CHAPTER 3: TIME-DOMAIN ANALYSIS OF DISCRETE-TIME SYSTEMS

linear, time-invariant, discrete-time (LTID)

3.1 Introduction

discrete-time signal: sequence of numbers

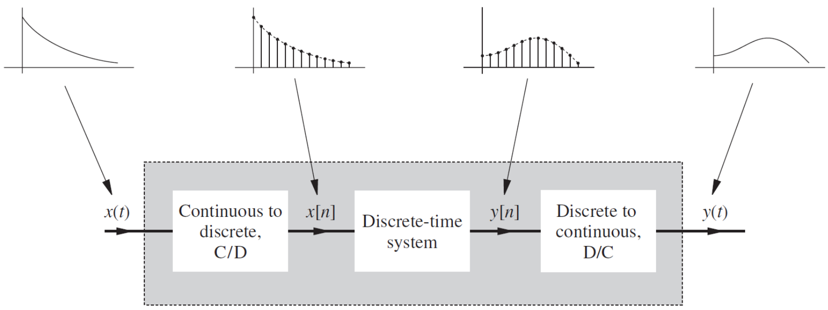


Figure 3.2 Processing a continuous-time signal by means of a discrete-time system.

continuous-time exponential $x(t) = e^{-t}$ sampled every $T = 0.1$ seconds results in a discrete-time signal $x(nT)$:

$$x(nT) = e^{-nT} = e^{-0.1n}$$

3.1-1 Size of a Discrete-Time Signal

Size measured by energy:

$$E_x = \sum_{n=-\infty}^{\infty} |x[n]|^2 \qquad (3.1.1\text{-}1)$$

if E_x is finite, signal is **energy signal**. Else, measured by signal power:

$$P_x = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{-N}^{N} |x[n]|^2 \qquad (3.1.1\text{-}2)$$

*2N+1 samples in interval from -N to N

3.2 Signal Operations

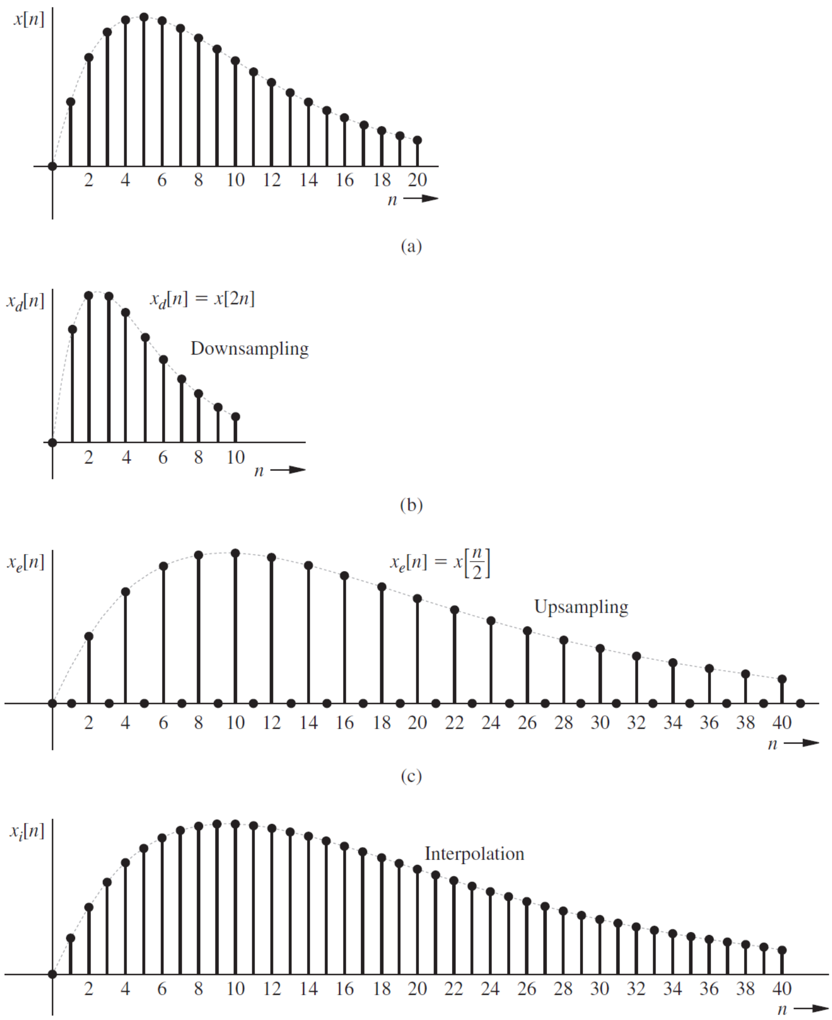
Shifting (by M units)

