Engr 65 - Circuit Theory

Problem Set #10

- Use fraction expansion to find the time-domain waveform $f(t)$ corresponding to the following transform:

1. a) $F(s) = 8/(s+1)(s+3)$

i. ↳ $\frac{8}{(s+1)(s+3)} = \frac{A}{s+1} + \frac{B}{s+3}$

↳ $8 = A(s+3) + B(s+1)$; Consider $s = -3, -1$

ii. $s = -3$; ↳ $A(-3+3) + B(-3+1) = 8$

↳ $-2B = 8 \Rightarrow B = -4$ ✓

iii. $s = -1$ ↳ $A(-1+3) + B(-1+1) = 8$

↳ $A(2) + 0 = 8$

↳ $A = 4$ ✓

iv. Apply Laplace Inverse

$\mathcal{L}^{-1}[F(s)] = \left[\frac{4}{s+1} - \frac{4}{s+3}\right]\mathcal{L}^{-1}$

↳ $4\mathcal{L}^{-1}\left(\frac{1}{s+1}\right) - 4\mathcal{L}^{-1}\left(\frac{1}{s+3}\right)$

↳ $f(t) = 4e^{-t} - 4e^{-3t}$

↳ $4(e^{-t} - e^{-3t})$ ✓

B. $F(s) = \frac{8}{s(s^2+4s+8)} = 8\left(\frac{A}{s} + \frac{B}{(s+2-j2)} + \frac{C}{(s+2+j2)}\right)$

i ↳ $a = 8/s^2+4s+8$; $s=0$ $=1$

$b = 8/s(s+2+j2)$; $s = -2+j2$ $= 8/s(-2+j2+2+j2)$ $= j0.5 - 0.5$

$c = 8/s(s+2+j2)$; Consider that $B = C$ probably

↳ $C = -0.5 + j0.5$

ii. $F(s) = 1/s + (0.5+j0.5)/s+2-j2 + (0.5-j0.5)/s+2+j2$

i. Laplace table ↳ $u(t) + (-0.5+j(0.5))\mathcal{L}^{-1}\, 1/s+2-j2 + (-0.5-j0.5)\, 1/s+2+j2$

↳ $u(t) + (-0.5+j(0.5))e^{-(2-j2)t}u(t) + (-0.5-j(0.5))e^{-(2+j2)t}u(t)$

Simplify ↳ $u(t) + (0.5+j(0.5))e^{-2t}e^{j2}u(t) + (-0.5-j(0.5))e^{-2t}e^{-j2t}u(t)$

Distribute ↳ $u(t) - 0.5e^{-2t}e^{j2t}u(t) + j(0.5)e^{-2t}e^{j2t}u(t) - 0.5e^{-2t}e^{-j2t}u(t) - j(0.5)e^{-2t}e^{-j2t}u(t)$

associate ↳ $u(t) - 0.5e^{-2t}\left(\frac{e^{j2t}+e^{-j2t}}{2}\right)u(t) + j0.5\left(\frac{e^{2t}-e^{-j2t}}{2j}\right)u(t)$

↳ Thus, referring to table ↳ $f(t) = u(t) - e^{-2t}\cos(2t)u(t) - e^{-2t}\sin(2t)u(t)$

c. $f(s) = s/(s+1)(s+2)^2$

i. ↳ $f(s) = A/(s+1) + B/(s+2) + C/(s+2)^2$

↳ $s = A(s+2)^2 + B(s+1)(s+2) + C(s+1)$

ii. ↳ $s=-1$; $A(-1+2)^2 + B(-1+1)(-1+2) + C(-1+1) = 2$

↳ $s=-2 \Rightarrow C=-2$

iv. Recall. $A = B^* = -2$

↳ $f(s) = 2/s+1 - 2/s+2 - 2/(s+2)^2$

v. ↳ $f(x) = 2e^{-t}u(t) - 2e^{-2t}u(t) - 2te^{-2t}u(t)$ ✓

Refer to

Laplace

c. Initial and final values of waveform

to following transform

d) $F_1(s) = 100 \dfrac{(s+3)}{s(s+5)(s+20)}$

$\lim_{s\to\infty} s \dfrac{100(s+3)}{s(s+5)(s+20)}$

transform property ↳ $\lim_{s\to\infty} \dfrac{s(1+3/s)100}{s(1+5/s)s(1+20/s)} = \infty$

