Assignment 15

K.A. Raja Babu

Download all python codes from

https://github.com/ka−raja−babu/Matrix−Theory/tree/main/Assignment15

and latex-tikz codes from

https://github.com/ka−raja−babu/Matrix−Theory/tree/main/Assignment15

1 Question No. 2.1(a)(Optimization)

Find the absolute maximum and absolute minimum value of $f(x) = 4x - \frac{1}{2}x^2, x \in \left[-2, \frac{9}{2}\right]$.

2 Solution

Lemma 2.1. *A function f(x) is said to be convex if following inequality is true for $\lambda \in [0,1]$:*

$$\lambda f(x_1) + (1-\lambda) f(x_2) \geq f(\lambda x_1 + (1-\lambda)x_2) \quad (2.0.1)$$

Given :

$$f(x) = 4x - \frac{1}{2}x^2, x \in \left[-2, \frac{9}{2}\right] \quad (2.0.2)$$

Checking convexity of $f(x)$:

$$\lambda\left(4x_1 - \frac{1}{2}x_1^2\right) + (1-\lambda)\left(4x_2 - \frac{1}{2}x_2^2\right) \geq 4(\lambda x_1 + (1-\lambda)x_2) - \frac{1}{2}(\lambda x_1 + (1-\lambda)x_2)^2 \quad (2.0.3)$$

resulting in

$$x_1^2\left(\frac{\lambda^2-\lambda}{2}\right) + x_2^2\left(\frac{\lambda^2-\lambda}{2}\right) + 2x_1x_2\left(\frac{\lambda-\lambda^2}{2}\right) \geq 0 \quad (2.0.4)$$

$$\implies \left(\frac{\lambda^2-\lambda}{2}\right)\left(x_1^2 + x_2^2 - 2x_1x_2\right) \geq 0 \quad (2.0.5)$$

$$\implies -\frac{1}{2}\lambda(1-\lambda)(x_1-x_2)^2 \geq 0 \quad (2.0.6)$$

$$\implies \frac{1}{2}\lambda(1-\lambda)(x_1-x_2)^2 \leq 0 \quad (2.0.7)$$

Hence,using lemma 2.1, given $f(x)$ is a concave function .

1) For Maxima :
Using gradient ascent method,

$$x_{n+1} = x_n + \alpha \nabla f(x_n) \quad (2.0.8)$$

$$\implies x_{n+1} = x_n + \alpha(4-x) \quad (2.0.9)$$

Taking $x_0 = -2, \alpha = 0.001$ and precision= 0.00000001,values obtained using python are:

$$\boxed{\text{Maxima} = 7.999999999950196 \approx 8} \quad (2.0.10)$$

$$\boxed{\text{Maxima Point} = 3.9999900196756437 \approx 4} \quad (2.0.11)$$

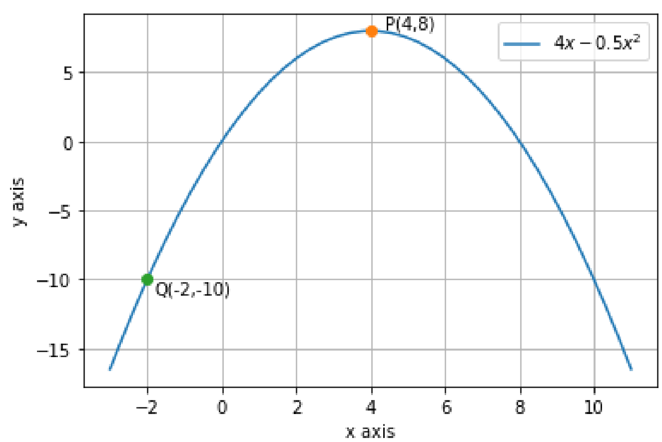

Fig. 2.1: $f(x) = 4x - 0.5x^2$

2) For Minima :

x	$f(x)$
-2	-10
4	8
4.5	7.875

TABLE 2.1: Value of $f(x)$

Critical point is given by

$$\nabla f(x) = 0 \tag{2.0.12}$$

$$\implies x = 4 \tag{2.0.13}$$

and,end points are $x = -2$ and $x = 4.5$.
Using table 2.1,

$$\boxed{\text{Minima} = -10} \tag{2.0.14}$$

$$\boxed{\text{Minima Point} = -2} \tag{2.0.15}$$