Introducere

Friday, 22 October 2021 10.00

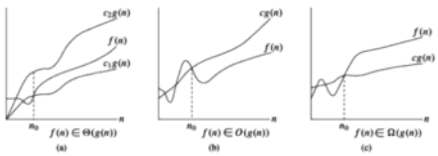
→ d.p.v. ← timp
memorie

→ depinde ← de dim. input (n)
de formatul inputului (ex. search în vector sortat vs. nesortat)

→ cazuri ← cel mai nefavorabil
mediu
cel mai favorabil

Notații de complexitate

Friday, 29 October 2021 10.13



$f \in O(g(n))$ $(f \le g)$ $f \in o(g(n))$ $(f < g)$ strict

$f \in \Omega(g(n))$ $(f \ge g)$ $f \in \omega(g(n))$ $(f > g)$

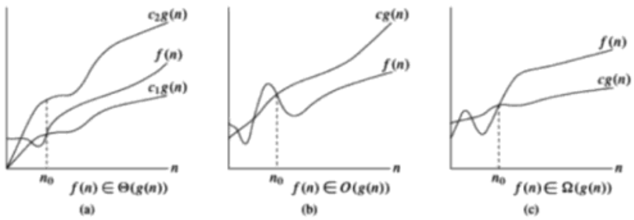
$f \in \Theta(g(n))$ $(f = g)$

- $O(g(n)) = \{f : N \to R_+ \mid \exists c \in R_+, c > 0$ și $\exists n_0 \in N$ a.î. $\forall n \ge n_0, 0 \le f(n) \le c g(n)\}$
- $\Omega(g(n)) = \{f : N \to R_+ \mid \exists c \in R_+, c > 0$ și $\exists n_0 \in N$ a.î. $\forall n \ge n_0, 0 \le c g(n) \le f(n)\}$
- $\Theta(g(n)) = \{f : N \to R_+ \mid \exists c_1, c_2 \in R_+^*$ și $\exists n_0 \in N$ a.î. $\forall n \ge n_0,$ $0 \le c_1 g(n) \le f(n) \le c_2 g(n)\}$ (obs: $c_1 \le c_2$)

- $o(g(n)) = \{f : N \to R_+ \mid \exists c \in R_+, c > 0$ și $\exists n_0 \in N$ a.î. $\forall n \ge n_0, f(n) < c g(n)\}$
- $\omega(g(n)) = \{f : N \to R_+ \mid \exists c \in R_+, c > 0$ și $\exists n_0 \in N$ a.î. $\forall n \ge n_0, 0 \le c g(n) < f(n)\}$

Notatii cu limita

Friday, 29 October 2021 10.28



- $\lim_{n \to \infty} \frac{f(n)}{g(n)} = L, \ 0 \le L < \infty \Rightarrow f(n) \in O(g(n))$
- $\lim_{n \to \infty} \frac{f(n)}{g(n)} = L, \ 0 < L < \infty \Rightarrow f(n) \in \Theta(g(n))$
- $\lim_{n \to \infty} \frac{f(n)}{g(n)} = L, \ 0 < L \le \infty \Rightarrow f(n) \in \Omega(g(n))$
- $\lim_{n \to \infty} \frac{f(n)}{g(n)} = L, \ L = 0 \Rightarrow f(n) \in o(g(n))$
- $\lim_{n \to \infty} \frac{f(n)}{g(n)} = L, \ L = \infty \Rightarrow f(n) \in \omega(g(n))$

Ex. tranzitivitate

Friday, 29 October 2021 10.35

- tranzitivitate $(a \le b,\ b \le c \Rightarrow a \le c)$

$f(n) \in O(g(n)),\ g(n) \in O(h(n)) \Rightarrow f(n) \in O(h(n))$

$f(n) \in O(g(n)) \Leftrightarrow \exists c_1 \in \mathbb{R}_+^*, \exists n_0^1$ a.î. $\forall n \ge n_0^1$ $f(n) \le c_1 g(n)$

$g(n) \in O(h(n)) \Leftrightarrow \exists c_2 \in \mathbb{R}_+^*, \exists n_0^2$ a.î. $\forall n \ge n_0^2$ $g(n) \le c_2 h(n)$

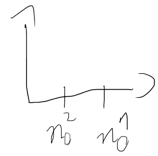
$$\begin{matrix} f(n) \le c_1 g(n) \\ g(n) \le c_2 h(n) \end{matrix} \Rightarrow f(n) \le \underbrace{c_1 c_2}_{} h(n) \overset{?}{\Rightarrow} f(n) \in O(h(n))$$

$c_3 = c_1 \cdot c_2$

$c_3 > 0\ (c_1, c_2 > 0)$

$\Rightarrow$ aleg $n_0^3 = \max(n_0^1, n_0^2)$

sau



$\exists c_3 = c_1 \cdot c_2 \in \mathbb{R}_+^*, \exists n_0^3 = \max(n_0^1, n_0^2)$ a.î. $\forall n \ge n_0^3$, $f(n) \le c_3 \cdot h(n) \Rightarrow f(n) \in O(h(n))$

