Algoritmi recursivi

Friday, 5 November 2021 10.02

$$T(n) = cost\_recursiv(n) + cost\_nerecursiv(n)$$

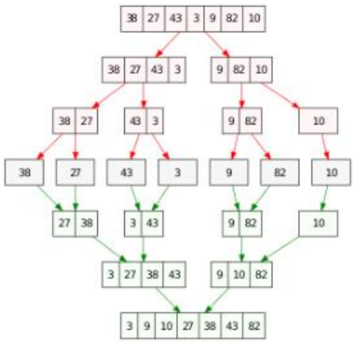
Merge sort

$$T(n) = 2T\left(\frac{n}{2}\right) + n$$

Algorithm 1 Pseudocod mergesort

```
function MERGE(V1[1..n], V2[1..m])
    Let R[1..n + m]
    i = 1, j = 1
    while i < n and j < m do
        if V1[i] < V2[j] then R[i + j] = V1[i + +]
        else R[i + j] = V2[j + +]
        end if
    end while
    while i < n do
        R[i + j] = V1[i + +]
    end while
    while j < m do
        R[i + j] = V2[j + +]
    end while
end function
```

```
function MERGESORT(V[1..n], start, end)
    if start ≥ end then return
    end if
    middle = (start + end)/2
    MergeSort(V, start, middle)
    MergeSort(V, middle, end)
    return Merge(V[start..middle], V[middle..end])
end function
```



Metoda iteratiei - merge sort (intuitie)

Friday, 5 November 2021 10.38

$T(n) = 2T\left(\frac{n}{2}\right) + n$

$T\left(\frac{n}{2}\right) = 2T\left(\frac{n}{4}\right) + \frac{n}{2}$

$T\left(\frac{n}{4}\right) = 2T\left(\frac{n}{8}\right) + \frac{n}{4}$

$\vdots$

$T\left(\frac{n}{2^k}\right) = 2T\left(\frac{n}{2^{k+1}}\right) + \frac{n}{2^k}$

$\vdots$

$T\left(\frac{n}{2^h}\right) = O(1) = 1$

caz de baza $T(1) \Rightarrow \frac{n}{2^h} = 1$

$2^h = n \Rightarrow h = \log n$

$T(n) = 2T\left(\frac{n}{2}\right) + n =$

$= 2\left[2T\left(\frac{n}{4}\right) + \frac{n}{2}\right] + n$

$= 4T\left(\frac{n}{4}\right) + 2n =$

$= 4\left[2T\left(\frac{n}{8}\right) + \frac{n}{4}\right] + 2n = 8T\left(\frac{n}{8}\right) + 3n$

$\vdots$

$= 2^k T\left(\frac{n}{2^k}\right) + k \cdot n$

$\vdots$

$= 2^h T(1) + hn = 2^{\log n} + \log n \cdot n =$

$= \log n + n \log n = O(n \log n)$

Metoda iteratiei (mai usor)

Friday, 5 November 2021 10.46

$$T(n) = 2\cancel{T\left(\frac{n}{2}\right)} + n$$

$$\cancel{2T\left(\frac{n}{2}\right)} = 2\left[2T\left(\frac{n}{4}\right) + \frac{n}{2}\right] = \cancel{4T\left(\frac{n}{4}\right)} + n$$

$$\cancel{4T\left(\frac{n}{4}\right)} = 8\cancel{T\left(\frac{n}{8}\right)} + n$$

$$8\cancel{T\left(\frac{n}{8}\right)} = 16\cancel{T\left(\frac{n}{16}\right)} + n$$

$$\vdots$$

$$\cancel{2^{k-1}T\left(\frac{n}{2^{k-1}}\right)} = 2^{k} T\left(\frac{n}{2^k}\right) + n$$

$$+$$

$$T(n) = k \cdot n + 2^k T\left(\frac{n}{2^k}\right) = \log n \cdot n + 2^{\log n} \cdot 1 = \Theta(n \log n)$$

$$k = \log n \Rightarrow T\left(\frac{n}{2^k}\right) = T(1) = 1$$

Ex

Friday, 5 November 2021 10.52

$T(n) = 2\cancel{T\left(\frac{n}{3}\right)} + \log n$

$2\cancel{T\left(\frac{n}{3}\right)} = 4\cancel{T\left(\frac{n}{9}\right)} + 2\log\frac{n}{3}$

