ENGR 065 Circuit Theory

Laboratory 10

Transient Response of RLC Circuits

Objectives

- Observe the transient responses of RLC circuits with SPICE simulator.
- Apply Laplace transform to predict behavior of RLC circuits.
- Learn how R, L, and C affect the behavior of RLC circuits.

Equipment and components

- A computer
- SPICE software

Preliminary Work

- Review what you have learned in the previous labs about online SPICE simulator. We will use this tool to analyze the behavior of RLC circuits.
- Review Lectures 21 (RLC Circuit), 23 and 24 (Laplace Transform and its application to RLC Circuit) . Review also Chapters 8.3 (Source-Free Series RLC), 8.5 (Step Response of a Series RLC Circuit) and 15 from the textbook [1].

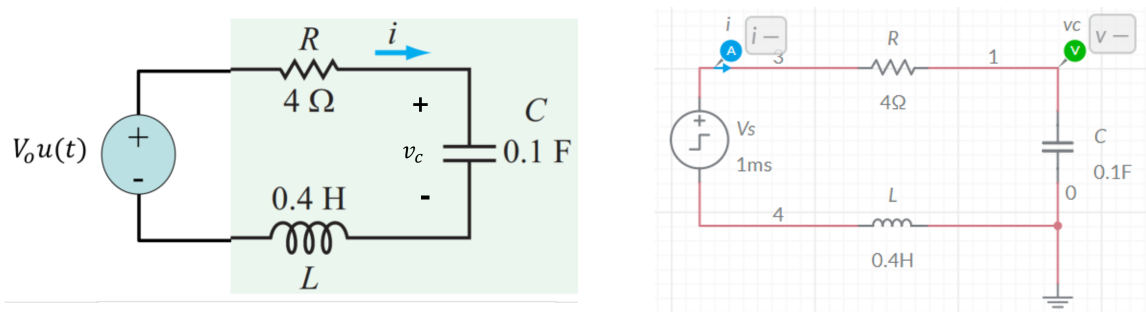
Procedure

Part A

Consider the following RLC circuit, composed of a resistor $R = 4\Omega$ in series with a

- capacitor $C = 0.1F$,
- inductor $L = 0.4H$ and
- step voltage source $v_s(t) = V_o u(t)$, with $V_0 = 1.6V$.

For $t < 0^-$, the capacitor voltage (v_c) is $0V$ and the inductor current (i) is 0A.



a) Circuit diagram

b) SPICE implementation

Figure 1: Series RLC circuit.

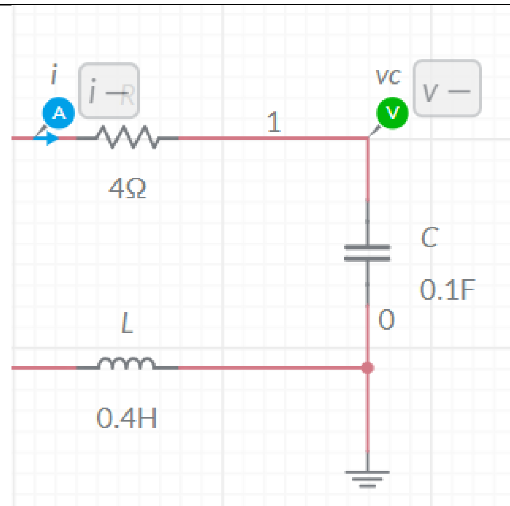
1. Complete the following Table for the RLC parameterization presented in Figure 1:

	Case I: R=4Ω, $C = 0.1F$
Laplace transform of the inductor current: $I(s) = \mathcal{L}\{i(t)\}$	
Inductor current(time domain): $i(t)$ Note: $t > 0^-$	
Neper Frequency (α)	
Resonant Frequency (ω_0)	
Poles* of $I(s)$	
Type of Poles (Real, Complex or Purely Imaginary?)	

* Suppose $I(s) = N(s)/D(s)$; the poles of $I(s)$ are the roots of $D(s) = 0$ (check Chap 15.4 of [1] for details)

2. Build the above RLC circuit in the SPICE simulator. Consider the following instructions:

<table>
<tr>
<td>
First, insert the

- resistor,
- inductor
- capacitor
- ground
- voltage probe (for the capacitor voltage $v_c(t)$)
- current probe (for the circuit current)
</td>
<td>


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<tr>
<td>
Next, insert the step voltage source.

