ENGR 065: Circuit Theory

Problem Set #10

Read Chapters 15 and 16 from [1] and then solve the following problems.

- *Hint 1: you can use the Laplace tables (included in Appendix) to solve the problems*
- *Hint 2: watch the [Lecture 26 – Extra video] to understand how to perform partial fraction expansion of functions withs repeated poles.*

Problem 1 (25%) Use partial fraction expansion to find the time-domain waveform $f(t)$ corresponding to the following transforms:

a) $F(s) = \frac{8}{(s+1)(s+3)}$

b) $F(s) = \frac{8}{s(s^2+4s+8)}$

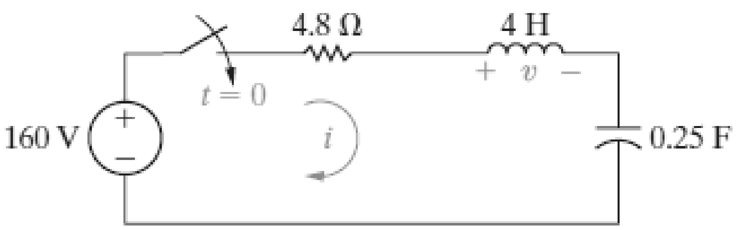
c) $F(s) = \frac{s}{(s+1)(s+2)^2}$

Additionally, find the initial and final values of the waveforms corresponding to the following transforms

d) $F_1(s) = 100\frac{(s+3)}{s(s+5)(s+20)}$

e) $F_2(s) = \frac{s-200}{s(s+50)}$

Problem 2 (20%):

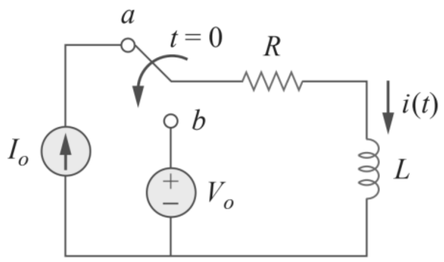


a) The energy stored in the circuit is zero at the time when the switch is closed ($t = 0$). Find $I(s)$.

b) compute $i(t)$

Hint: *the 160V source and the switch can be approximated with a single voltage source* $(160u(t))$.

Problem 3 (20%):



The switch moves to position b at $t = 0$.

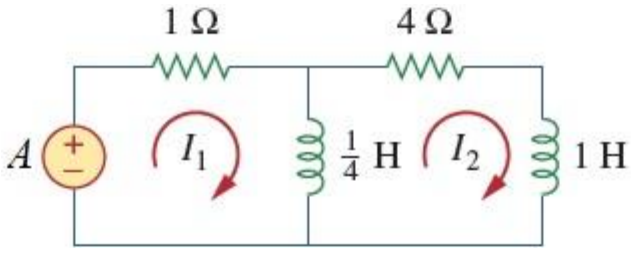
a) Find $I(s)$

b) Use the final-value theorem to determine the value of $i(t)$ as $t \to \infty$.

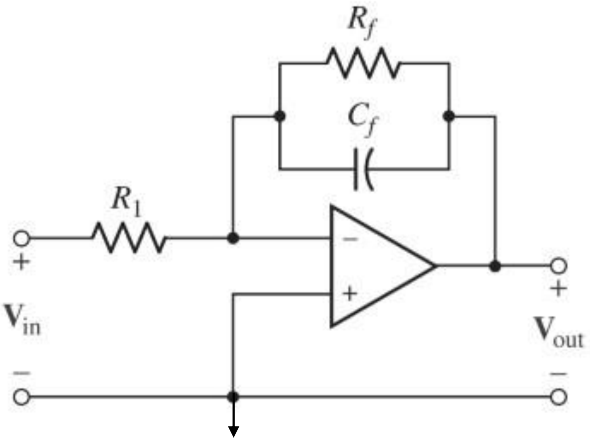
c) Determine $i(t)$.

