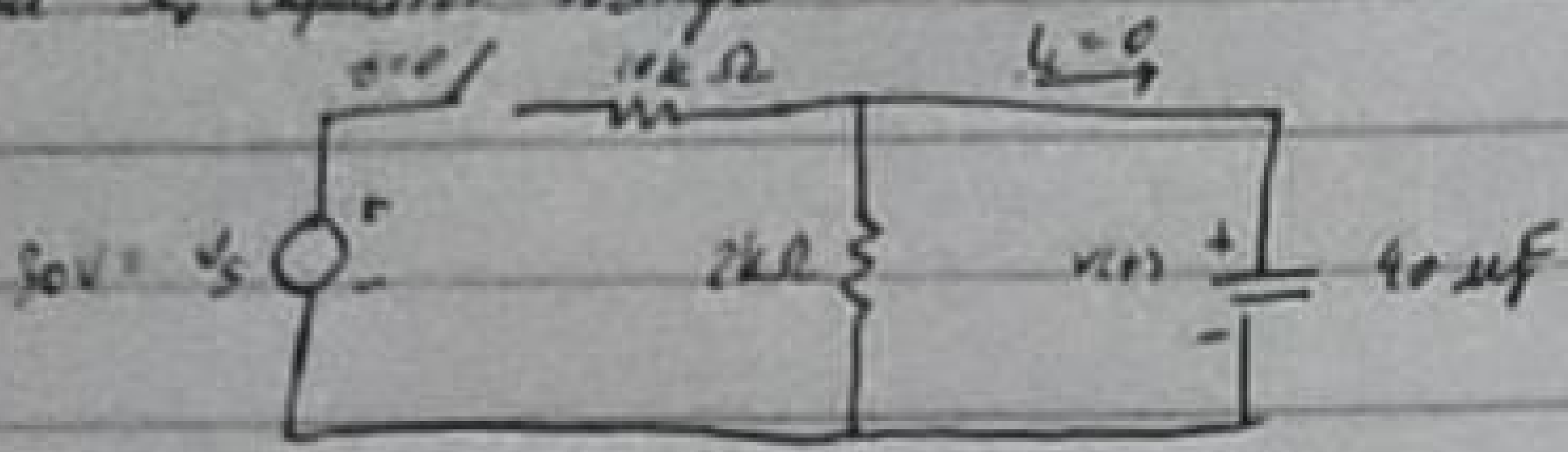
Engr 065

H.W No. 5

1. Consider at t=0 circuit opens. Taken $V_s = 30V$

a. find ↳ Capacitor Voltage

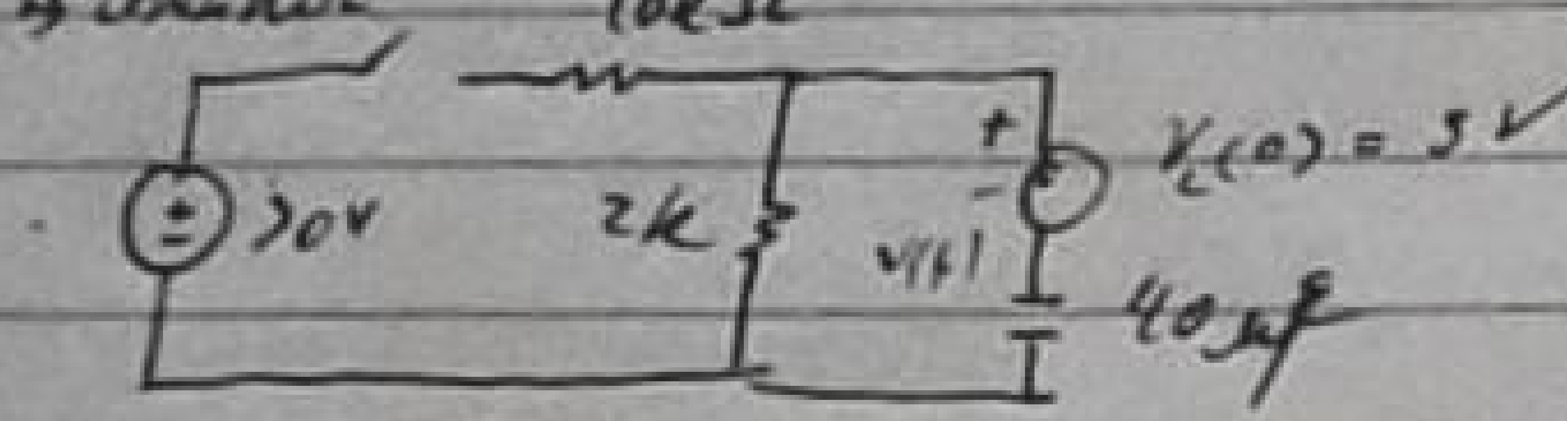


i. ↳ Ideal Cond - Capacitator is open circt.