ii. Consider $\dfrac{100(0+3)}{(0+5)(0+20)} = \dfrac{300}{100} = 3$ ✓

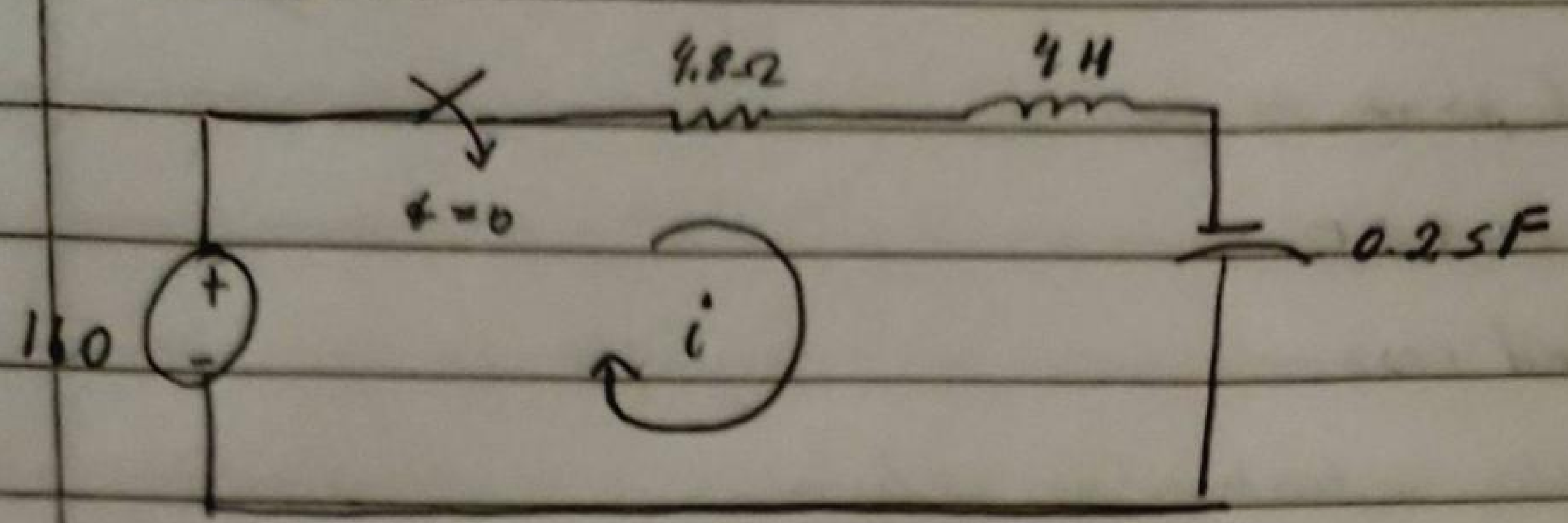
e. $f_2(s) = \dfrac{s-200}{s(s+50)}$

↳ $\lim_{s\to\infty} \dfrac{s(s-200)}{s(s+50)} = \dfrac{s-200}{s+50}$

↳ $\lim_{s\to} \dfrac{s(1-200/s)}{s(1+50/s)} = \dfrac{1-\frac{200}{\infty}}{1+50/\infty} = 1$

↳ ii. Consider $\lim_{x\to\infty} \dfrac{0-200}{0+50} = -4$ ✓

Problem 2



a. The energy stored in the circuit is zero at the time when switch is closed ($t=0$). Find $I(s)$

Laplace • ↳ for Circuit Convert Voltage from $v(t) \to V(s)$ domain

$v(t) = 1/v(s)$ $\quad v(t) = 160 \Rightarrow V(s) = 160/s$

• Capacitor $= 1/0.25$ F Inductor $= 4H$

↳ formula $I(s) = V(s) / R+L+C \quad = \dfrac{160}{4s^2 + 4.8s + 4}$ ✓

b. Compute $i(t)$

i ↳ $\mathcal{L}^{-1}\left(\dfrac{160}{4s^2+4.8s+4}\right) = \mathcal{L}^{-1}\left(\dfrac{40}{s^2+1.2s+1}\right)$