Problem 4 (20%):

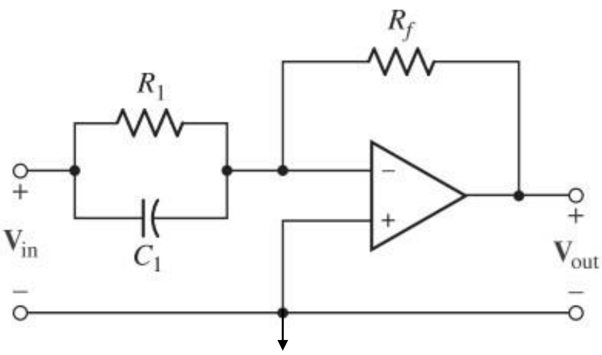
Find $I_2(s)$ in the circuit below. Assume A = 15u(t) V and zero initial currents in the inductors.



<u>Problem 5 (15%):</u> find the output voltage $V_{out}(s)$ for the following two circuits (assume zero initial conditions):



a)



b)

Appendix: Laplace transforms and properties

Table 1 – Laplace Transform Pairs

Type	$f(t)$ $(t > 0^-)$	$F(s)$
(impulse)	$\delta(t)$	1
(step)	$u(t)$	$\frac{1}{s}$
(ramp)	t	$\frac{1}{s^2}$
(exponential)	e^{-at}	$\frac{1}{s+a}$
(sine)	$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$
(cosine)	$\cos \omega t$	$\frac{s}{s^2+\omega^2}$
(damped ramp)	te^{-at}	$\frac{1}{(s+a)^2}$
(damped sine)	$e^{-at}\sin \omega t$	$\frac{\omega}{(s+a)^2+\omega^2}$
(damped cosine)	$e^{-at}\cos \omega t$	$\frac{s+a}{(s+a)^2+\omega^2}$

Table 2 – Laplace Transform Pairs (useful for Partial Fraction Expansion)

Pair Number	Nature of Roots	$F(s)$	$f(t)$
1	Distinct real	$\frac{K}{s+a}$	$Ke^{-at}u(t)$
2	Repeated real	$\frac{K}{(s+a)^2}$	$Kte^{-at}u(t)$
3	Distinct complex	$\frac{K}{s+\alpha-j\beta}+\frac{K^*}{s+\alpha+j\beta}$	$2\lvert K\rvert e^{-\alpha t}\cos(\beta t+\theta)u(t)$
4	Repeated complex	$\frac{K}{(s+\alpha-j\beta)^2}+\frac{K^*}{(s+\alpha+j\beta)^2}$	$2t\lvert K\rvert e^{-\alpha t}\cos(\beta t+\theta)u(t)$

Note:

- In pairs 1 and 2, K is a real quantity;
- In pairs 3 and 4, K is a complex quantity ($|K|e^{j\theta}$)

Table 3 – Laplace Properties

Operation	$f(t)$	$F(s)$
Multiplication by a constant	$Kf(t)$	$KF(s)$
Addition/subtraction	$f_1(t) + f_2(t) - f_3(t) + \cdots$	$F_1(s) + F_2(s) - F_3(s) + \cdots$
First derivative (time)	$\frac{df(t)}{dt}$	$sF(s) - f(0^-)$
Second derivative (time)	$\frac{d^2f(t)}{dt^2}$	$s^2F(s) - sf(0^-) - \frac{df(0^-)}{dt}$
nth derivative (time)	$\frac{d^nf(t)}{dt^n}$	$s^nF(s) - s^{n-1}f(0^-) - s^{n-2}\frac{df(0^-)}{dt} - s^{n-3}\frac{df^2(0^-)}{dt^2} - \cdots - \frac{d^{n-1}f(0^-)}{dt^{n-1}}$
Time integral	$\int_0^t f(x)\,dx$	$\frac{F(s)}{s}$
Translation in time	$f(t-a)u(t-a), a > 0$	$e^{-as}F(s)$
Translation in frequency	$e^{-at}f(t)$	$F(s+a)$
Scale changing	$f(at), a > 0$	$\frac{1}{a}F\left(\frac{s}{a}\right)$
First derivative (s)	$tf(t)$	$-\frac{dF(s)}{ds}$
nth derivative (s)	$t^nf(t)$	$(-1)^n\frac{d^nF(s)}{ds^n}$
s integral	$\frac{f(t)}{t}$	$\int_s^\infty F(u)\,du$

References

[1] C. Alexander and M. Sadiku "Fundamentals of Electric Circuits", 7th Edition, 2021, McGraw-Hill