This source is available in the “sources library” (see figure on the right)
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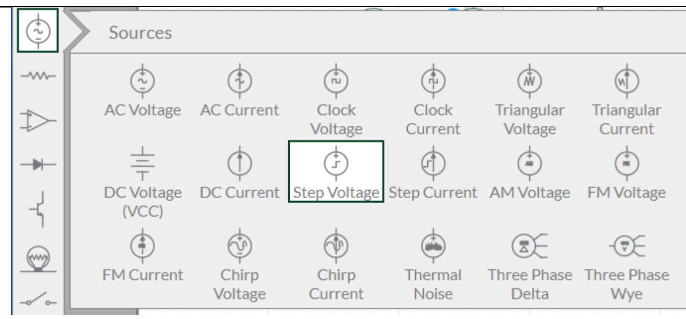
<td>


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<td>
Change the amplitude of the step voltage to 1.6V
</td>
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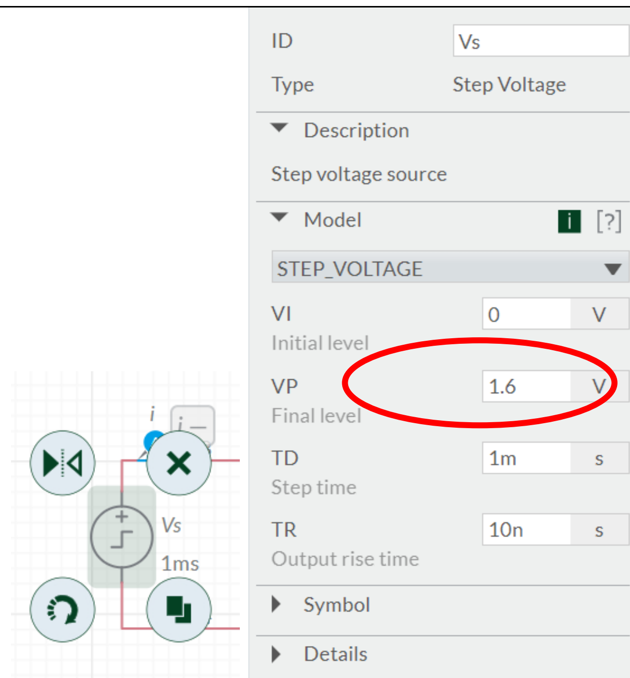

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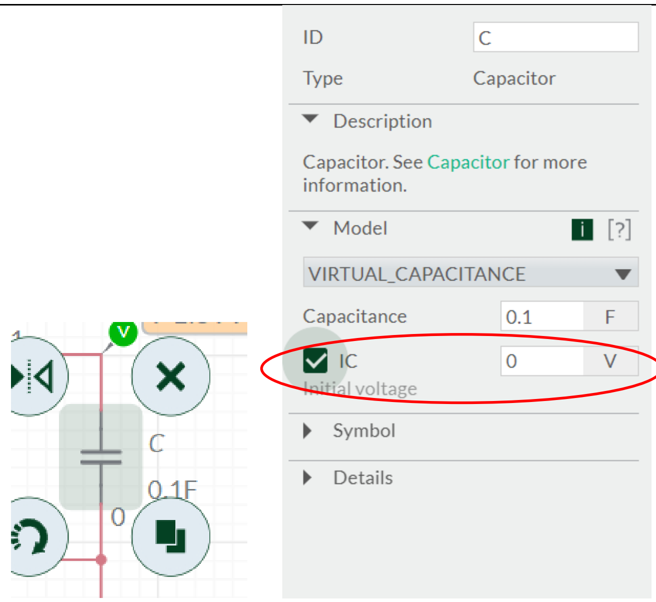
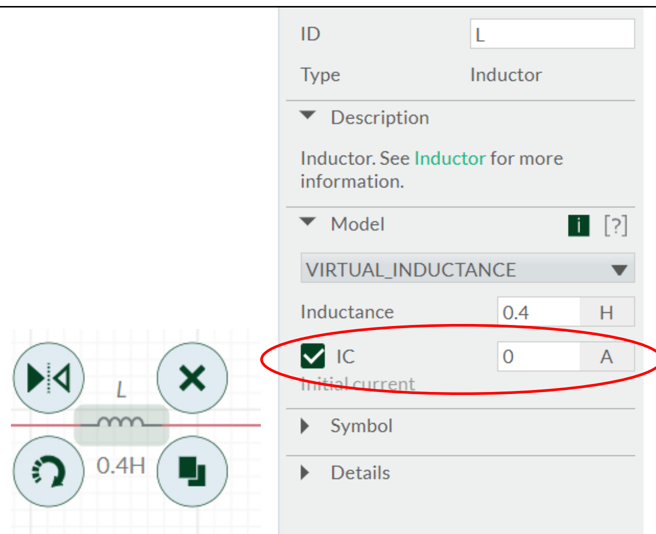
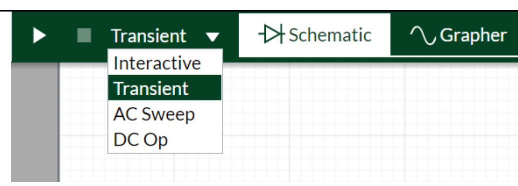
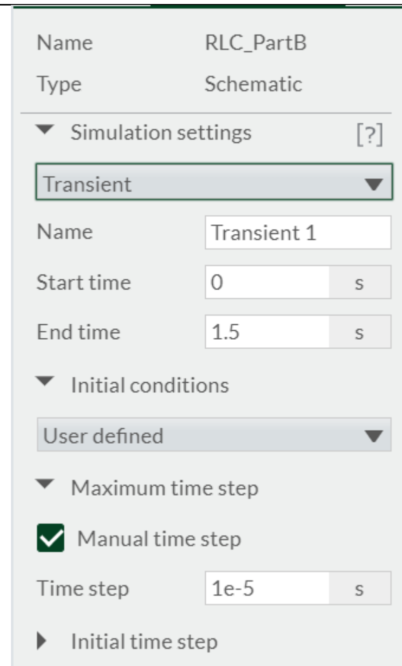
</table>