$4\cancel{T\left(\frac{n}{9}\right)} = 8\cancel{T\left(\frac{n}{27}\right)} + 4\log\frac{n}{9}$

$\vdots$

$2^{k-1}\cancel{T\left(\frac{n}{3^{k-1}}\right)} = 2^{k}T\left(\frac{n}{3^k}\right) + 2^{k-1}\log\frac{n}{3^{k-1}}$ +

Caz de bază:

$\frac{n}{3^k} = 1 \Rightarrow k = \log_3 n$

$T\left(\frac{n}{3^k}\right) = T(1) = 1$

$T(n) = \log n + 2\log\frac{n}{3} + 4\log\frac{n}{9} + \dots + 2^{\log_3 n - 1}\log\frac{n}{3^{\log_3 n - 1}} + 2^{\log_3 n} \cdot 1$

$T(n) = \log n + 2\log n - 2\log 3 + 4\log n - 4\log 9 + \dots + 2^{\log_3 n - 1}\log n - 2^{\log_3 n - 1}\log(3^{\log_3 n - 1}) + n^{\log_3 2}$

$T(n) = n^{\log_3 2} + \log n + 2\log n + 4\log n + \dots + 2^{\log_3 n - 1}\log n - 2\log 3 - 4\log 9 - \dots - 2^{\log_3 n - 1}\log(3^{\log_3 n - 1})$

$T(n) = n^{\log_3 2} + \log n[1 + 2 + 4 + \dots + 2^{\log_3 n - 1}] - 2\log 3 - 4\cdot 2\log 3 - 8\cdot 3\log 3 - 16\cdot 4\log 3 - \dots - 2^{\log_3 n - 1}\cdot(\log_3 n - 1)\cdot\log 3$

$T(n) = n^{\log_3 2} + \log n[2^{\log_3 n} - 1] - \sum_{k=1}^{\log_3 n - 1} 2^k \cdot k \cdot \log 3 =$

$\log 3 \cdot \sum_{k=1}^{\log_3 n - 1} k \cdot 2^k =$

$= \log 3 \cdot 2\left[2^{\log_3 n - 1}(\log_3 n - 1) - 2^{\log_3 n - 1} + 1\right]$

$T(n) = n^{\log_3 2} + n^{\log_3 2}\log n - \log n - \cancel{2}\log 3\left[\frac{n^{\log_3 2}}{\cancel{2}}\cdot(\log_3 n - 1) - \frac{n^{\log_3 2}}{\cancel{2}} + 1\right]$

$= n^{\log_3 2}[1 + \log n] - \log n + \cancel{\log 3} - \log 3\, n^{\log_3 2}\log_3 n - \cancel{\log 3} + \log 3 \cdot n^{\log_3 2}$

$= n^{\log_3 2}[1 + \log n - \log 3\,\log_3 n + \log 3] - \log n$

$= n^{\log_3 2}\cdot c_1 + n^{\log_3 2}\left[\log n - \cancel{\log 3}\cdot\frac{\log n}{\cancel{\log} 3}\right] - \log n$

