

Assignment 12

K.A. Raja Babu

Download all python codes from

<https://github.com/ka-raja-babu/Matrix-Theory/tree/main/Assignment12/Codes>

and latex-tikz codes from

<https://github.com/ka-raja-babu/Matrix-Theory/tree/main/Assignment12>

and,

$$4x + 20y \geq 460 \quad (2.0.6)$$

$$\Rightarrow -x - 5y \leq -115 \quad (2.0.7)$$

and,

$$6x + 4y \leq 300 \quad (2.0.8)$$

$$\Rightarrow 3x + 2y \leq 150 \quad (2.0.9)$$

1 QUESTION No. 2.36

(Diet problem) A dietician has to develop a special diet using two foods P and Q. Each packet (containing 30 g) of food P contains 12 units of calcium, 4 units of iron, 6 units of cholesterol and 6 units of vitamin A. Each packet of the same quantity of food Q contains 3 units of calcium, 20 units of iron, 4 units of cholesterol and 3 units of vitamin A. The diet requires at least 240 units of calcium, at least 460 units of iron and at most 300 units of cholesterol. How many packets of each food should be used to minimise the amount of vitamin A in the diet ? What is the minimum amount of vitamin A ?

2 SOLUTION

| Component | P | Q | Requirement |
|-------------|----------|----------|------------------|
| Calcium | 12 units | 3 units | ≥ 240 units |
| Iron | 4 units | 20 units | ≥ 460 units |
| Cholesterol | 6 units | 4 units | ≤ 300 units |
| Vitamin A | 6 units | 3 units | |

TABLE 2.1: Diet Requirements

Let the number of packets of food P be x and the number of packets of food Q be y such that

$$x \geq 0 \quad (2.0.1)$$

$$y \geq 0 \quad (2.0.2)$$

According to the question,

$$12x + 3y \geq 240 \quad (2.0.3)$$

$$\Rightarrow -4x - y \leq -80 \quad (2.0.4)$$

$$(2.0.5)$$

\therefore Our problem is

$$\min_{\mathbf{x}} Z = (6 \ 3) \mathbf{x} \quad (2.0.10)$$

$$s.t. \quad \begin{pmatrix} -4 & -1 \\ -1 & -5 \\ 3 & 2 \end{pmatrix} \mathbf{x} \leq \begin{pmatrix} -80 \\ -115 \\ 150 \end{pmatrix} \quad (2.0.11)$$

$$-\mathbf{x} \leq \mathbf{0} \quad (2.0.12)$$

Lagrangian function is given by

$L(\mathbf{x}, \lambda)$

$$\begin{aligned} &= (6 \ 3) \mathbf{x} + \left\{ [(-4 \ -1) \mathbf{x} + 80] \right. \\ &+ [(-1 \ -5) \mathbf{x} + 115] + [(3 \ 2) \mathbf{x} - 150] \\ &+ [(-1 \ 0) \mathbf{x}] + [(0 \ -1) \mathbf{x}] \left. \right\} \lambda \end{aligned} \quad (2.0.13)$$

where,

$$\lambda = \begin{pmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ \lambda_4 \\ \lambda_5 \\ \lambda_6 \end{pmatrix} \quad (2.0.14)$$

Now,

$$\nabla L(\mathbf{x}, \lambda) = \begin{pmatrix} 6 + (-4 \ -1 \ 3 \ -1 \ 0) \lambda \\ 3 + (-1 \ -5 \ 2 \ 0 \ -1) \lambda \\ (-4 \ -1) \mathbf{x} + 80 \\ (-1 \ -5) \mathbf{x} + 115 \\ (3 \ 2) \mathbf{x} - 150 \\ (-1 \ 0) \mathbf{x} \\ (0 \ -1) \mathbf{x} \end{pmatrix} \quad (2.0.15)$$

\therefore Lagrangian matrix is given by

$$\begin{pmatrix} 0 & 0 & -4 & -1 & 3 & -1 & 0 \\ 0 & 0 & -1 & -5 & 2 & 0 & -1 \\ -4 & -1 & 0 & 0 & 0 & 0 & 0 \\ -1 & -5 & 0 & 0 & 0 & 0 & 0 \\ 3 & 2 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \mathbf{x} \\ \lambda \end{pmatrix} = \begin{pmatrix} -6 \\ -3 \\ -80 \\ -115 \\ 150 \\ 0 \\ 0 \end{pmatrix} \quad (2.0.16)$$

