Homework Set: HW4
Momentum, Impulse, and 2D Collisions

Problem 1: Perfectly Inelastic 2D Collision + Impulse + Post-Collision Motion

Two low-friction carts collide on a horizontal air table and stick together (perfectly inelastic collision). Take +x to be east and +y to be north. External horizontal forces during the collision are negligible.

Cart A has mass $m_A = 0.45\,\text{kg}$ and moves at speed $6.2\,\text{m/s}$ at an angle of 25° *north of east* just before impact.

Cart B has mass $m_B = 0.70\,\text{kg}$ and moves at speed $4.5\,\text{m/s}$ at an angle of 40° *south of east* just before impact.

After they stick, the combined carts slide off the air table onto a rough surface and come to rest after traveling a distance of $3.5\,\text{m}$. Assume the rough surface provides a kinetic friction force with coefficient μ_k (unknown). Use $g = 9.81\,\text{m/s}^2$.

(a) **Vector momentum setup.** Write the initial velocity vectors $\vec{v}_A$ and $\vec{v}_B$ in component form (x- and y-components).

(b) **Post-collision velocity (2D).** Use conservation of momentum in x and y to find the combined velocity vector $\vec{v}_f$ immediately after the collision. Report:

- (i) v_{fx} and v_{fy},
- (ii) the speed v_f,
- (iii) the direction as an angle measured from +x (east), with correct quadrant.

(c) **Kinetic energy change.** Calculate the total kinetic energy before the collision and just after the collision, then determine:

- (i) the kinetic energy lost in the collision, $\Delta K = K_i - K_f$,
- (ii) the fraction of kinetic energy lost, $\Delta K / K_i$.

(d) **Impulse and average collision force.** The sticking collision lasts $\Delta t = 0.035\,\text{s}$.

- (i) Find the impulse vector on cart A, $\vec{J}_A$. Note $J = \Delta p = F \Delta t$.
- (ii) Find the impulse vector on cart B, $\vec{J}_B$.

(iii) Hence find the average force on each cart during the collision, $\vec{F}_{\text{avg}} = \vec{J}/\Delta t$ (give components).

(e) **Friction estimate from stopping distance.** After the collision, the combined carts (mass $m_A + m_B$) enter the rough surface with initial speed v_f (from part b) and stop after 3.5 m. Assuming kinetic friction is the only horizontal force on the rough surface, determine μ_k.

(f) **Direction reasoning (short explanation).** In 1–2 sentences, explain why the final direction in part (b) must lie between the initial directions of A and B.

Problem 2: Perfectly Elastic 2D Collision (Both Objects Initially Moving)

Two pucks collide on a frictionless horizontal surface. The collision is perfectly elastic. Take +x east and +y north. External horizontal forces are negligible during the collision.

Puck 1 has mass $m_1 = 0.35\,\text{kg}$ and initial velocity $\vec{v}_{1i} = \langle 5.8,\, 0\rangle$ m/s (due east).

Puck 2 has mass $m_2 = 0.50\,\text{kg}$ and initial velocity $\vec{v}_{2i} = \langle 0,\, 3.4\rangle$ m/s (due north).

After the collision, puck 1 is observed to move at speed 4.2 m/s at 30° *north of east*.

(a) **Component form of the known final velocity.** Write puck 1's final velocity $\vec{v}_{1f}$ in component form.

(b) **Use momentum conservation (2D).** Apply conservation of momentum in x and y to express $\vec{v}_{2f}$ in terms of known quantities, then compute the numerical components v_{2fx} and v_{2fy}.

(c) **Elastic condition check (energy conservation).** For a perfectly elastic collision, kinetic energy is conserved.

(i) Compute K_i (total kinetic energy before).

(ii) Compute K_f (total kinetic energy after) using your $\vec{v}_{2f}$ from part (b).

(iii) State whether your results are consistent with an elastic collision (to reasonable rounding).

(d) **Center of mass velocity.** Compute the center-of-mass velocity vector:

$$\vec{v}_{\text{cm}} = \frac{m_1\vec{v}_{1i} + m_2\vec{v}_{2i}}{m_1 + m_2}.$$

Then verify that using *final* velocities gives the same $\vec{v}_{\text{cm}}$.

(e) **Collision in the CM frame (conceptual + short calculation).** Define velocities in the CM frame by $\vec{v}' = \vec{v} - \vec{v}_{\text{cm}}$.

(i) Compute $\vec{v}'_{1i}$ and $\vec{v}'_{1f}$.

(ii) Compute their magnitudes $|\vec{v}'_{1i}|$ and $|\vec{v}'_{1f}|$ and comment on what you notice.