$$\therefore \; V_c(0) = \frac{V_s(2k\Omega)}{10k + 2k} = \frac{60}{12} = 5V$$

b. find v(t) for t ≥ 0

i. ↳ Consider



↳ Note that such circt complies to the RC Condition; thus, decrease exp. from i_0 value.

ii. ↳ refer to $v(t) = V_c(0)e^{-t/\tau}$; $V_c(0) = 5V$; $\tau = RC = 2k \cdot 40\mu F(10^{-6}) = 0.08$

iii. ↳ Thus, $v(t) = \underline{5e^{-t/0.08}}$ ✓

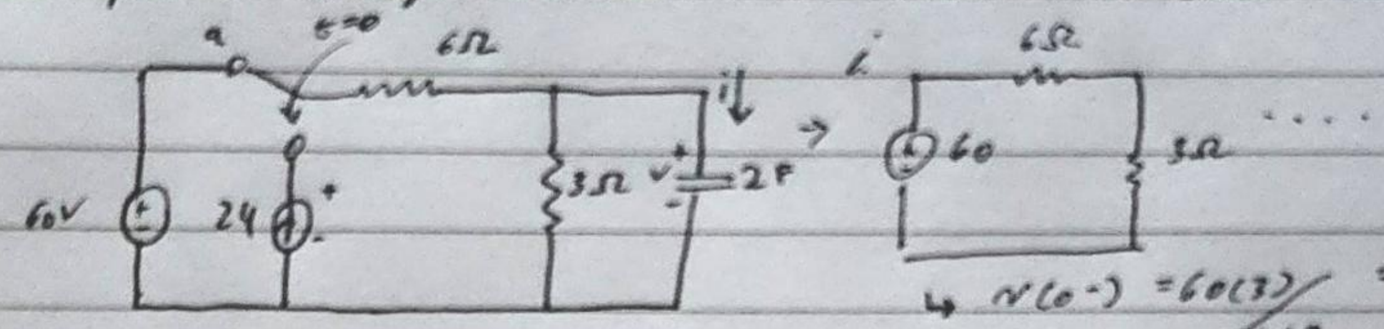
thus, because exp. form is value

ii. ↳ solve to $v(t) = v_c(0)\, e^{-t/\tau}$; $v_c(0) = 5V$; $\tau = RC = 2k \cdot 40\mu F (10^{-6}) = 0.08$

iii. ↳ Thus $v(t) = 5\, e^{-t/0.08}$ ✓

2. Consider switch position move to position b after a long time at $t=0$.

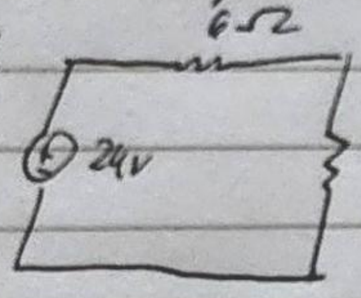
a. the Capacitor Voltage $v(t)$ for $t<0$ · Assume DC Cond.



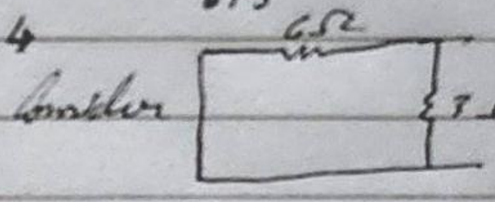
↳ $v(0^-) = 60\left(\frac{3}{6+3}\right) = 20V$; Consider DC Condition

