# FOR 306
# Forest Measurements

## Standing Tree Volume

# Tree Volume

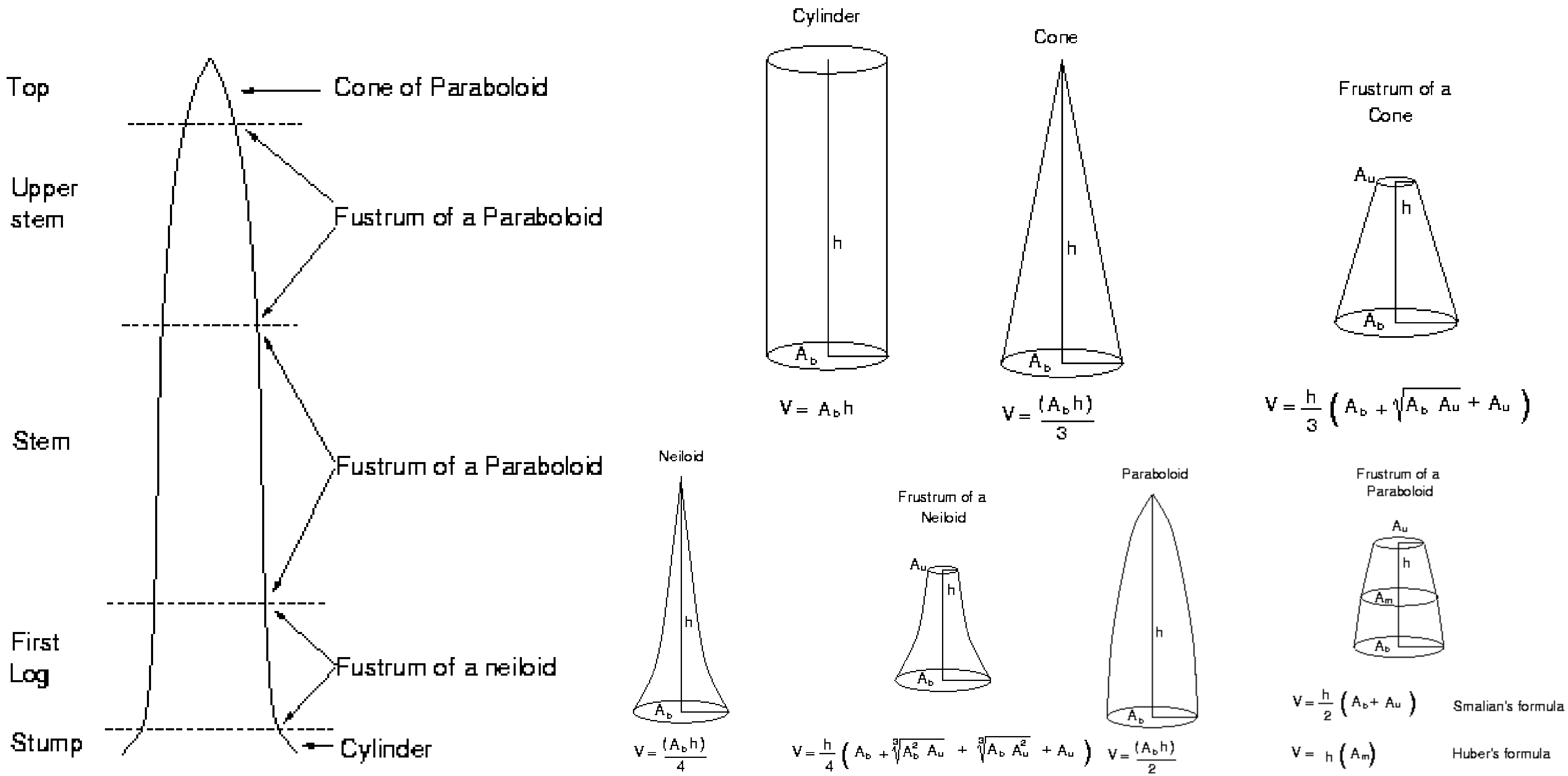
- Tree Stem – most important section of the tree in respect with usable wood
- Tree consists on four wooden parts:
  - Roots
  - Stump – lower part of the stem left after harvest
  - Stem
  - Branches
- Economic development – determination of volume/weight of tree and its component
- Units for volume: cu.ft /bd.ft or cubic meters

