P8106 Midterm - Code

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Exploratory Analysis

Loading in Data

```
load("dat1.RData")
load("dat2.RData")

dat1 <- dat1 %>% janitor::clean_names()
dat2 <- dat2 %>%janitor::clean_names()
```

Producing Summary Table

Training and test data have the same distribution of demographic characteristics; there is a difference in time since vaccination and log-transformed antibody levels between training and test data

```
# Combining data for summary table, data cleaning
dat1_com <- dat1 %>% mutate(set = "Training Data")
dat2_com <- dat2 %>% mutate(set = "Testing Data")
dat <- dat1_com %>%
  rbind(dat2 com) %>%
  rename(days_vaccinated = time) %>%
  mutate(race = as.character(race), smoking = as.character(smoking)) %>%
  mutate(race = case_match(
        race, "1" ~ "White", "2" ~ "Asian", "3" ~ "Black", "4" ~ "Hispanic"),
         gender = case_match(gender, 1 ~ "Male", 0 ~ "Female"),
         smoking = case_match(
           smoking, "0" ~ "Never", "1" ~ "Former", "2" ~ "Current"))
# Summary table
dat %>% select(!id) %>%
  tbl_summary(
   by = set,
   label = list(age = "Age", gender = "Gender", race = "Race", smoking = "Smoking",
                 height = "Height (cm)", weight = "Weight (kg)", bmi = "BMI",
                 diabetes = "Diabetes", hypertension = "Hypertension",
                 sbp = "Systolic Blood Pressure (mmHg)", ldl = "LDL Cholesterol (mg/dL)",
                 days vaccinated = "Time Since Vaccinated (days)",
                 log_antibody = "Log-Transformed Antibody Level")) %>%
  add_overall() %>% add_p() %>%
  modify_caption("Summary of Patient Testing and Training Data (N=6000)") %>%
  as_gt() %>% tab_options(table.font.size = 10)
```

Table 1: Summary of Patient Testing and Training Data (N=6000)

Characteristic	Overall $N = 6,000^{1}$	Testing Data $N = 1,000^{1}$	Training Data $N = 5{,}000^{1}$	$\mathbf{p} ext{-}\mathbf{value}^2$
Age	60.0 (57.0, 63.0)	60.0 (57.0, 63.0)	60.0 (57.0, 63.0)	0.9
Gender				0.7
Female	3,082 (51%)	509 (51%)	2,573 (51%)	
Male	2,918 (49%)	491 (49%)	2,427 (49%)	
Race				0.6
Asian	333 (5.6%)	55 (5.5%)	278 (5.6%)	
Black	1,235 (21%)	199 (20%)	1,036 (21%)	
Hispanic	548 (9.1%)	83 (8.3%)	465 (9.3%)	
White	3,884 (65%)	663 (66%)	3,221 (64%)	
Smoking				0.8
Current	589 (9.8%)	103 (10%)	486 (9.7%)	
Former	1,800 (30%)	296 (30%)	1,504 (30%)	
Never	3,611 (60%)	601 (60%)	3,010 (60%)	
Height (cm)	170.1 (166.1, 174.2)	170.2 (166.1, 174.2)	170.1 (166.1, 174.3)	0.7
Weight (kg)	80 (75, 85)	80 (75, 84)	80 (75, 85)	0.8
BMI	27.60 (25.80, 29.50)	27.60 (25.80, 29.60)	27.60 (25.80, 29.50)	0.9
Diabetes	929 (15%)	157 (16%)	772 (15%)	0.8
Hypertension	2,754 (46%)	456 (46%)	2,298 (46%)	0.8
Systolic Blood Pressure (mmHg)	130 (124, 135)	130 (124, 135)	130 (124, 135)	0.3
LDL Cholesterol (mg/dL)	110 (96, 124)	112 (96, 124)	110 (96, 124)	0.4
Time Since Vaccinated (days)	116 (82, 152)	171 (140, 205)	106 (76, 138)	< 0.001
Log-Transformed Antibody Level	10.06 (9.65, 10.45)	9.93 (9.50, 10.32)	10.09 (9.68, 10.48)	< 0.001

¹ Median (Q1, Q3); n (%)

Histograms of Differing Variables by Training and Test Set

```
# Antibody level
plot_sets <- dat %>%
  ggplot(aes(x = log_antibody,
             fill = set,
             color = set)) +
  geom_density(alpha = 0.3, linewidth = 1) +
  labs(x = "Log-Transformed Antibody Level",
       y = "Density",
       title = "Figure 1: Distribution of Log-Transformed Antibody Level, by Data Set") +
  theme_minimal()
# Time since vaccination (days)
plot_days <- dat %>%
  ggplot(aes(x = days_vaccinated,
            fill = set,
            color = set)) +
  geom_density(alpha = 0.3, linewidth = 1) +
  labs(x = "Time Since Vaccinated (days)",
       y = "Density",
       title = "Figure 2: Distribution of Days Since Vaccination, by Data Set") +
  theme minimal()
plot_sets
```

