# P8106 Midterm - Report

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#### Introduction

In this project, our team explored the dataset collected from a study on evaluating antibody responses to a newly authorized vaccine. The primary outcome of interest is the log-transformed antibody level measured via dried blood spots. The dataset includes a range of demographic and clinical predictors such as age, gender, race/ethnicity, smoking status, BMI, chronic conditions, and time since vaccination.

Our goal is to develop a predictive model that characterizes how these factors influence antibody responses and asses how well this model generalizes to a new independent dataset collected at a later time point. By doing so, we hope to identify key predictors of antibody levels and evaluate the robustness/generalizability of our model across different dataset.

## **Exploratory Analysis**

Our full combined dataset includes 6,000 patients and contains demographic and health information, time since the patient received the vaccine, and log-transformed antibody level. There are two different subsets of data: data on 5,000 patients was initially collected for model training, and data on 1,000 additional patients was independently collected several months later for model testing and evaluation.

Patients in both datasets have similar demographic and health characteristics (Table 1), but patients from the second collected dataset have a greater time since receiving the vaccine (an additional few months), and therefore slightly lower observed log-transformed antibody levels (Figures 1 and 2). Because we are more likely to observe lower antibody levels from patients in the testing dataset, it's possible that this difference may impact the prediction performance of our models, which are trained using the initial dataset. After plotting the most correlated quantitative variables versus log-transformed antibody level, we can see that the fitted line for the testing data is always flatter than the line of the training data, indicating a weaker relationship between these variables and the response (Figures 7-9).

Across gender and smoking status, there were very slight differences in the observed antibody levels. Women had slightly greater antibody responses than men overall (Figure 3 and Table 2), while current smokers had slightly lower antibody responses than former and never-smokers (Figure 5 and Table 4). There were no observed differences in antibody responses across race (Figure 4 and Table 3). The quantitative variables that were most correlated with log-transformed antibody level were BMI, weight, and age. It's important to note that several predictors are also correlated with each other, such as BMI and weight, BMI and height, and SBP and age (Figure 6), which will impact variable selection.

# **Model Training**

In this analysis, we trained three different models: Multiple Linear Regression (MLR), LASSO Regression, and Multivariate Adaptive Regression Splines (MARS). We ultimately selected MARS as the final model. The following sections provides each step in the model training process, from pre-processing to final model selection.

### **Data Pre-processing**

- We ensured that there were no missing values in the training data, missing data were imputed or removed.
- Continuous variables were untouched, while categorical variables were converted to factor types (such as race, gender, smoking).
- The response variable, log antibody, was log-transformed to normalize its distribution and reduce skewness.

# Multiple Linear Regression (MLR) Model

We started by fitting a MLR model with all available predictors in the dataset and the model was fit using ordinary least squares regression (OLS).

The model was trained using the lm() function and the training process involved fitting the model to the data, estimating the regression coefficients for each predictor, and computing the residuals. The code below was used:

```
mlr_model <- lm(log_antibody ~ ., data = dat1)</pre>
```

The coefficients were estimated through OLS regression, and the residuals were checked for normality. The model was trained on the entire training dataset, and no regularization was applied.

#### LASSO Model

To address potential multi-collinearity and perform feature selection, we used LASSO Regression, applying L1 regularization to shrink the coefficients of less important features to zero. The LASSO model was trained using the glmnet package.

Since LASSO is sensitive to differences in scale among predictor variables, numerical predictors were standardized before training to ensure fair comparison across variables. The preProcess() function from the caret package was used. The same transformations were applied to the validation and test datasets to maintain consistency.

The training procedure involved:

- 1. Creating a matrix of predictor variables (x) and a vector of the response variable (y)
- 2. Using cross-validation to select the best lambda (regularization parameter) based on the model's performance. The model with the lowest cross-validation error was used for evaluation.

```
lasso_model <- cv.glmnet(x_train, y_train, alpha = 1)
best_lambda <- lasso_model$lambda.min
lasso_final <- glmnet(x_train, y_train, alpha = 1, lambda = best_lambda)</pre>
```

## Multivariate Adaptive Regression Splines (MARS) Model

Non-linear regression MARS model automatically selects the best interactions and non-linear transformations of predictors. We first trained the MARS model without tuning, the used cross-validation to finetune it and determine the optimal number of terms and the degree of interactions.

