

# Homework Set: Lectures 3 & 4

## Kinematics and Newton's Second Law

### Instructions:

- Show all work clearly. Box or highlight final answers.
- Use appropriate units and directions (e.g., “to the right”, “up the incline”).
- Unless otherwise stated, take  $g = 9.8 \text{ m/s}^2$  and neglect air resistance.

### Problem 1: Multi-Step Static Equilibrium — Hanging Sign with Wind

A uniform rectangular sign of mass  $m = 25 \text{ kg}$  hangs at rest from two cables attached to a wall, as shown. The left cable makes an angle of  $35^\circ$  above the horizontal, and the right cable makes an angle of  $55^\circ$  above the horizontal. A steady horizontal wind exerts a force of magnitude  $60 \text{ N}$  on the sign *toward the right*.

- (a) Draw a clear free-body diagram (FBD) of the sign, labeling:
  - The weight  $W$ ,
  - The tensions  $T_1$  and  $T_2$  in the left and right cables,
  - The wind force  $F_{\text{wind}}$ .
- (b) Write the equilibrium equations for the  $x$ - and  $y$ -components of the forces.
- (c) Solve for the tensions  $T_1$  and  $T_2$  in the two cables.
- (d) Which cable is under greater tension? Give a brief explanation based on geometry and your results.

### Problem 2: Block on Rough Horizontal Surface with Angled Pull

A box of mass  $m = 18 \text{ kg}$  rests on a horizontal surface. The coefficient of kinetic friction between the box and the floor is  $\mu_k = 0.25$ . A person pulls on the box with a rope that makes an angle of  $30^\circ$  above the horizontal, exerting a constant force of magnitude  $F$ .

- Draw a free-body diagram of the box, labeling all forces (including the normal force and friction).
- Express the normal force  $N$  in terms of  $m$ ,  $g$ , and  $F$ .
- Write the  $x$ - and  $y$ -component equations from Newton's Second Law *during motion*, in terms of  $F$ ,  $N$ , and the acceleration  $a$ .
- Suppose the box moves at constant speed once it gets going. Find the *specific* value of  $F$  that makes the acceleration zero.
- The person now increases the pulling force to  $F = 120\text{ N}$  at the same angle. Find the resulting acceleration of the box.

(Hint: Friction is kinetic once the box is moving:  $f_k = \mu_k N$ .)

### Problem 3: Velocity–Time Graph, Displacement, and Forces

A cart of mass  $2.0\text{ kg}$  moves along a straight track under the influence of varying horizontal forces. Its velocity as a function of time is shown qualitatively below:

- From  $t = 0$  to  $t = 2.0\text{ s}$ , the velocity increases linearly from  $0$  to  $4.0\text{ m/s}$ .
- From  $t = 2.0\text{ s}$  to  $t = 5.0\text{ s}$ , the velocity remains constant at  $4.0\text{ m/s}$ .
- From  $t = 5.0\text{ s}$  to  $t = 7.0\text{ s}$ , the velocity decreases linearly from  $4.0\text{ m/s}$  to  $0$ .

Assume motion is along the  $+x$  direction.

- Sketch the velocity–time graph on a set of axes and label all key points.
- Using the graph, determine the total displacement of the cart between  $t = 0$  and  $t = 7.0\text{ s}$ .
- Determine the average velocity over the entire interval  $0 \leq t \leq 7.0\text{ s}$ .
- Find the acceleration (constant) in each of the three time intervals:  $0 - 2\text{ s}$ ,  $2 - 5\text{ s}$ , and  $5 - 7\text{ s}$ .
- If the cart experiences negligible friction, sketch a *qualitative* horizontal force–time graph  $F_x(t)$  over the same time interval. Explain how its shape relates to the acceleration found in part (d).

(Hint: Remember that area under the  $v$ – $t$  graph gives displacement, and  $F = ma$ .)

## Problem 4: Projectile Off a Cliff with Required Launch Speed

A projectile is launched from the edge of a cliff of height 45 m above level ground. It is launched with an initial velocity making an angle of  $40^\circ$  above the horizontal. The projectile must land at a point on the ground that is horizontally 60 m from the base of the cliff.

Assume no air resistance.

- Draw a diagram of the situation, labeling the initial velocity components  $v_{0x}$  and  $v_{0y}$ , the cliff height, and the horizontal range.
- Write the horizontal and vertical position equations as functions of time  $t$ , using  $x = 0$  and  $y = 0$  at the base of the cliff directly below the launch point.
- Write an equation that expresses the condition that the projectile lands at  $(x, y) = (60 \text{ m}, 0)$ .
- Use your equations to *solve algebraically* for the required launch speed  $v_0$  (in terms of  $g$  and known numbers). Show algebra carefully. (You may leave the final answer in symbolic form or evaluate numerically.)
- For your found  $v_0$ , determine the *time of flight* to reach the ground.

## Problem 5: Two-Mass System with Friction on an Incline

A block of mass  $m_1 = 6.0 \text{ kg}$  sits on a rough incline that makes an angle of  $30^\circ$  above the horizontal. The coefficient of kinetic friction between  $m_1$  and the incline is  $\mu_k = 0.20$ . The block is connected by a light string over a frictionless pulley to a hanging block of mass  $m_2$ . The string is parallel to the incline on the  $m_1$  side.

You observe experimentally that when the system is released from rest,  $m_2$  moves *downward* and  $m_1$  moves *up the incline* with a constant acceleration of magnitude  $1.2 \text{ m/s}^2$ .

- Draw separate free-body diagrams for  $m_1$  and  $m_2$ , labeling all forces (including weight components, normal force, friction, and tension).
- For  $m_1$ , write Newton's Second Law along the incline (parallel direction), taking *up the incline* as positive.
- For  $m_2$ , write Newton's Second Law in the vertical direction, taking downward as positive.
- Use the two equations to solve for the unknown tension  $T$  and the unknown mass  $m_2$ . (You may solve algebraically first, then substitute numerical values.)
- Suppose now that  $m_2$  is kept the same, but the coefficient of kinetic friction is increased to  $\mu_k = 0.35$ . Predict qualitatively:
  - Which direction will the system accelerate (if at all)?
  - Will the magnitude of the acceleration be larger, smaller, or zero compared to the original case? Justify using your force equations.