SUMMARY

General Addition Rule: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Rule of the Complement: $P(\overline{A}\,) = 1 - P(A)$

General Multiplication Rule: $P(A \cap B) = P(A)P(B \mid A)$ if $P(A) \neq 0$
$= P(B)P(A \mid B)$ if $P(B) \neq 0$

Conditional Probability: $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)}$ if $P(B) \neq 0$

C3: PROBABILITY

1. [def] probability
    a. allows quantifiable variability in experimental outcome
    b. cannot be predicted with certainty
2. how is probability determined
3. principles of probability

3.1 Sample Spaces & Events

sample space: a set of possible outcomes

discrete sample space: has finitely many or countable infinity of elements

continuous sample space: sample space that constitute a continuum

event: subset of a sample

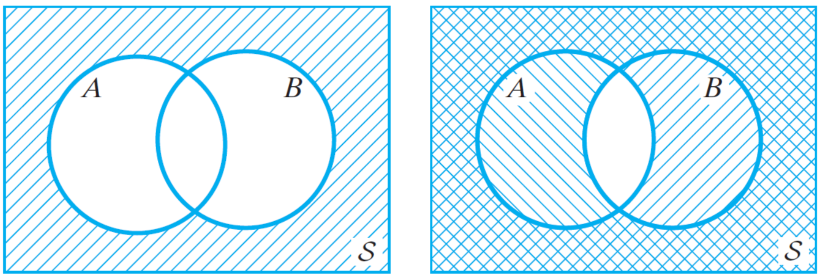
- **mutually exclusive events**: sets with no common elements
- **unions**: $A \cup B = S$

  subset S contains all elements in A AND B
- **intersections**: $A \cap B = S$

  subset S contains all elements COMMON to both A and B
- **complements**: $\bar{A} + A = S$

  subset of S containing all elements not in A

e.g. $(A \bar{\cup} B) = \bar{A} \cap \bar{B}$



$(\overline{A \cup B})$ $\qquad$ $\overline{A} \cap \overline{B}$

3.2 Counting

[thm] **fundamental theorem of counting**: if sets $A_1, A_2, \ldots, A_k$ contain, respectively, $n_1, n_2, \ldots, n_k$ elements, there are $n_1 \times n_2 \times \ldots \times n_k$ ways of choosing the first element of A_1, then an element of $A_2, \ldots, A_k$.

permutation: the set of arrangements of when r objects are chosen from a set of n distinct objects

factorial notation: product of consecutive integers

$$
\begin{gathered}
n! = n(n-1)(n-2)\ldots 2 \cdot 1 \\
0! = 1
\end{gathered}
$$

[thm] number of **permutations** of r objects selected from a set of n distinct objects is

$$
\begin{aligned}
{}_nP_r &= n(n-1)(n-2)\ldots(n-r+1) \\
&= \frac{n!}{(n-r)!}
\end{aligned} \tag{3.2-1}
$$

for $r = 1, 2, \ldots, n$

[thm] number of **combinations** of n objects taken r at a time is

$$
\begin{aligned}
{}_nC_r &= \binom{n}{r} \\
&= \frac{{}_nP_r}{r!} \\
&= \frac{n!}{r!(n-r)!} \\
&= \frac{n(n-1)(n-2)\ldots(n-r+1)}{r!}
\end{aligned} \tag{3.2-2}
$$

3.3 Probability

classical probability concept: if there are m equally likely possibilities, of which one must occur, and s is regarded as favourable outcomes, then probability of success is given by $\frac{s}{m}$.

frequency interpretation: probability of an event/outcome is proportion of the times the event will occur in the long run of repeated experiments.

$$
r_N = \frac{\text{Number of occurances of A in N trials}}{\text{N}}
$$

subjective probabilities express strength of one's belief w.r.t. uncertainties involved.

3.4 Axioms Probability

probability: a set of **additive set functions**

- a set function that assigns to each subset A of a finite sample space S the number of elements in A, written $N(A)$.

conditions of the additive set function defining $P(A)$:

1. $0 \leq P(A) \leq 1$ for each event A in S.
   $P(A) = \frac{s}{m}$, where $0 \leq s \leq m < 1$

2. $P(S) = 1$.

