ENGR 065: Circuit Theory

Problem Set #9

Read Chapter 15 from [1] and then solve the following problems.

Note: you can use the Laplace tables (included in Appendix) to solve these problems.

Problem 1: Find the Laplace transform of the following signals:

a) $v(t) = 8e^{-5t},\ t > 0^-$
b) $v(t) = (10e^{-1000t} - 5)u(t)$
c) $v(t) = e^{-2t} + 4t - u(t), t > 0^-$
d) $v(t) = 2u(t) + 2\sin(2t) - 2\cos(2t)\,, t > 0^-$

Recall: $u(t)$ is a step function.

Problem 2

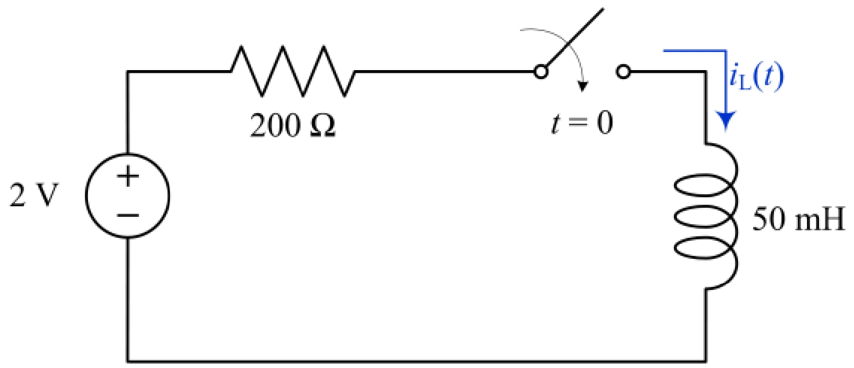
a**)** Let $v_1(t) = V_A e^{-\alpha t}u(t)$, where V_A and α are positive constants. Compute the Laplace transform of $\int_{0^-}^{t} V_A e^{-\alpha x}dx$

b) If $i(t) = 30e^{-1200t}u(t)\ mA,$ find the Laplace transform of $v(t) = 0.1\frac{di(t)}{dt}$ [V]

c) Compute the Laplace transform of $v(t)$, when

$$\frac{d^2}{dt^2}v(t) + 4\frac{dv(t)}{dt} + 3v(t) = 5e^{-2t},\ \ v(0^-) = -2\ V,\quad \frac{dv}{dt}(0) = 2\ \ V/s$$

Problem 3: The switch has been open for a long time. At $t = 0$ the switch is closed.



a) Determine the initial current $i_L(0)$.

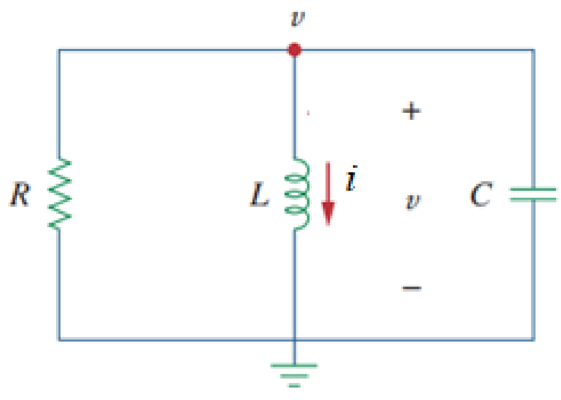
b) Find the differential equation for $i_L(t)$

c) Transform the equation into s domain and obtain $I_L(s)$.

Problem 4: Consider the following RLC circuit.

a) Compute the Laplace transform of the capacitor voltage v. Assume the following initial conditions $i(0^{-}) = a,\ v(0^{-}) = b$.

b) Determine the time-domain expression for the capacitor voltage $v(t), t \geq 0$. Assume a very large resistance $R = \infty$ and $b = 0$.



Appendix: Laplace transforms and properties

Type	$f(t)\ (t > 0^-)$	$F(s)$
(impulse)	$\delta(t)$	1
(step)	$u(t)$	$\frac{1}{s}$
(ramp)	t	$\frac{1}{s^2}$
(exponential)	e^{-at}	$\frac{1}{s+a}$
(sine)	$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$
(cosine)	$\cos \omega t$	$\frac{s}{s^2+\omega^2}$
(damped ramp)	te^{-at}	$\frac{1}{(s+a)^2}$
(damped sine)	$e^{-at}\sin \omega t$	$\frac{\omega}{(s+a)^2+\omega^2}$
(damped cosine)	$e^{-at}\cos \omega t$	$\frac{s+a}{(s+a)^2+\omega^2}$

Operation	$f(t)$	$F(s)$
Multiplication by a constant	$Kf(t)$	$KF(s)$
Addition/subtraction	$f_1(t) + f_2(t) - f_3(t) + \cdots$	$F_1(s) + F_2(s) - F_3(s) + \cdots$
First derivative (time)	$\frac{df(t)}{dt}$	$sF(s) - f(0^-)$
Second derivative (time)	$\frac{d^2f(t)}{dt^2}$	$s^2F(s) - sf(0^-) - \frac{df(0^-)}{dt}$
nth derivative (time)	$\frac{d^nf(t)}{dt^n}$	$s^nF(s) - s^{n-1}f(0^-) - s^{n-2}\frac{df(0^-)}{dt} - s^{n-3}\frac{df^2(0^-)}{dt^2} - \cdots - \frac{d^{n-1}f(0^-)}{dt^{n-1}}$
Time integral	$\int_0^t f(x)\,dx$	$\frac{F(s)}{s}$
Translation in time	$f(t-a)u(t-a), a > 0$	$e^{-as}F(s)$
Translation in frequency	$e^{-at}f(t)$	$F(s+a)$
Scale changing	$f(at), a > 0$	$\frac{1}{a}F\left(\frac{s}{a}\right)$
First derivative (s)	$tf(t)$	$-\frac{dF(s)}{ds}$
nth derivative (s)	$t^nf(t)$	$(-1)^n\frac{d^nF(s)}{ds^n}$
s integral	$\frac{f(t)}{t}$	$\int_s^\infty F(u)\,du$

References

[1] C. Alexander and M. Sadiku "Fundamentals of Electric Circuits", 7th Edition, 2021, McGraw-Hill