Report

Abstract

This research aims to analyse academic performance data from a public school and verify the effectiveness of regression analysis methods in predicting student outcomes. The study focuses on three critical test scores: Math, Reading, and Writing. After tidying the data, various models were developed to assess the impact of multiple student characteristics and contextual factors on these test scores. The final models are selected using stepwise methods. This analysis aims to: 1. Construct Predictive Models: develop robust models for Math, Reading, and Writing scores using an extensive set of predictors. 2. Identify Key Predictors: Determine which predictors significantly influence each test score and explore potential interaction effects among these predictors. 3. Model Comparisons: evaluate the optimal prediction models for each test score and assess the potential for leveraging one test score to improve the prediction of another.

results

The resulting linear regression equations provide insights into future trends and identify the most influential variables affecting academic performance. This study contributes valuable information for designing targeted educational interventions and policies to enhance student success.

Introduction, Background, and Context

Education is the cornerstone of both individual and societal success. It opens doors of opportunity for intellect, and imagination, and enables each of its receivers to contribute to a progressive society with an improved quality of life. Governments and societies thus have a vested interest in ensuring that learners receive quality education. However, achieving this goal involves addressing the complex challenge of identifying the key components of quality education. Beyond cognitive ability, factors

such as socio-economic status, parental involvement, and school environment have recently been appreciated to significantly shape student performance. Despite current literature, there still lacks an absolute model to predict educational outcomes given certain determinants. Understanding those determinants, what they are, how they influence educational outcomes, and how they influence each other is crucial to educators, policymakers, and researchers aiming to design interventions and policies to enhance student performance. This report aims to contribute to the existing body of literature delving deeper into the complexities that impact academic performance. Using a dataset from a public school and analyzing three critical test scores: Math, Reading, and Writing, alongside a wide range of student characteristics and contextual factors, this analysis ultimately seeks to identify key predictors of academic success and understand how various factors interact to influence educational outcomes.

Methods

Data Cleaning and Exploration

The full dataset includes data from 948 students. We created summary tables to describe the distributions of all variables, stratified by gender (Table 1-3). To further investigate the distribution of test scores by subject, we created visualizations including stratified density and boxplots, and a correlation matrix of the test scores (Fig. 1-3) . Most variables were missing between 15 to 100 values, spread evenly across the students. Students with missing data were included in all exploratory analysis and visualizations, and were excluded from regression models when necessary. We found a handful of score outliers for each subject, but chose to leave these points in, as they seemed like valid data points (and not errors in data entry). Other minor data cleaning was performed.

Model Development, Diagnostics and Selection

To begin, we analyzed the covariates to identify those with high correlations. Focusing on the categorical variables, we performed a series of chi-square tests and one-way ANOVA to determine whether the covariates were dependent (Table 4-5). The p-values from these tests helped us identify pairs of variables with the weakest associations, allowing us to select five variables that were most independent of each other. These variables formed the basis of our initial linear model for predicting

individual scores while using stepwise-regression. Additionally, we utilized forward selection. The missing data were dropped because imputed values for categorical variables may complicate the interpretability of the result and it adds uncertainty that needs to be accounted for.

Math Score Prediction

With forward selection, we obtained seven significant predictors. Subsequently,we visualized math score between pairs of categorical variables. Among the pairs where the mean values score are associated within sub-categories, we tested for interaction, and found that weekly study hours and parental marital status has significant combined effect (Fig. 4). Upon examining diagnostic plots for the selected model, the residual plot revealed greater variance for lower values as well as deviation from normality in Q-Q plot (Fig. 5, Table 6), suggesting that the data do not align with the model's assumptions. To address this, we applied a transformation of $(Math\ Score + 1)^{1.3}$, as determined by box-cox, to the math scores variable, improving its compatibility with the model (Fig. 6-7, Table 7-9). Step-wise regression also resulted in the same model. Based on the Mallow's Cp and adjusted R^2, an optimal model for math score is suggested to have 15-17 main effect parameters. This aligns with our model selected from forward and backward model selection. LASSO suggests taking number of siblings into account (Fig. 8, Table 10). So, we decided to cross-validate and compare performances between model with and without number of sibling as a modifier.

Writing Score Prediction

Using forward selection, we obtained seven significant covariates to predict writing score, namely test preparation, gender, lunch type, parent education, ethnic group, parent marital status and sports practice status. We visualized writing score between pairs of categorical variables. Among the pairs where the mean values score are associated within sub-categories, we tested for interaction, and found that signigicant interaction coefficient for `gender:wkly_study_hours` and `lunch_type:wkly_study_hours`(Fig. 9, Table 10-11). The same end model was selected upon step-wise regression selection. The residuals were followed homoscedascticity, mean zero assumptions, but deviated from normality assumptions. Therefore, box-cox transformation was used to identify the optimal transformation, and model was re-fitted using $\sqrt(Writing\ Score)^3$(Fig. 10-12, Table 13). The models were further investigated using test based criteria. Based on the Mallow's Cp and

adjusted R^2, an optimal model for math score will should 13-18 main effect parameters (Fig. 13). This aligns with our model selected from forward and step-wise model selection. LASSO suggests taking number of siblings into account (Table 14). So, we decided to cross-validate and compare performances between model with and without number of sibling as a modifier to predict writing score as well.

Reading Score Prediction

We implemented step-wise regression to deduce significant covariates for modeling `reading score`. We note that most of the variables listed are categorical as mentioned prior. Therefore, we incorporated factor reveling in order to give some dimensions. First, we checked for non-linearity in the `reading_score` and apply transformation if need be. As seen above the after Y transformation where $Y^* = (reading\_score + 1)^{1.44}$, the residuals follow normality, homoscedascity and mean zero looking at the diagnostic plots (Fig. 14-15, Table 15-17). Next, we produced two additional models by incorporating additional covariates and testing for potential interactions. Three models were selected for cross-validation (Fig. 16-17, Table 18-19).