# Determination of Volume-cu.units

- Direct determination of volume of parts of tree requires a precise identification of respective part
- Generally, the volume of any part of a tree obtained by felling the tree and cut the stems or its limb into sections
- Approaches to determine the volume in cubic units
  - Formulas – stem is represented by a solid of revolution
  - Displacement – water
  - Integration – generalization of formulas
  - Graphical methods
  - Height accumulation
- Bark volume

# Volume determined using formulas

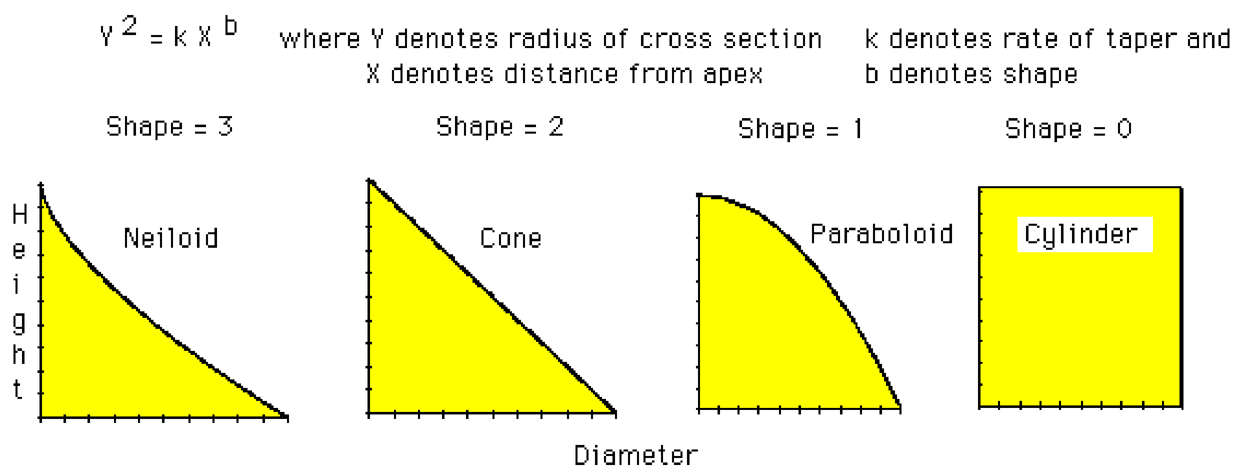
- Assume that the a part of the tree (stem, crown etc) can be represented by a solid of revolution



# FORMULAS OF SOLIDS OF REVOLUTION

$$Radius^2 = k(\text{distance from apex})^b = k(h_{total} - h_{on\ stem})^b$$

| Shape | b | Formula |
|---|---|---|
| Cylinder | 0 | Radius = ct. |
| Paraboloid (3rd degree) | 2/3 | Radius = k x $(h_{total} - h_{on\ stem})^{1/3}$ |
| Quadratic Paraboloid (2nd degree) | 1 | Radius = k x $(h_{total} - h_{on\ stem})^{1/2}$ |
| Conoid | 2 | Radius = k x $(h_{total} - h_{on\ stem})$ |
| Neiloid | 3 | Radius = k x $(h_{total} - h_{on\ stem})^{3/2}$ |