Considering λ_1, λ_2 as only active multiplier,

$$\begin{pmatrix} 0 & 0 & -4 & -1 \\ 0 & 0 & -1 & -5 \\ -4 & -1 & 0 & 0 \\ -1 & -5 & 0 & 0 \end{pmatrix} \begin{pmatrix} \mathbf{x} \\ \lambda \end{pmatrix} = \begin{pmatrix} -6 \\ -3 \\ -80 \\ -115 \end{pmatrix} \quad (2.0.17)$$

resulting in,

$$\begin{pmatrix} \mathbf{x} \\ \lambda \end{pmatrix} = \begin{pmatrix} 0 & 0 & -4 & -1 \\ 0 & 0 & -1 & -5 \\ -4 & -1 & 0 & 0 \\ -1 & -5 & 0 & 0 \end{pmatrix}^{-1} \begin{pmatrix} -6 \\ -3 \\ -80 \\ -115 \end{pmatrix} \quad (2.0.18)$$

$$\Rightarrow \begin{pmatrix} \mathbf{x} \\ \lambda \end{pmatrix} = \begin{pmatrix} 0 & 0 & \frac{-5}{19} & \frac{1}{19} \\ 0 & 0 & \frac{1}{19} & \frac{-4}{19} \\ \frac{-5}{19} & \frac{1}{19} & 0 & 0 \\ \frac{1}{19} & \frac{-4}{19} & 0 & 0 \end{pmatrix} \begin{pmatrix} -6 \\ -3 \\ -80 \\ -115 \end{pmatrix} \quad (2.0.19)$$

$$\Rightarrow \begin{pmatrix} \mathbf{x} \\ \lambda \end{pmatrix} = \begin{pmatrix} 15 \\ 20 \\ \frac{27}{19} \\ \frac{6}{19} \end{pmatrix} \quad (2.0.20)$$

$$\therefore \lambda = \begin{pmatrix} \frac{27}{19} \\ \frac{6}{19} \end{pmatrix} > \mathbf{0}$$

\therefore Optimal solution is given by

$$\mathbf{x} = \begin{pmatrix} 15 \\ 20 \end{pmatrix} \quad (2.0.21)$$

$$Z = \begin{pmatrix} 6 & 3 \end{pmatrix} \mathbf{x} \quad (2.0.22)$$

$$= \begin{pmatrix} 6 & 3 \end{pmatrix} \begin{pmatrix} 15 \\ 20 \end{pmatrix} \quad (2.0.23)$$

$$= 150 \quad (2.0.24)$$

By using cvxpy in python ,

$$\mathbf{x} = \begin{pmatrix} 14.99999999 \\ 20.00000001 \end{pmatrix} \quad (2.0.25)$$

$$Z = 150.00000001 \quad (2.0.26)$$

Hence , $x = 15$ packets of food P and $y = 20$ packets of food Q should be used to minimise the

amount of vitamin A in the diet and the minimum amount of vitamin A is $Z = 150$ units .

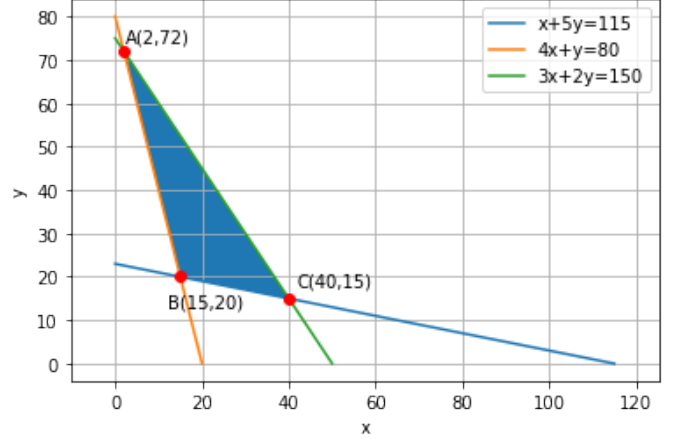


Fig. 2.1: Diet Problem