ii. ↳ Consider $s^2 + 1.2s + 1$

↳ $s^2 + 1.2s + 0.36 + 0.64$

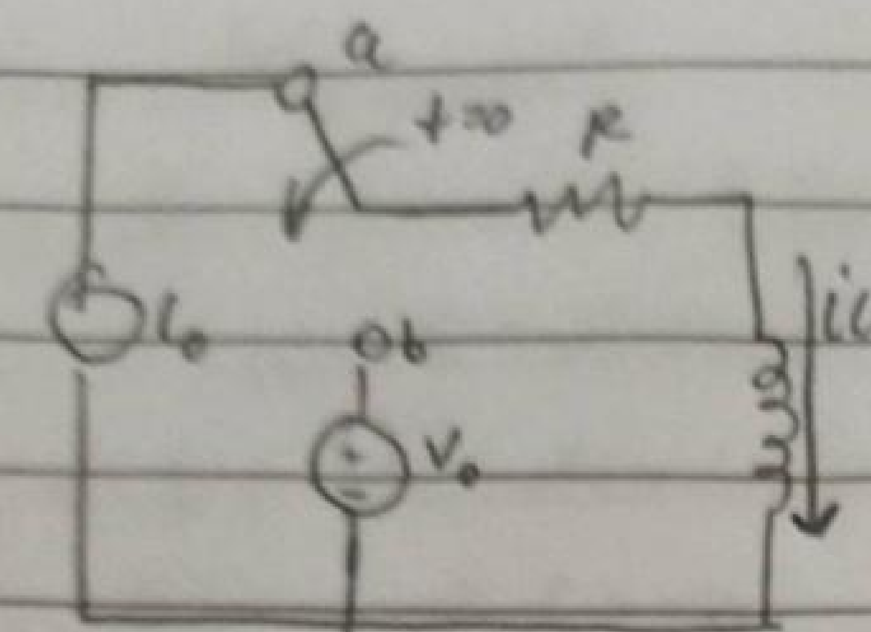
↳ $(s+0.6)^2 + 0.64 \Rightarrow (s+0.6)^2 + 0.8^2$

↳ $\dfrac{50 \cdot 0.8}{(s+0.6)^2 + 0.8^2} \overset{\mathcal{L}\{\cdot\}}{\Longleftrightarrow} 50\left[\mathcal{L}^{-1}\left(\dfrac{0.8}{(s+0.6^2)+0.8^2}\right)\right]$

• refer to Laplace transform table ↳ $\dfrac{w_0}{s+a^2+w_0^2} \overset{\mathcal{L}'}{=} e^{-at}\sin(wt)$

iii. ↳ Therefore, $i(t) = 50e^{-0.6t}\sin(0.8t)$ ✓

3.