Ex simetrie

Friday, 29 October 2021 10.47

Simetrie $(a=b \Rightarrow b=a)$

$f(n) \in \Theta(g(n)) \Leftrightarrow g(n) \in \Theta(f(n))$

$" \Rightarrow " \; f(n) \in \Theta(g(n)) \Rightarrow g(n) \in \Theta(f(n)) \rightarrow \exists c_3, c_4 \in \mathbb{R}_+^*, \exists n_0^2 \text{ a.î. } \forall n \geq n_0, c_3 f(n) \leq g(n) \leq c_4 f(n)$

$\exists c_1, c_2 \in \mathbb{R}_+^*, \exists n_0^1 \text{ a.î. } \forall n \geq n_0^1 \; c_1 g(n) \leq f(n) \leq c_2 g(n)$

$c_1 g(n) \leq f(n) \; | : c_1 \neq 0, c_1 > 0$

$g(n) \leq \frac{1}{c_1} f(n) \Rightarrow c_4 = \frac{1}{c_1}$

$c_4 \neq 0, c_4 > 0$

$f(n) \leq c_2 g(n) \; | : c_2, c_2 > 0, c_2 \neq 0$

$g(n) \geq \frac{1}{c_2} f(n) \Rightarrow c_3 = \frac{1}{c_2}; c_3 \neq 0, c_4 > 0$

$n_0^2 = n_0^1$

Insertion sort

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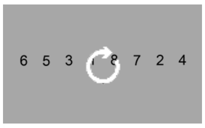
Insertion sort



Sorted partial result			Unsorted data
$\leq x$	$> x$	x	…

becomes

Sorted partial result			Unsorted data
$\leq x$	x	$> x$	…

Exemplu de analiză de complexitate: insertion-sort

```
function INSERTION-SORT(v)                  ▷ Nr. execuții/instrucțiune
    for j ← 2 to n do                       ▷ n
        elem ← v[j]                         ▷ n-1
        i ← j − 1                           ▷ n-1
        while i > 0 and elem < v[i] do      ▷ S1
            v[i + 1] ← v[i]                 ▷ S2
            i ← i − 1                       ▷ S2
        end while
        v[i + 1] ← elem                     ▷ n-1
    end for
end function
```

- Cazul favorabil?
  v sortat
  nu se intră în for

$n-2+1 = n-1$

$T(n) = 5\cdot(n-1) = 5n-5 \in O(n)\ (\in \Theta(n))$

- Cazul defavorabil
  v sortat invers
  n-1 ori în while
  la it. $j \Rightarrow$ de j ori în while

$\Rightarrow 1+2+3+\dots+n = \sum_{j=2}^{n}(j-1) = n(n-1)/2 = \frac{n^2-n}{2} \in O(n^2)$
$(\in \Theta(n^2))$

Ex complexitate cod

Friday, 29 October 2021 11.11

```
(a)  for (int i=1; i<c; i++) do
         //operații cu complexitate O(1)
     end for
```

$\Rightarrow O(c) \Rightarrow O(1)$ $\quad (O(3) = O(10) = O(2047) \ldots = O(1))$

c-ct

```
(b)  for (int i=1; i<=n; i+ = c) do
         //operații cu complexitate O(1)
     end for
```

$\Rightarrow O\left(\frac{n}{c}\right) = O(n)$

```
(c)  for (int i=n; i>0; i− = c) do
         for (int j=i+1; j<=n; j+ = c) do
             //operații cu complexitate O(1)
         end for
     end for
```

$\Rightarrow O\left(\frac{n^2}{c^2}\right) = O(n^2)$

```
(e)  for (int i=1; i<n; i* =2) do
         for (int j=n; j>0; j/ =2) do
             for (int k=j; k<n; k++) do
                 ... //număr constant c2 de operații
             end for
         end for
     end for
```

$\to O(\log n)$

$\to O(\log n)$

$\to O(n)$

$\Rightarrow O(n \log^2 n)$

for $(i=1; i<n; i*=2) \Rightarrow O(\log n)$

Ex.

Friday, 29 October 2021 11.27

a) $5 \cdot \log n + 6 \in O(n)$

$\frac{5 \cdot \log n + 6}{n} = 5 \cdot \frac{\log n}{n} + \frac{6}{n} \to 0 \Rightarrow f(n) \in O(n)$ (chiar $o(n)$)

b) $n^2 \in \Theta(n^2+n)$

$\frac{n^2}{n^2+n} \to 1 \neq 0$, finit $\Rightarrow f(n) \in \Theta(n)$

$\exists c_1, c_2 \in \mathbb{R}_+^*, \exists n_0$ a.î. $\forall n \geq n_0 \quad c_1(n^2+n) \leq n^2 \leq c_2(n^2+n)$

$c_1 n^2 + c_1 n \leq n^2 \leq c_2 n^2 + c_2 n$

$n^2 \leq c_2 n^2 + c_2 n$ $\quad n^2 \leq n^2 + n$

$c_1 < 1$ $\quad \frac{1}{2}(n^2+n) \leq n^2 \quad | \cdot 2$ $\quad c_2 \geq 1, c_2 = 1$ $\quad n \geq 0 \ \forall n \geq 0$

$c_1 = \frac{1}{2}$ $\quad 2n^2 \geq n^2 + n$

$n^2 \geq n \ \forall n \geq 1$ (de dem. $n^2 - n \geq 0$ pt. $n \geq 1$)

Am $c_1 = \frac{1}{2}, c_2 = 1$

$n_0 = 1$