# Student Information

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# Answer 1

**a)** $A \to B$ and $\neg(A \wedge \neg B)$

| A | B | $\neg B$ | $A \to B$ | $(A \wedge \neg B)$ | $\neg(A \wedge \neg B)$ |
|---|---|---|---|---|---|
| T | T | F | T | F | T |
| T | F | T | F | T | F |
| F | T | F | T | F | T |
| F | F | T | T | F | T |

Table 1: Truth Table for $A \to B$ and $\neg(A \wedge \neg B)$

The truth table shows that $A \to B$ and $\neg(A \wedge \neg B)$ are equivalent.

**b)** $A \leftrightarrow B$ and $(\neg A \vee B) \wedge (\neg B \vee A)$

| A | B | $\neg A$ | $\neg B$ | $A \leftrightarrow B$ | $(\neg A \vee B)$ | $(\neg B \vee A)$ | $(\neg A \vee B) \wedge (\neg B \vee A)$ |
|---|---|---|---|---|---|---|---|
| T | T | F | F | T | T | T | T |
| T | F | F | T | F | F | T | F |
| F | T | T | F | F | T | F | F |
| F | F | T | T | T | T | T | T |

Table 2: Truth Table for $A \leftrightarrow B$ and $(\neg A \vee B) \wedge (\neg B \vee A)$

The truth table shows that $A \leftrightarrow B$ and $(\neg A \vee B) \wedge (\neg B \vee A)$ are equivalent.

**c)** $A \to (\neg A \to B)$ and 1

| A | $\neg A$ | B | $\neg A \to B$ | $A \to (\neg A \to B)$ | 1 |
|---|---|---|---|---|---|
| T | F | T | T | T | T |
| T | F | F | T | T | T |
| F | T | T | T | T | T |
| F | T | F | F | T | T |

Table 3: Truth Table for $A \to (\neg A \to B)$ and 1

The truth table shows that $A \to (\neg A \to B)$ and 1 are equivalent.

**d)** $(A \vee \neg B) \rightarrow C$ and $(\neg A \wedge B) \vee C$

| A | B | C | $\neg A$ | $\neg B$ | $A \vee \neg B$ | $\neg A \wedge B$ | $(A \vee \neg B) \rightarrow C$ | $(\neg A \wedge B) \vee C$ |
|---|---|---|---|---|---|---|---|---|
| T | T | T | F | F | T | F | T | T |
| T | T | F | F | F | T | F | F | F |
| T | F | T | F | T | T | F | T | T |
| T | F | F | F | T | T | F | F | F |
| F | T | T | T | F | F | T | T | T |
| F | T | F | T | F | F | T | T | T |
| F | F | T | T | T | T | F | T | T |
| F | F | F | T | T | T | F | F | F |

Table 4: Truth Table for $(A \vee \neg B) \rightarrow C$ and $(\neg A \wedge B) \vee C$

The truth table shows that $(A \vee \neg B) \rightarrow C$ and $(\neg A \wedge B) \vee C$ are equivalent.

# Answer 2

**a)** $A \wedge (\neg A \rightarrow A)$

$$\begin{aligned} A \wedge (\neg A \rightarrow A) &\equiv A \wedge (\neg\neg A \vee A) && \text{(a)} \\ &\equiv A \wedge (A \vee A) && \text{(b)} \\ &\equiv A \wedge A && \text{(c)} \\ &\equiv A && \text{(d)} \end{aligned}$$

**b)** $(A \rightarrow B) \rightarrow ((A \rightarrow \neg B) \rightarrow \neg A)$

$$\begin{aligned} (A \rightarrow B) \rightarrow ((A \rightarrow \neg B) \rightarrow \neg A) &\equiv (\neg A \vee B) \rightarrow ((\neg A \vee \neg B) \rightarrow \neg A) && \text{(a)} \\ &\equiv (\neg A \vee B) \rightarrow (\neg(\neg A \vee \neg B) \vee \neg A) && \text{(b)} \\ &\equiv (\neg A \vee B) \rightarrow ((A \wedge B) \vee \neg A) && \text{(c)} \\ &\equiv (\neg A \vee B) \rightarrow ((A \vee \neg A) \wedge (B \vee \neg A)) && \text{(d)} \\ &\equiv (\neg A \vee B) \rightarrow (T \wedge (B \vee \neg A)) && \text{(e)} \\ &\equiv (\neg A \vee B) \rightarrow (B \vee \neg A) && \text{(f)} \\ &\equiv (\neg A \vee B) \rightarrow (\neg A \vee B) && \text{(g)} \\ &\equiv T && \text{(h)} \end{aligned}$$

**c)** $(A \to (B \vee \neg C)) \wedge \neg A \wedge B$

$$
\begin{aligned}
(A \to (B \vee \neg C)) \wedge \neg A \wedge B &\equiv (\neg A \vee (B \vee \neg C)) \wedge \neg A \wedge B && \text{(a)} \\
&\equiv (\neg A \vee B \vee \neg C) \wedge \neg A \wedge B && \text{(b)}
\end{aligned}
$$

# Answer 3

(1) $\neg A \wedge B$

(2) $\neg(B \wedge C)$

(3) $C \vee D$

(4) $\neg(\neg A \to D)$

(5) $\neg A$ From (1)

(6) $B$ From (1)

(7) $\neg A$ From (4)

(8) $\neg D$ From (4)

(9) $C$ | $D$ (closed) Alternatives from (3)

(10) $\neg B$ (closed) | $\neg C$ (closed) Alternatives from (2)

This semantic tableaux shows that $\neg A \wedge B, \neg(B \wedge C), C \vee D, \neg(\neg A \to D)$ are not mutually consistent.