Assignment 16

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Download all python codes from

```
https://github.com/ka−raja−babu/Matrix−Theory/
    tree/main/Assignment16
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and latex-tikz codes from

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https://github.com/ka−raja−babu/Matrix−Theory/
    tree/main/Assignment16
```

1 Question No. 8.2(GATE Probability)

Consider a binary digital communication system with equally likely 0's and 1's. When binary 0 is transmitted the voltage at the detector input can lie between the level -0.25V and +0.25V with equal probability: when binary 1 is transmitted, the voltage at the detector can have any value between 0 and 1V with equal probability. If the detector has a threshold of 0.2V (i.e., if the received signal is greater than 0.2V, the bit is taken as 1), the average bit error probability is

1) 0.15 2) 0.2 3) 0.05 4) 0.5

2 Solution

Let the random variable for bit 0 and bit 1 be $X \in [-0.25, 0.25]$ and $Y \in [0, 1]$ respectively .

It is given that X follows uniform distribution .

$\therefore$ PDF of X is given by

$$p_X(x) = \begin{cases} \frac{1}{0.5} & -0.25 \leq X \leq 0.25 \\ 0 & \text{otherwise} \end{cases} \tag{2.0.1}$$

$$= \begin{cases} 2 & -0.25 \leq X \leq 0.25 \\ 0 & \text{otherwise} \end{cases} \tag{2.0.2}$$

and,PDF of Y is given by

$$p_Y(y) = \begin{cases} 1 & 0 \leq Y \leq 1 \\ 0 & \text{otherwise} \end{cases} \tag{2.0.3}$$

$\because$ Threshold of the detector is given as 0.2V .

$\therefore$ Using eq. (2.0.2), bit error probability for bit 0 is given by

$$\text{BEP}_0 = \Pr(0.2 \leq X \leq 0.25) = 2(0.25 - 0.2) = 0.1 \tag{2.0.4}$$

and,using eq. (2.0.3), bit error probability for bit 1 is given by

$$\text{BEP}_1 = \Pr(0 \leq Y \leq 0.2) = 1(0.2 - 0) = 0.2 \tag{2.0.5}$$

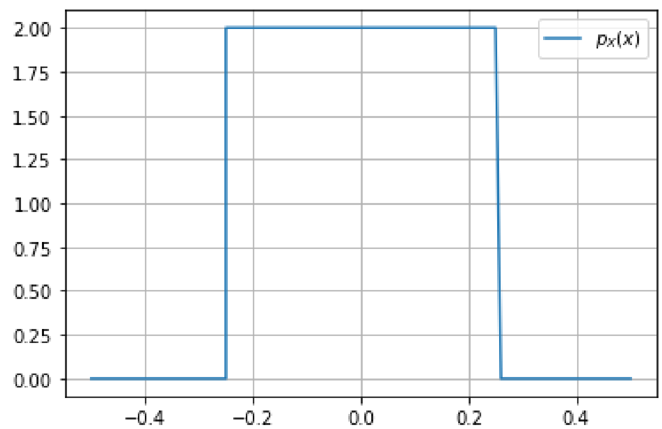

Fig. 2.1: PDF of X

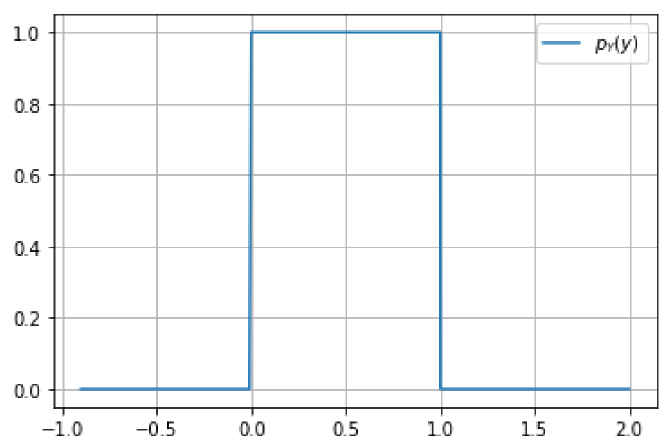

Fig. 2.2: PDF of Y

Hence,using eq. (2.0.4) and eq. (2.0.5) ,average bit error probability is given by

$$
\begin{aligned}
\text{BEP}_{avg} &= \frac{1}{2}\left(\text{BEP}_0 + \text{BEP}_1\right) && (2.0.6)\\
&= \frac{1}{2}\left(0.1 + 0.2\right) && (2.0.7)\\
&= \boxed{0.15} && (2.0.8)
\end{aligned}
$$