Engr 063

Homework No. 9

a) $v(t) = 8e^{-5t}$, $t > 0^-$

↳ refer to Laplace table

$$8e^{-5t}u(t) \xrightarrow{\mathcal{L}\{\cdot\}} \frac{8}{s+5} \checkmark$$

b) $v(t) = (10e^{-1000t} - 5)\,u(t)$

refer to Laplace ↳ $10e^{-1000t}u(t) \Longrightarrow \frac{10}{s+1000} - \frac{5}{s}$

↳ partial expansion $= \frac{10s - 5(s+1000)}{s(s+1000)}$

↳ $\frac{5s - 5000}{s(s+1000)}$ ✓

c. $v(t) = e^{-2t} + 4t - u(t)$, $t > 0^-$

↳ $\mathcal{L}\{\cdot\} \rightarrow \frac{1}{s+2} + \frac{4}{s^2} - \frac{1}{s}$ ✓

d. $v(t) = 2u(t) + 2\sin(2t) - 2\cos(2t)$, $t > 0^-$

↳ $2(u(t) + \sin(2t) - 2\cos(2t) \xleftarrow{\mathcal{L}\{\cdot\}} 2\left(\frac{1}{s} + \frac{2}{s^2+4} - \frac{s}{s^2+4}\right)$

↳ $2\left(\frac{1}{s} + \frac{2}{s^2+4} - \frac{s}{s^2+4}\right) = \frac{s^2 + 4 + 2s - s^2}{s(s^2+4)} = \left(\frac{4 + 2s}{s(s^2+4)}\right)2$

↳ $\frac{4(2+s)}{s(s^2+4)}$ ✓

2. Laplace transformation $v_1(t) = V_A e^{-\alpha t}$; v_a & α are positive $= V_1(s) = \frac{V_a}{s+a}$

Laplace trans. $\int_{0^-}^{t} V_A e^{-\alpha x}\,dx \xLeftarrow{\mathcal{L}\{\cdot\}} V_A \int_{0^-}^{t} e^{-\alpha x}$

table ↳ $V_a\left(\frac{1}{s+\alpha}\right)$ ✓

2b. $i(t) = 30e^{-1200t}\,u(t)$

↳ $30e^{-1200t} \xrightarrow{\mathcal{L}\{\cdot\}} 30(\mathcal{L}(e^{-1200t})\,u(t)$

↳ $30\left(\frac{1}{s+1200}\right)$

ii. Given $v(t) = 0.1\,di(t)/dt \xrightarrow{\mathcal{L}\{\cdot\}} 0.1\left[\frac{di(t)}{dt}\right]$; result from table $\mathcal{L}(t) = 1$

↳ $V(s) = 0.1sI(s)$; $I_s = \frac{30}{s+1200}$

↳ Thus, $V(s) = \frac{3s}{s+1200}$ ✓

2c. $\frac{d^2}{dt^2}v(t) + 4\frac{dv(t)}{dt} + 3v(t) = 5e^{-2t}$, $v(0^-) = -2V$, $\frac{dv}{dt} = 2\,V/s$

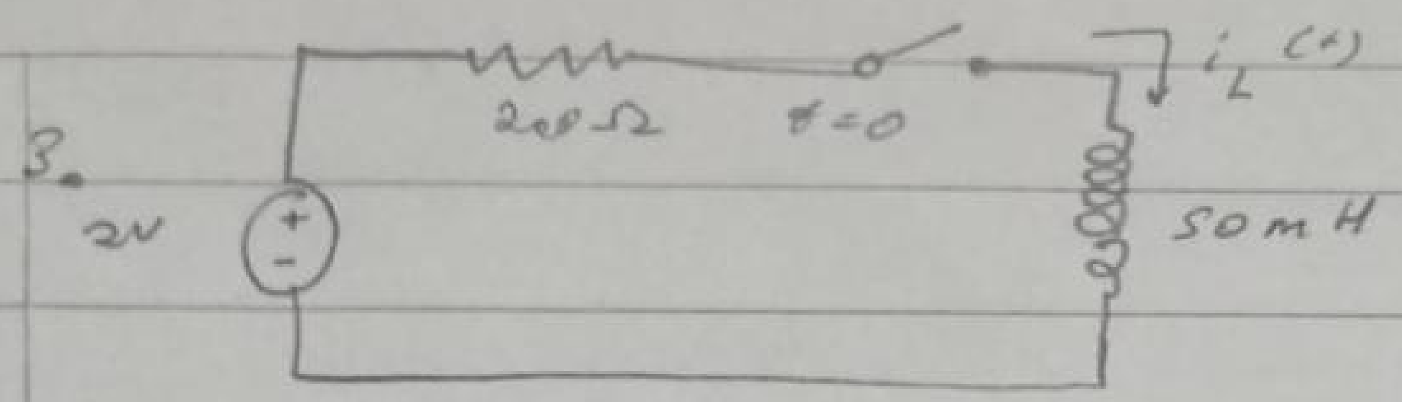
↳ $\mathcal{L}\left(\frac{d^2}{dt^2}v(t) + 4\frac{dv(t)}{dt} + 3v(t) = 5\mathcal{L}[e^{-2t}]\right.$

quadratic ↳ $s^2V(s) - sV(0) - v'(0) + 4[sV(s) - v(0^+)] + 3V_s = \frac{5}{s+2}$

expansion ↳ $(s^2 + 4s + 3)V(s) + 2s - 2 + 8 = \frac{5}{s+2}$

↳ $V(s) = \frac{5}{(s+2)(s+1)(s+3)} - \frac{2(s+3)}{(s+1)(s+3)}$ ✓

3.

