

Student Information

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Answer 1

p	q	r	$(p \wedge q) \rightarrow r$	$q \vee \neg q$
T	T	T	T	T
T	T	F	F	T
T	F	T	T	T
T	F	F	T	T
F	T	T	T	T
F	T	F	T	T
F	F	T	T	T
F	F	F	T	T

Table 1: Truth table for the given premises and the conclusion

p	q	r	$(p \wedge q) \rightarrow r$	$q \vee \neg q$
T	T	T	T	T
X	X	X	X	X
T	F	T	T	T
T	F	F	T	T
X	X	X	X	X
X	X	X	X	X
X	X	X	X	X
X	X	X	X	X

Table 2: Table with the eliminated rows that do not satisfy premises

p	q	r	$(p \wedge q) \rightarrow r$	$q \vee \neg q$
T	T	T	T	T
X	X	X	X	X
T	F	T	T	T
X	X	X	X	X
F	T	T	T	T
X	X	X	X	X
F	F	T	T	T
X	X	X	X	X

Table 3: Table with the eliminated rows that do not satisfy the conclusion

Since the set of the remaining rows in the Table 2 is not a subset of the remaining rows in the Table 3, r is not logically entailed by the supplied premises.

Answer 2

- | | | |
|----|---|--------------------|
| 1. | $p \Rightarrow q$ | Premise |
| 2. | $q \Rightarrow r$ | Premise |
| 3. | $(q \Rightarrow r) \Rightarrow (p \Rightarrow (q \Rightarrow r))$ | II |
| 4. | $p \Rightarrow (q \Rightarrow r)$ | Modus Ponens: 3, 2 |
| 5. | $(p \Rightarrow (q \Rightarrow r)) \Rightarrow ((p \Rightarrow q) \Rightarrow (p \Rightarrow r))$ | ID |
| 6. | $(p \Rightarrow q) \Rightarrow (p \Rightarrow r)$ | Modus Ponens: 5, 4 |
| 7. | $p \Rightarrow r$ | Modus Ponens: 6, 1 |
| 8. | $(p \Rightarrow r) \Rightarrow ((p \Rightarrow \neg r) \Rightarrow \neg p)$ | CR |
| 9. | $(p \Rightarrow \neg r) \Rightarrow \neg p$ | Modus Ponens: 8, 7 |

Answer 3

1.	$\neg\neg p$	Premise
2.	$(\neg p \Rightarrow \neg p) \Rightarrow ((\neg p \Rightarrow \neg\neg p) \Rightarrow p)$	CR
3.	$\neg p \Rightarrow ((\neg p \Rightarrow \neg p) \Rightarrow \neg p)$	II
4.	$\neg p \Rightarrow (\neg p \Rightarrow \neg p)$	II
5.	$(\neg p \Rightarrow ((\neg p \Rightarrow \neg p) \Rightarrow \neg p)) \Rightarrow ((\neg p \Rightarrow (\neg p \Rightarrow \neg p)) \Rightarrow (\neg p \Rightarrow \neg p))$	ID
6.	$(\neg p \Rightarrow (\neg p \Rightarrow \neg p)) \Rightarrow (\neg p \Rightarrow \neg p)$	Modus Ponens: 5, 3
7.	$\neg p \Rightarrow \neg p$	Modus Ponens: 6, 4
8.	$(\neg p \Rightarrow \neg\neg p) \Rightarrow p$	Modus Ponens: 7, 2
9.	$\neg\neg p \Rightarrow (\neg p \Rightarrow \neg\neg p)$	II
10.	$\neg p \Rightarrow \neg\neg p$	Modus Ponens: 9, 1
11.	p	Modus Ponens: 10, 8

Answer 4

$$\begin{aligned}
 & \neg((p \vee q \Rightarrow r) \Rightarrow (p \Rightarrow (q \Rightarrow r))) \\
 \text{I} \quad & \neg(\neg(\neg(p \vee q) \vee r) \vee (\neg p \vee (\neg q \vee r))) \\
 \text{N} \quad & \neg(\neg\neg(p \vee q) \wedge \neg r) \wedge \neg(\neg p \vee (\neg q \vee r)) \\
 & ((\neg p \wedge \neg q) \vee r) \wedge (p \wedge (q \wedge \neg r)) \\
 \text{D} \quad & ((\neg p \vee r) \wedge (\neg q \vee r)) \wedge (p \wedge (q \wedge \neg r)) \\
 & (\neg p \vee r) \wedge (\neg q \vee r) \wedge p \wedge q \wedge \neg r \\
 \text{O} \quad & \{\neg p, r\} \\
 & \{\neg q, r\} \\
 & \{p\} \\
 & \{q\} \\
 & \{r\} \\
 & \{\neg r\} \\
 & \{\}
 \end{aligned}$$

Here, we showed the negative of the given sentence is unsatisfiable. Thus, the sentence is valid.

Answer 5

The set of clauses is:

$$\begin{aligned} &(\neg p \vee q \vee s) \\ &(p \vee s \vee t) \\ &(p \vee s \vee \neg t) \\ &(p \vee \neg s \vee \neg t) \\ &(p \vee \neg s \vee t) \\ &(p \vee q \vee \neg s) \\ &(p \vee \neg q \vee s) \end{aligned}$$

In this set of clauses there are neither tautological clauses nor clauses with pure literals. So, we can not simplify the set of clauses. We can not find a unit clause either. So, we have to choose a literal to assign. Let's choose p .

After assigning T to p , we have:

$$\begin{aligned} &(\neg T \vee q \vee s) \\ &(T \vee s \vee t) \\ &(T \vee s \vee \neg t) \\ &(T \vee \neg s \vee \neg t) \\ &(T \vee \neg s \vee t) \\ &(T \vee q \vee \neg s) \\ &(T \vee \neg q \vee s) \end{aligned}$$

Now, after eliminating tautological clauses, and removing $\neg T$ from the first clause as it does not have any effect, we have:

$$q \vee s$$

After assigning T to q or s , it can be seen that this set of clauses are satisfiable.