Assignment 15

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Download all python codes from

https://github.com/ka−raja−babu/Matrix−Theory/tree/main/Assignment15/Codes

and latex-tikz codes from

https://github.com/ka−raja−babu/Matrix−Theory/tree/main/Assignment15

1 Question No. 1.4(Optimization)

Maximize $Z = 3x + 4y$ subject to constraints : $x + y \leq 4, x \geq 0, y \geq 0$.

2 Solution

Our problem can be interpreted as

$$\max_{\mathbf{x}} Z = \begin{pmatrix} 3 & 4 \end{pmatrix} \mathbf{x} \tag{2.0.1}$$

$$s.t. \quad \begin{pmatrix} 1 & 1 \end{pmatrix} \mathbf{x} \leq 4 \tag{2.0.2}$$

$$-\mathbf{x} \leq \mathbf{0} \tag{2.0.3}$$

Lagrangian function is given by

$$\begin{aligned} &L(\mathbf{x}, \boldsymbol{\lambda}) \\ &= \begin{pmatrix} -3 & -4 \end{pmatrix} \mathbf{x} + \left\{ \left[\begin{pmatrix} 1 & 1 \end{pmatrix} \mathbf{x} - 4 \right] \right. \\ &+ \left. \left[\begin{pmatrix} -1 & 0 \end{pmatrix} \mathbf{x} \right] + \left[\begin{pmatrix} 0 & -1 \end{pmatrix} \mathbf{x} \right] \right\} \boldsymbol{\lambda} \end{aligned} \tag{2.0.4}$$

where,

$$\boldsymbol{\lambda} = \begin{pmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{pmatrix} \tag{2.0.5}$$

Now,

$$\nabla L(\mathbf{x}, \boldsymbol{\lambda}) = \begin{pmatrix} -3 + \begin{pmatrix} 1 & -1 & 0 \end{pmatrix} \boldsymbol{\lambda} \\ -4 + \begin{pmatrix} 1 & 0 & -1 \end{pmatrix} \boldsymbol{\lambda} \\ \begin{pmatrix} 1 & 1 \end{pmatrix} \mathbf{x} - 4 \\ \begin{pmatrix} -1 & 0 \end{pmatrix} \mathbf{x} \\ \begin{pmatrix} 0 & -1 \end{pmatrix} \mathbf{x} \end{pmatrix} \tag{2.0.6}$$

$\therefore$ Lagrangian matrix is given by

$$\begin{pmatrix} 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & 0 & -1 \\ 1 & 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \mathbf{x} \\ \boldsymbol{\lambda} \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \\ 4 \\ 0 \\ 0 \end{pmatrix} \tag{2.0.7}$$

Considering λ_1, λ_2 as only active multiplier,

$$\begin{pmatrix} 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \mathbf{x} \\ \boldsymbol{\lambda} \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \\ 4 \\ 0 \end{pmatrix} \tag{2.0.8}$$

resulting in,

$$\begin{pmatrix} \mathbf{x} \\ \boldsymbol{\lambda} \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}^{-1} \begin{pmatrix} 3 \\ 4 \\ 4 \\ 0 \end{pmatrix} \tag{2.0.9}$$

$$\implies \begin{pmatrix} \mathbf{x} \\ \boldsymbol{\lambda} \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ -1 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 3 \\ 4 \\ 4 \\ 0 \end{pmatrix} \tag{2.0.10}$$

$$\implies \begin{pmatrix} \mathbf{x} \\ \boldsymbol{\lambda} \end{pmatrix} = \begin{pmatrix} 0 \\ 4 \\ 4 \\ 1 \end{pmatrix} \tag{2.0.11}$$

$\because \boldsymbol{\lambda} = \begin{pmatrix} 4 \\ 1 \end{pmatrix} \succ \mathbf{0}$

$\therefore$ Optimal solution is given by

$$\mathbf{x} = \begin{pmatrix} 0 \\ 4 \end{pmatrix} \tag{2.0.12}$$

$$Z = \begin{pmatrix} 3 & 4 \end{pmatrix} \mathbf{x} \tag{2.0.13}$$

$$= \begin{pmatrix} 3 & 4 \end{pmatrix} \begin{pmatrix} 0 \\ 4 \end{pmatrix} \tag{2.0.14}$$

$$= 16 \tag{2.0.15}$$

By using cvxpy in python ,

$$\mathbf{x} = \begin{pmatrix} 2.25279985e-08 \\ 3.99999997e+00 \end{pmatrix} \approx \begin{pmatrix} 0 \\ 4 \end{pmatrix} \tag{2.0.16}$$

$$Z = 15.99999994 \approx 16 \tag{2.0.17}$$

Hence, $\boxed{x = 0}$ and $\boxed{y = 4}$ will maximize Z and the maximum value of Z is $\boxed{Z = 16}$.

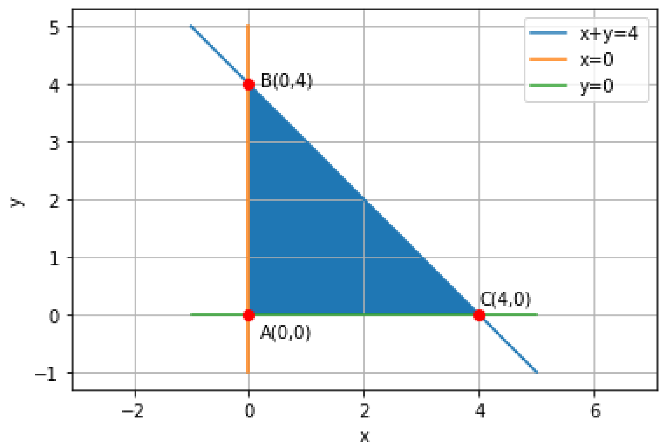

Fig. 2.1: Feasible Region