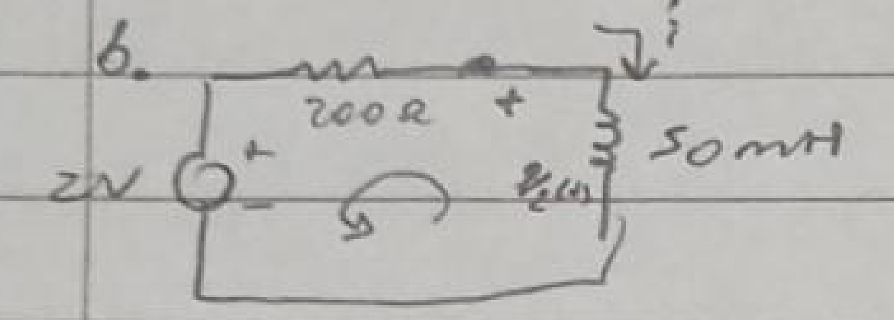


a. find $i_L(0)$

Given that $t=0$ circuit is open

thus $i_L(0) = 0$ Amp

b.



↳ KVL :

$2 - 200\, i_L(t) - V_L(t) = 0$

$\hookrightarrow 2 - 200 i_L - L \dfrac{di(t)}{dt} = 0$

$\hookrightarrow 0.05 \dfrac{di(t)}{dt} - 200 i_L = 2$

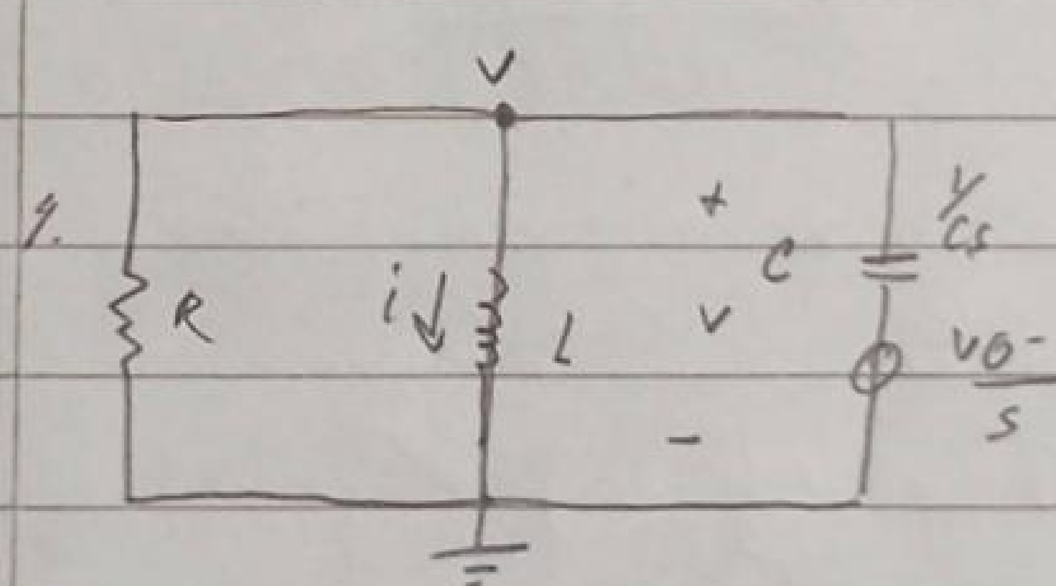
$\hookrightarrow \dfrac{di_L(t)}{dt} - 4000 i_L = 40$ ✓

2c. S-domain recall $[S I_L(s) - I_L(0)$ - lecture

$\hookrightarrow [S I_L(s) - I_L(0)] + 400 i_L(s) - 40 = 0$

$\hookrightarrow S I_L(s) + 400 I_L(s) = 40 \; ; \; i_L(s) = \dfrac{40}{s+400}$ ✓

4.



Given $i_0^- = a$ $v_0^- = b$

a. KCL @ A : $\dfrac{v(s)}{R} + \dfrac{v(s) + La}{Ls} + \dfrac{v(s) - b/s}{(1/Cs)} = 0$

Simplify $\hookrightarrow vs\left(\dfrac{1}{R} + \dfrac{1}{Ls} + Cs\right) = \dfrac{-La}{La} + bC$

LCD $\hookrightarrow vs\left(\dfrac{Ls + R + LRCs^2}{RLS}\right) = \dfrac{bLCS - La}{Ls}$

$\hookrightarrow vs = \dfrac{bLCS - La}{\cancel{Ls}} \cdot \dfrac{R\cancel{Ls}}{Ls + R + LRCs^2} = \dfrac{R(bLCs - La)}{Ls + R + LRCs^2}$ ✓

b. Given $R \to \infty$; $b = 0$

Derive time-domain expression

$\hookrightarrow v(s) = \dfrac{-La}{LCs^2 + 1 + Ls/R} = \dfrac{-La}{LCs^2 + 1}$

$\hookrightarrow V(s) = \dfrac{-a/C}{s^2 + \frac{1}{\sqrt{LC}}^2)}$

$\hookrightarrow v(t) = -(a/C)\sqrt{LC}\,\sin(1/LC)\; ; \; t \geq 0$ ✓