The best parameters were selected as follows:

- nprune = 10: The final model had 10 terms (lowest Generalized Cross Validation score)
- degree = 1: The degree of interaction was set to 1, which considers only pairwise interactions between features.

#### Results

After testing three different models — Multiple Linear Regression (MLR), LASSO Regression, and Multivariate Adaptive Regression Splines (MARS) — we selected the final model based on performance evaluated on an independent test dataset (dat2). The decision was guided by two key metrics: Root Mean Squared Error (RMSE), which measures average prediction error (lower is better), and Adjusted R-squared, which accounts for model complexity and estimates the proportion of variation in antibody response explained by the model (higher is better).

The MARS model achieved the best performance, with an RMSE of 0.533 and an adjusted R-squared of 0.169, indicating that it explained about 17% of the variation in antibody response in the test dataset. In contrast, the Multiple Linear Regression model had a higher RMSE of 0.568 and a lower adjusted R-squared of 0.149, while the LASSO model, which automatically selects features, had the same RMSE as MLR (0.568) but a substantially lower adjusted R-squared of only 0.048. These results suggest that MARS not only provided the most accurate predictions but also captured more meaningful variation in the outcome compared to the other two models.

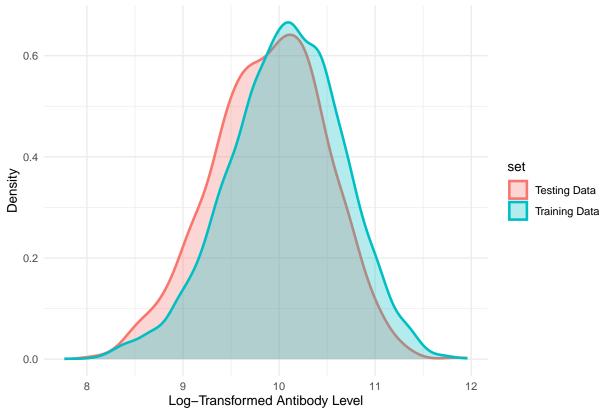
The final MARS model included both main effects and non-linear terms selected through internal tuning, allowing it to account for relationships that aren't strictly linear. For example, rather than assuming that BMI affects antibody levels in a straight line, MARS can capture thresholds or changes in direction — such as the effect of BMI being stronger in certain age groups or tapering off at higher values. This flexibility likely contributed to its improved performance.

Although adjusted R-squared is traditionally reserved for linear models, we adapted the formula for the MARS model by using the number of selected basis functions to represent model complexity. While this provides a rough but interpretable estimate of fit, we emphasize that RMSE is a more appropriate metric of predictive accuracy in non-linear settings like MARS. Nonetheless, reporting adjusted R-squared alongside RMSE allows for meaningful comparison across models using a consistent standard.

Table 1: Summary of Patient Testing and Training Data (N=6000)