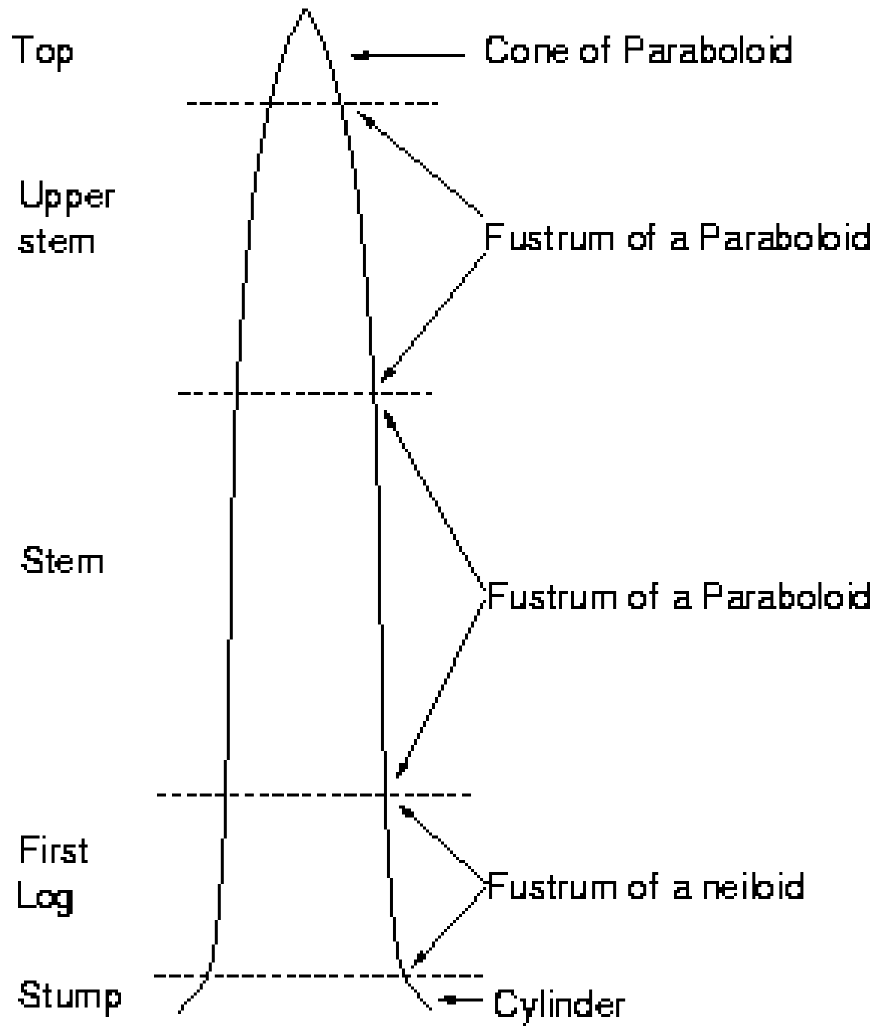
# Volume of Solids of Revolution

Solid of revolution = geometric solid

| **Solid of revolution** | **Volume** |
|---|---|
| Cylinder | $A_b h$ |
| Paraboloid | $½×A_b h$ |
| Cone | $1/×3A_b h$ |
| Neiloid | $1/4×A_b h$ |

# Tree Shape and Volume

- **Top**: cone (paraboloid rare)
- **Main bole**: frustum of paraboloid
- **Butt-log (first log)**: frustum of neiloid