Select the capacitor. Make sure the initial voltage is 0V by marking the IC Field of the Capacitor Model (see figure on the right)	
Select the inductor. Make sure the initial current is $0A$ by marking the IC Field of the Inductor Model (see figure on the right)	
Select Transient simulation	

On the right side of your window, select the option "Document" Use the following settings ○ End Time =1.5s ○ Initial conditions = ***User defined*** ○ Maximum time step/ Manual time step =$1e-5$s	
Run the simulation and inspect the graphical results available in the window "Grapher"	

3. Use the SPICE simulator to complete the following table

	Variable	**Case I: R=4Ω** $\boldsymbol{C = 0.1F}$
1	maximum value of inductor current $i(t)$	
2	maximum energy that the inductor stores	
3	maximum value of the capacitor voltage $v_c(t)$	
4	maximum energy that the capacitor stores	

4. Discuss what happens to $i(t)$ and $v_c(t)$ as $t \to \infty$. (Hint: recall that a capacitor behaves as an open circuit in the presence of a constant terminal voltage)

Part B

In the second part, you will analyze the behavior of the RLC circuit when the values of R and C are modified.

5. Complete the following table for the new values of R and C:

	Case II	Case III
Resistance R	**0 Ω**	**0 Ω**
Capacitance C	$\mathbf{0.1}\boldsymbol{F}$	$\mathbf{0.01}\boldsymbol{F}$
Laplace transform of the inductor current: $I(s) = \mathcal{L}\{i(t)\}$		
Inductor current, time domain: $i(t)$	(*)	
Neper Frequency (α)		
Resonant Frequency (ω_0)		
Poles of $I(s)$		
Type of Poles (Real, Complex or Purely Imaginary?)		

(*) there are multiple ways of computing the inverse Laplace Transform for case II. One simple approach is to use the attached Laplace table. Another option is to employ the partial fraction expansion technique (Chapter 15.4 from [1]).

6. Simulate the circuit with the new values of R and C. Complete the following table.
   - Note: for $R = 0\Omega$ you can remove the resistance from the circuit or use a very small resistance (e.g. $1n\Omega$)

	Case II	Case III
Resistance R	**0 Ω**	**0 Ω**
Capacitance C	$\mathbf{0.1}\boldsymbol{F}$	$\mathbf{0.01}\boldsymbol{F}$
maximum value of inductor current $i(t)$		
maximum energy that the inductor stores		
maximum value of the capacitor voltage $v_c(t)$		
maximum energy that the capacitor stores		

7. Check the validity of the time-domain current formula $i(t)$ by comparing the theoretical values (obtained through the inverse Laplace transform) and the simulated values (obtained from the SPICE simulator) when $t = 0.5s$. Complete the following table.

	Case I	Case II	Case III
Resistance R	4Ω	$0\ \Omega$	$0\ \Omega$
Capacitance C	$0.1F$	$0.1F$	$0.01F$
Theoretical value for $i(t = 0.5s)$			
Simulated value for $i(t = 0.5s)$			

8. For cases I,II, and III, discuss
   - what happens to $i(t)$ and $v_c(t)$ as $t \to \infty$
   - what type of poles lead to oscillations in $i(t)$ and $v_c(t)$?

9. For Case II and III, explain
   - what happens to $i(t)$ and $v_c(t)$ when C is decreased from 0.1F to 0.01F
   - the role of the resonant frequency ω_0 in the circuit's oscillations
   - what happens to the maximum amount of energy that the capacitor is able to store when C is decreased from 0.1F to 0.01F

10. Consider the case $R = 0\Omega$. Determine the capacitance value C that produces oscillations in $i(t)$ with a frequency of $10Hz$. Check your results using the SPICE simulator

Questions and conclusions

- Summarize your findings in the lab report.

References

[1] [1] C. Alexander and M. Sadiku "Fundamentals of Electric Circuits", 7th Edition, 2021, McGraw-Hill

Appendix: Laplace transforms and properties

Type	$f(t)\ (t > 0^-)$	$F(s)$
(impulse)	$\delta(t)$	1
(step)	$u(t)$	$\frac{1}{s}$
(ramp)	t	$\frac{1}{s^2}$
(exponential)	e^{-at}	$\frac{1}{s+a}$
(sine)	$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$
(cosine)	$\cos \omega t$	$\frac{s}{s^2+\omega^2}$
(damped ramp)	te^{-at}	$\frac{1}{(s+a)^2}$
(damped sine)	$e^{-at}\sin \omega t$	$\frac{\omega}{(s+a)^2+\omega^2}$
(damped cosine)	$e^{-at}\cos \omega t$	$\frac{s+a}{(s+a)^2+\omega^2}$
(damped sine, 2nd formulation)	$\frac{\omega_n}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \sin \omega_n \sqrt{1-\zeta^2}\, t$	$\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$