$$x_s[n] = x[n-M]$$

Time Reversal

$$x_r[n] = x[-n]$$

Sampling Rate Alteration: Downsampling, Upsampling, Interpolation



Downsampling: Compression by factor M

$$x_d[n] = x[Mn], M \in \mathbb{N}^+$$

Interpolated signal

$$x_e[n] = \begin{cases} x[n/L] & n = 0, \pm L, \pm 2L, \dots \\ 0 & \text{otherwise} \end{cases}$$

Upsampling: L times that of $x[n]$: general sequence:

$$x_e[n] = x[0], \underbrace{0, 0, \dots, 0, 0,}_{L-1 \text{ zeros}} x[1], \underbrace{0, 0, \dots, 0, 0,}_{L-1 \text{ zeros}} x[2], \underbrace{0, 0, \dots, 0, 0,}_{L-1 \text{ zeros}} \dots$$

3.3 Discrete-Time Signal Models

3.3-1 Discrete-Time Impulse Function $\delta[n]$

Unit impulse sequence: Kronecker delta

$$\delta[n] = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases} \qquad (3.3.1\text{-}1)$$

3.3-2 Discrete-Time Unit Step Function $u[n]$

$$u[n] = \begin{cases} 1 & \text{for } n \geq 0 \\ 0 & \text{for } n < 0 \end{cases}$$

3.3-3 Discrete-Time Exponential γ^n

continuous-time exponential $e^{\lambda t}$ can be alternatively expressed as:

$$e^{\lambda t} = \gamma^t \qquad (\gamma = e^{\lambda} \text{ or } \lambda = \ln \gamma)$$

discrete:

$$\gamma^n = e^{\lambda n} \qquad (\gamma = e^{\lambda} \text{ or } \lambda = \ln \gamma)$$

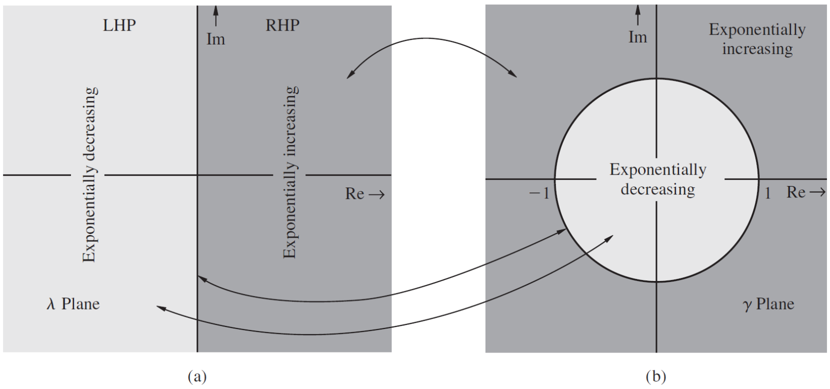


Figure 3.8 The λ plane, the γ plane, and their mapping.

e.g. signal $e^{\lambda n}$ where λ lies on left half-plane ($\lambda = a + jb, a < 0$), exponential decay

$$\gamma = e^{\lambda} = e^{a+jb} = e^a e^{jb}$$
$$|\gamma| = |e^a||e^{jb}| = e^a, \text{ because } |e^{jb}| = 1$$