²Wilcoxon rank sum test; Pearson's Chi-squared test

Figure 1: Distribution of Log-Transformed Antibody Level, by Data Set

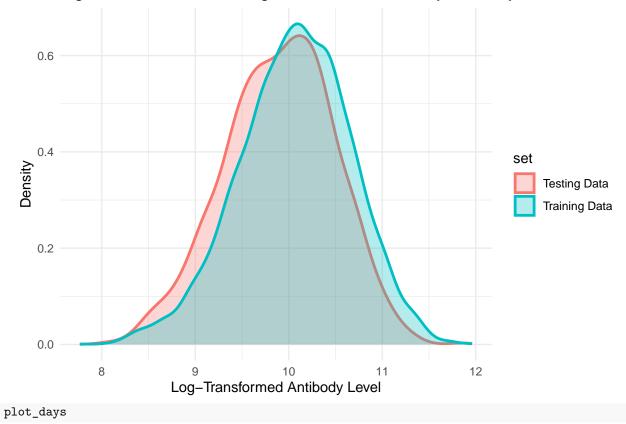
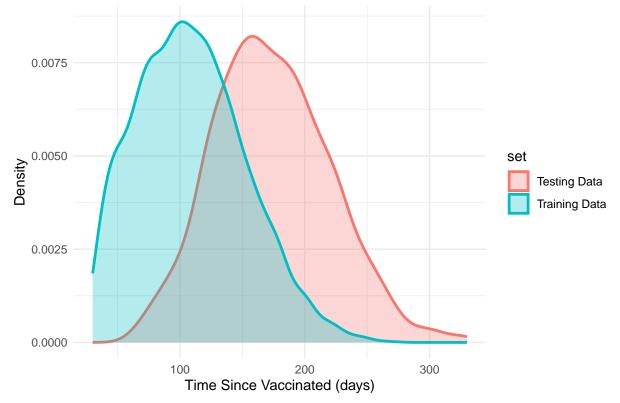
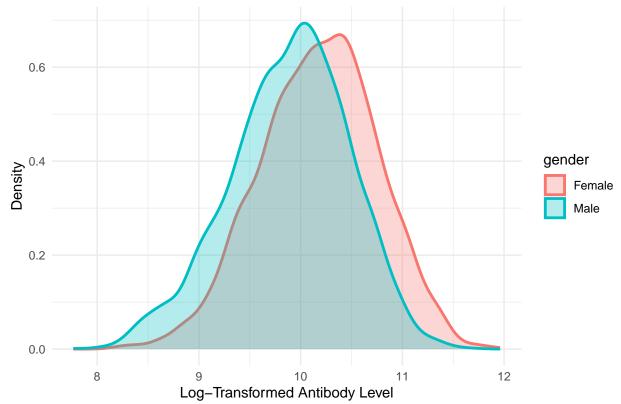


Figure 2: Distribution of Days Since Vaccination, by Data Set



Plots of Log-Transformed Antibody Level, by Categorical Variables

Figure 3: Distribution of Log-Transformed Antibody Level, by Gender



```
strip_markdown <- function(x) {gsub("\\*\\*", "", x)}

dat %>% select(gender, log_antibody) %>%
   tbl_summary(by = gender) %>% add_p() %>%
   modify_caption("Log-Transformed Antibody Level, by Gender") %>%
   as_kable() %>%
   footnote(general_title = "", general = "Median (Q1, Q3), Wilcoxon Rank Sum Test") %>%
   strip_markdown()
```

Table 2: Log-Transformed Antibody Level, by Gender

Characteristic	Female $N = 3,082$	$\mathrm{Male\ N} = 2{,}918$	p-value
log_antibody	$10.20\ (9.79,\ 10.58)$	9.93 (9.51, 10.30)	< 0.001

Median (Q1, Q3), Wilcoxon Rank Sum Test

Figure 4: Distribution of Log-Transformed Antibody Level, by Race

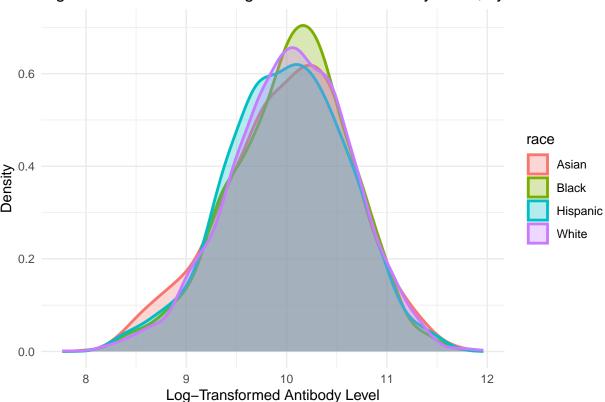


Table 3: Log-Transformed Antibody Level, by Race

Characteristic	Asian $N = 333$	Black $N = 1,235$	Hispanic $N = 548$	White $N = 3,884$	p-value
log_antibody	10.06 (9.62, 10.44)	10.08 (9.65, 10.44)	10.03 (9.61, 10.42)	10.06 (9.65, 10.46)	0.4

