

Assignment 16

K.A. Raja Babu

Download all python codes from

<https://github.com/ka-raja-babu/Matrix-Theory/tree/main/Assignment16/Codes>

and latex-tikz codes from

<https://github.com/ka-raja-babu/Matrix-Theory/tree/main/Assignment16>

Now, Joint PMF of X, Y is given by

$$p_{XY}(x, y) = \begin{cases} \frac{(0.25-0.2)}{(1+0.25)} & x = 0, y = 1 \\ \frac{(0.2-0)}{(1+0.25)} & x = 1, y = 0 \\ \frac{(0.25+0.25)}{(1+0.25)} & x = 0, y = 0 \\ \frac{(1-0.2)}{(1+0.25)} & x = 1, y = 1 \end{cases} \quad (2.0.3)$$

$$= \begin{cases} 0.04 & x = 0, y = 1 \\ 0.16 & x = 1, y = 0 \\ 0.36 & x = 0, y = 0 \\ 0.64 & x = 1, y = 1 \end{cases} \quad (2.0.4)$$

1 QUESTION NO. 8.2(GATE PROBABILITY)

Consider a binary digital communication system with equally likely 0's and 1's. When binary 0 is transmitted the voltage at the detector input can lie between the level $-0.25V$ and $+0.25V$ with equal probability. When binary 1 is transmitted, the voltage at the detector can have any value between 0 and 1V with equal probability. If the detector has a threshold of $0.2V$ (i.e., if the received signal is greater than $0.2V$, the bit is taken as 1), the average bit error probability is

- 1) 0.15 2) 0.2 3) 0.05 4) 0.5

2 SOLUTION

Let $X \in \{0, 1\}$ be the transmitted symbol and $Y \in \{0, 1\}$ be the detected symbol.

PMF of X is given by

$$p_X(x) = \begin{cases} \frac{(0.25+0.25)}{(1+0.25)} & x = 0 \\ \frac{(1-0)}{(1+0.25)} & x = 1 \end{cases} \quad (2.0.1)$$

$$= \begin{cases} 0.4 & x = 0 \\ 0.8 & x = 1 \end{cases} \quad (2.0.2)$$

Now, bit error probability for $X = 0$ is given by

$$P_{e0} = \Pr(Y = 1|X = 0) = \frac{\Pr[(Y = 1)(X = 0)]}{\Pr(X = 0)} \quad (2.0.5)$$

$$= \frac{0.04}{0.4} \quad (2.0.6)$$

$$= 0.01 \quad (2.0.7)$$

and, bit error probability for $X = 1$ is given by

$$P_{e1} = \Pr(Y = 0|X = 1) = \frac{\Pr[(Y = 0)(X = 1)]}{\Pr(X = 1)} \quad (2.0.8)$$

$$= \frac{0.16}{0.8} \quad (2.0.9)$$

$$= 0.02 \quad (2.0.10)$$

Now, average bit error probability or bit error rate is given by

$$P_e = BER = P_0 P_{e0} + P_1 P_{e1} \quad (2.0.11)$$

\therefore Symbols 0 and 1 are equally likely.

$\therefore P_0$ and P_1 is given by

$$P_0 = P_1 = \frac{1}{2} \quad (2.0.12)$$

Hence, average bit error probability is given by

$$P_e = \frac{1}{2}(P_{e0} + P_{e1}) \quad (2.0.13)$$

$$= \frac{1}{2}(0.01 + 0.02) \quad (2.0.14)$$

$$= \boxed{0.15} \quad (2.0.15)$$

∴ Option (1) 0.15 is the correct answer.

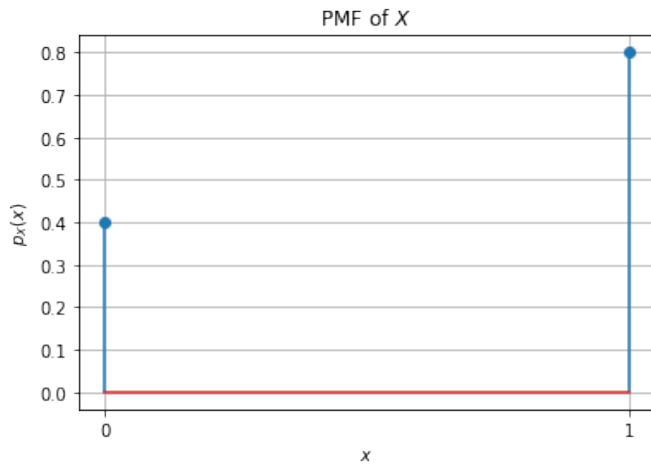


Fig. 2.1: PMF of X

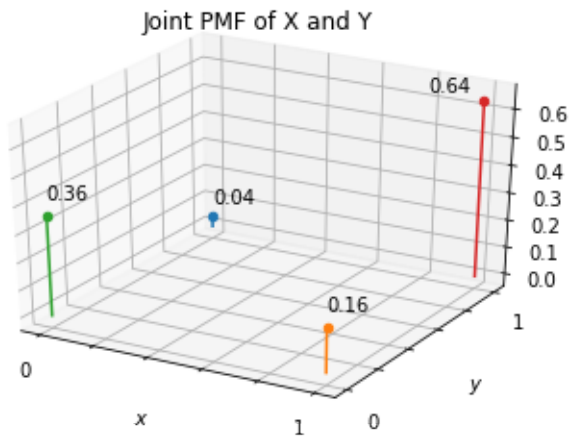


Fig. 2.2: Joint PMF of X, Y