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Assignment 16

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Download all python codes from

https://github.com/ka-raja-babu/Matrix-Theory/ tree/main/Assignment16/Codes

and latex-tikz codes from

https://github.com/ka-raja-babu/Matrix-Theory/ tree/main/Assignment16

1 Question No. 8.2(GATE Probability)

Consider a binary digital communication system with equally likely 0's and 1's. When binary 0 is transmitted the voltage at the detector input can lie between the level -0.25V and +0.25V with equal probability. When binary 1 is transmitted, the voltage at the detector can have any value between 0 and 1V with equal probability. If the detector has a threshold of 0.2V (i.e., if the received signal is greater than 0.2V, the bit is taken as 1), the average bit error probability is

2 Solution

Let $X \in \{0, 1\}$ be the transmitted symbol and $Y \in$ {0, 1} be the detected symbol.

PMF of X is given by

$$p_X(x) = \begin{cases} \frac{(0.25 + 0.25)}{(1 + 0.25)} & x = 0\\ \frac{(1 - 0)}{(1 + 0.25)} & x = 1 \end{cases}$$

$$= \begin{cases} 0.4 & x = 0\\ 0.8 & x = 1 \end{cases}$$
(2.0.1)

Now, Joint PMF of X, Y is given by

$$p_{XY}(x,y) = \begin{cases} \frac{(0.25 - 0.2)}{(1+0.25)} & x = 0, y = 1\\ \frac{(0.2 - 0)}{(1+0.25)} & x = 1, y = 0\\ \frac{(0.2 + 0.25)}{(1+0.25)} & x = 0, y = 0\\ \frac{(1 - 0.2)}{(1+0.25)} & x = 1, y = 1 \end{cases}$$
(2.0.3)

$$= \begin{cases} 0.04 & x = 0, y = 1\\ 0.16 & x = 1, y = 0\\ 0.36 & x = 0, y = 0\\ 0.64 & x = 1, y = 1 \end{cases}$$
 (2.0.4)

Now, bit error probability for X = 0 is given by

$$P_{e0} = p_{Y|X=0}(1) = \frac{p_{XY}(0,1)}{p_X(0)}$$

$$= \frac{0.04}{0.4}$$
(2.0.5)

$$=\frac{0.04}{0.4}\tag{2.0.6}$$

$$= 0.01$$
 (2.0.7)

and, bit error probability for X = 1 is given by

$$P_{e1} = p_{Y|X=1}(0) = \frac{p_{XY}(1,0)}{p_X(1)}$$

$$= \frac{0.16}{0.8}$$
(2.0.8)

$$= \frac{0.16}{0.8}$$
 (2.0.9)
= 0.02 (2.0.10)

Hence, average bit error probability is given by

$$P_e = \frac{1}{2}(P_{e0} + P_{e1}) \tag{2.0.11}$$

$$=\frac{1}{2}(0.01+0.02)\tag{2.0.12}$$

$$= \boxed{0.15} \tag{2.0.13}$$

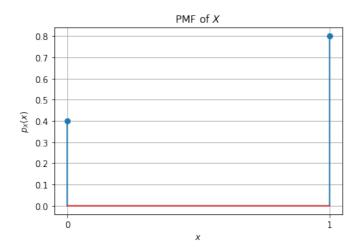


Fig. 2.1: PMF of *X*

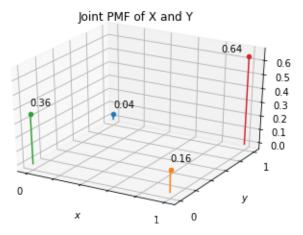


Fig. 2.2: Joint PMF of X, Y