Median (Q1, Q3), Kruskal-Wallis Rank Sum Test

Figure 5: Distribution of Log-Transformed Antibody Level, by Smoking

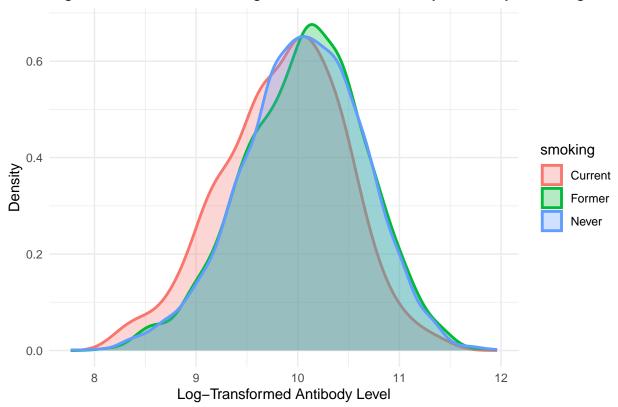


Table 4: Log-Transformed Antibody Level, by Smoking Status

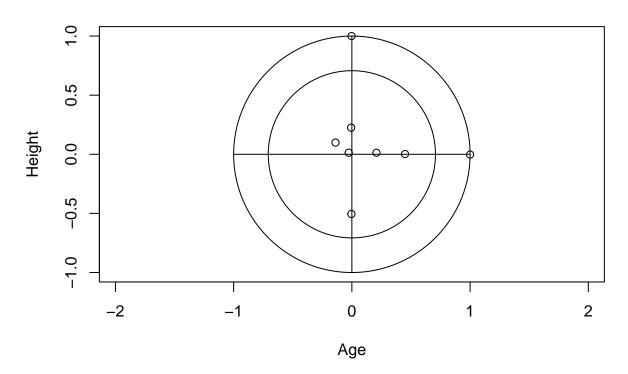
Characteristic	Current $N = 589$	Former $N = 1,800$	Never $N = 3,611$	p-value
log_antibody	9.91 (9.46, 10.28)	10.10 (9.66, 10.48)	10.07 (9.68, 10.46)	< 0.001

Median (Q1, Q3), Kruskal-Wallis Rank Sum Test

Correlation Matrix of Numerical Variables

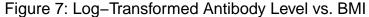
```
cor matrix <- dat %>%
  select(age, height, weight, bmi, sbp, ldl, days_vaccinated, log_antibody) %>%
 rename("Age" = age,
         "Height" = height,
         "Weight" = weight,
         "BMI" = bmi,
         "SBP" = sbp,
         "LDL" = 1d1,
         "Days Vaccinated" = days_vaccinated,
         "Log(Antibody)" = log_antibody) %>%
  cor()
cor_plot <- corrplot(cor_matrix,</pre>
                     main = "Figure 6: Correlation Matrix of Numerical Variables",
                     mar=c(0,0,1,0), cex.main = 1,
                     method = "color",
                     addCoef.col = "black",
                     tl.col = "black",
                     number.cex = 0.8,
                     tl.srt = 45,
                     order = 'original',
                     diag = F)
```

Figure 6: Correlation Matrix of Numerical Variables



Plots of Log-Transformed Antibody Level vs. Selected Numerical Variables

```
# Antibody level vs. BMI
plot_bmi <- dat %>% ggplot(aes(x = bmi, y = log_antibody, fill = set, color = set)) +
  geom_point(alpha = 0.3, size = 2) +
  geom_smooth(method = "lm") +
  labs(y = "Log-Transformed Antibody Level", x = "BMI",
       title = "Figure 7: Log-Transformed Antibody Level vs. BMI") +
  theme minimal()
# Antibody level vs. Weight
plot_weight <- dat %>%
  ggplot(aes(x = weight, y = log_antibody, fill = set, color = set)) +
  geom_point(alpha = 0.3, size = 2) +
  geom smooth(method = "lm") +
  labs(y = "Log-Transformed Antibody Level", x = "Weight",
       title = "Figure 8: Log-Transformed Antibody Level vs. Weight") +
  theme minimal()
# Antibody level vs. Age
plot_age <- dat %>% ggplot(aes(x = age, y = log_antibody, fill = set, color = set)) +
  geom_point(alpha = 0.3, size = 2) + geom_smooth(method = "lm") +
  labs(y = "Log-Transformed Antibody Level", x = "Age",
       title = "Figure 9: Log-Transformed Antibody Level vs. Age") +
  theme_minimal()
plot_bmi
```



