

## HW 6-7 Sols

### Prog 1 # The Euclidean alg

```
def gcd(y, x):  
    while x>0:  
        oldx=x  
        x=y%x  
        y=oldx  
        #print(oldx,x,y)  
    return oldx  
print(gcd(287,92))
```

### Prog2 # Adding binary (Note: reads the binary backwards)

```
import time  
def add_bin(list1,list2):  
    n=len(list1)  
    lst=[0]*(n+1)  
    c=0  
    for i in range(n):  
        lst[i]=(list1[i]+list2[i]+c)%2  
  
        c=(list1[i]+list2[i]+c)//2 # c is the carry  
    lst[n]=c  
    return lst
```

## prog 4 Karatsuba

```
1. import math
2. global count # counts recursive calls
3. count = 0
4. x=123456789101112131415161718192012223242526272829303132333435363
5. y=1357911131517192123252729313335373941434547495153555759616365676
6. #x=1234
7. #y=5678
8.
9. def kara(x, y):
10.     #print("x y = ", x, y)
11.     global count # Counts recursive calls
12.     count += 1
13.
14.     n = max(int(math.log10(x+.001))+1,int(math.log10(y+.001))+1)
15.     # The .001 is a kludge to fix when x=0
16.     if x<10 or y<10:
17.         return x*y
18.     else:
19.         frontx, backx = int(x//(10**(n//2))), x % int((10**(n//2)))
20.         fronty, backy = int(y//(10**(n//2))), y % int((10**(n//2)))
21.         z2 = kara(frontx, fronty)
22.         z0 = kara(backx, backy)
23.         z1 = kara(frontx+backx, fronty+backy)-z2-z0
24.
25.         return 10**(2*(n//2))*z2+10**((n//2))*z1+z0
26.
27.
28. print(kara(x,y),'\n')
29.
30. print("recursive calls = ",count,'\n') # Prints recursive calls
31.
32. print(x*y)
```

1676433481817705266129514737418074430686007117601914669622664784839603755332117192534  
14749453310762762797651012520861917800388

recursive calls = 1525

HW 6-7 Sols

2) b)  $101 - 100 = 1 = \boxed{\text{gcd}}$

c)  $\text{gcd}(1001, 1331) = \text{gcd}(330, 1001) =$   
 $\text{gcd}(11, 330) = \boxed{11}$

d)  $\text{gcd}(1529, 14039) = \text{gcd}(278, 1529)$   
 $= \text{gcd}(139, 278) \Rightarrow \boxed{139}$

e)  $\text{gcd}(1529, 14038) = \text{gcd}(277, 1529)$   
 $\rightarrow (144, 277) \rightarrow (133, 144) \rightarrow (11, 133) \rightarrow (1, 11)$

$\boxed{1}$

f)  $\text{gcd}(11111, 11111) \rightarrow \boxed{\text{gcd}}$

3)  $(21, 34) \xrightarrow[1]{} (13, 21) \xrightarrow[2]{} (8, 13) \xrightarrow[3]{} (5, 8) \xrightarrow[4]{} (3, 5)$

$\xrightarrow[5]{} (2, 3) \xrightarrow[6]{} (1, 2)$

$\boxed{6}$

4)  $(34, 55) \rightarrow (21, 34) \rightarrow (13, 21) \rightarrow (8, 13) \rightarrow (5, 8)$

$\rightarrow (3, 5) \rightarrow (2, 3) \rightarrow (1, 2)$

$\boxed{7}$

# HW 6-7

18) a)  $22 \rightarrow 10110 \rightarrow \boxed{010110}$

b)  $31 \rightarrow 11111 \rightarrow \boxed{011111}$

c)  $-7 \rightarrow 111 \rightarrow 000111 \rightarrow \boxed{111000}$

d)  $-19 \rightarrow 10011 \rightarrow 010011 \rightarrow \boxed{101100}$

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19) a)  $11001 \rightarrow -(1001) \rightarrow -(0110) \rightarrow \boxed{-6}$

b)  $01101 \rightarrow \boxed{13}$

c)  $10001 \rightarrow -(1110) \rightarrow \boxed{-14}$

d)  $11111 \rightarrow -(1111) = \boxed{0}$

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24) a)  $22 \rightarrow \boxed{010110}$

b)  $31 \rightarrow \boxed{011111}$

c)  $n=6 \quad 2^5 - 7 = 25 \quad \text{so } 2\text{'s compl.} = \boxed{111001}$

d)  $2^5 - 19 = 13 \quad \Rightarrow \quad 2\text{'s compl.} = \boxed{101101}$

Notice how you get from 1's complement to 2's complement.

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25) a)  $11001 \rightarrow 11000 = -(0111) = \boxed{-7}$

b)  $01101 \rightarrow \boxed{13}$

c)  $10001 \rightarrow -(0000) \rightarrow -(1111) = \boxed{-15}$

*↑  
1's*

d)  $11111 \rightarrow -(1110) \rightarrow -(0001) = \boxed{-1}$

*↑  
1's*

39) See back of text

Prob K is interesting:a) Should be 3 calls for each level. ie for 2 -4 digits it would be 1 call for then 3 calls plus 3 more calls for each of 3 calls for  $1 + 3 + 9 = 13$  calls but its more complicated since sometimes the adding of two n digit numbers will have sums with more digits. ie a few extra calls are required.

b) with two 8 digit numbers it should be  $1+3+9+3^8$

c) in general it would be  $1 + 3 + 9 + \dots + 3^{\log(n)}$  This is a geometric series  
 $1 + r + r^2 + \dots + r^n = (1 - r^{n+1}) / (1 - r)$ .

so we get  $1 + 3 + 9 + \dots + 3^{\log(n)} = (1 - 3^{\log(n) + 1}) / (1 - 3)$