

# Linear Search

## Linear Search


index	0	1	2	3	4	5	6
array	-INF	8	2	9	3	4	INF

## Linear Search

index	0	1	2	3	4	5	6
array	-INF	8	2	9	3	4	INF

Where is the index of 3 ?

## Linear Search



index	0	1	2	3	4	5	6
array	-INF	8	2	9	3	4	INF

Where is the index of 3 ?

## Linear Search



index	0	1	2	3	4	5	6
array	-INF	8	2	9	3	4	INF

Where is the index of 3 ?

## Linear Search



index	0	1	2	3	4	5	6
array	-INF	8	2	9	3	4	INF

Where is the index of 3 ?


## Linear Search



index	0	1	2	3	4	5	6
array	-INF	8	2	9	3	4	INF

Where is the index of 3 ?

## Linear Search




index	0	1	2	3	4	5	6
array	-INF	8	2	9	3	4	INF

Where is the index of 3 ?



## Linear Search



index	0	1	2	3	4	5	6
array	-INF	8	2	9	3	4	INF

The index of 3 is 4


$O(n)$

## Linear Search

index	0	1	2	3	4	5	6
array	-INF	1	4	7	12	15	INF

Where is the index of 12 ?

## Linear Search



index	0	1	2	3	4	5	6
array	-INF	1	4	7	12	15	INF

Where is the index of 12 ?

## Linear Search



index	0	1	2	3	4	5	6
array	-INF	1	4	7	12	15	INF

Where is the index of 12 ?

## Linear Search



index	0	1	2	3	4	5	6
array	-INF	1	4	7	12	15	INF

Where is the index of 12 ?


## Linear Search



index	0	1	2	3	4	5	6
array	-INF	1	4	7	12	15	INF

Where is the index of 12 ?

## Linear Search




index	0	1	2	3	4	5	6
array	-INF	1	4	7	12	15	INF

Where is the index of 12 ?



## Linear Search



index	0	1	2	3	4	5	6
array	-INF	1	4	7	12	15	INF

The index of 12 is 4

$O(n)$

# Binary Search

## Binary Search

index	0	1	2	3	4	5	6	7	8	9	10	11
array	-INF	1	4	7	12	15	18	23	28	29	35	INF

Where is the index of 23 ?

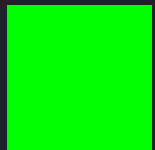
## Binary Search

index	0	1	2	3	4	5	6	7	8	9	10	11
array	-INF	1	4	7	12	15	18	23	28	29	35	INF

Where is the index of 23 ?



Value < 23



Value  $\geq$  23

## Binary Search

	l					mid						r
	▼					▼						▼
index	0	1	2	3	4	5	6	7	8	9	10	11
array	-INF	1	4	7	12	15	18	23	28	29	35	INF

Where is the index of 23 ?

1.let  $l = 0$  ,  $r = 11$

2.let  $mid = (l + r) / 2$

3.If  $array[mid] \geq 23$  then  $r = mid$   
else  $array[mid] < 23$  then  $l = mid + 1$

4.Repeat 2-3 until  $l = r$

## Binary Search

	l					mid						r
	▼					▼						▼
index	0	1	2	3	4	5	6	7	8	9	10	11
array	-INF	1	4	7	12	15	18	23	28	29	35	INF

Where is the index of 23 ?




1.let  $l = 0$  ,  $r = 11$

2.let  $mid = (l + r) / 2$

3.If  $array[mid] \geq 23$  then  $r = mid$   
else  $array[mid] < 23$  then  $l = mid + 1$

4.Repeat 2-3 until  $l = r$

## Binary Search

	l							mid		r		
												
index	0	1	2	3	4	5	6	7	8	9	10	11
array	-INF	1	4	7	12	15	18	23	28	29	35	INF

Where is the index of 23 ?

1.let  $l = 0$  ,  $r = 11$

2.let  $mid = (l + r) / 2$

3.If  $array[mid] \geq 23$  then  $r = mid$   
else  $array[mid] < 23$  then  $l = mid + 1$

4.Repeat 2-3 until  $l = r$



## Binary Search

							l	mid	r			
							▼	▼	▼			
index	0	1	2	3	4	5	6	7	8	9	10	11
array	-INF	1	4	7	12	15	18	23	28	29	35	INF

Where is the index of 23 ?


