

# Assignment 14

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Download all python codes from

<https://github.com/ka-raja-babu/Matrix-Theory/tree/main/Assignment14>

and

and latex-tikz codes from

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## 1 QUESTION No. 6.17

A person plays a game of tossing a coin thrice. For each head, he is given Rs 2 by the organiser of the game and for each tail, he has to give Rs 1.50 to the organiser. Let  $X$  denote the amount gained or lost by the person. Show that  $X$  is a random variable and exhibit it as a function on the sample space of the experiment.

## 2 SOLUTION

Let  $X_1, X_2, X_3$  be the three tosses of the coin and  $X$  be the total amount such that

$$X = X_1 + X_2 + X_3 \quad (2.0.1)$$

where

$$X_i = \{2, -1.5\} \quad (2.0.2)$$

From eq.(2.0.1), value of  $X$  is given by

$$X = x \in \{-4.5, -1, 2.5, 6\} \quad (2.0.3)$$

Let PMF of  $X$  be

$$p_X(x) = ax^2 + bx + c \quad (2.0.4)$$

Now,  $\Pr(X = x)$  satisfies PMF such that

$$a(-4.5)^2 + b(-4.5) + c = \frac{1}{8} \quad (2.0.5)$$

$$\Rightarrow 162a - 36b + 8c = 1 \quad (2.0.6)$$

and

$$a(-1)^2 + b(-1) + c = \frac{3}{8} \quad (2.0.7)$$

$$\Rightarrow 8a - 8b + 8c = 3 \quad (2.0.8)$$

$$a(6)^2 + b(6) + c = \frac{1}{8} \quad (2.0.9)$$

$$\Rightarrow 288a + 48b + 8c = 1 \quad (2.0.10)$$

From eq.(2.0.6), eq.(2.0.8) and eq.(2.0.10),

$$\begin{pmatrix} 162 & -36 & 8 \\ 8 & -8 & 8 \\ 288 & 48 & 8 \end{pmatrix} \mathbf{a} = \begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix} \quad (2.0.11)$$

$$\Rightarrow \mathbf{a} = \begin{pmatrix} -\frac{1}{98} \\ \frac{3}{196} \\ \frac{157}{392} \end{pmatrix} \quad (2.0.12)$$

Hence, PMF of  $X$  is given by

$$p_X(x) = \left(\frac{-1}{98}\right)x^2 + \left(\frac{3}{196}\right)x + \left(\frac{157}{392}\right) \quad (2.0.13)$$

$$\Rightarrow p_X(x) = \frac{1}{392}(-4x^2 + 6x + 157) \quad (2.0.14)$$

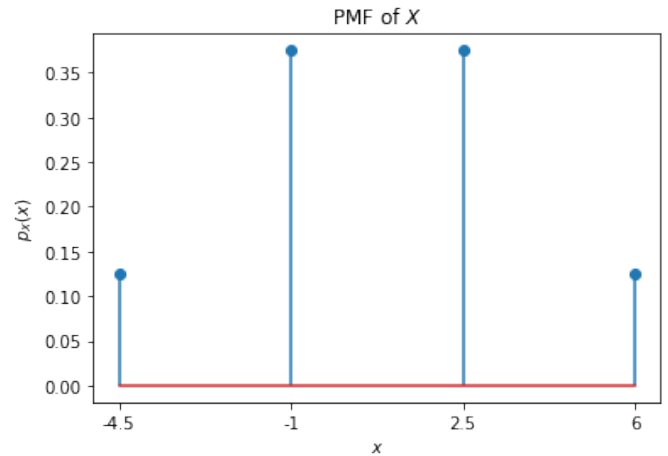


Fig. 2.1: PMF of  $X$