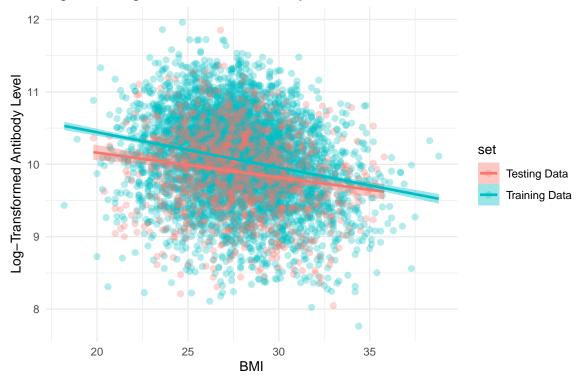
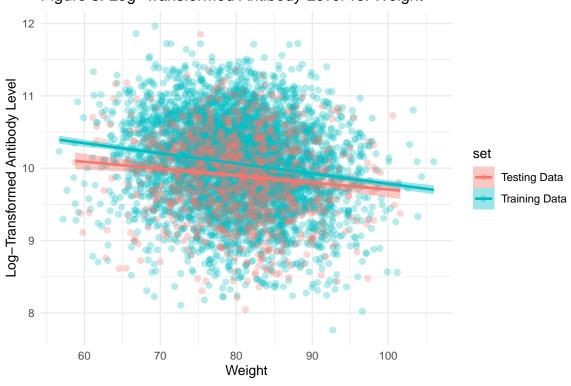
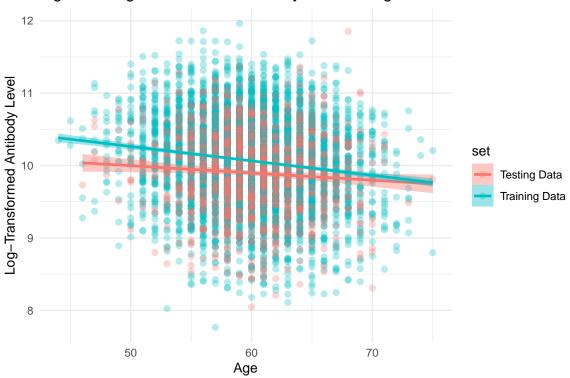


Figure 8: Log-Transformed Antibody Level vs. Weight



plot_age

Figure 9: Log-Transformed Antibody Level vs. Age



Model Selection and Training

Since the response variable (log_antibody) is continuous, this project will consider the following models:

- Multiple Linear Regression (MLR) as a baseline.
- LASSO Regression to improve predictive performance by selecting important predictors.
- MARS model allow remain in regression but also capture nonlinear effects

After comparing model performance, the best model will be based on cross-validation results.

Data Pre-processing

```
# Converting categorical variables into factors
dat1 <- dat1 %>%
  mutate(
    gender = factor(gender, levels = c(0, 1), labels = c("Female", "Male")),
   race = factor(race, levels = c(1, 2, 3, 4), labels = c("White", "Asian", "Black", "Hispanic")),
   smoking = factor(smoking, levels = c(0, 1, 2), labels = c("Never", "Former", "Current")),
   diabetes = factor(diabetes),
   hypertension = factor(hypertension)
dat2 <- dat2 %>%
  mutate(
   gender = factor(gender, levels = c(0, 1), labels = c("Female", "Male")),
   race = factor(race, levels = c(1, 2, 3, 4), labels = c("White", "Asian", "Black", "Hispanic")),
   smoking = factor(smoking, levels = c(0, 1, 2), labels = c("Never", "Former", "Current")),
   diabetes = factor(diabetes),
   hypertension = factor(hypertension)
sum(is.na(dat1))
## [1] 0
sum(is.na(dat2))
## [1] 0
dat1 <- dat1 %>% select(-id)
dat2 <- dat2 %>% select(-id)
```