**Frustum**: portion of a solid between two parallel planes

**Log**: 16 feet long section of stem

# Volume Approximations for Frustum of Geometric Solids

*Frustum of Paraboloid*

$$Huber's\ formula: V = A \times L$$

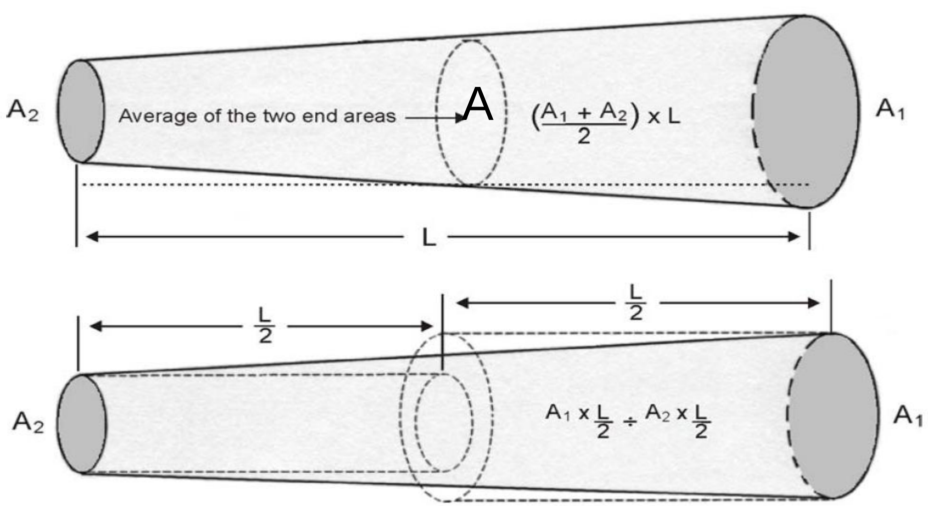
$$Smalian's\ formula: V = \frac{A_1 + A_2}{2} \times L$$

*Frustum of Neiloid*

$$V = \frac{L}{4} \times (A_1 + \sqrt[3]{A_1^2 A_2} + \sqrt[3]{A_1 A_2^2} + A_2)$$

*Frustum of cone, neiloid or paraboloid*

$$Newton's\ formula: V = \frac{A_1 + 4A + A_2}{6} \times L$$



# EXAMPLE

Determine the volume of a log (section of a tree 16 feet long) that has the larger diameter 20” and the smaller diameter 18”. Assume that that diameter at the middle of the log is the average between the larger and smaller diameter.

**Solution**:

According to the position along the stem the volume can be:

| Position on stem | Geometric solid | Smalian’s formula | Huber ‘s formula | Newton's formula |
|---|---|---|---|---|
| Butt-log | Neiloid | | | |
| Upper logs | Paraboloid | | | |

Diameter at the middle of the log:

# Volume Determination using Displacement & Integration

- **Water Displacement** – the most accurate method of measuring the volume of irregular solids
- Xylometer – tank used to measure tree volume
- **Integration**: determine the volume of a solid with known cross sectional area $V = \int_a^b A(X)dX$
- Requires the expression of the profile of the tree by an equation – knowledge of taper
- If the stem is assumed to be obtained from a solid of revolution then the cross sectional area is a circle and the stem volume is $V = \pi \int_a^b Y^2 dX$

# Volume Determination using Graphical Method

- Graphical method of determination of tree volume involves measuring diameter at various positions along the stem, plotting the data, and joining the points by curve lines giving a diametral outline of the stem
- The curve obtained by joining the points are called taper curve or stem profile
- Stem volume can be calculated in several ways:
    1. Use diameter at selected positions along the stem from the taper curve to calculate stem volume by summing the volumes of sections using Smalian‘s or Huber's formula.
    2. A program calculates the volume of the section between each set of successive points and accumulates the sums

# Comparison of the Formulas

- Water displacement (Young et al, 1967)
  - Average error for 8'- and 16'- softwood logs
    - 0% for Newton's formulae,
    - +9% for Smalian's formulae
    - -3.5% for Huber's formulae
  - No significant errors for 4' logs
- Graphical techniques (Miller, 1959)
  - Average error for 16' hardwood logs
    - 2% for Newton's formulae,
    - 12% for Smalian's formulae
    - -5% for Huber's formulae

# Bark Volume

- Bark volume –10-20% of unpeeled volume
- Bark –factor method – determine the bark volume as the difference between the unpeeled and peeled volume
- Bark thickness needs to be accurately determined

$b = 0.5\times(dob-dib)$

- Between dib and dob there is a direct relationship

$dob = k\times dib \rightarrow k$ is called bark factor

- Bark factor varies between 0.87-0.93
- Bark factor varies with species, age and site
- Volume bark [%] = $(1-k^2)\times 100$

# Volume table

- Volume table – tabulated description of the average volume of a tree by one or more tree dimensions
- Most common tree dimensions used to determine the volume of a tree are:
  - Dbh
  - Height
  - Tree form
- Results supplied in board feet or cubic feet

