Assignment 13

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Download all python codes from

https://github.com/ka-raja-babu/Matrix-Theory/tree/main/Assignment13

and latex-tikz codes from

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1 Question No. 2.9

A laboratory blood test is 99% effective in detecting a certain disease when it is in fact, present. However, the test also yields a false positive result for 0.5% of the healthy person tested (i.e. if a healthy person is tested, then, with probability 0.005, the test will imply he has the disease). If 0.1 percent of the population actually has the disease, what is the probability that a person has the disease given that his test result is positive?

2 Solution

Let the input variables $X \in \{0, 1\}$ and $Y \in \{0, 1\}$ be defined according to the table 2.1.

Input Variable	Value	Description
X	0	Absence of disease in test
	1	Presence of disease in test
Y	0	Absence of disease in reality
	1	Presence of disease in reality

TABLE 2.1: Input Variables

Given data of the question is presented in the table 2.2.

S.No.	Expression	Value
1.	$\Pr(X=1 Y=1)$	99 100
2.	$\Pr(X=1 Y=0)$	$\frac{0.5}{100} = \frac{1}{200}$
3.	Pr(Y=1)	$\frac{0.1}{100} = \frac{1}{1000}$
4.	Pr(Y=0)	$\frac{99.9}{100} = \frac{999}{1000}$

TABLE 2.2: Given Data

Hence, probability that a person has the disease given that his test result is positive, is given by

$$Pr(Y = 1|X = 1) = \frac{Pr(X = 1|Y = 1)Pr(Y = 1)}{\sum_{i=0}^{1} Pr(X = 1|Y = i)Pr(Y = i)}$$

$$= \frac{\frac{99}{100} \times \frac{1}{1000}}{\frac{1}{200} \times \frac{999}{1000} + \frac{99}{100} \times \frac{1}{1000}}$$

$$= \frac{22}{122}$$
(2.0.3)