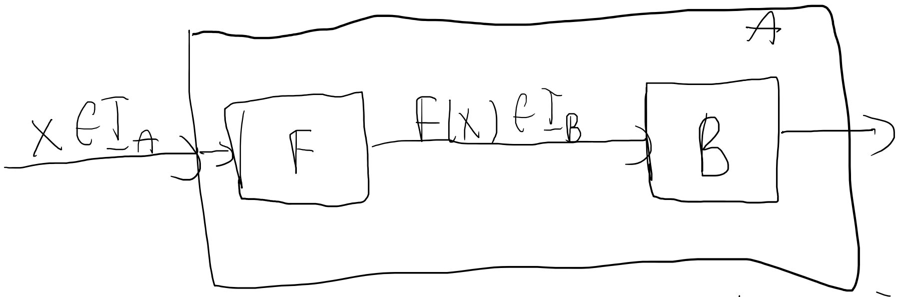
Reduceri Turing

Thursday, October 21, 2021 9:51 PM

$A \leq_T B \iff \exists F$ a.î. $\forall x \in I_A \Rightarrow F(x) \in I_B$

a.î. $A(x) = 1 \iff B(F(x)) = 1$



$\leq_T$ relație de ordine („A la fel de ușoară sau mai ușoară ca B")

$A \leq_T B$, $B \in R \Rightarrow A \in R$

$A \leq_T B$, $A \notin R \Rightarrow B \notin R$, $B \in RE$ sau NRE

$A \leq_T B$, $A \in NRE \Rightarrow B \in NRE$

$\boxed{HALT \in REVR}$ dacă vreau să dem. $B \in REVR$

$HALT \leq_T B$

Exercitii 1

Friday, 22 October 2021 10.22

$A \le_T B$, $B \le_T C \Rightarrow A \le_T C$

- $A \in RE \setminus R$, $C \in R$ F
- $C \in RE$, $A \cup B \in RE$ A
- $A \in R$, $B \in RE$, $C \in NRE$ A
- $B \in R$, $\overline{A} \in R$ ($B \in R \Rightarrow A \in R \Rightarrow \overline{A} \in R$) A
- $C \in RE \setminus R$, $A \in R$ A
- $B \in RE$, $B \cap C \in RE$ A/F !! dacă $C \in RE$
- $C \in R$, $\overline{B} \in NRE$ F
- $B \in R \Rightarrow \overline{C} \in R$? A/F dacă $C \in R$

$A \in R$, $\exists\, P_A(x) = \begin{cases} 1, & x \in A \\ 0, & x \notin A \end{cases}$

$\Rightarrow P_{\overline{A}}(x) = \begin{cases} 0, & x \in A \\ 1, & x \notin A \end{cases}$

$P_{\overline{A}}(x) = !P_A(x) \Rightarrow \overline{A} \in R$

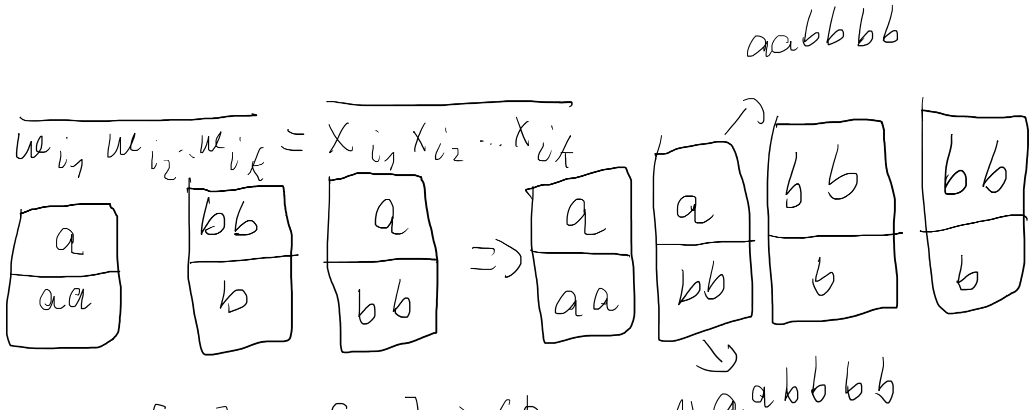
Post correspondance problem

$W = [w_1, w_2, \ldots, w_n]$

$X = [x_1, x_2, \ldots, x_n]$

$\exists$ k și $i_1, i_2, \ldots, i_k$ a.î. $w_{i_1} w_{i_2} \ldots w_{i_k} = x_{i_1} x_{i_2} \ldots x_{i_k}$

$W = [a, bb, a]$



$X = [aa, b, bb]$

1-PCP: 1 singur simbol, $W = [a]$, $X = [aa]$ $\Rightarrow$ $\in R$ (nu are sol)

alt ex.: $W = [a, bb, ab, abc]$, $X = [bad, bcd, cd, d]$ $\in R$ (nu are sol)

PCP $\in$ RE\R   HALT $\leq_T$ PCP

MPCP: soluțiile trb. să înceapă cu 1

HALT $\leq_T$ MPCP $\leq_T$ PCP

MPCP

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$MPCP \leq_T PCP$

$MPCP(W, X) \xrightarrow{F} PCP(W', X')$

$\exists F$ a.î. $MPCP(W, X) = 1 \Leftrightarrow PCP(F(W, X)) = 1$

F :

- la W, adaug * după fiecare simbol (W = [ab] => W' = [a* b*]
- la X, adaug * înainte de fiecare simbol (X = [ab] => X' = [*a*b]
- la W_1, adaug * la început (W = [ab, bc] => W' = [*a*b*, b*c*]
- adaug $ la finalul W (W = [a] => W' = [*a*, $]
  *$ la finalul X (X = [a] => X' = [*a, *$]

Dem.

