

Homework Set: Lectures 3 & 4

Kinematics and Newton's Second Law

Instructions:

- Show all work clearly. Box or highlight final answers.
- Use appropriate units and directions (e.g., “to the right”, “up the incline”).
- Unless otherwise stated, take $g = 9.8 \text{ m/s}^2$ and neglect air resistance.

Problem 1: Multi-Step Static Equilibrium — Hanging Sign with Wind

A uniform rectangular sign of mass $m = 25 \text{ kg}$ hangs at rest from two cables attached to a wall, as shown. The left cable makes an angle of 35° above the horizontal, and the right cable makes an angle of 55° above the horizontal. A steady horizontal wind exerts a force of magnitude 60 N on the sign *toward the right*.

(a) Draw a clear free-body diagram (FBD) of the sign, labeling:

- The weight W ,
- The tensions T_1 and T_2 in the left and right cables,
- The wind force F_{wind} .

(b) Write the equilibrium equations for the x - and y -components of the forces.

(c) Solve for the tensions T_1 and T_2 in the two cables.

(d) Which cable is under greater tension? Give a brief explanation based on geometry and your results.

Problem 2: Block on Rough Horizontal Surface with Angled Pull

A box of mass $m = 18 \text{ kg}$ rests on a horizontal surface. The coefficient of kinetic friction between the box and the floor is $\mu_k = 0.25$. A person pulls on the box with a rope that makes an angle of 30° above the horizontal, exerting a constant force of magnitude F .

- (a) Draw a free-body diagram of the box, labeling all forces (including the normal force and friction).
- (b) Express the normal force N in terms of m , g , and F .
- (c) Write the x - and y -component equations from Newton's Second Law *during motion*, in terms of F , N , and the acceleration a .
- (d) Suppose the box moves at constant speed once it gets going. Find the *specific* value of F that makes the acceleration zero.
- (e) The person now increases the pulling force to $F = 120\text{ N}$ at the same angle. Find the resulting acceleration of the box.

(Hint: Friction is kinetic once the box is moving: $f_k = \mu_k N$.)

Problem 3: Velocity–Time Graph, Displacement, and Forces

A cart of mass 2.0 kg moves along a straight track under the influence of varying horizontal forces. Its velocity as a function of time is shown qualitatively below:

- From $t = 0$ to $t = 2.0\text{ s}$, the velocity increases linearly from 0 to 4.0 m/s .
- From $t = 2.0\text{ s}$ to $t = 5.0\text{ s}$, the velocity remains constant at 4.0 m/s .
- From $t = 5.0\text{ s}$ to $t = 7.0\text{ s}$, the velocity decreases linearly from 4.0 m/s to 0 .

Assume motion is along the $+x$ direction.

- (a) Sketch the velocity–time graph on a set of axes and label all key points.
- (b) Using the graph, determine the total displacement of the cart between $t = 0$ and $t = 7.0\text{ s}$.
- (c) Determine the average velocity over the entire interval $0 \leq t \leq 7.0\text{ s}$.
- (d) Find the acceleration (constant) in each of the three time intervals: $0 - 2\text{ s}$, $2 - 5\text{ s}$, and $5 - 7\text{ s}$.
- (e) If the cart experiences negligible friction, sketch a *qualitative* horizontal force–time graph $F_x(t)$ over the same time interval. Explain how its shape relates to the acceleration found in part (d).

(Hint: Remember that area under the v – t graph gives displacement, and $F = ma$.)

Problem 4: Projectile Off a Cliff with Required Launch Speed

A projectile is launched from the edge of a cliff of height 45 m above level ground. It is launched with an initial velocity making an angle of 40° above the horizontal. The projectile must land at a point on the ground that is horizontally 60 m from the base of the cliff.

Assume no air resistance.

- (a) Draw a diagram of the situation, labeling the initial velocity components v_{0x} and v_{0y} , the cliff height, and the horizontal range.
- (b) Write the horizontal and vertical position equations as functions of time t , using $x = 0$ and $y = 0$ at the base of the cliff directly below the launch point.
- (c) Write an equation that expresses the condition that the projectile lands at $(x, y) = (60 \text{ m}, 0)$.
- (d) Use your equations to *solve algebraically* for the required launch speed v_0 (in terms of g and known numbers). Show algebra carefully. (You may leave the final answer in symbolic form or evaluate numerically.)
- (e) For your found v_0 , determine the *time of flight* to reach the ground.

Problem 5: Two-Mass System with Friction on an Incline

A block of mass $m_1 = 6.0 \text{ kg}$ sits on a rough incline that makes an angle of 30° above the horizontal. The coefficient of kinetic friction between m_1 and the incline is $\mu_k = 0.20$. The block is connected by a light string over a frictionless pulley to a hanging block of mass m_2 . The string is parallel to the incline on the m_1 side.

You observe experimentally that when the system is released from rest, m_2 moves *downward* and m_1 moves *up the incline* with a constant acceleration of magnitude 1.2 m/s^2 .

- (a) Draw separate free-body diagrams for m_1 and m_2 , labeling all forces (including weight components, normal force, friction, and tension).
- (b) For m_1 , write Newton's Second Law along the incline (parallel direction), taking *up the incline* as positive.
- (c) For m_2 , write Newton's Second Law in the vertical direction, taking downward as positive.
- (d) Use the two equations to solve for the unknown tension T and the unknown mass m_2 . (You may solve algebraically first, then substitute numerical values.)
- (e) Suppose now that m_2 is kept the same, but the coefficient of kinetic friction is increased to $\mu_k = 0.35$. Predict qualitatively:
 - Which direction will the system accelerate (if at all)?
 - Will the magnitude of the acceleration be larger, smaller, or zero compared to the original case? Justify using your force equations.