# Assignment 18

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Download all python codes from

https://github.com/ka–raja–babu/Matrix–Theory/tree/main/Assignment18

and latex-tikz codes from

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## 1 Question No. 14.8(Markov Chain)

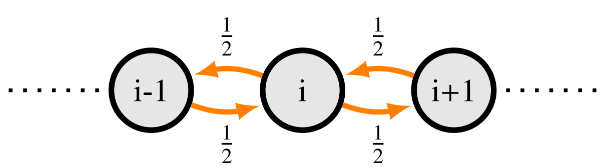
Consider a simple symmetric random walk on integers, where from every state $i$ you move to $i-1$ and $i+1$ with the probability half each. Then which of the following are true?

1) The random walk is aperiodic
2) The random walk is irreducible
3) The random walk is null recurrent
4) The random walk is positive recurrent

## 2 Solution

Let us define a Markov Chain for the given simple symmetric random walk with states $\{i-1, i, i+1\}$ .

**Markov chain diagram**




State transition matrix $P$ can be defined as:

$$P = \begin{array}{c c} & \begin{array}{ccc} i-1 & i & i+1 \end{array} \\ \begin{array}{c} i-1 \\ i \\ i+1 \end{array} & \begin{bmatrix} 0 & \frac{1}{2} & \frac{1}{4} \\ \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{4} & \frac{1}{2} & 0 \end{bmatrix} \end{array} \tag{2.0.1}$$

1) From State Transition Matrix $P$,

$$p_{mn} = 0 \tag{2.0.2}$$

where

$$m, n = \{i-1, i, i+1\} \quad \& \quad m = n \tag{2.0.3}$$

$\because$ There is no self-transition in the chain.
$\therefore$ Random Walk is not aperiodic.

2) From State Transition Matrix $P$,

$$p_{mn} > 0 \tag{2.0.4}$$

where

$$m, n = \{i-1, i, i+1\} \quad \& \quad m \neq n \tag{2.0.5}$$

$\because$ All states communicate with each other .
$\therefore$ Random Walk is irreducible.

3) Let $p = \frac{1}{2}$ be the probability to move from state $i$ to state $i+1$ and $q = \frac{1}{2}$ be the probability to move from state $i$ to state $i-1$ .
Then,the excepted time of getting back to $i$ $\forall$ $i$ is given by

$$E(\tau_{ii}) = \frac{1}{|p-q|} \tag{2.0.6}$$
$$= \frac{1}{0} \tag{2.0.7}$$
$$= \infty \tag{2.0.8}$$

$\therefore$ Random Walk is null recurrent .

Hence, $\boxed{Options \quad (2),(3)}$ are true .


Fig. 2.1: Random Walk on Integers