University of California Merced
School of Engineering
Department of Electrical Engineering

ENGR 065 Circuit Theory

Lab 10: Transient response of RLC Circuit

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Section

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Objective

The objective of this lab is to further dive into the concept and application of how resistors, inductors and capacitors all intertwine and influence the behavior of an RLC circuit. The behavior of such circuits will be analyzed using Laplace transformation, and for means of application, SPICE simulator, will be employed to study the technicality of this topic. Theoretical analysis will utilize laplace and inverse laplace transformation, while simulated value using software will allow for the competence of the two values to establish correlation which indicate validity in terms of both theory and application.

Introduction

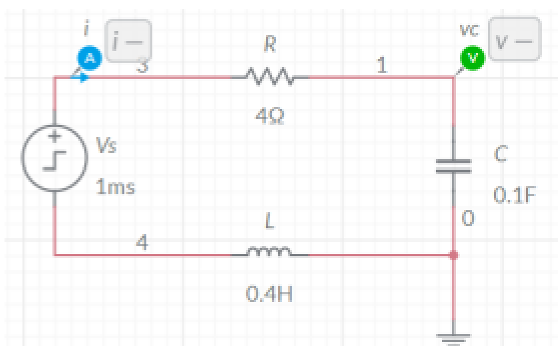
A deeper dive into the topic of RLC circuits are make-models consisting of resistors, inductors and capacitors - these components all interconnect are prescribed to facilitate the accumulation of current within an element and equally, or through weighted distribution skew the currents to the rest of the circuit. In theory, this form of circuit is known as the second-order circuit, meaning a second-order differential equation may be applied to deduce a desired behavioral value within the circuit. Consider that the RLC components are linked in terms of parallel or in series; therefore, by presentation of topology, the model of circuit can denote empirical laws which must be considered when observing an RLC circuit. Note that an RLC circuit connected in series, are the current running through the resistors, inductor and capacitors are the same, if voltage of resistor is within the same phase and the current, then voltage is in the same phase as I, and we know Vs is the vector sum of (Vl - Vc)Vr = Vs. Furthermore, by means of theoretical analysis, consider given an circuit within an s-domain, we must convert it into the time domain - first by integration, then by means of Laplace, then through minute computation of assignment variables, the equation can be rewritten and with application of inverse Lapalce, we will be able to find the induction current within the time domain which will be shown within this lab.

Procedure

The circuit model assigned to this lab is a simple RLC circuit consisting of one step voltage source, one resistor, capacitor and inductor. By using SPICE, start by dragging into the workspace the needed component. Recall the shape of a circuit ; thus, first the voltage source will be on the right side, the resistor will be on top, the capacitor will be adjacent to the step voltage while the inductor will be parallel with the resistor. Now, using connection wire, form atop the step voltage go up and fold which will connect and run through the resistor and fold, once more, through the capacitor and into the ground. From the ground, the connection wire will fold and run through the induction and back into the step voltage. Then the first procedure is by means of Laplace transformation theorem and basic circuit analysis formula, to find the s-domain transformation into the time domain, the inductor value, the Neper frequency, the resonant frequency, and the induction current of the poles within the S-domain. The second circuit model for this lab relies off the first one, but consider that the step voltage to be changed to 1.6 V, changing the IC field for the capacitor of the simulation to 0V, make the IC field for the inductor 0 A, changing the setting to transient and setting the ending time to 1.5S, the initial condition as defined by user, and the maximum time step is 1e^-5s.

<u>Data and Model Presentation</u>

<u>Part A:</u>



	Case I: R=4Ω, $C = 0.1F$
Laplace transform of the inductor current: $I(s) = \mathcal{L}\{i(t)\}$	(4/(S^2 +10S +25) = 4te^-5t
Inductor current(time domain): $i(t)$ Note: $t > 0^-$	4Tw^-5t
Neper Frequency (α)	5
Resonant Frequency (ω_0)	5
Poles* of $I(s)$	-5,5
Type of Poles (Real, Complex or Purely Imaginary?)	The pole of this circuit is real

Spice Simulation:

	Variable	Case I: R=4Ω $C = 0.1F$
1	maximum value of inductor current $i(t)$	294.3 mA
2	maximum energy that the inductor stores	0.017
3	maximum value of the capacitor voltage $v_c(t)$	1.592 V
4	maximum energy that the capacitor stores	0.128

Part B:

	Case II	Case III
Resistance R	0 Ω	0 Ω
Capacitance C	$0.1F$	$0.01F$
Laplace transform of the inductor current: $I(s) = \mathcal{L}\{i(t)\}$	4/(S^2 +25)	4/(S^2 + 250)
Inductor current, time domain: $i(t)$	(*) 4/sin(5t)	4/srt(250)(sin(sqrt(250)t))
Neper Frequency (α)	0	0
Resonant Frequency (ω_0)	5	sqrt(250)
Poles of $I(s)$	-5,5	+,- sqrt(250)
Type of Poles (Real, Complex or Purely Imaginary?)	Complex	Purely imaginary

Part C:

	Case II	Case III
Resistance R	**0 Ω**	**0 Ω**
Capacitance C	$\mathbf{0.1}\boldsymbol{F}$	$\mathbf{0.01}\boldsymbol{F}$
maximum value of inductor current $i(t)$	800	252.9
maximum energy that the inductor stores	0.017	0.017
maximum value of the capacitor voltage $v_c(t)$	3.32	3.23
maximum energy that the capacitor stores	0.126	0.0126

Part D:

	Case I	Case II	Case III
Resistance R	**4Ω**	**0 Ω**	**0 Ω**
Capacitance C	$\mathbf{0.1}\boldsymbol{F}$	$\mathbf{0.1}\boldsymbol{F}$	$\mathbf{0.01}\boldsymbol{F}$
Theoretical value for $i(t = 0.5s)$	0.164 A	0.410 A	0.227 A
Simulated value for $i(t = 0.5s)$	165.89 mA	459.78 mA	254.7 ma

Data Discussion

In order to compute for the Laplace time domain convert within the first table, consider applying the formula I(o) = (V0/L) / (s^2 + R/Ls + I/LC), and given the circuit's input being that Vo is 1.6, R is 4 , L = 0.4h, C is 0.1 F we can use to substitute it into the equation which will derived the the following output, I = ((1.6)/0.4))/s^2+(4/0.4)s+(1/0.004) = 4/(s^2+10s+25) - then, utilizing laplace transform identity, the final result should be 4te^-5t. Next, by simplification through means of algebra, we will be able to derive the poles through condensing the denominator of the transformed function and solve for X, yielding -5 and 5. Since the poles output is not infinity and no-imaginary number, the poles are evidently complex numbers. Part B table shows the values recorded at each element in order to compare with the case one that we have theoretically solved for. After plugin in the values for case one, both the simulation and theoretical data are in close proximity indicating credibility. The second index of the lab is given the resistance and capacitance, the goal is to again determine the data required as per instruction within the first procure. Hence, given at R is 0, L is 0.4, C is 0.1, we can apply such into an equation and applying laplace transform will reap I(t) = 4/5 sin(5t) and consider that in this case alpha is R/2L which is 0 radian, the poles can be computed via p = alpha + jB = t,-5 which indicated that the poles in case two are also -,+5, a complex number. For case three, The same procedure will be applied which outputs i(t) = 4/sqrt(250)(sin(sqrt(250)t)) and the poles as =,- sqrt(250), a purely imaginary number. Lastly, using SPICE, the model and adjustment mentioned within the procedure were made and the results were as follows. By plugging in the value given for the resistance, inductor and capacitor; the value for data in regard to the theoretical and simulation are aligned which concludes that the objective of this lab has suffice - laplace transformation analysis of RLC circuit has been proof through the lab's work.

Conclusion

Conclusively, the analysis of the circuit behavior for RLC circuit model within this lab comprises both application via simulation model construction using SPICE, and the theoretical analysis using mathematical properties to break down the circuit and transform its state from one domain to another has been sufficient. By means of applying RLC circuit behavior theorem; ergo, recalling that domain transformation for capacitors are cs into 1/cs and by taking the KVl of the circuit we are able to derived two equation which we can set to reference with the given resistor and capacities, we are asked to compute for the inductance which is assigned the formula I(o) = (V0/L) / (s^2 + R/Ls + I/LC). By application of this formula, the equation can further be simplified and by means of laplace transformation and laplace inverse properties, we are able to find the desired value for the induction current for the converted domain state. This procedure was repeated recursively through many cases with various values of capacitance and resistor which served as the theoretical component to this lab. By means of spice, we are able to compare our data to the simulated data for the circuit from SPICE. Evidently, the result from the two data are comparably aligned to each other which shows that whether by means of theoretical analysis or actual application, the theorem of RLC is proofed.