# Type of Volume Tables

- Local volume tables
  - Gives the volume of a tree only as a function of dbh
  - Restricted to a species and a confined area
- Standard volume tables
  - Tree volume is a function of dbh and height
  - Height can be total height or merchantable height
  - Prepared for individual species and locations
- Form class volume table
  - Tree volume is a function of dbh, height and tree form
  - Tree form expressed as Girard form class or form quotient
  - Height can be total height or merchantable height

# LOCAL&STANDARD VOLUME TABLES

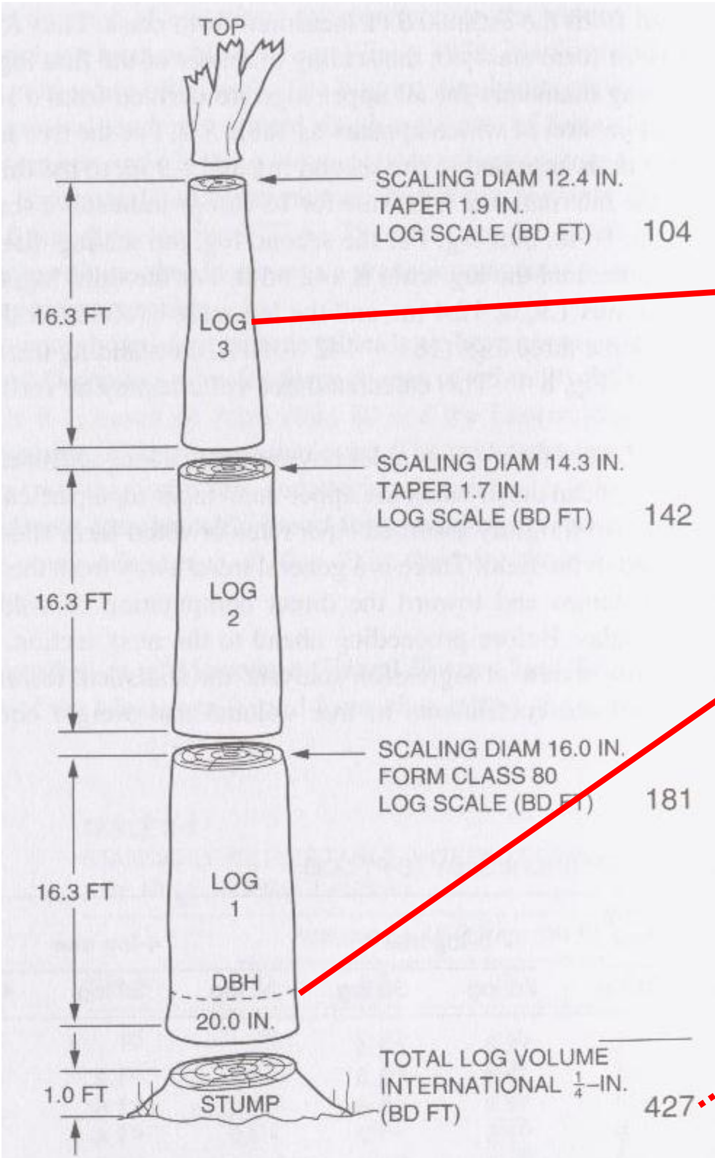
**TABLE 6-6. Example of a Local Volume Table for Yellow Poplar (*Liriodendron tulipifera*) in Stark County, Ohio**[a]

| Dbh Outside Bark (in.) | Volume Per Tree (board feet) | Merchantable Length (ft) |
|---|---|---|
| 10 | 30 | 19.5 |
| 11 | 50 | 23 |
| 12 | 70 | 26.5 |
| 13 | 95 | 30 |
| 14 | 125 | 33 |
| 15 | 155 | 36.5 |
| 16 | 190 | 40 |
| 17 | 235 | 43 |
| 18 | 285 | 45.5 |
| 19 | 345 | 48 |
| 20 | 405 | 51 |
| 21 | 480 | 53.5 |
| 22 | 555 | 56 |
| 23 | 635 | 58 |