Model Validation

To evaluate its performance, we conducted 10-fold cross-validation using training datasets, and selected the model based on adjusted R^2 and RMSE values as well as the correlation between predicted and observed outcome. Among the two models each that were selected for math score and writing score, models with out sibling counts have lower root mean square error, higher R^2 has similar predictive ability as the more complex model (Table 20-21, Fig. 18-19). Similarly, the model 2 (with one interaction term) had the best performanceFig. 20) in predicting the reading score.

Results

Our dataset includes data from 948 students. A summary of their demographic and academic data can be found in Tables 1, 2, and 3 in the Appendix. We found that most students were the oldest children in their families, had between one to three siblings, rode the bus to school, and had standard (not free/reduced) lunch. The distributions for Math, Reading, and Writing test scores all

Table 1: Summary of Student Demographic Variables (N=948)

Characteristic	**Overall** N = 948[1]	**female** N = 488[1]	**male** N = 460[1]
Ethnic Group			
group A	80 (9.0%)	32 (7.0%)	48 (11%)
group B	171 (19%)	91 (20%)	80 (19%)
group C	277 (31%)	156 (34%)	121 (28%)
group D	237 (27%)	117 (25%)	120 (28%)
group E	124 (14%)	63 (14%)	61 (14%)
Unknown	59	29	30
Parents' Education			
some high school	163 (18%)	86 (18%)	77 (18%)
associate's degree	198 (22%)	101 (22%)	97 (23%)
bachelor's degree	104 (12%)	57 (12%)	47 (11%)
high school	176 (20%)	83 (18%)	93 (22%)
master's degree	55 (6.1%)	33 (7.1%)	22 (5.1%)
some college	199 (22%)	107 (23%)	92 (21%)
Unknown	53	21	32
Lunch Type			
standard	617 (65%)	309 (63%)	308 (67%)
free/reduced	331 (35%)	179 (37%)	152 (33%)
Parents' Marital Status			
single	213 (24%)	120 (26%)	93 (21%)
divorced	146 (16%)	75 (16%)	71 (16%)
married	516 (57%)	255 (55%)	261 (60%)
widowed	24 (2.7%)	12 (2.6%)	12 (2.7%)
Unknown	49	26	23
Practice Sport			
never	112 (12%)	57 (12%)	55 (12%)
regularly	343 (37%)	177 (37%)	166 (37%)
sometimes	477 (51%)	249 (52%)	228 (51%)
Unknown	16	5	11
Oldest Child (Yes/No)	604 (66%)	321 (68%)	283 (64%)
Unknown	30	13	17
# Siblings	2 (1, 3)	2 (1, 3)	2 (1, 3)
Unknown	46	28	18
Transport Means			
school_bus	509 (60%)	268 (61%)	241 (60%)
private	337 (40%)	174 (39%)	163 (40%)
Unknown	102	46	56

[1] n (%); Median (Q1, Q3)

had similar distributions (Fig. 1), were slightly left-skewed. These distributions were also relatively consistent after grouping by the amount of hours students spent studying weekly (<5, 5-10, >10 hours) (Fig. 2). Within each subject, the median score increased as the amount of time spent studying increased, with the lowest study group having the lowest scores in all subjects. Between the three study groups, there was a statistically significant difference in the distributions (Table 3). Finally, Math, Reading, and Writing scores were all heavily correlated (Fig. 3). All three scores have the same main effect parameters - weekly study hours, test preparation status, lunch type, parents' education and marital status, ethnic group and gender. However, they differ in the presence of some interaction terms. The combined effect of parents' marital status and weekly study hours is significant in predicting math and reading score, while weekly study hours' combined effect with male student and with lunch type is significantly associated with writing score.

Table 2: Summary of Student Academic Variables (N=948)

Characteristic	Overall N = 948[1]	female N = 488[1]	male N = 460[1]
Test Prep			
completed	322 (36%)	162 (35%)	160 (37%)
none	571 (64%)	297 (65%)	274 (63%)
Unknown	55	29	26
Weekly Study Hours			
< 5	253 (28%)	131 (28%)	122 (28%)
5-10	508 (56%)	263 (56%)	245 (56%)
> 10	150 (16%)	76 (16%)	74 (17%)
Unknown	37	18	19
Math Score	66 (56, 76)	64 (53, 74)	69 (59, 79)
Reading Score	70 (59, 80)	73 (63, 83)	65 (55, 76)
Writing Score	68 (57, 79)	74 (63, 83)	64 (53, 74)

[1] n (%); Median (Q1, Q3)

Table 3: Academic Variables by Time Spent Studying (N=911)

Characteristic	Overall N = 911[1]	< 5 N = 253[1]	5-10 N = 508[1]	> 10 N = 150[1]	p-value[2]
Test Prep					0.12
completed	314 (37%)	76 (32%)	175 (37%)	63 (43%)	
none	545 (63%)	160 (68%)	300 (63%)	85 (57%)	
Unknown	52	17	33	2	
Math Score	66 (56, 76)	62 (55, 73)	67 (56, 78)	70 (57, 79)	<0.001
Reading Score	70 (59, 80)	66 (57, 77)	71 (60, 80)	71 (60, 80)	0.025
Writing Score	68 (57, 79)	66 (55, 76)	70 (58, 79)	71 (58, 79)	0.020

[1] n (%); Median (Q1, Q3)
[2] Pearson's Chi-squared test; Kruskal-Wallis rank sum test