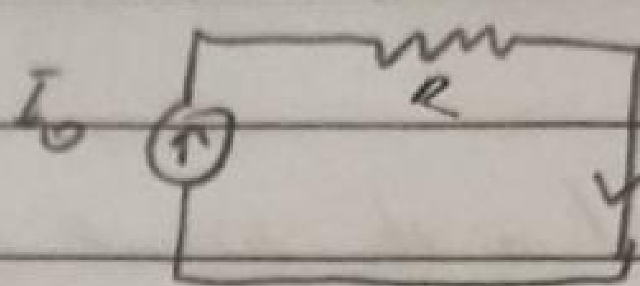


· Switch points to position b at t = 0

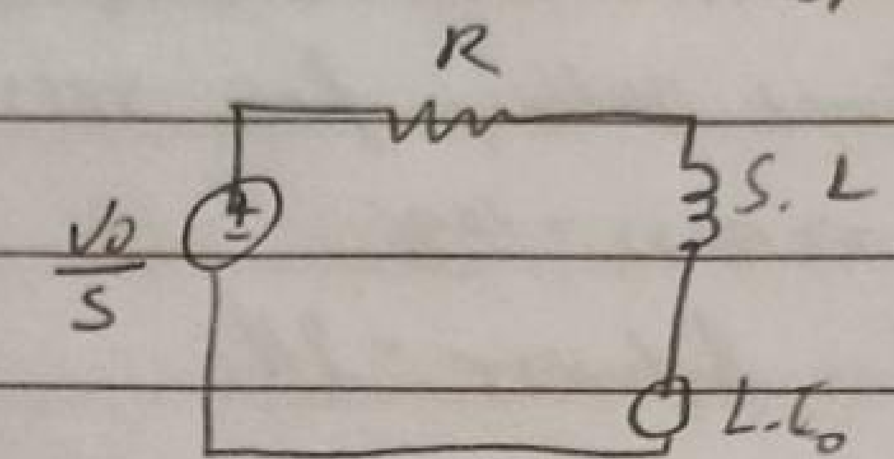
a) find $I(s)$

· Consider inductor are short

$i(0) = I_0$



$\hookrightarrow$ translate into s-domain



· KVL

$\hookrightarrow I(s)(R + SL) = LI_0 + \frac{V_0}{S}$

$\hookrightarrow I(s) = \frac{I_0 \cdot L}{R+SL} + \frac{V_0}{S(SL+R)}$

· By partial fraction

$\hookrightarrow I(s) = \frac{I_0 \cdot L}{S+R/L} + \frac{V_0}{S(S+R/L)}$

$\hookrightarrow \frac{V_0}{S(S+R/L)L} = \frac{A}{S} + \frac{B}{S+R/L}$

$\hookrightarrow \frac{V_0}{L} = A(S+R/L) + B \cdot S$, $S = -R/L$

$\hookrightarrow \frac{V_0}{L} = B(\frac{-R}{L}) + 0$

$\hookrightarrow B = -V_0/R$

$\cdot I(s) = \frac{I_0}{(S+R/L)} + \frac{V_0}{R}(\frac{1}{S}) - \frac{V_0}{R}(\frac{1}{S+R/L})$

$\mathcal{L}(s^{-1}) \hookrightarrow I_0 e^{-t R/L} + V_0/R - V_0/R \cdot e^{-t R/L}$

b. final value theorem of $I(s)$

$i(t) = \lim_{s\to 0} S(I(s))$

$\hookrightarrow \lim_{s\to 0} \left(\frac{S \cdot I_0}{S+R/L}\right) + \lim_{s\to 0} \frac{S \cdot V_0}{S(L)(S+R/L)}$

$\hookrightarrow i(t) = \frac{0 \cdot I_0}{0+R/L} + \frac{V_0}{L(0+R/L)}$

$\hookrightarrow i(\infty) = \frac{V_0}{R}$ ✓

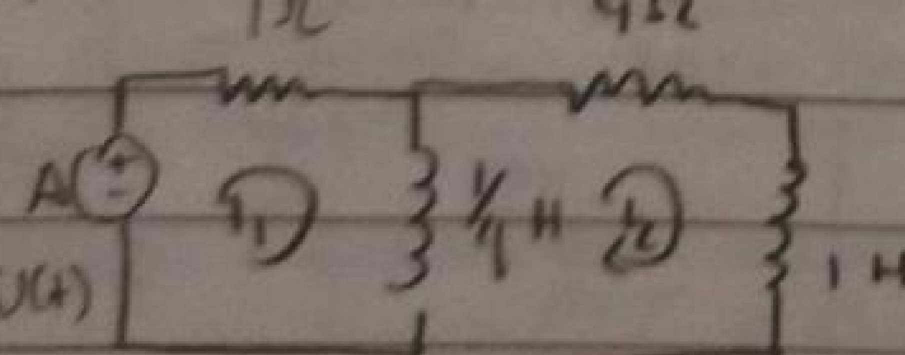
c. $\tau = L/R$

by application of part A

$\hookrightarrow i(t) = (I_0 - V_0/R)e^{-t/\tau} + \frac{V_0}{R}$ ✓

4. Find $I_2(s)$. Assume $A = 15u(t)$ V & zero initial current in the inductor

ii.