"=>" MPCP(W, X) = 1 => PCP(F(W, X)) = 1
are sol. $1, i_1, \dots, i_k$ => $1, i_1, \dots, i_k, i_{final}$ sol în PCP

"<=" PCP(F(W, X)) = 1 => MPCP(W, X) = 1
are sol. $1, i_1, \dots, i_k, i_{final}$ => $1, i_1, \dots, i_k$ sol în MPCP

Gramatici independente de context

Friday, 22 October 2021 11.08

GIC

V - neterminali

T - terminali

S - simbol start

P - reguli

$S \to A$

$A \to aB \mid e \to e$ = nimic

$B \to aAb$

$S \to A \to aB \to aaAb \to$
$\to aaaBb \to aaaaAb \to$
$\to aaaab$

P_i - problema intersecției a două GIC $L(G_1) \cap L(G_2) \neq \emptyset$

$PCP \leq_T P_i \Rightarrow P_i \in RE$

VID

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VID : P nu se oprește pe niciun input ?

$\overline{VID}$: P se oprește pe orice input $\quad VID(p) = 1 \Leftrightarrow \overline{VID}(p') = 0$

$PO \leq_T \overline{VID}$

$\exists F$ a.î. $PO(p, w) = 1 \Leftrightarrow \overline{VID}(F(p, w)) = 1 \quad (VID(F(p, w)) = 0)$

pt. $PO \to P$

pt. $\overline{VID} \to P'$

$P'(X)$ {

return $P(W)$;

}

" $\Rightarrow$ " $PO(P, w) = 1 \Rightarrow P(w) \neq \perp \Rightarrow P'(x) \neq \perp \; \forall X$

$\Rightarrow \overline{VID}(P') = 1 \Leftrightarrow VID(P) = 0$

" $\Leftarrow$ " $\overline{VID}(P') = 1 \Rightarrow P'(x) \neq \perp \; \forall X \Rightarrow P(w) \neq \perp$

$\Rightarrow PO(P, w) = 1$

P5

Friday, 22 October 2021 11.29

$P_5(P)$: Se oprește P cu input 5? , $P_5 \in RE \setminus R$

$PO \leq_T P_5$

$\exists F, F(P, w) \to P'$ a.î. $PO(P, w) = 1 \Leftrightarrow P_5(P') = 1$

```
P'(x){
    return P(w);
}
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- " $\Rightarrow$ " $PO(P, w) = 1 \Rightarrow P(w) \neq \perp \Rightarrow P'(x) \neq \perp \ \forall x$
  $\Rightarrow P'(5) \neq \perp \Rightarrow P_5(P') = 1$
- " $\Leftarrow$ " $P_5(P') = 1 \Rightarrow P'(5) \neq \perp \Rightarrow P(w) \neq \perp \Rightarrow PO(P, w) = 1$

EQP

Friday, 22 October 2021 11.37

$EQP(P_1, P_2)$: se opresc P_1 și P_2 pe ac. input-uri?

$PO \leq_T EQP \Rightarrow EQP \in RE \setminus R$

$\exists F$ a.î. $F(P, u) \to P_1, P_2$

$P_1(x) = P_2(x) = P(u)$ $\quad P_1(x)$ și $P_2(x) \neq \perp \; \forall x \Rightarrow EQP(P_1, P_2) = 1$

"$\Rightarrow$" $PO(P, u) = 1 \Rightarrow P(u) \neq \perp \Rightarrow P_1(x) \neq \perp \Leftrightarrow P_2(x) \neq \perp \Rightarrow P(u) \neq \perp \Leftrightarrow P(u) \neq \perp$

"$\Leftarrow$" $EQP(P_1, P_2) = 1 \Rightarrow$

"Greșit!"

$P_1(x) = P(u)$

$P_2(x) = \text{return } 1;$

• "$\Rightarrow$" $PO(P, u) = 1 \Rightarrow P(u) \neq \perp \Rightarrow P_1(x) \neq \perp \; \forall x$, $P_2(x) \neq \perp \; \forall x \Rightarrow EQP(P_1, P_2) = 1$

• "$\Leftarrow$" $EQP(P_1, P_2) = 1 \Rightarrow P_1(x) \neq \perp \Rightarrow P(u) \neq \perp \Rightarrow PO(P, u) = 1$
$P_2(x) \neq \perp \; \forall x$