Training multiple linear regression model

```
# train control
set.seed(123)
ctrl1 = trainControl(method = "cv", number = 10)
mlr_model <- lm(log_antibody ~ ., data = dat1)
summary(mlr_model)
##
## Call:
## lm(formula = log_antibody ~ ., data = dat1)
##
## Residuals:</pre>
```

```
Median
                1Q
## -2.14396 -0.35840 0.02944 0.37802 1.65090
## Coefficients:
                  Estimate Std. Error t value Pr(>|t|)
                26.6751961 2.3149812 11.523 < 2e-16 ***
## (Intercept)
                -0.0205979  0.0019385  -10.626  < 2e-16 ***
## age
## genderMale
                -0.2974929 0.0155977 -19.073 < 2e-16 ***
## raceAsian
                -0.0060422 0.0344613 -0.175
                                               0.8608
## raceBlack
                -0.0075295 0.0196815 -0.383
                                               0.7021
## raceHispanic
                -0.0417571 0.0273309 -1.528
                                               0.1266
## smokingFormer
                 0.0219907
                            0.0173992
                                      1.264
                                               0.2063
## smokingCurrent -0.1934834  0.0269576  -7.177  8.15e-13 ***
                ## height
## weight
                                      5.987 2.29e-09 ***
                 0.0859034 0.0143481
## bmi
                 ## diabetes1
                 0.0112795 0.0215643
                                      0.523
                                               0.6010
## hypertension1 -0.0179106 0.0260931
                                     -0.686
                                               0.4925
                                      0.890
                                               0.3733
                 0.0015181 0.0017049
## sbp
## ldl
                -0.0001645 0.0004028 -0.409
                                               0.6829
## time
                -0.0003011 0.0001795 -1.677
                                               0.0936 .
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 0.5503 on 4984 degrees of freedom
## Multiple R-squared: 0.1513, Adjusted R-squared: 0.1488
## F-statistic: 59.25 on 15 and 4984 DF, p-value: < 2.2e-16
##using caret to see the differece in model result
mlr_fit <- train(</pre>
 log_antibody ~ .,
 data = dat1,
 method = "lm",
 trControl = ctrl1
summary(mlr_fit)
##
## Call:
## lm(formula = .outcome ~ ., data = dat)
## Residuals:
##
                1Q
                     Median
## -2.14396 -0.35840 0.02944 0.37802 1.65090
## Coefficients:
                  Estimate Std. Error t value Pr(>|t|)
## (Intercept)
                26.6751961 2.3149812 11.523 < 2e-16 ***
## age
                -0.0205979 0.0019385 -10.626
                                             < 2e-16 ***
## genderMale
                -0.2974929 0.0155977 -19.073
                                             < 2e-16 ***
## raceAsian
                -0.0060422 0.0344613 -0.175
                                              0.8608
## raceBlack
                -0.0075295 0.0196815 -0.383
                                               0.7021
## raceHispanic -0.0417571 0.0273309 -1.528
                                               0.1266
## smokingFormer 0.0219907 0.0173992
                                      1.264
                                               0.2063
```

```
## smokingCurrent -0.1934834 0.0269576 -7.177 8.15e-13 ***
## height -0.0821381 0.0135622 -6.056 1.49e-09 ***
## weight
                 0.0859034 0.0143481 5.987 2.29e-09 ***
                -0.2977935  0.0412612  -7.217  6.10e-13 ***
## bmi
## diabetes1
                 0.0112795 0.0215643 0.523 0.6010
## hypertension1 -0.0179106 0.0260931 -0.686 0.4925
                0.0015181 0.0017049 0.890 0.3733
## sbp
                -0.0001645 0.0004028 -0.409 0.6829
## ldl
## time
                -0.0003011 0.0001795 -1.677 0.0936 .
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 0.5503 on 4984 degrees of freedom
## Multiple R-squared: 0.1513, Adjusted R-squared: 0.1488
## F-statistic: 59.25 on 15 and 4984 DF, p-value: < 2.2e-16
# Evaluating model performance on validation data
mlr_pred <- predict(mlr_model, newdata = dat2)</pre>
mlr_rmse <- sqrt(mean((mlr_pred - dat2$log_antibody)^2))</pre>
mlr_adj_r2 <- summary(mlr_model)$adj.r.squared</pre>
mlr_rmse
## [1] 0.568318
mlr_adj_r2
## [1] 0.1487817
mlr_caret_pred <- predict(mlr_fit, newdata = dat2)</pre>
mlr caret rmse <- sqrt(mean((mlr caret pred - dat2$log antibody)^2))
mlr_adj_r2_caret <- summary(mlr_fit)$adj.r.squared</pre>
mlr_caret_rmse
## [1] 0.568318
mlr adj r2 caret
## [1] 0.1487817
```

