

# Assignment 15

K.A. Raja Babu

Download all python codes from

<https://github.com/ka-raja-babu/Matrix-Theory/tree/main/Assignment15/Codes>

and latex-tikz codes from

<https://github.com/ka-raja-babu/Matrix-Theory/tree/main/Assignment15>

∴ Lagrangian matrix is given by

$$\begin{pmatrix} 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & 0 & -1 \\ 1 & 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \mathbf{x} \\ \lambda \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \\ 4 \\ 0 \\ 0 \end{pmatrix} \quad (2.0.7)$$

Considering  $\lambda_1, \lambda_2$  as only active multiplier,

$$\begin{pmatrix} 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \mathbf{x} \\ \lambda \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \\ 4 \\ 0 \end{pmatrix} \quad (2.0.8)$$

resulting in,

$$\begin{pmatrix} \mathbf{x} \\ \lambda \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}^{-1} \begin{pmatrix} 3 \\ 4 \\ 4 \\ 0 \end{pmatrix} \quad (2.0.9)$$

$$\Rightarrow \begin{pmatrix} \mathbf{x} \\ \lambda \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ -1 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 3 \\ 4 \\ 4 \\ 0 \end{pmatrix} \quad (2.0.10)$$

$$\Rightarrow \begin{pmatrix} \mathbf{x} \\ \lambda \end{pmatrix} = \begin{pmatrix} 0 \\ 4 \\ 4 \\ 1 \end{pmatrix} \quad (2.0.11)$$

$$\because \lambda = \begin{pmatrix} 4 \\ 1 \end{pmatrix} > \mathbf{0}$$

∴ Optimal solution is given by

$$\mathbf{x} = \begin{pmatrix} 0 \\ 4 \end{pmatrix} \quad (2.0.12)$$

$$Z = \begin{pmatrix} 3 & 4 \end{pmatrix} \mathbf{x} \quad (2.0.13)$$

$$= \begin{pmatrix} 3 & 4 \end{pmatrix} \begin{pmatrix} 0 \\ 4 \end{pmatrix} \quad (2.0.14)$$

$$= 16 \quad (2.0.15)$$

## 1 QUESTION NO. 1.4(OPTIMIZATION)

Maximize  $Z = 3x + 4y$  subject to constraints :  
 $x + y \leq 4, x \geq 0, y \geq 0$ .

## 2 SOLUTION

Our problem can be interpreted as

$$\max_{\mathbf{x}} Z = \begin{pmatrix} 3 & 4 \end{pmatrix} \mathbf{x} \quad (2.0.1)$$

$$s.t. \quad \begin{pmatrix} 1 & 1 \end{pmatrix} \mathbf{x} \leq 4 \quad (2.0.2)$$

$$-\mathbf{x} \leq \mathbf{0} \quad (2.0.3)$$

Lagrangian function is given by

$$\begin{aligned} L(\mathbf{x}, \lambda) &= \begin{pmatrix} -3 & -4 \end{pmatrix} \mathbf{x} + \left\{ \left[ \begin{pmatrix} 1 & 1 \end{pmatrix} \mathbf{x} - 4 \right] \right. \\ &\quad \left. + \left[ \begin{pmatrix} -1 & 0 \end{pmatrix} \mathbf{x} \right] + \left[ \begin{pmatrix} 0 & -1 \end{pmatrix} \mathbf{x} \right] \right\} \lambda \end{aligned} \quad (2.0.4)$$

where,

$$\lambda = \begin{pmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{pmatrix} \quad (2.0.5)$$

Now,

$$\nabla L(\mathbf{x}, \lambda) = \begin{pmatrix} -3 + \begin{pmatrix} 1 & -1 & 0 \end{pmatrix} \lambda \\ -4 + \begin{pmatrix} 1 & 0 & -1 \end{pmatrix} \lambda \\ \begin{pmatrix} 1 & 1 \end{pmatrix} \mathbf{x} - 4 \\ \begin{pmatrix} -1 & 0 \end{pmatrix} \mathbf{x} \\ \begin{pmatrix} 0 & -1 \end{pmatrix} \mathbf{x} \end{pmatrix} \quad (2.0.6)$$

By using cvxpy in python ,

$$\mathbf{x} = \begin{pmatrix} 2.25279985e-08 \\ 3.99999997e+00 \end{pmatrix} \approx \begin{pmatrix} 0 \\ 4 \end{pmatrix} \quad (2.0.16)$$

$$Z = 15.99999994 \approx 16 \quad (2.0.17)$$

Hence,  $x = 0$  and  $y = 4$  will maximize  $Z$  and the maximum value of  $Z$  is  $Z = 16$  .

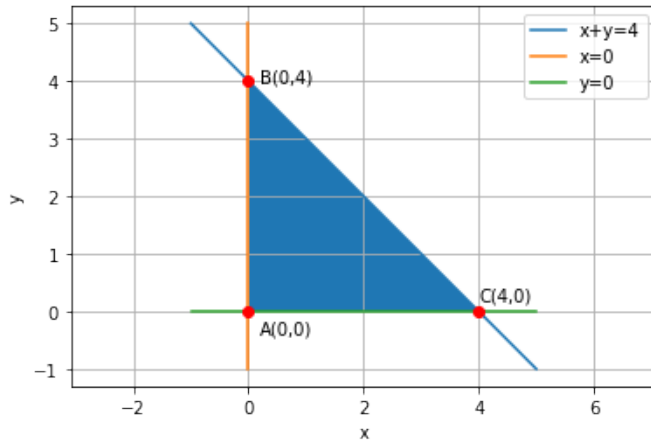


Fig. 2.1: Feasible Region