Assignment 18

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Download all python codes from

```
https://github.com/ka−raja−babu/Matrix−Theory/
    tree/main/Assignment18
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and latex-tikz codes from

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https://github.com/ka−raja−babu/Matrix−Theory/
    tree/main/Assignment18
```

1 Question No. 14.8(Markov Chain)

Consider a simple symmetric random walk on integers, where from every state i you move to $i-1$ and $i+1$ with the probability half each. Then which of the following are true?

1) The random walk is aperiodic
2) The random walk is irreducible
3) The random walk is null recurrent
4) The random walk is positive recurrent

2 Solution

Definition 1 (Aperiodicity)**.** *A random walk defined by a Markov chain having state space S and state transition matrix P ,is said to be aperiodic if there exists self-transition in the chain such that*

$$p_{ii}^n > 0 \quad for \quad i \in S, n \in Z^+ \tag{2.0.1}$$

Definition 2 (Irreducibility)**.** *A random walk defined by a Markov chain having state space S and state transition matrix P ,is said to be irreducible if all states communicate with each other such that*

$$p_{ij}^n > 0 \quad for \quad i, j \in S, n \in Z^+ \tag{2.0.2}$$

Definition 3 (Positive and Null Recuurancy)**.** *A random walk defined by a Markov chain having state space S ,is said to be positive recurrent if the excepted time to return to state i $\forall$ $i \in S$ is finite such that*

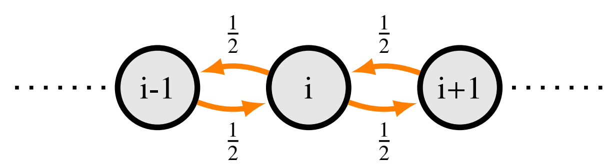
$$E(\tau_{ii}) < \infty \tag{2.0.3}$$

and,is said to be null recurrent if the excepted time to return to state i $\forall$ $i \in S$ is infinite such that

$$E(\tau_{ii}) = \infty \tag{2.0.4}$$

Let us define a Markov Chain for the given simple symmetric random walk with states $\{i-1, i, i+1\}$.

Markov chain diagram



State transition matrix P can be defined as:

$$P = \begin{array}{c c} & \begin{array}{ccc} i-1 & i & i+1 \end{array} \\ \begin{array}{c} i-1 \\ i \\ i+1 \end{array} & \begin{bmatrix} 0 & \frac{1}{2} & \frac{1}{4} \\ \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{4} & \frac{1}{2} & 0 \end{bmatrix} \end{array} \tag{2.0.5}$$

1) From State Transition Matrix P,
$$p_{mm} = 0 \tag{2.0.6}$$
where
$$m = \{i-1, i, i+1\} \tag{2.0.7}$$
$\because$ There is no self-transition in the chain.
$\therefore$ Random Walk is not aperiodic.
2) From State Transition Matrix P,
$$p_{mn} > 0 \tag{2.0.8}$$
where
$$m, n = \{i-1, i, i+1\} \tag{2.0.9}$$
$\because$ All states communicate with each other .
$\therefore$ Random Walk is irreducible.
3) Let $p = \frac{1}{2}$ be the probability to move from state i to state $i+1$ and $q = \frac{1}{2}$ be the probability to move from state i to state $i-1$.
Then,the excepted time of getting back to i $\forall$

i is given by

$$E(\tau_{ii}) = \frac{1}{|p-q|} \tag{2.0.10}$$
$$= \frac{1}{0} \tag{2.0.11}$$
$$= \infty \tag{2.0.12}$$

$\therefore$ Random Walk is null recurrent .
Hence, $\boxed{Options \quad (2),(3)}$ are true .

Fig. 2.1: Random Walk on Integers