Training LASSO regression model

Standardizing numerical variables for LASSO using glmnet

```
num_vars <- c("age", "height", "weight", "bmi", "sbp", "ldl", "time") #only continuous variable
preprocess_params <- preProcess(dat1[, num_vars], method = c("center", "scale"))
dat1[, num_vars] <- predict(preprocess_params, dat1[, num_vars])
dat2[, num_vars] <- predict(preprocess_params, dat2[, num_vars])

# Preparing the data matrices for glmnet
x_train <- model.matrix(log_antibody ~ ., dat1)[, -1] # Removing the intercept
y_train <- dat1$log_antibody

x_valid <- model.matrix(log_antibody ~ ., dat2)[, -1]
y_valid <- dat2$log_antibody

set.seed(123)
lasso_model <- cv.glmnet(x_train, y_train, alpha = 1) # LASSO with cross validation</pre>
```

```
best_lambda <- lasso_model$lambda.min</pre>
lasso_final <- glmnet(x_train, y_train, alpha = 1, lambda = best_lambda) # final model is based on opti
# predicting with LASSO on validation data
lasso pred <- predict(lasso final, newx = x valid)</pre>
lasso_rmse <- sqrt(mean((lasso_pred - y_valid)^2))</pre>
lasso_rmse
## [1] 0.5683693
ss_total_lasso <- sum((y_valid - mean(y_valid))^2)</pre>
ss_res_lasso <- sum((lasso_pred - y_valid)^2)</pre>
r_squared_lasso <- 1 - ss_res_lasso / ss_total_lasso
n <- length(y_valid)</pre>
p <- length(coef(lasso_final)) - 1</pre>
lasso_adj_r2 <- 1 - ((1 - r_squared_lasso) * (n - 1) / (n - p - 1))
system.time(glmnet(x_train, y_train, alpha = 1, lambda = best_lambda))
##
      user system elapsed
##
     0.002
            0.000 0.001
using caret package for LASSO to compare the difference between the two methodologies
(packages)
## using caret for LASSO prediction
set.seed(123)
lasso_fit=train(y=y_train,
                   x=x_train,
                   method = "glmnet",
                   tuneGrid = expand.grid(alpha = 1,
                                           lambda = \exp(\text{seq}(2, -6, \text{length} = 1000))),
                   preProcess = c("center", "scale"),
                   trControl = ctrl1)
## Warning in nominalTrainWorkflow(x = x, y = y, wts = weights, info = trainInfo,
## : There were missing values in resampled performance measures.
# selected tuning parameter and test error
lasso best lambda = lasso fit$bestTune$lambda
predy2_lasso_fit = predict(lasso_fit, newdata = x_valid)
lasso_rmse = sqrt(mean((y_valid - predy2_lasso_fit)^2))
lasso_best_lambda
## [1] 0.0046664
lasso_rmse
## [1] 0.5735065
ss_total_lasso <- sum((y_valid - mean(y_valid))^2)</pre>
ss_res_lasso <- sum((predy2_lasso_fit - y_valid)^2)</pre>
r squared lasso <- 1 - ss res lasso / ss total lasso
n <- length(y_valid)</pre>
p <- length(coef(lasso_final)) - 1</pre>
```

```
lasso_adj_r2 <- 1 - ((1 - r_squared_lasso) * (n - 1) / (n - p - 1))
lasso_adj_r2
## [1] 0.03108439</pre>
```

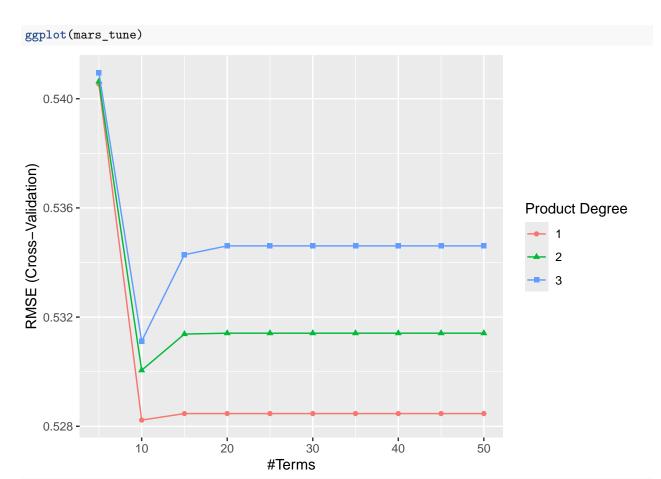
Training MARS model

```
mars_model <- earth(log_antibody ~ ., data = dat1)</pre>
summary(mars_model)
## Call: earth(formula=log_antibody~., data=dat1)
##
##
                     coefficients
## (Intercept)
                       10.8474469
## genderMale
                       -0.2962905
## smokingCurrent
                       -0.2051269
## h(-0.215005-age)
                        0.0726888
## h(age- -0.215005) -0.1034029
## h(bmi- -1.46481)
                       -0.2327459
## h(0.0216075-bmi)
                       -0.1710074
## h(-1.19454-time)
                       -1.4557278
## h(time- -1.19454)
                       -0.0978688
## Selected 9 of 10 terms, and 5 of 15 predictors
## Termination condition: RSq changed by less than 0.001 at 10 terms
## Importance: bmi, genderMale, time, age, smokingCurrent, raceAsian-unused, ...
## Number of terms at each degree of interaction: 1 8 (additive model)
                    RSS 1384.431
## GCV 0.2787787
                                     GRSq 0.2166152
                                                       RSq 0.2216218
# predicting with MARS on validation data
mars_pred <- predict(mars_model, newdata = dat2)</pre>
mars_rmse <- sqrt(mean((mars_pred - dat2$log_antibody)^2))</pre>
mars_rmse
```

[1] 0.5327718

• The MARS model achieves the lowest RMSE. Therefore MARS will be used as the preferred model for predicting log_antibody. Although further fine-tuning and additional feature exploration could further enhance the model's predictive power.