i. Convert Circuit into s-domain

$\hookrightarrow 15u(t) \to 15/s$

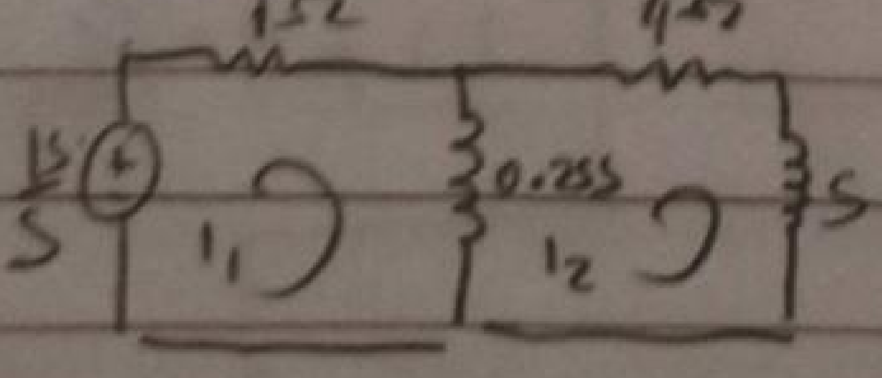
$I_1 \to I_1(s)$

$I_2 \to I_2(s)$

$1/4 H \to 1/4 \cdot S = 0.25S$

$1H \to 1 \cdot S = S$

transform $\hookrightarrow$



iii. · KVL I_1

$\hookrightarrow -\frac{15}{S} + 1(I_1(s)) + 0.25S[I_1(s) - I_2(s)] = 0$

$\hookrightarrow (1 + 0.25S)I_1(s) - 0.25S\, I_2(s) = 15/s$

· KVL I_2

$\hookrightarrow 4(I_2(s)) + S \cdot I_2(s) + 0.25S[I_2(s) - I_1(s)]$

$\hookrightarrow (4 + 1.25S)I_2(s) = 0.25S\, I_1(s)$

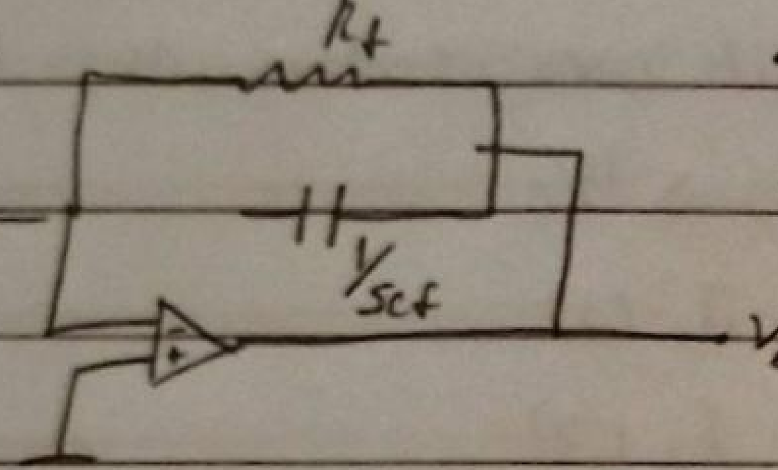
$\hookrightarrow$ Thus, $I_1(s) = \left(\frac{4+1.25S}{0.25S}\right) I_2(s)$

iv. from Eq 1 & Eq 2

$\hookrightarrow (1+0.25S)\left(\frac{4+1.25S}{0.25S}\right)I_2(s) - 0.25S\, I_2(s) = 15/s$

$\hookrightarrow I_2(s) = \frac{12}{S^2 + 7.2S + 12.8}$ A ✓

Problem No. 5

i. 

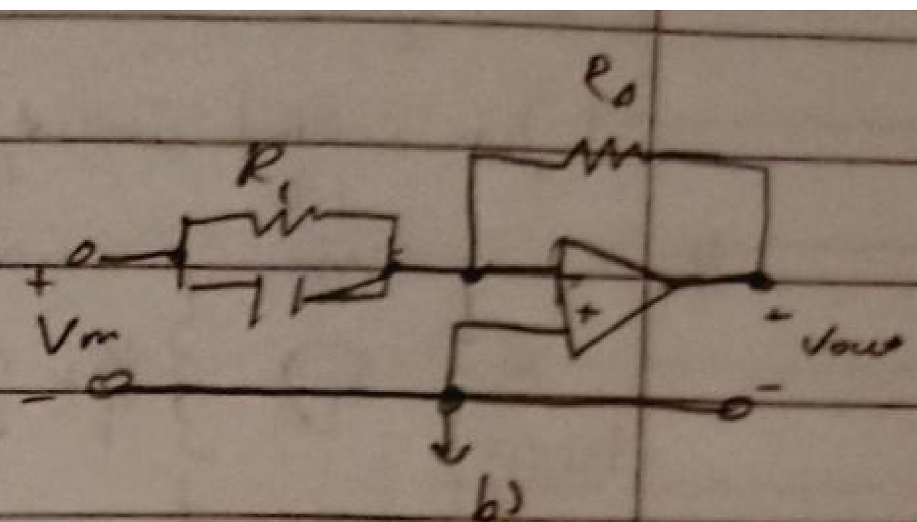
• $C \Rightarrow 1/Cs$

• $V_+ = V_- = 0V$

↳ $R_1 = V_i / R_i$

↳ KVL: $V_o = -I \cdot Z_{eq}$

$$V_o = V_i / R_i \times \frac{R_f \cdot 1/sC_f}{R_f + 1/sC_f} \; ; \; V_o = \frac{-V_i}{R_1} \times \frac{R_f}{1 + sC_f R_f} \checkmark$$

5b. 

i ↳ $I = V_i / Z_{eq} = \dfrac{V_i}{\dfrac{R_1 \cdot 1/sC_1}{R_1 + 1/sC_1}}$

ii. ↳ $\dfrac{V_i / R_1}{sR_1 C_1 + 1} = \dfrac{V_i (1 + sC_1 R_1)}{R_1}$

↳ $V_o = -I R_f = \dfrac{V_i}{R_1} (1 + sC_1 R_1) R_f$

Thus,

$$V_{out} = -V_i \frac{R_f}{R_1} (1 + sC_1 R_1) \checkmark$$