1

Assignment 14

K.A. Raja Babu

Download all python codes from

https://github.com/ka-raja-babu/Matrix-Theory/tree/main/Assignment14

and latex-tikz codes from

https://github.com/ka-raja-babu/Matrix-Theory/ tree/main/Assignment14

1 **QUESTION No. 6.17**

A person plays a game of tossing a coin thrice. For each head, he is given Rs 2 by the organiser of the game and for each tail,he has to give Rs 1.50 to the organiser. Let X denote the amount gained or lost by the person. Show that X is a random variable and exhibit it as a function on the sample space of the experiment.

2 Solution

Let X_1, X_2, X_3 be the three tosses of the coin and X be the total amount such that

$$X = X_1 + X_2 + X_3 \tag{2.0.1}$$

where

$$X_i = \{2, -1.5\} \tag{2.0.2}$$

From eq.(2.0.1), value of X is given by

$$X = x \in \{-4.5, -1, 2.5, 6\}$$
 (2.0.3)

Let PMF of X be

$$p_X(x) = ax^2 + bx + c (2.0.4)$$

Now,Pr(X = x) satisfies PMF such that

$$a(-4.5)^2 + b(-4.5) + c = \frac{1}{8}$$
 (2.0.5)

$$\implies 162a - 36b + 8c = 1$$
 (2.0.6)

and

$$a(-1)^2 + b(-1) + c = \frac{3}{8}$$
 (2.0.7)

$$\implies 8a - 8b + 8c = 3 \tag{2.0.8}$$

and

$$a(6)^2 + b(6) + c = \frac{1}{8}$$
 (2.0.9)

$$\implies 288a + 48b + 8c = 1$$
 (2.0.10)

From eq.(2.0.6),eq.(2.0.8) and eq.(2.0.10),

$$\begin{pmatrix} 162 & -36 & 8 \\ 8 & -8 & 8 \\ 288 & 48 & 8 \end{pmatrix} \mathbf{a} = \begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix}$$
 (2.0.11)

$$\implies \mathbf{a} = \begin{pmatrix} \frac{-1}{98} \\ \frac{3}{196} \\ \frac{157}{392} \end{pmatrix} \tag{2.0.12}$$

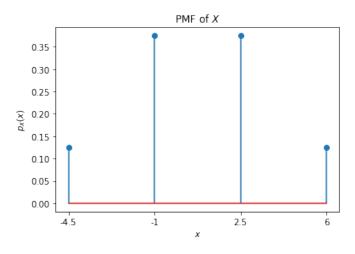


Fig. 2.1: PMF of *X*

Hence,PMF of X is given by

$$p_X(x) = \left(\frac{-1}{98}\right)x^2 + \left(\frac{3}{196}\right)x + \left(\frac{157}{392}\right)$$
 (2.0.13)

$$\implies p_X(x) = \frac{1}{392} \left(-4x^2 + 6x + 157 \right)$$
 (2.0.14)

For x = 2.5,PMF is given by

$$p_X(x) = \frac{1}{392} \left[-4(2.5)^2 + 6(2.5) + 157 \right]$$

$$\implies p_X(x) = \frac{3}{8}$$
(2.0.15)

Hence, $p_X(x)$ is true for all possible values of x.