Assignment 14

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Download all python codes from

```
https://github.com/ka−raja−babu/Matrix−Theory/
    tree/main/Assignment14
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and latex-tikz codes from

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    tree/main/Assignment14
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1 Question No. 6.17

A person plays a game of tossing a coin thrice. For each head, he is given Rs 2 by the organiser of the game and for each tail,he has to give Rs 1.50 to the organiser. Let X denote the amount gained or lost by the person. Show that X is a random variable and exhibit it as a function on the sample space of the experiment.

2 Solution

Let X_1, X_2, X_3 be the three tosses of the coin and X be the total amount such that

$$X = X_1 + X_2 + X_3 \tag{2.0.1}$$

where

$$X_i = \{2, -1.5\} \tag{2.0.2}$$

From eq.(2.0.1),value of X is given by

$$X = x \in \{-4.5, -1, 2.5, 6\} \tag{2.0.3}$$

Let p be the number of occurrence of particular value of x in X such that

$$p = {}^nC_r \tag{2.0.4}$$

where

$$n = \text{No. of tosses} \tag{2.0.5}$$

$$r = \text{No. of 2 or -1.5 of } X_i \text{ required to get } x \tag{2.0.6}$$

Hence,p is given by

$$p = \{{}^3C_3, {}^3C_2, {}^3C_1, {}^3C_0\} \tag{2.0.7}$$

$$\implies p = \{1, 3, 3, 1\} \tag{2.0.8}$$

Using eq.(2.0.8),probability of X is given by

$$\Pr(X = x) = \left\{\frac{1}{8}, \frac{3}{8}, \frac{3}{8}, \frac{1}{8}\right\} \tag{2.0.9}$$

Let PMF of X be

$$p_X(x) = ax^2 + bx + c \tag{2.0.10}$$

Now,$\Pr(X = x)$ satisfies PMF such that

$$a(-4.5)^2 + b(-4.5) + c = \frac{1}{8} \tag{2.0.11}$$

$$\implies 162a - 36b + 8c = 1 \tag{2.0.12}$$

and

$$a(-1)^2 + b(-1) + c = \frac{3}{8} \tag{2.0.13}$$

$$\implies 8a - 8b + 8c = 3 \tag{2.0.14}$$

and

$$a(6)^2 + b(6) + c = \frac{1}{8} \tag{2.0.15}$$

$$\implies 288a + 48b + 8c = 1 \tag{2.0.16}$$

From eq.(2.0.12),eq.(2.0.14) and eq.(2.0.16),

$$\begin{pmatrix} 162 & -36 & 8 \\ 8 & -8 & 8 \\ 288 & 48 & 8 \end{pmatrix} \mathbf{a} = \begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix} \tag{2.0.17}$$

$$\implies \mathbf{a} = \begin{pmatrix} \frac{-1}{98} \\ \frac{3}{196} \\ \frac{157}{392} \end{pmatrix} \tag{2.0.18}$$

Hence,PMF of X is given by

$$p_X(x) = \left(\frac{-1}{98}\right)x^2 + \left(\frac{3}{196}\right)x + \left(\frac{157}{392}\right) \tag{2.0.19}$$

$$\implies \boxed{p_X(x) = \frac{1}{392}\left(-4x^2 + 6x + 157\right)} \tag{2.0.20}$$

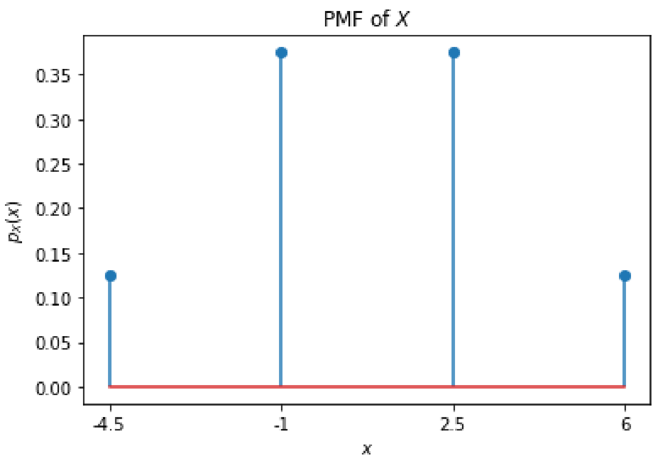

Fig. 2.1: PMF of X

For $x = 2.5$,PMF is given by

$$p_X(2.5) = \frac{1}{392}\left[-4(2.5)^2 + 6(2.5) + 157\right] \tag{2.0.21}$$

$$\implies p_X(2.5) = \frac{3}{8} \tag{2.0.22}$$

Hence,$p_X(x)$ is true for all possible values of x .