$= n^{\log_3 2}\cdot c_1 + n^{\log_3 2}\cdot\log n - \log n$

0.63

$= n^{\log_3 2} \cdot c_1 + n^{\log_3 2} \cdot \log n - \log n$

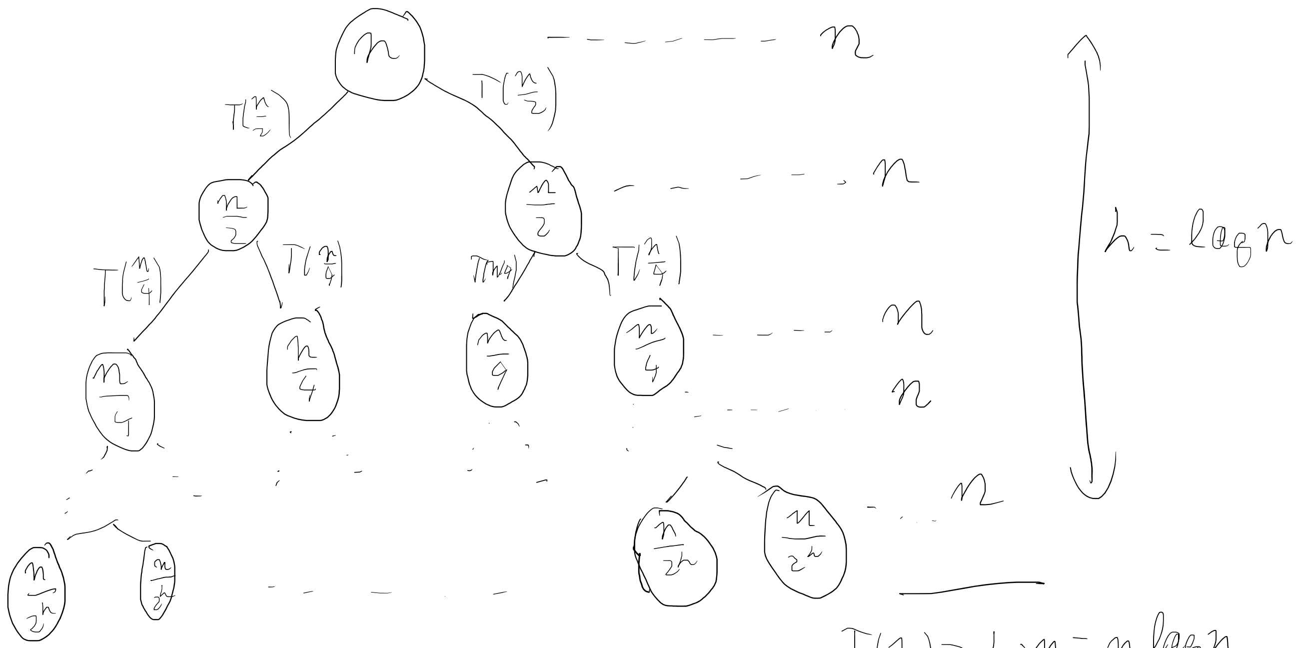
$\log_3 2 \simeq 0.63$

$T(n) \simeq n^{0.63} \cdot c_1 + \boxed{n^{0.63} \cdot \log n} - \log n \in \Theta(n^{0.63} \log n)$

Metoda arborelui de recurenta

Friday, 5 November 2021 11.22

$$T(n) = 2T\left(\frac{n}{2}\right) + n$$



$$\frac{n}{2^h} = 1 \Rightarrow h = \log n$$

$$T(n) = h \cdot n = n \log n$$

$$O(n \log n)$$

Fibonnaci

Friday, 5 November 2021 11.29

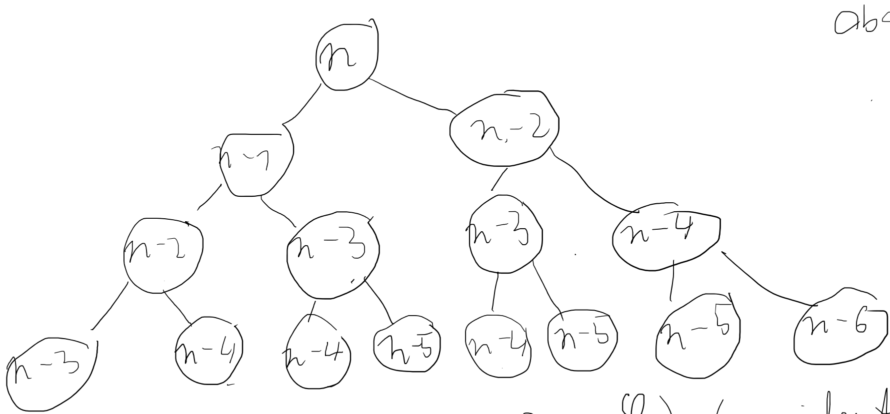
$$Fib(n) = Fib(n-1) + Fib(n-2)$$

$$T(n) = T(n-1) + T(n-2) + O(1)$$

$2n\log n$? $2^{\log n} = n$?

obs: alg. ineficient

=> memoizare (sau bottom-up)