Figure 1: Distributions of Test Score by Subject

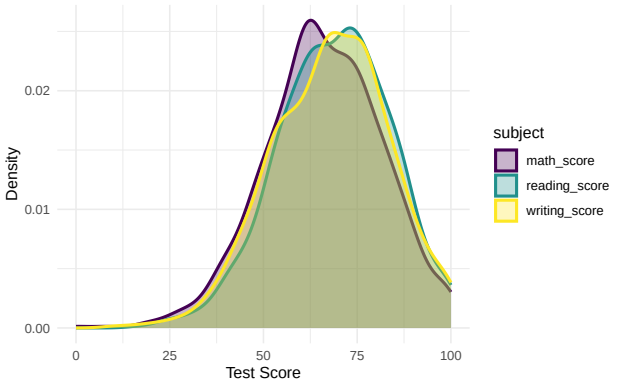
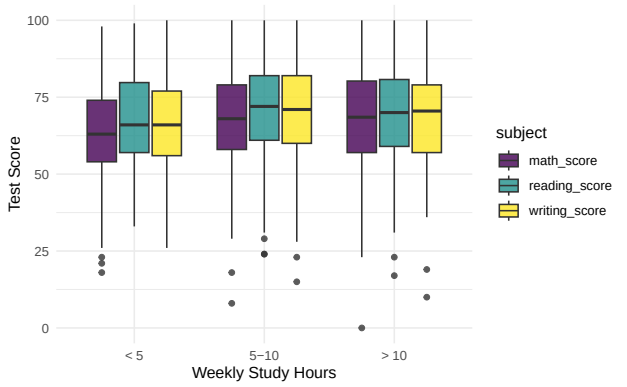


Figure 2: Distributions of Test Scores by Subject and Weekly Study H



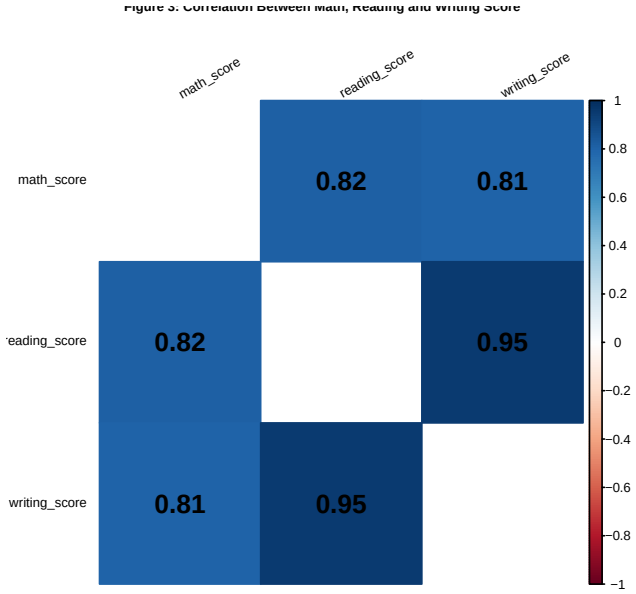


Table 4: Chi-Squared Test: Top 2 results (NS)

statistic	p.value	group
15.206	0.0553	ethnic_group:wkly_study_hours
9.385	0.0947	parent_educ:transport_means
9.282	0.0983	parent_educ:test_prep
7.462	0.1130	gender:ethnic_group
4.251	0.1190	test_prep:wkly_study_hours

Table 5: ANOVA: Number of Siblings v/s Other Covariates (<0.05)

term	df	statistic	p.value
is_first_child	1	16.366	5.68e-05
wkly_study_hours	2	3.024	4.91e-02