$P(S) = \frac{m}{m} = 1$

3. if A and B are mutually exclusive events in S, then

$$\begin{aligned} P(A \cup B) &= \frac{s_1}{m} + \frac{s_2}{m} \\ &= \frac{s_1 + s_2}{m} \\ &= P(A) + P(B) \end{aligned}$$

$$P(A) = \sum_{i=0}^{N} p_i$$

where N is the number of outcomes in A and p_i is the probability of the i^{th} outcome.

3.5 Elementary Theorems

[thm] if $A_1, A_2, \ldots, A_n$ are mutually exclusive in a sample space S, then

$$P(A_1 \cup A_2 \cup \ldots \cup A_n) = P(A_1) + P(A_2) + \ldots + P(A_n) \quad (3.5\text{-}1)$$

[thm] if A is an event in the finite sample space S, then $P(A)$ equals the sum of probabilities of the individual outcomes comprising A.

[thm] if A and B are any events in S, then

$$\begin{aligned} P(A \cup B) &= P(A) + P(B) - P(A \cap B) \\ \text{proof: } P(A \cup B) &= P(A \cap B) + P(A \cap \bar{B}) + P(\bar{A} \cap B) \\ &= [P(A \cap B) + P(A \cap \bar{B})] + [P(A \cap B) + P(\bar{A} \cap B)] - P(A \cap B) \\ &= P(A) + P(B) - P(A \cap B) \end{aligned}$$

mutually exclusive, thus $P(A \cap B) = 0$.

[thm] if A is any event in S, then $P(\bar{A}) = 1 - P(A)$.

3.6 Conditional Probability

if A and B are any events in S and $P(B) \neq 0$, the conditional probability of A given by B is

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \qquad (3.6\text{-}1)$$

[thm] if A and B are <u>independent</u> events in S, then

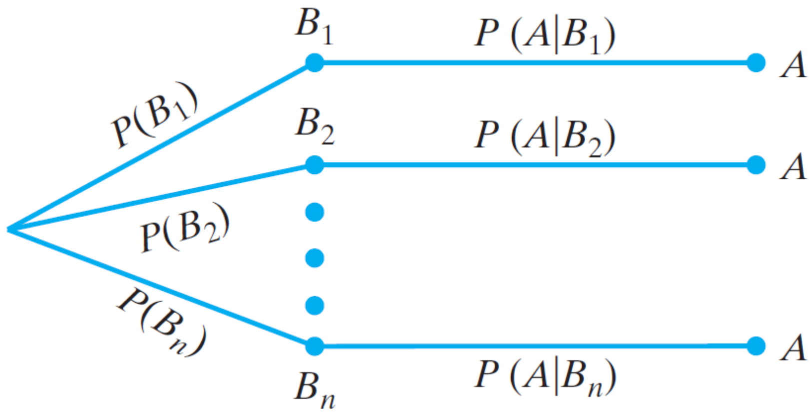
$$\begin{aligned} P(A \cap B) &= P(A) \cdot P(B|A) && \text{if } P(A) \neq 0 \\ &= P(B) \cdot P(A|B) && \text{if } P(B) \neq 0 \end{aligned}$$

[thm] 2 events A and B are independent events iff

$$P(A \cap B) = P(A) \cdot P(B)$$

3.7 Bayes' Theorem

$$P(A) = \sum_{i=1}^{n} P(B_i) \cdot P(A|B_i) \qquad (3.7\text{-}1)$$



[thm] **Bayes' Theorem** (rule of total probability): if $B_1, B_2, \ldots, B_n$ are mutually exclusive events of which one must occur, then

$$P(B_r|A) = \frac{P(B_r) \cdot P(A|B_r)}{\sum_{i=1}^{n} P(B_i) \cdot P(A|B_i)}$$

for $r = 1, 2, \ldots, n$