$\Rightarrow O(2^n)$

$\varphi = 1.61 = \frac{1+\sqrt{5}}{2} \Rightarrow Fib \in O(2^{\varphi})$ (evident și $O(2^n)$)

Inductie

P_n: $T(k) \in O(2^k)\ \forall k \le n$

$$T(n+1) = T(n) + T(n-1) + O(1) = O(2^n) + O(2^{n-1}) + O(1)$$

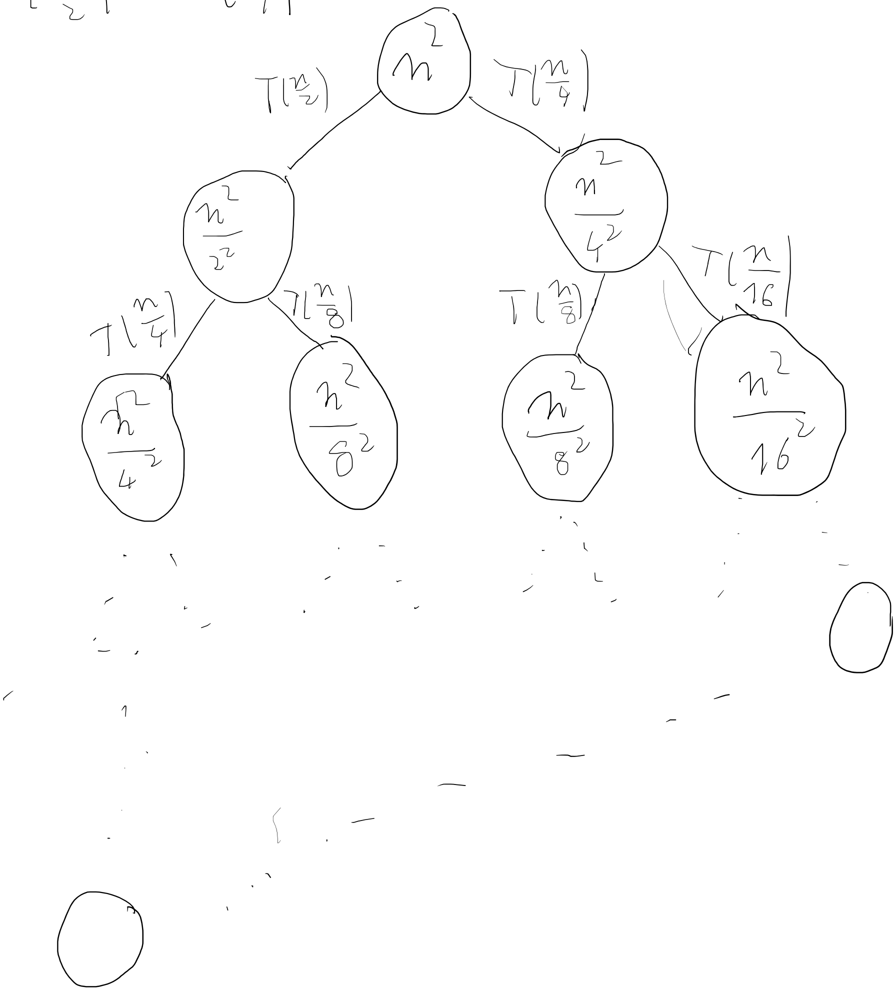
$$< O(2^n) + O(2^n) + O(1)$$

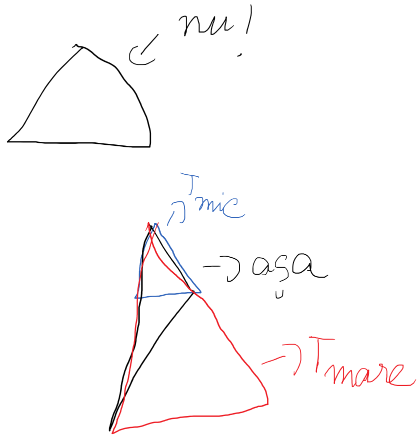
$$\Rightarrow O(2^{n+1})$$

Ex

Friday, 5 November 2021 11.40

$$T(n) = T\left(\frac{n}{2}\right) + T\left(\frac{n}{4}\right) + n^2$$





$$T_{mic} \leq T \leq T_{mare}$$