1.let  $l = 0$  ,  $r = 11$

2.let  $mid = (l + r) / 2$

3.If  $array[mid] \geq 23$  then  $r = mid$   
else  $array[mid] < 23$  then  $l = mid + 1$

4.Repeat 2-3 until  $l = r$

## Binary Search

	l mid r											
												
index	0	1	2	3	4	5	6	7	8	9	10	11
array	-INF	1	4	7	12	15	18	23	28	29	35	INF

Where is the index of 23 ?

1.let  $l = 0$  ,  $r = 11$

2.let  $mid = (l + r) / 2$

3.If  $array[mid] \geq 23$  then  $r = mid$   
else  $array[mid] < 23$  then  $l = mid + 1$

4.Repeat 2-3 until  $l = r$

## Binary Search

l mid r



index	0	1	2	3	4	5	6	7	8	9	10	11
array	-INF	1	4	7	12	15	18	23	28	29	35	INF

Where is the index of 23 ?

1. let  $l = 0$  ,  $r = 11$

2. let  $mid = (l + r) / 2$

3. If  $array[mid] \geq 23$  then  $r = mid$   
else  $array[mid] < 23$  then  $l = mid + 1$

4. Repeat 2-3 until  $l = r$

## Binary Search

l mid r



index	0	1	2	3	4	5	6	7	8	9	10	11
array	-INF	1	4	7	12	15	18	23	28	29	35	INF

The index of 23 is 7

1. let  $l = 0$  ,  $r = 11$

2. let  $mid = (l + r) / 2$

3. If  $array[mid] \geq 23$  then  $r = mid$   
else  $array[mid] < 23$  then  $l = mid + 1$

4. Repeat 2-3 until  $l = r$

## Binary Search

	l					mid						r
	▼					▼						▼
index	0	1	2	3	4	5	6	7	8	9	10	11
array	-INF	1	4	7	12	15	18	23	28	29	35	INF

Where is the index of 14 ?

1.let  $l = 0$  ,  $r = 11$

2.let  $mid = (l + r) / 2$

3.If  $array[mid] \geq 23$  then  $r = mid$   
else  $array[mid] < 23$  then  $l = mid + 1$

4.Repeat 2-3 until  $l = r$

## Binary Search

	l		mid			r						
	▼		▼			▼						
index	0	1	2	3	4	5	6	7	8	9	10	11
array	-INF	1	4	7	12	15	18	23	28	29	35	INF

Where is the index of 14 ?

1.let  $l = 0$  ,  $r = 11$

2.let  $mid = (l + r) / 2$

3.If  $array[mid] \geq 23$  then  $r = mid$   
else  $array[mid] < 23$  then  $l = mid + 1$

4.Repeat 2-3 until  $l = r$

## Binary Search

	<div>l      mid      r</div>											
	<div>▼      ▼      ▼</div>											
index	0	1	2	3	4	5	6	7	8	9	10	11
array	-INF	1	4	7	12	15	18	23	28	29	35	INF

Where is the index of 14 ?

1. let  $l = 0$  ,  $r = 11$

2. let  $mid = (l + r) / 2$

3. If  $array[mid] \geq 23$  then  $r = mid$   
else  $array[mid] < 23$  then  $l = mid + 1$

4. Repeat 2-3 until  $l = r$

## Binary Search

l mid r



index	0	1	2	3	4	5	6	7	8	9	10	11
array	-INF	1	4	7	12	15	18	23	28	29	35	INF

Where is the index of 14 ?

1.let  $l = 0$  ,  $r = 11$

2.let  $mid = (l + r) / 2$

3.If  $array[mid] \geq 23$  then  $r = mid$   
else  $array[mid] < 23$  then  $l = mid + 1$

4.Repeat 2-3 until  $l = r$



## Binary Search

l mid r



index	0	1	2	3	4	5	6	7	8	9	10	11
array	-INF	1	4	7	12	15	18	23	28	29	35	INF

Is the index of array[5] is 14 ?

