

Assignment 15

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Download all python codes from

<https://github.com/ka-raja-babu/Matrix-Theory/tree/main/Assignment15/Codes>

and latex-tikz codes from

<https://github.com/ka-raja-babu/Matrix-Theory/tree/main/Assignment15>

1 QUESTION NO. 2.1(A)(OPTIMIZATION)

Find the absolute maximum and absolute minimum value of $f(x) = 4x - \frac{1}{2}x^2$, $x \in \left(-2, \frac{9}{2}\right)$.

2 SOLUTION

Lemma 2.1. A function $f(x)$ is said to be convex if following inequality is true for $\lambda \in [0, 1]$:

$$\lambda f(x_1) + (1 - \lambda)f(x_2) \geq f(\lambda x_1 + (1 - \lambda)x_2) \quad (2.0.1)$$

Given :

$$f(x) = 4x - \frac{1}{2}x^2, x \in \left(-2, \frac{9}{2}\right) \quad (2.0.2)$$

Checking convexity of $f(x)$:

$$\lambda \left(4x_1 - \frac{1}{2}x_1^2\right) + (1 - \lambda) \left(4x_2 - \frac{1}{2}x_2^2\right) \geq \quad (2.0.3)$$

$$4(\lambda x_1 + (1 - \lambda)x_2) - \frac{1}{2}(\lambda x_1 + (1 - \lambda)x_2)^2$$

resulting in

$$x_1^2 \left(\frac{\lambda^2 - \lambda}{2}\right) + x_2^2 \left(\frac{\lambda^2 - \lambda}{2}\right) + 2x_1x_2 \left(\frac{\lambda - \lambda^2}{2}\right) \geq 0 \quad (2.0.4)$$

$$\Rightarrow \left(\frac{\lambda^2 - \lambda}{2}\right)(x_1^2 + x_2^2 - 2x_1x_2) \geq 0 \quad (2.0.5)$$

$$\Rightarrow -\frac{1}{2}\lambda(1 - \lambda)(x_1 - x_2)^2 \geq 0 \quad (2.0.6)$$

$$\Rightarrow \frac{1}{2}\lambda(1 - \lambda)(x_1 - x_2)^2 \leq 0 \quad (2.0.7)$$

Hence, using lemma 2.1, given $f(x)$ is a concave function .

1) For Maxima :

Using gradient ascent method,

$$x_{n+1} = x_n + \alpha \nabla f(x_n) \quad (2.0.8)$$

$$\Rightarrow x_{n+1} = x_n + \alpha(4 - x) \quad (2.0.9)$$

Taking $x_0 = -2$, $\alpha = 0.001$ and precision = 0.00000001, values obtained using python are:

$$\text{Maxima} = 7.999999999950196 \approx 8 \quad (2.0.10)$$

$$\text{Maxima Point} = 3.99999900196756437 \approx 4 \quad (2.0.11)$$

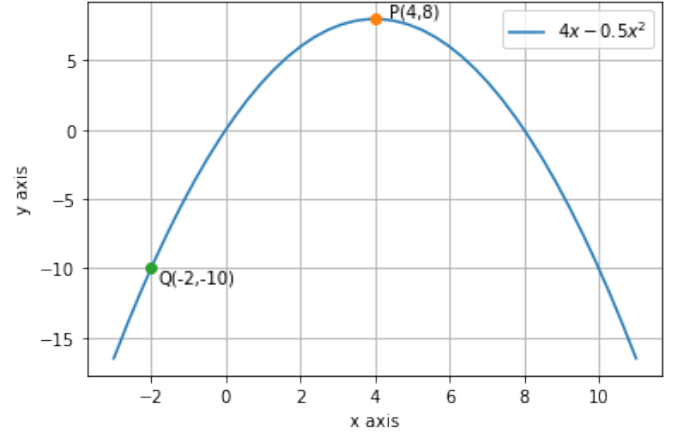


Fig. 2.1: $f(x) = 4x - 0.5x^2$

2) For Minima :

x	$f(x)$
-2	-10
4	8
4.5	7.875

TABLE 2.1: Value of $f(x)$

Critical point is given by

$$\nabla f(x) = 0 \quad (2.0.12)$$

$$\implies x = 4 \quad (2.0.13)$$

and, end points are $x = -2$ and $x = 4.5$.

Using table 2.1,

Minima = -10

(2.0.14)

Minima Point = -2

(2.0.15)