\hat{\mu} = \bar{x}$

σ^2: $\hat{\sigma}^2 = s^2$

p: $\hat{p} = x/n$

$\mu_1 - \mu_2$: $\hat{\mu}_1 - \hat{\mu}_2 = \bar{x}_1 - \bar{x}_2$

$p_1 - p_2$: $\hat{p}_1 - \hat{p}_2$

$\text{Bias} = E(\hat{\Theta}) - \theta$

0 when unbiased

Minimum Variance Unbiased Estimator (MVUE)

↳ unbiased estimators of θ w/ smallest variance

μ: MVUE = sample mean $\bar{X}$

// normal dist. w/ μ, σ^2 & random sample size n.

Standard Error

$\sigma_{\hat{\Theta}} = \sqrt{V(\hat{\Theta})}$

$\hat{\sigma}_{\hat{\Theta}}$ → estimation of std. error w/ unknown params.

$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}}$

$\hat{\sigma}_{\bar{X}} = \frac{s}{\sqrt{n}}$

Mean Squared Error

$MSE(\hat{\Theta}) = E(\hat{\Theta} - \theta)^2$

$MSE(\bar{X}) = E((\bar{X} - \mu)^2) = \left(\frac{\sigma}{\sqrt{n}}\right)^2$

Relative Efficiency

$\frac{MSE(\hat{\Theta}_1)}{MSE(\hat{\Theta}_2)}$

Methods of PE

k^{th} population moment $= E(X^k)$

k^{th} sample moment $= \frac{1}{n}\sum_{i=1}^{n} X_i^k$

Moment Estimators $\hat{\Theta}_1, \hat{\Theta}_2, \ldots, \hat{\Theta}_m$ found by

$$\begin{cases} \mu = \bar{X} \\ \mu^2 + \sigma^2 = \frac{1}{n}\sum_{i=1}^{n} X_i^2 \end{cases}$$

$\hat{\mu} = \bar{X}$

$$\hat{\sigma}^2 = \frac{\sum_{i=1}^{n} X_i^2 - \left(\frac{1}{n}\sum_{i=1}^{n} X_i\right)^2}{n} = \frac{\sum_{i=1}^{n}(X_i - \bar{X})^2}{n}$$

// not unbiased

Likelihood Function

$L(\theta) = f(x_1;\theta) \cdot f(x_2;\theta) \cdot \ldots \cdot f(x_n;\theta)$

↳ Maximum Likelihood Estimators

$\hat{\mu} = \bar{X}$ $\quad \hat{\sigma}^2 = \frac{1}{n}\sum_{i=1}^{n}(X_i - \bar{X})^2$ $\quad \frac{dL(\theta)}{d\theta} = 0$

1) $E(\hat{\Theta}) \simeq \theta$
2) $V(\hat{\Theta})$ nearly as small as variance obtained w/ any other estimator
3) $\hat{\Theta}$ has an approx. normal distribution

Invariance Property

if $\hat{\Theta}_1, \hat{\Theta}_2, \ldots, \hat{\Theta}_k$ are max. likelihood estimators w/ param $\theta_1, \theta_2, \ldots, \theta_k$, then max likelihood est. of any function $h(\theta_1, \theta_2, \ldots, \theta_k)$ of these parameters is the same function $h(\hat{\Theta}_1, \hat{\Theta}_2, \ldots, \hat{\Theta}_k)$ of estimators $\hat{\Theta}_1, \hat{\Theta}_2, \ldots, \hat{\Theta}_k$.

Statistical Inference

© Sampling distribution: probability distribution of a statistic

© Central Limit Thm:

if 1) $X_1, X_2, \ldots, X_n$ of rand. sample size n has μ & σ^2
2) $\bar{X}$ = sample mean

$Z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}}$ as $n \to \infty$ // approx. std. normal dist.

© Sampling Distribution of Means:

if 1) 2 independent populations w/ μ_1 & μ_2, σ_1^2 & σ_2^2
2) $\bar{X}_1$ & $\bar{X}_2$ are sample means

$Z = \frac{\bar{X}_1 - \bar{X}_2 - (\mu_1 - \mu_2)}{\sqrt{\sigma_1^2/n_1 + \sigma_2^2/n_2}}$ // approx. std. normal