MARS model tuning

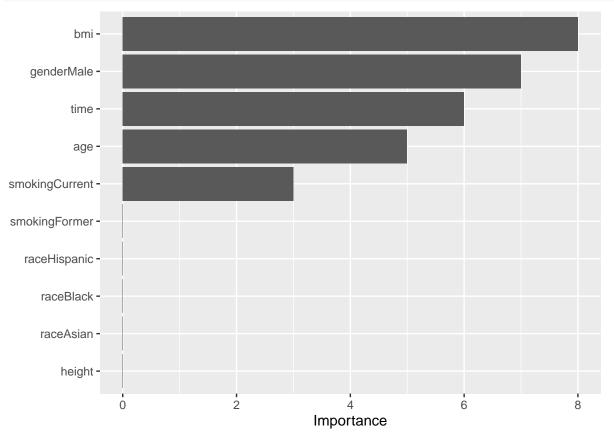


```
## Call: earth(x=matrix[5000,15], y=c(10.65,9.889,1...), keepxy=TRUE, degree=1,
##
               nprune=10)
##
##
                     coefficients
## (Intercept)
                       10.8474469
## genderMale
                       -0.2962905
## smokingCurrent
                       -0.2051269
## h(-0.215005-age)
                        0.0726888
## h(age- -0.215005)
                       -0.1034029
## h(bmi- -1.46481)
                       -0.2327459
## h(0.0216075-bmi)
                       -0.1710074
## h(-1.19454-time)
                       -1.4557278
## h(time- -1.19454)
                       -0.0978688
##
```

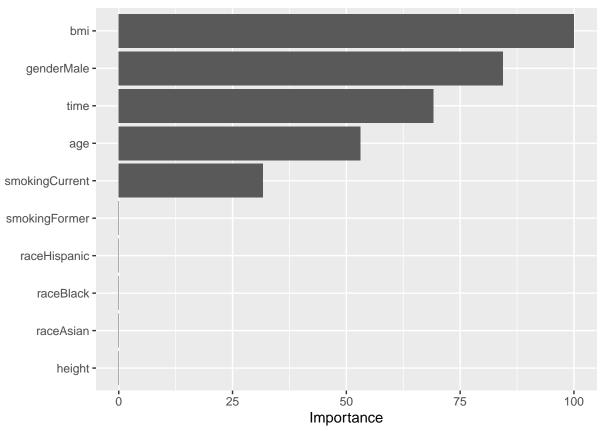
```
## Selected 9 of 10 terms, and 5 of 15 predictors (nprune=10)
## Termination condition: RSq changed by less than 0.001 at 10 terms
## Importance: bmi, genderMale, time, age, smokingCurrent, raceAsian-unused, ...
## Number of terms at each degree of interaction: 1 8 (additive model)
## GCV 0.2787787
                    RSS 1384.431
                                    GRSq 0.2166152
                                                       RSq 0.2216218
print(mars_model_tune$bestTune)
     nprune degree
##
## 1
         10
# Report the final model (regression function)
print(coef(mars_model_tune$finalModel))
         (Intercept) h(0.0216075-bmi) h(time--1.19454) h(-1.19454-time)
##
##
         10.84744693
                           -0.17100738
                                             -0.09786882
                                                                -1.45572777
          genderMale h(age- -0.215005) h(-0.215005-age)
##
                                                             smokingCurrent
         -0.29629045
                           -0.10340288
                                               0.07268881
                                                                -0.20512685
  h(bmi - 1.46481)
##
##
         -0.23274594
mars_tune_pred <- predict(mars_model_tune, newdata = dat2)</pre>
mars_tune_rmse <- sqrt(mean((mars_tune_pred - dat2$log_antibody)^2))</pre>
mars_tune_rmse
```

[1] 0.5327718

```
# variable importance plot for MARS
vip(mars_model_tune$finalModel, type = "nsubsets")
```



```
vip(mars_model_tune$finalModel, type = "rss")
```



```
# Compute R-squared
ss_total_mars <- sum((dat2$log_antibody - mean(dat2$log_antibody))^2)
ss_res_mars <- sum((mars_tune_pred - dat2$log_antibody)^2)
r_squared_mars <- 1 - ss_res_mars / ss_total_mars

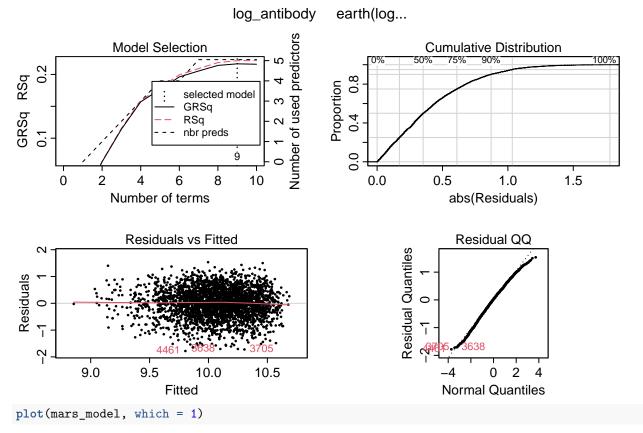
# Get n and number of selected terms (p)
n <- nrow(dat2)
p <- length(mars_model_tune$finalModel$selected.terms)

# Adjusted R-squared for MARS
mars_adj_r2 <- 1 - ((1 - r_squared_mars) * (n - 1) / (n - p - 1))

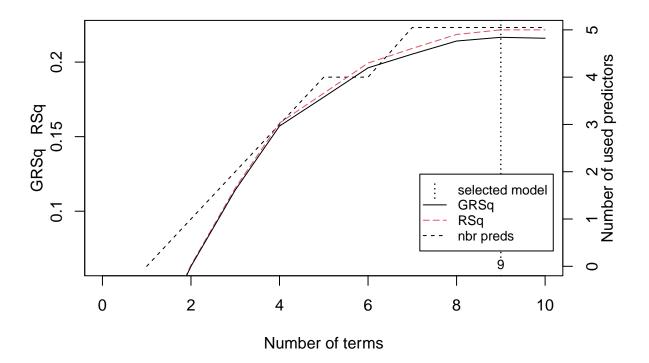
mars_adj_r2

## [1] 0.1689031
mars_tune_rmse

## [1] 0.5327718
plot(mars_model)</pre>
```

