Middle East Technical University | Department of Computer Engineering

CENG 487

Introduction to Quantum Computing

Fall 2023-2024

Assignment III

Due date: December 21, 23:59

Introduction

In this assignment you are supposed to solve two questions related to the algorithmic approaches mentioned in the course; Simon's Periodicity Algorithm and Shor Code. If the question does not ask for an explanation, you can directly provide the answer. The circuits will be implemented on IBM Quantum Computing. The necessary information regarding IBM Quantum Computing can be found in the first assignment. It will not be repeated here, please check that assignment for the clarification about the environment.

Part 1 - Simon's Periodicity Algorithm (50 pts)

Simon's Algorithm provides an exponential speed-up when determining the period c for functions where $f : \{0,1\}^n \rightarrow \{0,1\}^n$ and $f(x) = f(y) \longleftrightarrow x = y \oplus c$.

a) The functions defined are generally two-to-one. Give a specific c that would make the function one-to-one. *(5 pts)*

b) What is the minimum number of evaluations required for classical approach to determine whether the function is two-to-one or one-to-one for $n = 7$. *(5 pts)*

c) For the following function where $n = 3$, what is c? Explain your approach for finding c. *(5 pts)*

x	$f(x)$
000	000
001	001
010	001
011	000
100	100
101	101
110	101
111	100

d) Assume you run the circuit 8 times and get the following results for a function with $n = 6$;

'001000': 1
'111111': 1
'000100': 1
'110010': 2
'100110': 1
'000001': 1
'010100': 1

Can c be determined with these inputs? If yes, show your calculations step-by-step; if no, explain your reasoning. *(15 pts)*

e) Provide the circuit for Simon's Algorithm where $n = 4$ and $c =' 1010'$. Briefly explain the circuit. What would you change in the circuit if c is changed to $'0101'$? *(20 pts)*

Part 2 - Shor Code (50 pts)

a) What is "majority voting" in error correction? Describe how it is used in classical computation and if it can be used in quantum computing. If it can be used state how; if it can't be used state why. *(10 pts)*

b) Create a phase gate $P(\frac{2a-20}{a}\pi)$ where a **is the sum of the digits of your student ID**. Apply this phase gate as follows;

$$|\psi\rangle = H \cdot P \cdot H \cdot |0\rangle$$

What is the *cos* of the angle in P gate? What do you expect $|\psi\rangle$ to be when measured? Use the simulator with shots = 1024 and compare the probabilities. *(15 pts)*

c) Implement three different 9 qubit Shor circuits and test them on the simulator with shots = 1024 to see if your implementation corrects the error **for $|\psi\rangle$ (obtained in part 2b).** For the first circuit use only **NOT** gate after the encoding part, for the second circuit use only **Z** gate, and for the third circuit use both. Provide a screenshot of each circuit and results of the simulator. Briefly explain what each circuit does and analyze the results. *(25 pts)*

Submission

- Submit a single **PDF** file named eXXXXXXX.pdf, where XXXXXXX is your student ID.
- Hand-written solutions will **not be accepted**.
- We have a late policy with penalty. The penalty is $5 * day^2$ where *day* is the amount of days submitted late. After three days, late submissions will not be accepted.
- The assignments are for individual work. We have zero tolerance policy for cheating. People involved in cheating will be punished according to the university regulations.
- You may ask your questions by sending a mail to "mduymus@ceng.metu.edu.tr".