Assignment 16

K.A. Raja Babu

Download all python codes from

```
https://github.com/ka-raja-babu/Matrix-Theory/
    tree/main/Assignment16/Codes
```

and latex-tikz codes from

```
https://github.com/ka-raja-babu/Matrix-Theory/
    tree/main/Assignment16
```

1 Question No. 8.2(GATE Probability)

Consider a binary digital communication system with equally likely 0's and 1's. When binary 0 is transmitted the voltage at the detector input can lie between the level -0.25V and +0.25V with equal probability.When binary 1 is transmitted, the voltage at the detector can have any value between 0 and 1V with equal probability. If the detector has a threshold of 0.2V (i.e., if the received signal is greater than 0.2V, the bit is taken as 1), the average bit error probability is

1) 0.15
2) 0.2
3) 0.05
4) 0.5

2 Solution

Let $X \in \{0, 1\}$ be the transmitted symbol and $Y \in \{0, 1\}$ be the detected symbol.

PMF of X is given by

$$p_X(x) = \begin{cases} \frac{(0.25+0.25)}{(1+0.25)} & x = 0 \\ \frac{(1-0)}{(1+0.25)} & x = 1 \end{cases} \tag{2.0.1}$$

$$= \begin{cases} 0.4 & x = 0 \\ 0.8 & x = 1 \end{cases} \tag{2.0.2}$$

Now,Joint PMF of X, Y is given by

$$p_{XY}(x, y) = \begin{cases} \frac{(0.25-0.2)}{(1+0.25)} & x = 0, y = 1 \\ \frac{(0.2-0)}{(1+0.25)} & x = 1, y = 0 \\ \frac{(0.2+0.25)}{(1+0.25)} & x = 0, y = 0 \\ \frac{(1-0.2)}{(1+0.25)} & x = 1, y = 1 \end{cases} \tag{2.0.3}$$

$$= \begin{cases} 0.04 & x = 0, y = 1 \\ 0.16 & x = 1, y = 0 \\ 0.36 & x = 0, y = 0 \\ 0.64 & x = 1, y = 1 \end{cases} \tag{2.0.4}$$

Now, bit error probability for $X = 0$ is given by

$$P_{e_0} = \Pr(Y = 1|X = 0) = \frac{\Pr[(Y = 1)(X = 0)]}{\Pr(X = 0)} \tag{2.0.5}$$

$$= \frac{0.04}{0.4} \tag{2.0.6}$$

$$= 0.01 \tag{2.0.7}$$

and,bit error probability for $X = 1$ is given by

$$P_{e_1} = \Pr(Y = 0|X = 1) = \frac{\Pr[(Y = 0)(X = 1)]}{\Pr(X = 1)} \tag{2.0.8}$$

$$= \frac{0.16}{0.8} \tag{2.0.9}$$

$$= 0.02 \tag{2.0.10}$$

Now,average bit error probability or bit error rate is given by

$$P_e = BER = P_0 P_{e_0} + P_1 P_{e_1} \tag{2.0.11}$$

$\because$ Symbols 0 and 1 are equally likely.

$\therefore$ P_0 and P_1 is given by

$$P_0 = P_1 = \frac{1}{2} \tag{2.0.12}$$

Hence,average bit error probability is given by

$$P_e = \frac{1}{2}(P_{e_0} + P_{e_1}) \tag{2.0.13}$$
$$= \frac{1}{2}(0.01 + 0.02) \tag{2.0.14}$$
$$= \boxed{0.15} \tag{2.0.15}$$

$\therefore$ Option (1) 0.15 is the correct answer.

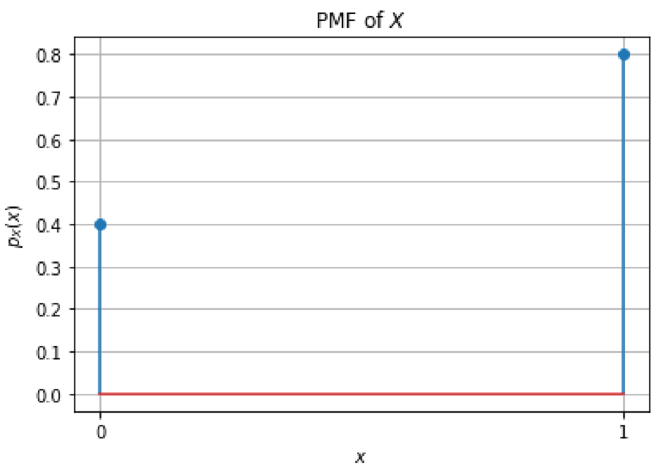

Fig. 2.1: PMF of X

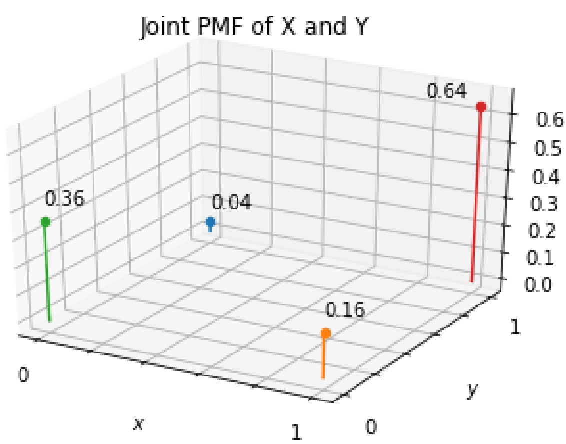

Fig. 2.2: Joint PMF of X, Y