**TABLE 6-7. Example of a Standard Volume Table, Using Board-Foot Volume, International $\frac{1}{4}$-in. Rule, for Red Oak (*Quercus rubra*) in Pennsylvania**[a]

| Dbh | Merchantable Height—Number of 16-ft Logs | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| (in.) | $\frac{1}{2}$ | 1 | $1\frac{1}{2}$ | 2 | $2\frac{1}{2}$ | 3 | $3\frac{1}{2}$ | 4 | $4\frac{1}{2}$ | 5 | $5\frac{1}{2}$ |
| 8 | 8 | 18 | 28 | 37 | 47 | 57 | | | | | |
| 9 | 11 | 23 | 35 | 48 | 60 | 73 | | | | | |
| 10 | 13 | 29 | 44 | 59 | 75 | 90 | 105 | 121 | | | |
| 11 | 17 | 35 | 54 | 72 | 91 | 109 | 128 | 146 | | | |
| 12 | 20 | 42 | 64 | 86 | 108 | 130 | 153 | 175 | 197 | | |
| 13 | 24 | 50 | 76 | 102 | 128 | 153 | 179 | 205 | 231 | | |
| 14 | 28 | 58 | 88 | 118 | 148 | 178 | 208 | 238 | 268 | 298 | 328 |
| 15 | 33 | 67 | 102 | 136 | 170 | 205 | 239 | 274 | 308 | 343 | 377 |
| 16 | 37 | 77 | 116 | 155 | 194 | 233 | 273 | 312 | 351 | 390 | 429 |
| 17 | | 87 | 131 | 175 | 219 | 264 | 308 | 352 | 396 | 441 | 485 |
| 18 | | 97 | 147 | 197 | 246 | 296 | 345 | 395 | 445 | 494 | 544 |

# Form Class Volume Tables




STANDARD VOLUME TABLE, INTERNATIONAL ¼-IN. RULE, FORM CLASS 80

| | Volume by 16-ft logs (bd ft) | | | |
|---|---|---|---|---|
| dbh (in.) | 1 | 2 | 3 | 4 |
| 10 | 39 | 63 | 80 | |
| 12 | 59 | 98 | 127 | 146 |
| 14 | 83 | 141 | 186 | 216 |
| 16 | 112 | 190 | 256 | 305 |
| 18 | 144 | 248 | 336 | 402 |
| 20 | 181 | 314 | 427 | 512 |
| 22 | 221 | 387 | 528 | 638 |
| 24 | 266 | 469 | 644 | 773 |
| 26 | 315 | 558 | 767 | 931 |
| 28 | 367 | 654 | 904 | 1,096 |
| 30 | 424 | 758 | 1,050 | 1,272 |
| 32 | 485 | 870 | 1,213 | 1,480 |
| 34 | 550 | 989 | 1,383 | 1,691 |
| 36 | 620 | 1,121 | 1,571 | 1,922 |
| 38 | 693 | 1,256 | 1,772 | 2,167 |
| 40 | 770 | 1,403 | 1,977 | 2,432 |

*Source:* Mesavage and Girard, 1946.