Characteristic	Overall $N = 6,000^{1}$	Testing Data $N = 1,000^{1}$	Training Data $N=5{,}000^{1}$	$\mathbf{p} ext{-}\mathbf{value}^2$
Age	60.0 (57.0, 63.0)	60.0 (57.0, 63.0)	60.0 (57.0, 63.0)	0.9
Gender		,	,	0.7
Female	3,082 (51%)	509 (51%)	2,573 (51%)	
Male	2,918 (49%)	491 (49%)	2,427 (49%)	
Race		, ,		0.6
Asian	333 (5.6%)	55 (5.5%)	278 (5.6%)	
Black	1,235 (21%)	199 (20%)	1,036 (21%)	
Hispanic	548 (9.1%)	83 (8.3%)	465 (9.3%)	
White	3,884 (65%)	663 (66%)	3,221 (64%)	
Smoking				0.8
Current	589 (9.8%)	103 (10%)	486 (9.7%)	
Former	1,800 (30%)	296 (30%)	1,504 (30%)	
Never	3,611 (60%)	601 (60%)	3,010 (60%)	
Height (cm)	170.1 (166.1, 174.2)	170.2 (166.1, 174.2)	170.1 (166.1, 174.3)	0.7
Weight (kg)	80 (75, 85)	80 (75, 84)	80 (75, 85)	0.8
BMI	27.60 (25.80, 29.50)	27.60 (25.80, 29.60)	27.60 (25.80, 29.50)	0.9
Diabetes	929 (15%)	157 (16%)	772 (15%)	0.8
Hypertension	2,754 (46%)	456 (46%)	2,298 (46%)	0.8
Systolic Blood Pressure (mmHg)	130 (124, 135)	130 (124, 135)	130 (124, 135)	0.3
LDL Cholesterol (mg/dL)	110 (96, 124)	112 (96, 124)	110 (96, 124)	0.4
Time Since Vaccinated (days)	116 (82, 152)	171 (140, 205)	106 (76, 138)	< 0.001
Log-Transformed Antibody Level	10.06 (9.65, 10.45)	9.93 (9.50, 10.32)	10.09 (9.68, 10.48)	< 0.001

Figure 1: Distribution of Log-Transformed Antibody Level, by Data Set



 $<sup>^{\</sup>rm I}$ Median (Q1, Q3); n (%)  $^{\rm 2}$ Wilcoxon rank sum test; Pearson's Chi-squared test