Figure 4: Interaction between Covariates (Significant in Pre

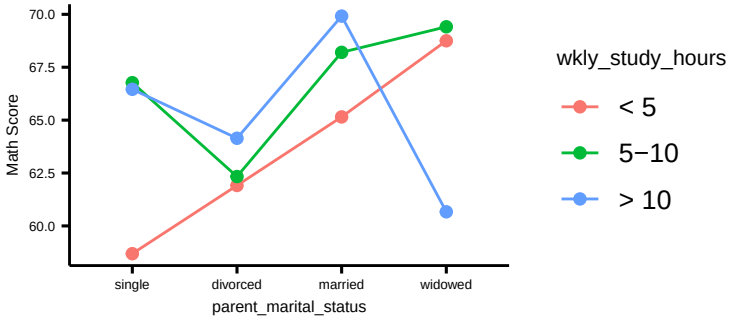


Figure 5: Model from Forward Selection

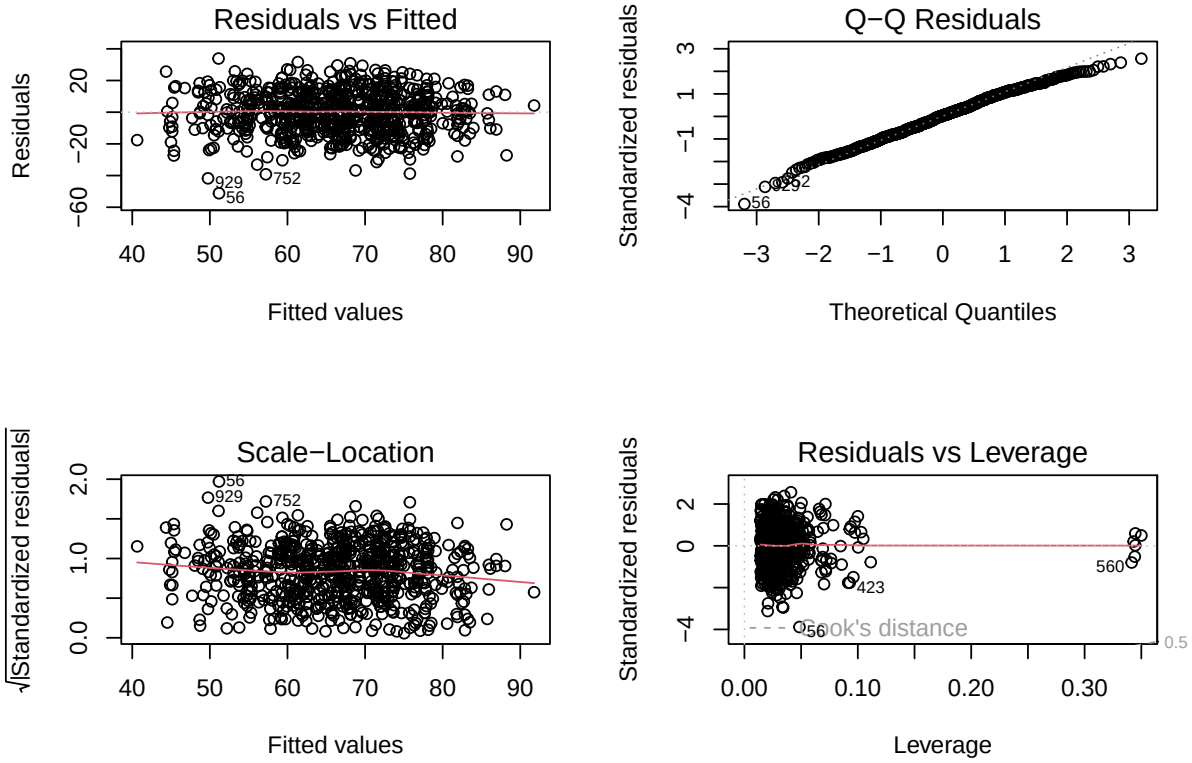


Table 6: Normality Test

statistic	p.value	method
0.9944663	0.010165	Shapiro-Wilk normality test

Figure 6: Box−cox Likelihood

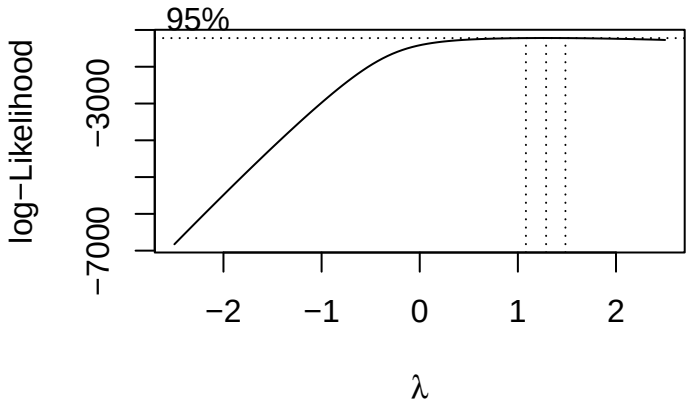


Figure 7: Math Model Assumptions After Transformation

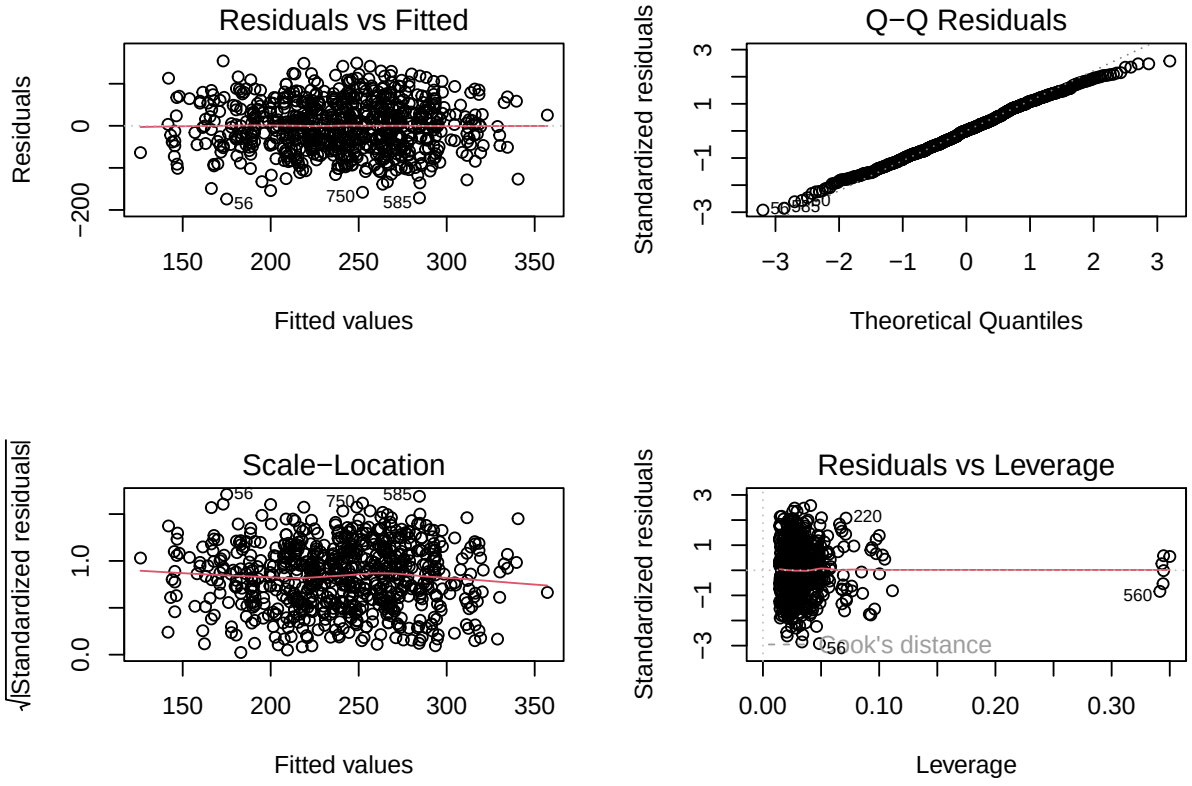


Table 7: Normality Test After Transformation

statistic	p.value	method
0.9958945	0.0558016	Shapiro-Wilk normality test

Table 8: Forward Selection: Coefficients

term	estimate	std.error	statistic	p.value
(Intercept)	59.45	2.95	20.17	0.00e+00
lunch_typefree/reduced	-12.25	1.07	-11.47	5.15e-28
test_prepnone	-6.14	1.07	-5.75	1.35e-08
gendermale	5.00	1.03	4.86	1.47e-06
ethnic_groupgroup B	0.57	2.07	0.28	7.82e-01
ethnic_groupgroup C	1.00	1.95	0.51	6.09e-01
ethnic_groupgroup D	4.93	1.97	2.50	1.27e-02
ethnic_groupgroup E	10.31	2.18	4.74	2.62e-06
parent_educassociate's degree	4.53	1.62	2.80	5.29e-03
parent_educbachelor's degree	5.49	1.89	2.91	3.78e-03
parent_educhigh school	-0.63	1.64	-0.39	7.00e-01
parent_educmaster's degree	7.08	2.35	3.01	2.69e-03
parent_educsome college	4.33	1.62	2.67	7.84e-03
parent_marital_statusdivorced	4.46	3.16	1.41	1.58e-01
parent_marital_statusmarried	8.06	2.31	3.48	5.26e-04
parent_marital_statuswidowed	14.38	8.10	1.78	7.62e-02
wkly_study_hours5-10	7.67	2.42	3.17	1.61e-03
wkly_study_hours> 10	10.12	3.19	3.17	1.56e-03
parent_marital_statusdivorced:wkly_study_hours5-10	-7.40	3.83	-1.93	5.37e-02
parent_marital_statusmarried:wkly_study_hours5-10	-5.97	2.87	-2.08	3.80e-02
parent_marital_statuswidowed:wkly_study_hours5-10	-8.55	9.14	-0.94	3.50e-01
parent_marital_statusdivorced:wkly_study_hours>10	-14.20	5.15	-2.76	5.96e-03