# Volume Table - Volume Equations

- Past: Tables → Present: Equations or Functions

INTERNATIONAL LOG RULE, ¼-IN. SAW KERF, FOR LOGS 8 TO 20 FT IN LENGTH

| Diameter (small end of log inside bark) (in.) | Length of log (ft) 8 | 10 | 12 | 14 | 16 | 18 | 20 | Diameter (in.) |
|---|---|---|---|---|---|---|---|---|
| | Volume (bd ft) | | | | | | | |
| 4 | .... | 5 | 5 | 5 | 5 | 5 | 10 | 4 |
| 5 | 5 | 5 | 10 | 10 | 10 | 15 | 15 | 5 |
| 6 | 10 | 10 | 15 | 15 | 20 | 25 | 25 | 6 |
| 7 | 10 | 15 | 20 | 25 | 30 | 35 | 40 | 7 |
| 8 | 15 | 20 | 25 | 35 | 40 | 45 | 50 | 8 |
| 9 | 20 | 30 | 35 | 45 | 50 | 60 | 70 | 9 |
| 10 | 30 | 35 | 45 | 55 | 65 | 75 | 85 | 10 |
| 11 | 35 | 45 | 55 | 70 | 80 | 95 | 105 | 11 |
| 12 | 45 | 55 | 70 | 85 | 95 | 110 | 125 | 12 |
| 13 | 55 | 70 | 85 | 100 | 115 | 135 | 150 | 13 |
| 14 | 65 | 80 | 100 | 115 | 135 | 155 | 175 | 14 |
| 15 | 75 | 95 | 115 | 135 | 160 | 180 | 205 | 15 |
| 16 | 85 | 110 | 130 | 155 | 180 | 205 | 235 | 16 |
| 17 | 95 | 125 | 150 | 180 | 205 | 235 | 265 | 17 |
| 18 | 110 | 140 | 170 | 200 | 230 | 265 | 300 | 18 |
| 19 | 125 | 155 | 190 | 225 | 260 | 300 | 335 | 19 |
| 20 | 135 | 175 | 210 | 250 | 290 | 330 | 370 | 20 |
| 21 | 155 | 195 | 235 | 280 | 320 | 365 | 410 | 21 |
| 22 | 170 | 215 | 260 | 305 | 355 | 405 | 455 | 22 |
| 23 | 185 | 235 | 285 | 335 | 390 | 445 | 495 | 23 |
| 24 | 205 | 255 | 310 | 370 | 425 | 485 | 545 | 24 |
| 25 | 220 | 280 | 340 | 400 | 460 | 525 | 590 | 25 |
| 26 | 240 | 305 | 370 | 435 | 500 | 570 | 640 | 26 |
| 27 | 260 | 330 | 400 | 470 | 540 | 615 | 690 | 27 |
| 28 | 280 | 355 | 430 | 510 | 585 | 665 | 745 | 28 |
| 29 | 305 | 385 | 465 | 545 | 630 | 715 | 800 | 29 |

$$
\begin{aligned}
Volume[BF] &= 0.0498LD^2 - 0.1607LD + \\
&+ 0.0498L^2DT + 0.0166L^3T^2 - \\
&- 0.0804L^2T - 0.1992LDT - \\
&- 0.0996L^2T^2 + 0.3214LT + 0.1328LT^2
\end{aligned}
$$

Grosenbaugh (1952)

# Types of Equations

- Classification based on # attributes:
  1. Local volume equations: **single entry eq.**
     - Equations based on a single variable (dbh)
  2. Standard volume equations:
     - Equations using DBH and height
  3. General volume equations: as standard eq. + tree form
     - No advantage in using tree form in addition to dbh and ht.

**multiple entry eq.**

- Classification based on # species:
  1. Species equations: for each species/group species with similar tree form
  2. Composite equations: for diverse species combinations (coniferous and broadleaves)
     - Corrections to address different stem taper

# Multiple – Entry Volume Equations

- Estimate volume of an individual tree using DBH and height, sometimes tree form
- Girard tree form
- Disadvantages in using the tree form
  - Tendency of rough estimate of form class rather than actually measuring it
  - Variation in upper-stem diameter cannot be described by the but-log taper
- Problems: 1% change in tree-form →3%change in merchantable volume

# Multiple – Entry Volume Equations

- Spur type:
  - Ponderosa pine: $V=-0.44670+0.00216\times DBH^2H$ [cu.ft]
  - Loblolly pine: $V=0.34864+0.00232\times DBH^2\times H$ [cu.ft]

<u>Example</u>: DBH=10” and H=50’→V=…….