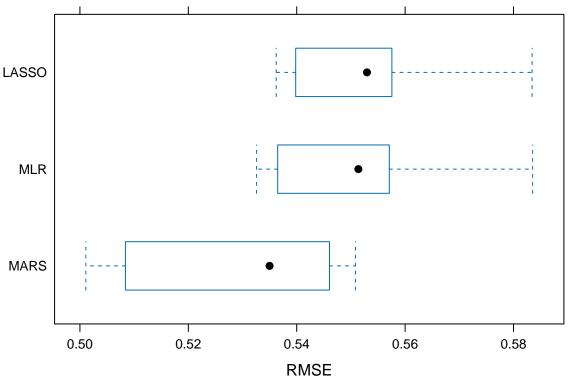


Model Selection



Results

```
res <- resamples(list(MLR = mlr_fit,
                      LASSO = lasso_fit,
                      MARS = mars_model_tune))
summary(res)
##
## Call:
## summary.resamples(object = res)
## Models: MLR, LASSO, MARS
## Number of resamples: 10
##
## MAE
##
              Min.
                     1st Qu.
                                Median
                                             Mean
                                                    3rd Qu.
                                                                 Max. NA's
         0.4261808\ 0.4320149\ 0.4340128\ 0.4390390\ 0.4425761\ 0.4726409
## MLR
## LASSO 0.4277022 0.4330544 0.4364975 0.4403869 0.4449154 0.4721228
                                                                         0
## MARS 0.3998101 0.4107959 0.4303208 0.4232841 0.4338504 0.4412629
                                                                         0
##
## RMSE
##
                     1st Qu.
                                Median
                                                    3rd Qu.
                                                                 Max. NA's
              Min.
                                             Mean
        0.5325907 0.5394231 0.5513718 0.5507854 0.5564032 0.5835238
## LASSO 0.5362151 0.5422491 0.5529600 0.5525148 0.5572990 0.5834269
                                                                         0
## MARS 0.5010943 0.5102697 0.5349891 0.5287651 0.5452837 0.5508336
##
## Rsquared
##
               Min.
                      1st Qu.
                                 Median
                                              Mean
                                                     3rd Qu.
                                                                  Max. NA's
         0.10033968 0.1319832 0.1534930 0.1479018 0.1622355 0.1906302
## LASSO 0.09477008 0.1342704 0.1433758 0.1428285 0.1502852 0.1912944
                                                                          0
## MARS 0.18035214 0.1837984 0.2135109 0.2156188 0.2370960 0.2847354
# boxplot of RMSE performance for all models
bwplot(res, metric = "RMSE")
```



```
# Summary table
model_perf <- tibble(</pre>
  Model = c("Multiple Linear Regression (MLR)",
            "LASSO Regression",
            "MARS (Final, Tuned)"),
  RMSE = c(mlr_rmse, lasso_rmse, mars_tune_rmse),
  `Adjusted R-squared` = c(mlr_adj_r2, lasso_adj_r2, mars_adj_r2),
  Notes = c("Baseline model; assumes linear relationships",
            "Performs variable selection via L1 regularization",
            "Best performance; captures non-linear effects")
model_perf %>%
  gt() %>%
  tab header(
    title = "Table: Model Performance on Independent Test Set (dat2)"
  ) %>%
  cols_label(
    Model = "Model",
    RMSE = "RMSE ↓",
    `Adjusted R-squared` = "Adjusted R-squared^",
    Notes = "Notes"
  fmt_number(columns = c(RMSE, `Adjusted R-squared`), decimals = 3) %>%
  tab_options(table.font.size = "small")
```

Table: Model Performance on Independent Test Set $(\mathrm{dat}2)$

Model	$\mathrm{RMSE}\downarrow$	Adjusted R-squared †	Notes
Multiple Linear Regression (MLR)	0.568	0.149	Baseline model; assumes linear relationships
LASSO Regression	0.574	0.031	Performs variable selection via L1 regularization
MARS (Final, Tuned)	0.533	0.169	Best performance; captures non-linear effects