term	estimate	std.error	statistic	p.value
parent_marital_statusmarried:wkly_study_hours>10	-8.22	3.78	-2.18	2.98e-02
parent_marital_statuswidowed:wkly_study_hours>10	-16.81	11.52	-1.46	1.45e-01

Table 9: Forward Selection: Model Summary

r.squared	adj.r.squared	sigma	statistic	p.value	df	logLik	AIC	BIC	deviance	df.residual	nobs
0.32	0.29	14	14	1.8e-43	23	-2879.706	5809	5924	126790.8	695	719

Figure 8: Test−based Procedures For Math Score

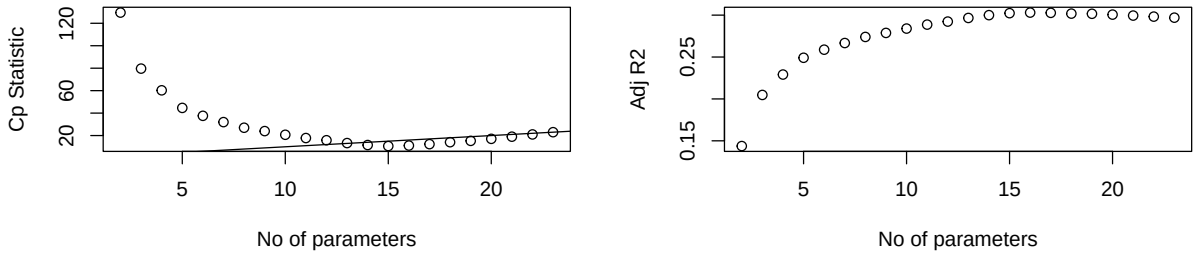


Table 10: LASSO Model For Math Score

term	step	estimate	lambda	dev.ratio
(Intercept)	1	65.269	0.05	0.004
nr_siblings	1	0.658	0.05	0.004

Figure 9: Interaction Between Covariates (Significant in Prediction)

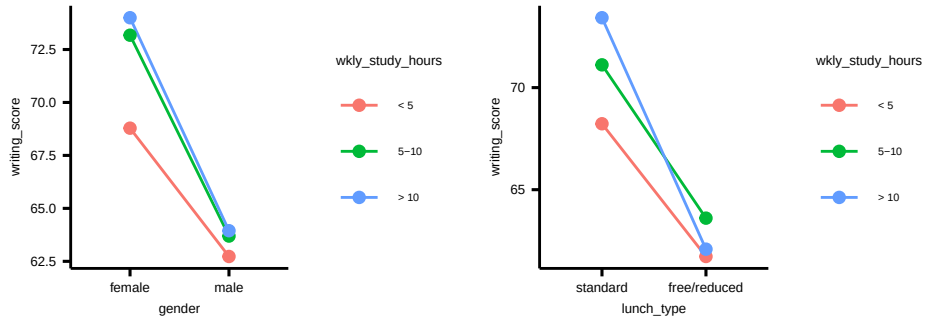


Table 11: Forward Selection: Coefficients

term	estimate	std.error	statistic	p.value
(Intercept)	568.09	35.27	16.11	1.00e-49
test_prepnone	-117.48	12.25	-9.59	1.61e-20
gendermale	-68.47	22.35	-3.06	2.27e-03
lunch_typefree/reduced	-97.08	22.94	-4.23	2.64e-05
parent_educassociate's degree	68.07	18.45	3.69	2.43e-04
parent_educbachelor's degree	99.07	21.40	4.63	4.39e-06
parent_educhigh school	-7.95	18.67	-0.43	6.71e-01
parent_educmaster's degree	151.74	27.12	5.60	3.19e-08
parent_educsome college	64.77	18.67	3.47	5.56e-04
ethnic_groupgroup B	-11.32	23.63	-0.48	6.32e-01
ethnic_groupgroup C	7.60	22.30	0.34	7.33e-01
ethnic_groupgroup D	65.57	22.66	2.89	3.93e-03
ethnic_groupgroup E	64.00	24.99	2.56	1.07e-02
parent_marital_statusdivorced	-25.76	18.76	-1.37	1.70e-01
parent_marital_statusmarried	46.44	14.25	3.26	1.18e-03
parent_marital_statuswidowed	63.62	39.53	1.61	1.08e-01
practice_sportregularly	44.60	19.25	2.32	2.08e-02
practice_sportsometimes	38.46	18.61	2.07	3.92e-02
wkly_study_hours5-10	58.65	21.93	2.68	7.65e-03