1. let  $l = 0$  ,  $r = 11$
2. let  $mid = (l + r) / 2$
3. If  $array[mid] \geq 23$  then  $r = mid$   
else  $array[mid] < 23$  then  $l = mid + 1$
4. Repeat 2-3 until  $l = r$
5. If  $array[l] == 23$  : found  
else : not found

## Binary Search

l mid r



index	0	1	2	3	4	5	6	7	8	9	10	11
array	-INF	1	4	7	12	15	18	23	28	29	35	INF

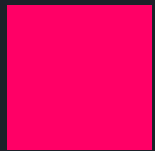
The index of array[5] is 15

1. let  $l = 0$  ,  $r = 11$
2. let  $mid = (l + r) / 2$
3. If  $array[mid] \geq 23$  then  $r = mid$   
else  $array[mid] < 23$  then  $l = mid + 1$
4. Repeat 2-3 until  $l = r$
5. If  $array[l] == 5$  : found  
else : not found

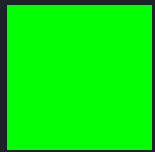
## Binary Search

index	0	1	2	3	4	5	6	7	8	9	10	11
array	-INF	1	4	7	12	15	18	23	28	29	35	INF

Where is the index of 23 ?



Value > 23



Value  $\leq$  23

## Binary Search

	l					mid						r
	▼					▼						▼
index	0	1	2	3	4	5	6	7	8	9	10	11
array	-INF	1	4	7	12	15	18	23	28	29	35	INF

Where is the index of 23 ?

1. let  $l = 0$  ,  $r = 11$
2. let  $mid = (l + r + 1) / 2$
3. If  $array[mid] \leq 23$  then  $l = mid$   
else  $array[mid] > 23$  then  $r = mid - 1$
4. Repeat 2-3 until  $l = r$
5. If  $array[l] == 23$  : found  
else : not found

## Binary Search

						l				mid				r
						▼				▼				▼
index	0	1	2	3	4	5	6	7	8	9	10	11		
array	-INF	1	4	7	12	15	18	23	28	29	35	INF		

Where is the index of 23 ?

1. let  $l = 0$  ,  $r = 11$
2. let  $mid = (l + r + 1) / 2$
3. If  $array[mid] \leq 23$  then  $l = mid$   
else  $array[mid] > 23$  then  $r = mid - 1$
4. Repeat 2-3 until  $l = r$
5. If  $array[l] == 23$  : found  
else : not found

## Binary Search

	<div>l                  mid                  r</div>											
	<div>▼                  ▼                  ▼</div>											
index	0	1	2	3	4	5	6	7	8	9	10	11
array	-INF	1	4	7	12	15	18	23	28	29	35	INF

Where is the index of 23 ?

1. let  $l = 0$  ,  $r = 11$
2. let  $mid = (l + r + 1) / 2$
3. If  $array[mid] \leq 23$  then  $l = mid$   
else  $array[mid] > 23$  then  $r = mid - 1$
4. Repeat 2-3 until  $l = r$
5. If  $array[l] == 23$  : found  
else : not found

## Binary Search

l mid r



index	0	1	2	3	4	5	6	7	8	9	10	11
array	-INF	1	4	7	12	15	18	23	28	29	35	INF

The index of 23 is 7 ?

1. let  $l = 0$  ,  $r = 11$
2. let  $mid = (l + r + 1) / 2$
3. If  $array[mid] \leq 23$  then  $l = mid$   
else  $array[mid] > 23$  then  $r = mid - 1$
4. Repeat 2-3 until  $l = r$
5. If  $array[l] == 23$  : found  
else : not found

# Upperbound & Lowerbound



Upperbound(l , r , val)

- first index in range l-r that value more than val

Lowerbound(l , r , val)

- first index in range l-r that value more than equal val

## Upper and Lower bound

```
int lowerbound (int L , int R , int val){  
  
    int l = L , r = R , mid ;  
  
    while(l ≠ r){  
  
        mid = (l + r) / 2 ;  
        if(arr[mid] ≥ val)r = mid ;  
        else l = mid + 1 ;  
    }  
  
    return l ;  
}
```

```
int upperbound (int L , int R , int val){  
  
    int l = L , r = R , mid ;  
  
    while(l ≠ r){  
  
        mid = (l + r) / 2 ;  
        if(arr[mid] > val)r = mid ;  
        else l = mid + 1 ;  
    }  
  
    return l ;  
}
```

## Upper and Lower bound

index	0	1	2	3	4	5	6	7	8	9	10	11
array	-INF	1	4	7	12	15	18	23	28	29	35	INF

Upperbound(0 , 11 , val)

lowerbound(0 , 11 , val)