thus, since $v(0^-) = 20$; $v(t) = 20V$ $v(0^-) = v(0^+)$

b. at $t=0$ in position b for



↳ $v_c(\infty) = \frac{24(3)}{6+3} = 8$; $v_c = 8V$ ✓



↳ $= \frac{6(3)}{6+3} = 2\Omega$

c. Capacitor Current $i(t>0)$

↳ $i(t) = C\frac{dv(t)}{dt} = 2\frac{d}{dt}(8 + 12 e^{-t/4})$

↳ $2(12)\left(-\frac{1}{4}\right) e^{-t/4}$

↳ $6 e^{-t/4}$ for $t>0$ ✓

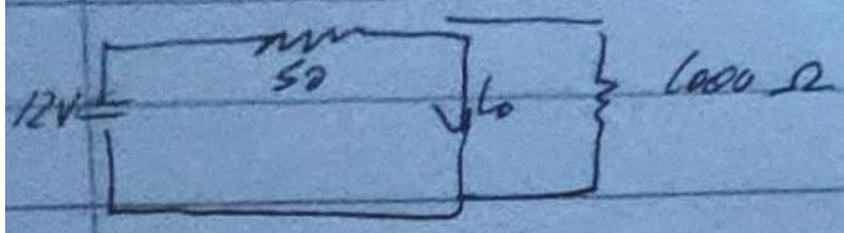
↳ $\tau = Req(C) = 2(2) = 4$

↳ Consider $v(t) = v_c(\infty) + (v_c(0) - v_c(\infty)) e^{-t/4}$

↳ $8 + [20 - 8] e^{-t/4}$

↳ $v(t) = 8 + 12 e^{-t/4}$ for v ✓

3. a. Initial inductor current



↳ $i_L(0) = 12/50 = 0.24A$

b. At $t=0$, switch opens while $t \geq 0$

i ↳ $i_L(\infty) = 0A$ Inductor is shorted

↳ $\tau = 2/1000 = .002\,s$

ii ↳ Consider $i(t) = i_L(\infty) + (i_L(0^+) - i_L(\infty))e^{-t/\tau}$; $\tau = .002$ sec

↳ $i_L(t) = 0 + (0.24 - 0)e^{-t/0.002}$

↳ $i_L(t) = 0.24e^{-500t}$ A ✓

c. Consider $v_o(t) = 2\,di_L(t)/dt$

↳ $v_o(t) = 2(0.24)\,d/dt\,e^{-500t}$

↳ $v_o(t) = -240e^{-500t}$ ✓ ✓

4. Consider Circuit given Capacitor defined as

$v(t) = 60e^{-1000t} - 30e^{-4000t}$, $t \geq 0$

a. Compute the resonant freq (ω_0), neper freq (α), and K

i ↳ $s_1 = -\alpha + \sqrt{\alpha^2 + \omega_0^2} = -1000 \Rightarrow -2\alpha = -5000 = 2500$ ✓ $= \alpha$

$s_2 = -\alpha + \sqrt{\alpha^2 - \omega_0^2} = -4000 \Rightarrow s - 1000 = -2000 + \sqrt{2500^2 - \omega_0^2}$

↳ $\omega_0 = 2000$ rad/sec ✓ $= \omega_0$

ii ↳ Consider $\alpha = R/2L = 2500$ iii. $\omega_0 = 1/\sqrt{LC}$

↳ $R = 50\,\Omega$ ✓ ↳ $1/\sqrt{10mL} = 2000$

↳ 25 mf ✓ $= C$ ✓

b. Compute $i_R(t)$. $i_C(t)$

↳ recall $v(t)/R = i_R(t)$

↳ $60e^{-1000t} - 90e^{-4000t}/50$

↳ recall $i_C(t) = C\,dv(t)/dt = 25\mu f\,d/dt\,(60e^{-1000t} - 30e^{-4000t})$

↳ $i_C(t) = -e^{-1000t} - 7e^{-4000t}A$, $t \geq 0$

5. Find Laplace transform of $f(t) = A\cos(\beta t)$

↳ Consider Euler

$f(t) = A/2 [e^{i\beta t} + e^{-i\beta t}]$

↳ $x(t) \to x(s)$

while $e^{s_0 t} x(t) \to x(s - s_0)$

↳ $A/2 (e^{i\beta t} + e^{-i\beta t}) \to A/2 (f(s - j\beta) + F(s + j\beta))$ ✓

6. Compute Laplace when $dv(t)/dt + 6v(t) = 4u(t)$, $v(0^-) = -3V$

↳ Consider $sv(s) - v(0^-) + 6v(s) = 4/s$; $dv(t)/dt + 6v(t) = 4u(t)$, $v(0^-) = -3V$

↳ $sv(s) + 3 + 6v(s) = 4/s$

↳ $v(s)(s+6) = 4/s - 3$

↳ $v_s = \frac{4 - 3s}{s(s+6)}$ ✓

7. Find Laplace transform of

$x(t) = -7u(t)$ where $u(t)$ is step function

↳ Consider $x(t) \to x(s)$

$u(t) \to 1/s$

Then $x(s) = -7/s$

$x(t) = -7u(t) \to -7/s$ ✓