term	estimate	std.error	statistic	p.value
wkly_study_hours> 10	80.58	29.73	2.71	6.88e-03
gendermale:wkly_study_hours5-10	-63.27	27.40	-2.31	2.13e-02
gendermale:wkly_study_hours> 10	-67.54	36.09	-1.87	6.17e-02
lunch_typefree/reduced:wkly_study_hours5-10	-6.36	28.40	-0.22	8.23e-01
lunch_typefree/reduced:wkly_study_hours> 10	-77.34	37.21	-2.08	3.80e-02

Table 12: Forward Selection: Model Summary

r.squared	adj.r.squared	sigma	statistic	p.value	df	logLik	AIC	BIC	deviance	df.residual	nobs
0.38	0.36	150	18	0	23	-4542.314	9135	9249	16022165	682	706

Figure 10: Writing Score Model from Forward Selection

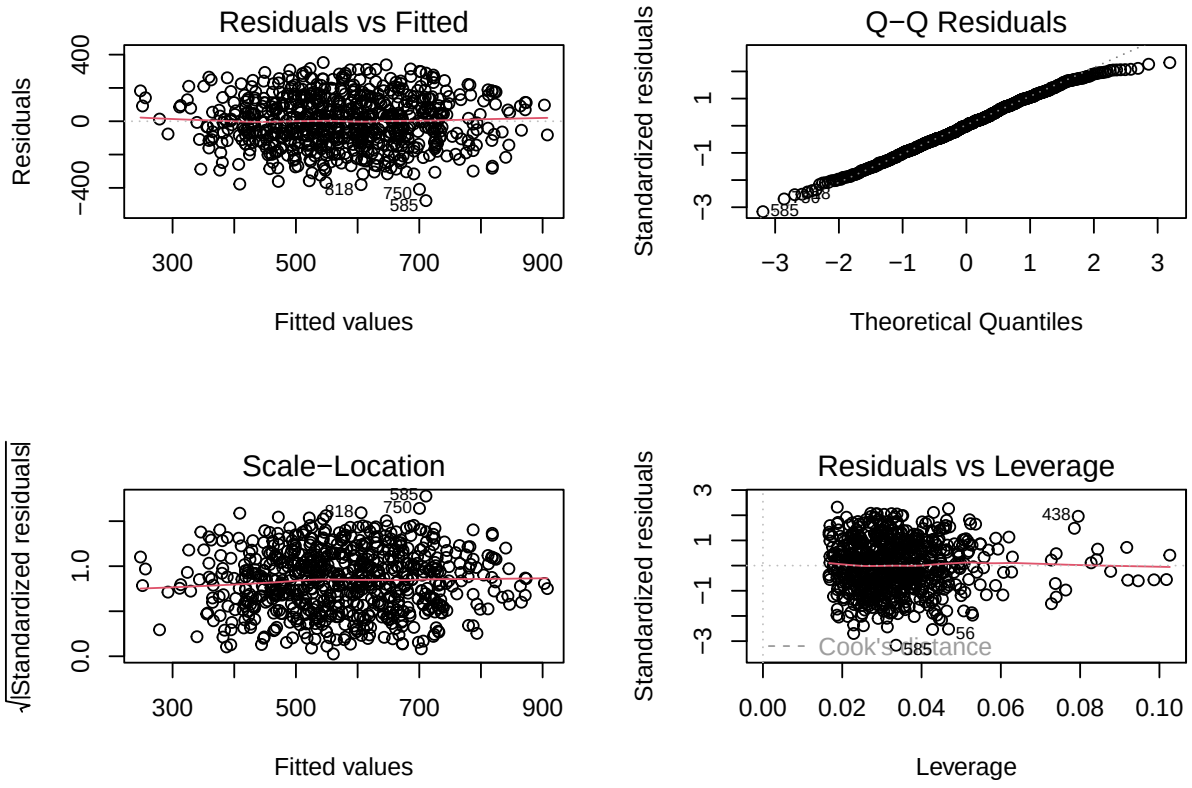


Table 13: Writing Score Model Normality Test

statistic	p.value	method
0.9897292	7.61e-05	Shapiro-Wilk normality test

Figure 11: Box−cox Likelihood

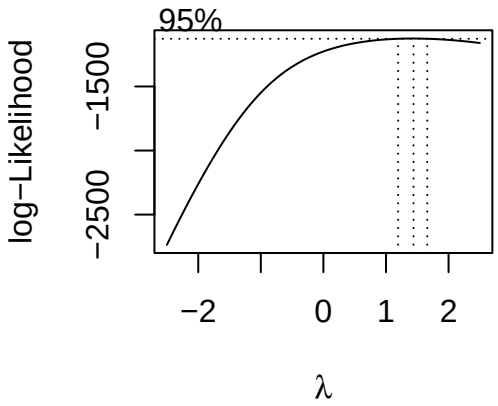


Figure 12: Assumptions for Writing Score Model After Transformation

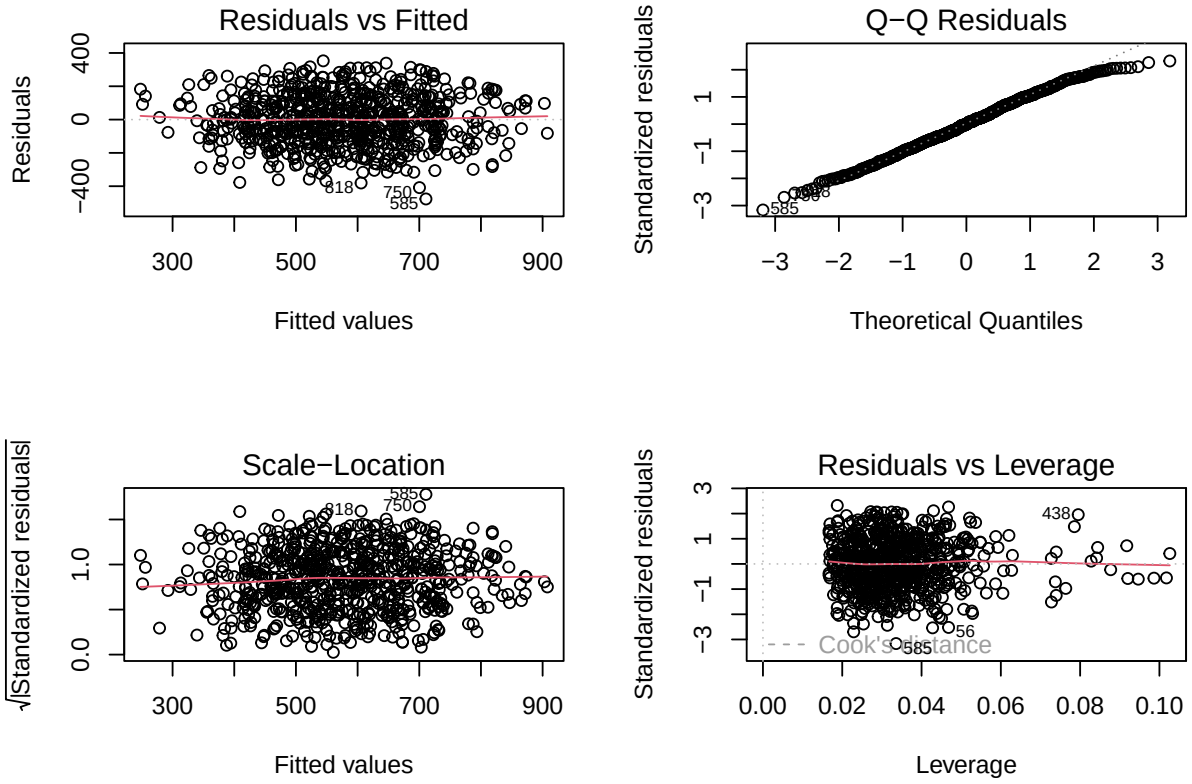


Figure 13: Test−based Procedures For Math Score

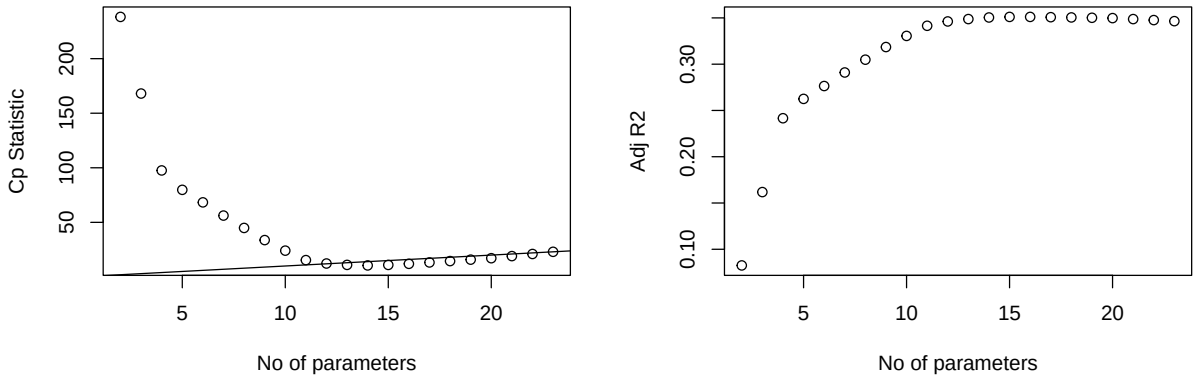


Table 14: LASSO Model For Writing Score

term	step	estimate	lambda	dev.ratio
(Intercept)	1	565.413	0.501	0.005
nr_siblings	1	8.268	0.501	0.005