Figure 2: Distribution of Days Since Vaccination, by Data Set

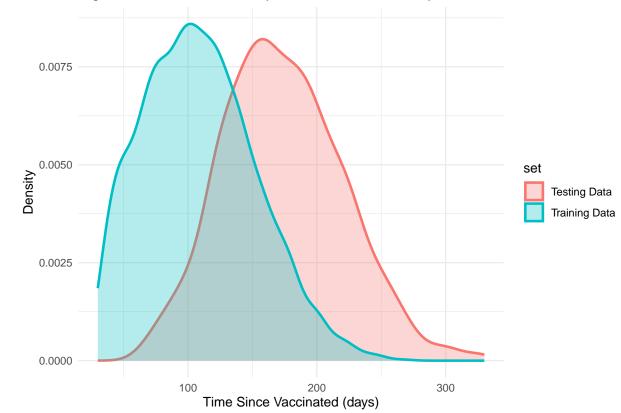


Figure 3: Distribution of Log-Transformed Antibody Level, by Gender

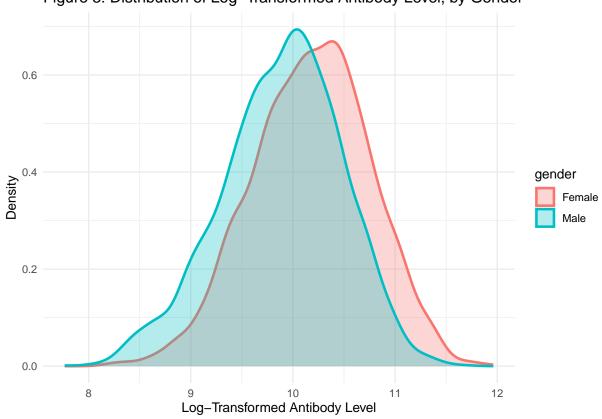


Figure 4: Distribution of Log-Transformed Antibody Level, by Race

o.6

Asian
Black
Hispanic
White

o.2

Log-Transformed Antibody Level

Figure 5: Distribution of Log-Transformed Antibody Level, by Smoking

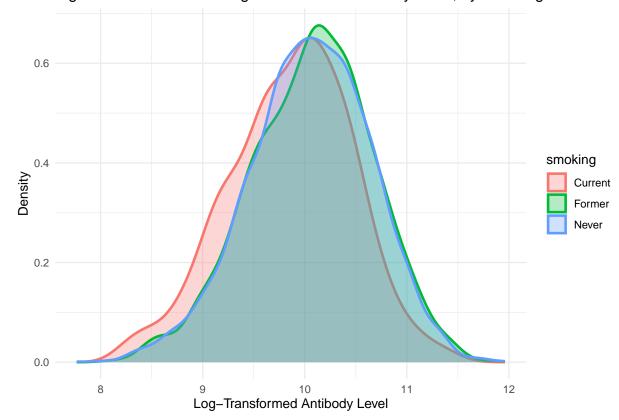


Table 2: Log-Transformed Antibody Level, by Gender

Characteristic	<b>Female</b> $N = 3,082^{1}$	<b>Male</b> $N = 2.918^{1}$	$\mathbf{p} ext{-}\mathbf{value}^2$
log_antibody	10.20 (9.79, 10.58)	9.93 (9.51, 10.30)	< 0.001

<sup>&</sup>lt;sup>1</sup>Median (Q1, Q3)

Table 3: Log-Transformed Antibody Level, by Race

Characteristic	Asian $N = 333^1$	Black $N = 1,235^1$	Hispanic $N = 548^1$	<b>White</b> $N = 3,884^{1}$	$\mathbf{p} ext{-}\mathbf{value}^2$
log_antibody	10.06 (9.62, 10.44)	10.08 (9.65, 10.44)	10.03 (9.61, 10.42)	10.06 (9.65, 10.46)	0.4

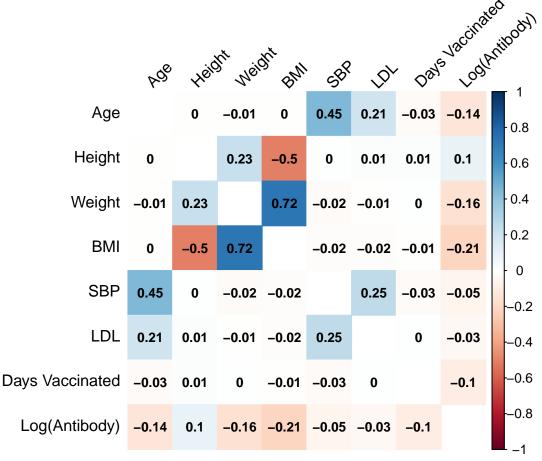
<sup>&</sup>lt;sup>1</sup>Median (Q1, Q3)

Table 4: Log-Transformed Antibody Level, by Smoking Status

Characteristic	Current $N = 589^1$	Former $N = 1,800^1$	Never $N = 3,611^1$	$\mathbf{p} ext{-}\mathbf{value}^2$
log_antibody	9.91 (9.46, 10.28)	$10.10 \ (9.66, \ 10.48)$	10.07 (9.68, 10.46)	< 0.001

<sup>&</sup>lt;sup>1</sup>Median (Q1, Q3)

**Figure 6: Correlation Matrix of Numerical Variables** 



<sup>&</sup>lt;sup>2</sup>Wilcoxon rank sum test

<sup>&</sup>lt;sup>2</sup>Kruskal-Wallis rank sum test

 $<sup>^2\</sup>mathrm{Kruskal\text{-}Wallis}$ rank sum test

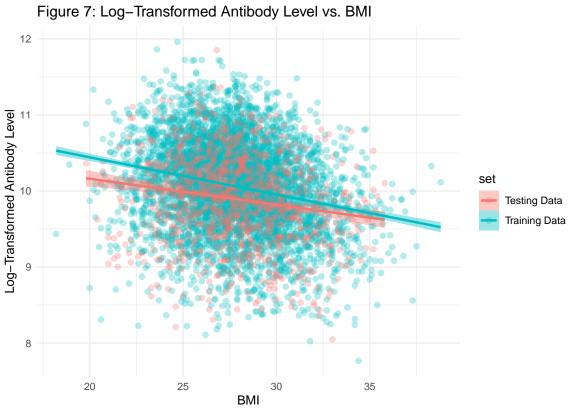


Figure 8: Log–Transformed Antibody Level vs. Weight

12

10

Set

Testing Data

Training Data

Log-Transformed Antibody Level