- Schumacher & Hall type (Schumacher & Hall, 1933)

– Loblolly pine: $V=6.315+0.124\times(DBH-5)^{1.737}(H-17)^{0.36}$ [cu.ft]

– Shortleaf pine: $V=7.7+0.1576\times(DBH-8)^{1.051}(H-4.5)^{1.261}$ [bd.ft]

<u>Example</u>: DBH=10” and H=50’→V=……

Note: DBH in inches and Height in feet

# Single-Entry Equations

- Development:
  - Multiple entry volume equations
  - Scaled measured felled trees

- Advantages:
  - Quick
  - Uniformity in timber estimations

# Constructing Single-Entry Equations

- From felled trees

$V=b_0+b_1 x\ DBH^2$

Yellow poplar

$V = -8.4166+0.2679 \times DBH^2$ [cu.ft]

- From multiple entry volume equations

$V = 786.74^{-1} \times DBH^{1.544} \times H^{1.29}$ – Schumacher & Hall type

$\ln(H)=b_0+b_1 x\ DBH^{-1}$ – Schumacher type

$\ln(H) = 5.21909 - 14.32872 / DBH$

$\ln(V)=-6.6679 + 1.544 \times \ln(DBH)+1.29 \times \ln(H)=$

$= -6.6679 + 1.544 \times \ln(DBH) + 1.29 \times (5.21909 - 14.32872 / DBH)$

$\ln(V)=0.0647+1.544 \times \ln(DBH)-18.484/DBH \longrightarrow$ $V = e^{0.0647+1.544\ln(DBH)-18.484/DBH}$

$V = 1.0668 \times DBH^{1.544} \times e^{-18.484/DBH}$

# Volume Eq. to Upper-Stem Diameter

- **Merchantable** - part of a tree that can be manufactured into a saleable product

- **Utilization standard**: <u>the dimensions </u>(stump height, top diameter, base diameter, and length) and <u>quality</u> of trees that are harvested

- Volume estimates for various **<u>couple</u>** of diameters representing different utilization standards

- Upper-stem diameter identifies the utilization standard of a tree

# Volume Eq. to Upper-Stem Diameter

- Volume estimates should be logically related:
  - sum of the parts sum up to the stem volume
- Method: volume – ratio equations

Example:

- Loblolly pine:

  $V=0.34864+0.00232 \times DBH^2 \times H$ [cu.ft]

  $R=1 - 0.32354(d^{3.1579}/DBH^{2.7115})$

- If DBH=10” and H=100’ then V=
- $R_{4''}=$
- $V_{4''}=$

# Volume Distribution in Trees

- Volume distribution along tree stem:
  - Improve volume estimates
  - Aid in estimating volume looses due to defects

| Usable length | % total volume in each log by position | | | |
|---|---|---|---|---|
| (16 ft logs) | 1st | 2nd | 3rd | 4th |
| 1 | 100 | | | |
| 2 | 59 | 42 | | |
| 3 | 42 | 33 | 25 | |
| 4 | 34 | 29 | 22 | 15 |

# Usable Volume of a Tree

- Honer et al. (1983) – procedure to determine products obtained from a particular tree

Example:

1. Tree with dbh=30” & h=90’
2. Top diam. sawlog: 12”
3. $d_{upper}$/dbh=12/30=0.4→%V=96.2
4. %V=96.22→$h_{upper}$/H=0.8
5. $h_{upper}$= 0.8×90 = 72”
6. #logs=(72-17.3)/16.3 + 1 = 4

   0.3 – trimming allowance

   First log

7. $h_{upper}$= 1 + 16.3 × 4=66.2
8. $h_{upper}$/H = 0.73
9. %V=93.9%

| Height Ratio, $h_u/H$ | Volume Percentage | Diameter Ratio, $d_u/D$ | Volume Percentage |
|---|---|---|---|
| 0.10 | 21.76 | 0.10 | 100.00 |
| 0.20 | 39.60 | 0.20 | 99.96 |
| 0.30 | 55.12 | 0.30 | 98.88 |
| 0.40 | 68.30 | 0.40 | 96.22 |
| 0.50 | 79.12 | 0.50 | 91.59 |
| 0.60 | 87.70 | 0.60 | 84.07 |
| 0.70 | 93.92 | 0.70 | 72.52 |
| 0.80 | 97.80 | 0.80 | 55.63 |
| 0.90 | 99.36 | 0.90 | 31.87 |

*Source*: Adapted from Honer et al. (1983).

**Reading: Chapter 6**