

# Homework Set: HW4

## Momentum, Impulse, and 2D Collisions

### **Problem 1: Perfectly Inelastic 2D Collision + Impulse + Post-Collision Motion**

Two low-friction carts collide on a horizontal air table and stick together (perfectly inelastic collision). Take  $+x$  to be east and  $+y$  to be north. External horizontal forces during the collision are negligible.

Cart A has mass  $m_A = 0.45\text{ kg}$  and moves at speed  $6.2\text{ m/s}$  at an angle of  $25^\circ$  *north of east* just before impact.

Cart B has mass  $m_B = 0.70\text{ kg}$  and moves at speed  $4.5\text{ m/s}$  at an angle of  $40^\circ$  *south of east* just before impact.

After they stick, the combined carts slide off the air table onto a rough surface and come to rest after traveling a distance of  $3.5\text{ m}$ . Assume the rough surface provides a kinetic friction force with coefficient  $\mu_k$  (unknown). Use  $g = 9.81\text{ m/s}^2$ .

- (a) **Vector momentum setup.** Write the initial velocity vectors  $\vec{v}_A$  and  $\vec{v}_B$  in component form ( $x$ - and  $y$ -components).
- (b) **Post-collision velocity (2D).** Use conservation of momentum in  $x$  and  $y$  to find the combined velocity vector  $\vec{v}_f$  immediately after the collision. Report:
  - (i)  $v_{fx}$  and  $v_{fy}$ ,
  - (ii) the speed  $v_f$ ,
  - (iii) the direction as an angle measured from  $+x$  (east), with correct quadrant.
- (c) **Kinetic energy change.** Calculate the total kinetic energy before the collision and just after the collision, then determine:
  - (i) the kinetic energy lost in the collision,  $\Delta K = K_i - K_f$ ,
  - (ii) the fraction of kinetic energy lost,  $\Delta K/K_i$ .
- (d) **Impulse and average collision force.** The sticking collision lasts  $\Delta t = 0.035\text{ s}$ .
  - (i) Find the impulse vector on cart A,  $\vec{J}_A$ . Note  $J = \Delta p = F\Delta t$ .
  - (ii) Find the impulse vector on cart B,  $\vec{J}_B$ .

- (iii) Hence find the average force on each cart during the collision,  $\vec{F}_{\text{avg}} = \vec{J}/\Delta t$  (give components).
- (e) **Friction estimate from stopping distance.** After the collision, the combined carts (mass  $m_A + m_B$ ) enter the rough surface with initial speed  $v_f$  (from part b) and stop after 3.5 m. Assuming kinetic friction is the only horizontal force on the rough surface, determine  $\mu_k$ .
- (f) **Direction reasoning (short explanation).** In 1–2 sentences, explain why the final direction in part (b) must lie between the initial directions of A and B.