

HW 6-7 Sols

Prog 1 # The Euclidean alg

```
def gcd(y, x):
    while x>0:
        oldx=x
        x=y%x
        y=oldx
        #print(oldx,x,y)
    return oldx
print(gcd(287,92))
```

Prog2 # Adding binary (Note: reads the binary backwards)

```
import time
def add_bin(list1,list2):
    n=len(list1)
    lst=[0]*(n+1)
    c=0
    for i in range(n):
        lst[i]=(list1[i]+list2[i]+c)%2

        c=(list1[i]+list2[i]+c)//2 # c is the carry
    lst[n]=c
    return lst
```

prog 4 Karatsuba

```
1. import math
2. global count # counts recursive calls
3. count = 0
4. x=1234567891011121314151617181920212223242526272829303132333435363
5. y=1357911131517192123252729313335373941434547495153555759616365676
6. #x=1234
7. #y=5678
8.
9. def kara(x, y):
10.     #print("x y = ", x, y)
11.     global count # Counts recursive calls
12.     count += 1
13.
14.     n = max(int(math.log10(x+.001))+1,int(math.log10(y+.001))+1)
15.     # The .001 is a kludge to fix when x=0
16.     if x<10 or y<10:
17.         return x*y
18.     else:
19.         frontx, backx = int(x//(10**(n//2))), x % int((10**(n//2)))
20.         fronty, backy = int(y//(10**(n//2))), y % int((10**(n//2)))
21.         z2 = kara(frontx, fronty)
22.         z0 = kara(backx, backy)
23.         z1 = kara(frontx+backx, fronty+backy)-z2-z0
24.
25.         return 10**(2*(n//2))*z2+10**(n//2)*z1+z0
26.
27.
28. print(kara(x,y), '\n')
29.
30. print("recursive calls = ",count, '\n') # Prints recursive calls
31.
32. print(x*y)
```

16764334818177052661295147374180744306860071176019146696226647848396037553321171925334
14749453310762762797651012520861917800388

recursive calls = 1525

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2) b) $101 - 100 = 1 = \underline{\text{gcd}}$

c) $\text{gcd}(1001, 1331) = \text{gcd}(330, 1001) =$
 $\text{gcd}(11, 330) = \underline{11}$

d) $\text{gcd}(1529, 14039) = \text{gcd}(278, 1529)$
 $= \text{gcd}(139, 278) \Rightarrow \underline{139}$

e) $\text{gcd}(1529, 14038) = \text{gcd}(277, 1529)$
 $\rightarrow (144, 277) \rightarrow (133, 144) \rightarrow (11, 133) \rightarrow (1, 11)$
 $\underline{11}$

f) $\text{gcd}(11111, 111111) \rightarrow \underline{11111}$

3) $(21, 34) \xrightarrow{1} (13, 21) \xrightarrow{2} (8, 13) \xrightarrow{3} (5, 8) \xrightarrow{4} (3, 5)$

$\xrightarrow{5} (2, 3) \xrightarrow{6} (1, 2)$

$\underline{6}$

4) $(34, 55) \rightarrow (21, 34) \rightarrow (13, 21) \rightarrow (8, 13) \rightarrow (5, 8)$
 $\rightarrow (3, 5) \rightarrow (2, 3) \rightarrow (1, 2)$

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18) a) $22 \rightarrow 10110 \rightarrow \boxed{010110}$

b) $31 \rightarrow 11111 \rightarrow \boxed{011111}$

c) $-7 \rightarrow 111 \rightarrow 000111 \rightarrow \boxed{111000}$

d) $-19 \rightarrow 10011 \rightarrow 010011 \rightarrow \boxed{101100}$

19) a) $11001 \rightarrow -(1001) \rightarrow -(0110) \rightarrow \boxed{-6}$

b) $01101 \rightarrow \boxed{13}$

c) $10001 \rightarrow -(1110) \rightarrow \boxed{-14}$

d) $11111 \rightarrow -(1111) = \boxed{0}$

24) a) $22 \rightarrow \boxed{010110}$

b) $31 \rightarrow \boxed{011111}$

c) $n=6 \quad 2^5 - 7 = 25$ so 2's compl. = 111001

d) $2^5 - 19 = 13 \Rightarrow$ 2's compl = 101101

Notice how you get from 1's complement to 2's complement

25) a) $11001 \rightarrow 11000 = -(0111) = \boxed{-7}$

↑
1's

b) $01101 \rightarrow \boxed{13}$

$$c) 10001 \rightarrow -(0000) \rightarrow -(1111) = \boxed{-15}$$

$$d) 11111 \rightarrow -(1110) \rightarrow -(0001) = \boxed{-1}$$

39) See back of text

Prob K is interesting: a) Should be 3 calls for each level. ie for 2-4 digits it would be 1 call for then 3 calls plus 3 more calls for each of 3 calls for $1 + 3 + 9 = 13$ calls but its more complicated since sometimes the adding of two n digit numbers will have sums with more digits. ie a few extra calls are required.

b) with two 8 digit numbers it should be $1 + 3 + 9 + 3^3$

c) in general it would be $1 + 3 + 9 + \dots + 3^{\log(n)}$ This is a geometric series
 $1 + r + r^2 + \dots + r^n = (1 - r^{n+1}) / (1 - r)$.

so we get $1 + 3 + 9 + \dots + 3^{\log(n)} = (1 - 3^{\log(n)+1}) / (1 - 3)$