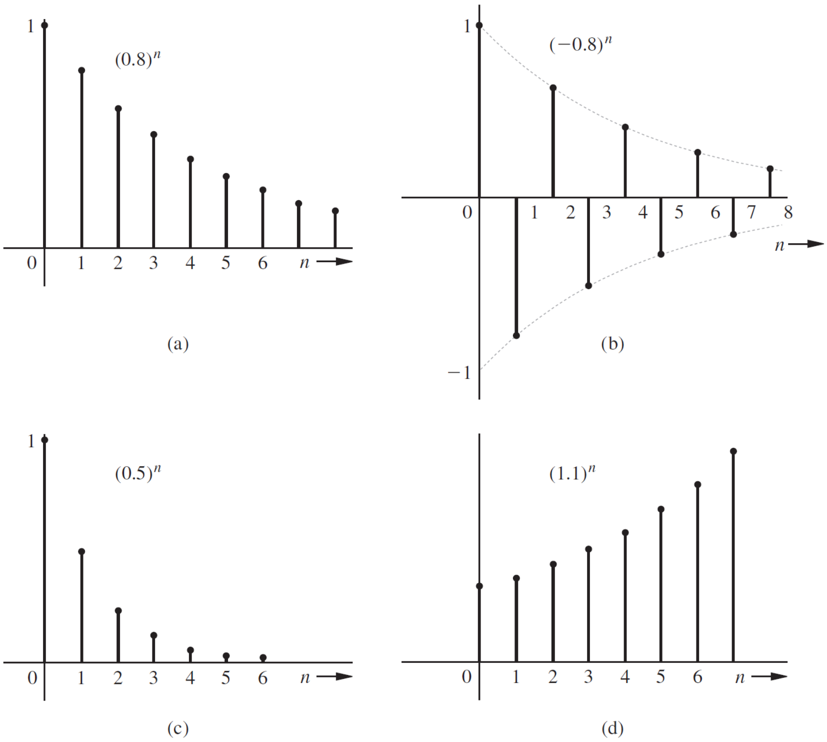


Figure 3.9 Discrete-time exponentials γ^n.

3.3-4 Discrete-Time Sinusoid $cos(\Omega n + \theta)$

General discrete-time sinusoid:

$$C\cos(\Omega n + \theta) = C\cos(2\pi F n + \theta) \text{ where } F = \Omega/2\pi$$

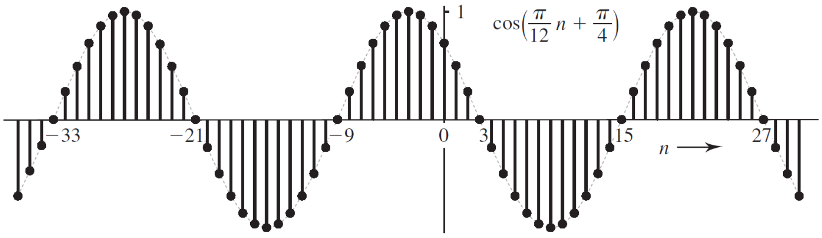


Figure 3.11 A discrete-time sinusoid $\cos\left(\frac{\pi}{12}n + \frac{\pi}{4}\right)$.

Sampled Continuous-Time Sinusoid Yields a Discrete-Time Sinusoid

A continuous-time sinusoid, $\cos wt$, sampled every T seconds yields a discrete-time sinusoid. Sample signal $x[n]$:

$$x[n] = \cos \omega nT = \cos \Omega n \text{ where } \Omega = \omega T$$

3.3-5 Discrete-Time Complex Exponential $e^{j\Omega n}$

$$e^{j\Omega n} = (\cos \Omega n + j \sin \Omega n)$$
$$e^{-j\Omega n} = (\cos \Omega n - j \sin \Omega n)$$

For $r = 1$ and $\theta = n\Omega$,

$$e^{j\Omega n} = re^{j\theta}$$