## Upper and Lower bound

index	0	1	2	3	4	5	6	7	8	9	10	11
array	-INF	1	4	7	12	15	18	23	28	29	35	INF

Upperbound(0 , 11 , 13)  $\rightarrow$  5

lowerbound(0 , 11 , 13)  $\rightarrow$  5

## Upper and Lower bound

index	0	1	2	3	4	5	6	7	8	9	10	11
array	-INF	1	4	7	12	15	18	23	28	29	35	INF

Upperbound(0 , 11 , 12)  $\rightarrow$  5

lowerbound(0 , 11 , 12)  $\rightarrow$  4

## Upper and Lower bound

index	0	1	2	3	4	5	6	7	8	9	10	11
array	-INF	1	2	2	2	3	3	7	9	9	10	INF

How many 3 are there ?

## Upper and Lower bound

index	0	1	2	3	4	5	6	7	8	9	10	11
array	-INF	1	2	2	2	3	3	7	9	9	10	INF

Solution 1 : Linear Search  $O(n)$

## Upper and Lower bound



index	0	1	2	3	4	5	6	7	8	9	10	11
array	-INF	1	2	2	2	3	3	7	9	9	10	INF

Solution 1 : Linear Search  $O(n)$



## Upper and Lower bound



index	0	1	2	3	4	5	6	7	8	9	10	11
array	-INF	1	2	2	2	3	3	7	9	9	10	INF

Solution 1 : Linear Search  $O(n)$

## Upper and Lower bound



index	0	1	2	3	4	5	6	7	8	9	10	11
array	-INF	1	2	2	2	3	3	7	9	9	10	INF

Solution 1 : Linear Search  $O(n)$

## Upper and Lower bound



index	0	1	2	3	4	5	6	7	8	9	10	11
array	-INF	1	2	2	2	3	3	7	9	9	10	INF

Solution 1 : Linear Search  $O(n)$

## Upper and Lower bound



index	0	1	2	3	4	5	6	7	8	9	10	11
array	-INF	1	2	2	2	3	3	7	9	9	10	INF

Solution 1 : Linear Search  $O(n)$

## Upper and Lower bound



index	0	1	2	3	4	5	6	7	8	9	10	11
array	-INF	1	2	2	2	3	3	7	9	9	10	INF

Solution 1 : Linear Search  $O(n)$

## Upper and Lower bound



index	0	1	2	3	4	5	6	7	8	9	10	11
array	-INF	1	2	2	2	3	3	7	9	9	10	INF

Solution 1 : Linear Search  $O(n)$

## Upper and Lower bound



index	0	1	2	3	4	5	6	7	8	9	10	11
array	-INF	1	2	2	2	3	3	7	9	9	10	INF

Solution 1 : Linear Search  $O(n)$

## Upper and Lower bound



index	0	1	2	3	4	5	6	7	8	9	10	11
array	-INF	1	2	2	2	3	3	7	9	9	10	INF

Solution 1 : Linear Search  $O(n)$




## Upper and Lower bound



index	0	1	2	3	4	5	6	7	8	9	10	11
array	-INF	1	2	2	2	3	3	7	9	9	10	INF

Solution 1 : Linear Search  $O(n)$

## Upper and Lower bound



index	0	1	2	3	4	5	6	7	8	9	10	11
array	-INF	1	2	2	2	3	3	7	9	9	10	INF

Solution 1 : Linear Search  $O(n)$


## Upper and Lower bound



index	0	1	2	3	4	5	6	7	8	9	10	11
array	-INF	1	2	2	2	3	3	7	9	9	10	INF

Solution 1 : Linear Search  $O(n)$

## Upper and Lower bound



index	0	1	2	3	4	5	6	7	8	9	10	11
array	-INF	1	2	2	2	3	3	7	9	9	10	INF

Solution 1 : Linear Search  $O(n)$

## Upper and Lower bound



index	0	1	2	3	4	5	6	7	8	9	10	11
array	-INF	1	2	2	2	3	3	7	9	9	10	INF

Solution 1 : Linear Search  $O(n)$


Count(3)  $\rightarrow$  2

## Upper and Lower bound

index	0	1	2	3	4	5	6	7	8	9	10	11
array	-INF	1	2	2	2	3	3	7	9	9	10	INF

Solution 2 : upper and lower bound  $O(\log_2 n)$

## Upper and Lower bound




index	0	1	2	3	4	5	6	7	8	9	10	11
array	-INF	1	2	2	2	3	3	7	9	9	10	INF

Solution 2 : upper and lower bound  $O(\log_2 n)$

Count(3)  $\rightarrow$  Upperbound(0 , 11 , 3) – Lowerbound(0 , 11 , 3)

## Upper and Lower bound



index	0	1	2	3	4	5	6	7	8	9	10	11
array	-INF	1	2	2	2	3	3	7	9	9	10	INF

Solution 2 : upper and lower bound  $O(\log_2 n)$

Count(3)  $\rightarrow 7 - 5 \rightarrow 2$



## Upper and Lower bound

```
lower_bound(arr , arr + n , val) ;  
upper_bound(arr , arr + n , val) ;  
//→ return pointer of array
```

```
lower_bound(vec.begin() , vec.end() , val) ;  
upper_bound(vec.begin() , vec.end() , val) ;  
//→ return iterator of vector
```

# DP Optimization

# Longest Increase Subsequence

## DP Optimization

index	0	1	2	3	4	5	6	5	6
array	3	1	4	7	2	6	9	3	10

**LIS(array) ?**

## DP Optimization

index	0	1	2	3	4	5	6	5	6
array	3	1	4	7	2	6	9	3	10

**LIS(array) → 5**

## DP Optimization

index	0	1	2	3	4	5	6	5	6
array	3	1	4	7	2	6	9	3	10

**LIS(array) → 5**

## DP Optimization

index	0	1	2	3	4	5	6	5	6
array	3	1	4	7	2	6	9	3	10

**LIS(array) → 5**

```
int LIS(){  
  
    int MAX = 0 ;  
  
    for(int i = 0 ; i < N ; i ++ ){  
  
        if(i == 0)dp[i] = 1;  
  
        else {  
            for(int j = 0 ; j < i ; j ++ ){  
                if(arr[i] > arr[j])dp[i] = max(dp[i] , dp[j] + 1) ;  
            }  
            MAX = max(MAX , dp[i]) ;  
        }  
  
    }  
  
    return MAX ;  
}
```



## DP Optimization

index	0	1	2	3	4	5	6	5	6
array	3	1	4	7	2	6	9	3	10
lis	3								

## DP Optimization

index	0	1	2	3	4	5	6	5	6
array	3	1	4	7	2	6	9	3	10
lis	1								

## DP Optimization

index	0	1	2	3	4	5	6	5	6
array	3	1	4	7	2	6	9	3	10
lis	1	4							

## DP Optimization

index	0	1	2	3	4	5	6	5	6
array	3	1	4	7	2	6	9	3	10
lis	1	4	7						

## DP Optimization

index	0	1	2	3	4	5	6	5	6
array	3	1	4	7	2	6	9	3	10
lis	1	2	7						

## DP Optimization

index	0	1	2	3	4	5	6	5	6
array	3	1	4	7	2	6	9	3	10
lis	1	2	6						

## DP Optimization

index	0	1	2	3	4	5	6	5	6
array	3	1	4	7	2	6	9	3	10
lis	1	2	6	9					

## DP Optimization

index	0	1	2	3	4	5	6	5	6
array	3	1	4	7	2	6	9	3	10
lis	1	2	3	9					



## DP Optimization

index	0	1	2	3	4	5	6	5	6
array	3	1	4	7	2	6	9	3	10
lis	1	2	3	9	10				

**LIS(array) -> 5**

## DP Optimization

index	0	1	2	3	4	5	6	5	6
array	3	1	4	7	2	6	9	3	10
lis	1	2	3	9	10				

**LIS(array) -> 5**

## DP Optimization

```
int LIS(){  
    vector<int>lis ;  
    for(int i = 0 ; i < N ; i ++ ){  
        if(i == 0)lis.push_back(arr[i]) ;  
        else {  
            auto it = lower_bound(lis.begin(),lis.end(),arr[i]);  
            if(it == lis.end())lis.push_back(arr[i]) ;  
            else *it = arr[i] ;  
        }  
    }  
    return lis.size() ;  
}
```