Table 15: Reading Score Model Normality Test

statistic	p.value	method
0.992667	0.0013157	Shapiro-Wilk normality test

14: Reading Score Model Box−Cox Like

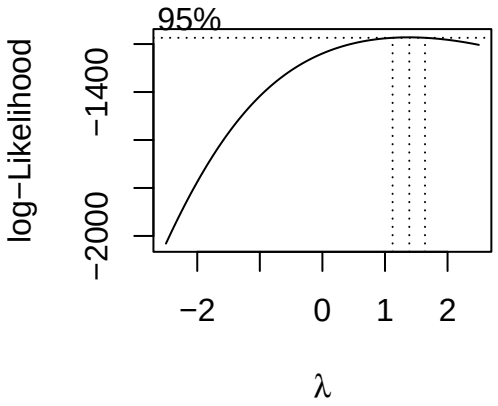


Figure 15: Reading Score Model 1 After Transformation

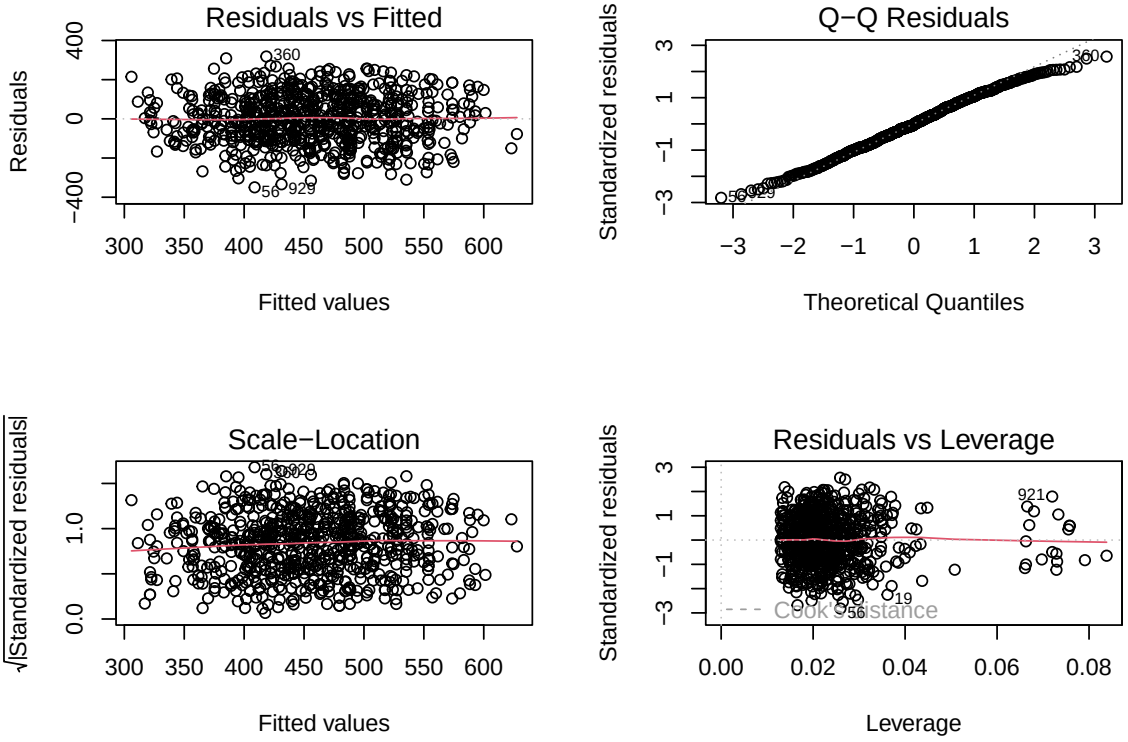


Table 16: Reading Score Model 1 - Normality Test After Transformation

statistic	p.value	method
0.9951259	0.0222055	Shapiro-Wilk normality test

Table 17: Reading Score: Model 1

term	estimate	std.error	statistic	p.value
(Intercept)	462.227	22.948	20.142	0.00e+00
gendermale	-65.952	9.470	-6.965	7.59e-12
ethnic_groupgroup B	-5.408	19.200	-0.282	7.78e-01
ethnic_groupgroup C	4.715	18.072	0.261	7.94e-01
ethnic_groupgroup D	34.147	18.324	1.863	6.28e-02
ethnic_groupgroup E	52.220	20.152	2.591	9.76e-03
parent_educassociate's degree	33.020	14.982	2.204	2.78e-02
parent_educbachelor's degree	52.783	17.467	3.022	2.60e-03
parent_educhigh school	-13.645	15.149	-0.901	3.68e-01
parent_educmaster's degree	72.752	21.774	3.341	8.78e-04
parent_educsome college	24.481	15.084	1.623	1.05e-01
test_prepnone	-65.914	9.896	-6.661	5.51e-11
parent_marital_statusdivorced	-15.475	15.196	-1.018	3.09e-01
parent_marital_statusmarried	32.690	11.613	2.815	5.02e-03
parent_marital_statuswidowed	37.303	31.434	1.187	2.36e-01
wkly_study_hours5-10	21.260	11.051	1.924	5.48e-02
wkly_study_hours> 10	7.719	14.597	0.529	5.97e-01

Table 18: Reading Score: Model 2

term	estimate	std.error	statistic	p.value
(Intercept)	455.328	26.016	17.502	0.00e+00
gendermale	-69.154	9.080	-7.616	8.59e-14
parent_educassociate's degree	36.032	14.301	2.520	1.20e-02
parent_educbachelor's degree	56.443	16.685	3.383	7.58e-04
parent_educhigh school	-13.273	14.458	-0.918	3.59e-01
parent_educmaster's degree	82.682	20.753	3.984	7.49e-05
parent_educsome college	26.521	14.343	1.849	6.49e-02
lunch_typefree/reduced	-77.506	9.430	-8.219	1.01e-15
ethnic_groupgroup B	-4.475	18.270	-0.245	8.07e-01
ethnic_groupgroup C	2.256	17.212	0.131	8.96e-01
ethnic_groupgroup D	35.107	17.432	2.014	4.44e-02
ethnic_groupgroup E	48.590	19.205	2.530	1.16e-02
test_prepnone	-66.606	9.424	-7.068	3.83e-12
parent_marital_statusdivorced	50.609	27.868	1.816	6.98e-02
parent_marital_statusmarried	75.210	20.430	3.681	2.50e-04
parent_marital_statuswidowed	135.675	71.503	1.897	5.82e-02
wkly_study_hours5-10	58.848	21.384	2.752	6.08e-03
wkly_study_hours> 10	92.628	28.144	3.291	1.05e-03
parent_marital_statusdivorced:wkly_study_hours5-10	-68.314	33.811	-2.020	4.37e-02
parent_marital_statusmarried:wkly_study_hours5-10	-50.385	25.341	-1.988	4.72e-02
parent_marital_statuswidowed:wkly_study_hours5-10	-105.332	80.681	-1.306	1.92e-01
parent_marital_statusdivorced:wkly_study_hours>10	-168.208	45.456	-3.700	2.32e-04

term	estimate	std.error	statistic	p.value
parent_marital_statusmarried:wkly_study_hours> 10	-92.343	33.329	-2.771	5.74e-03
parent_marital_statuswidowed:wkly_study_hours> 10	-183.445	101.686	-1.804	7.17e-02

Figure 16: Reading Score Model 2

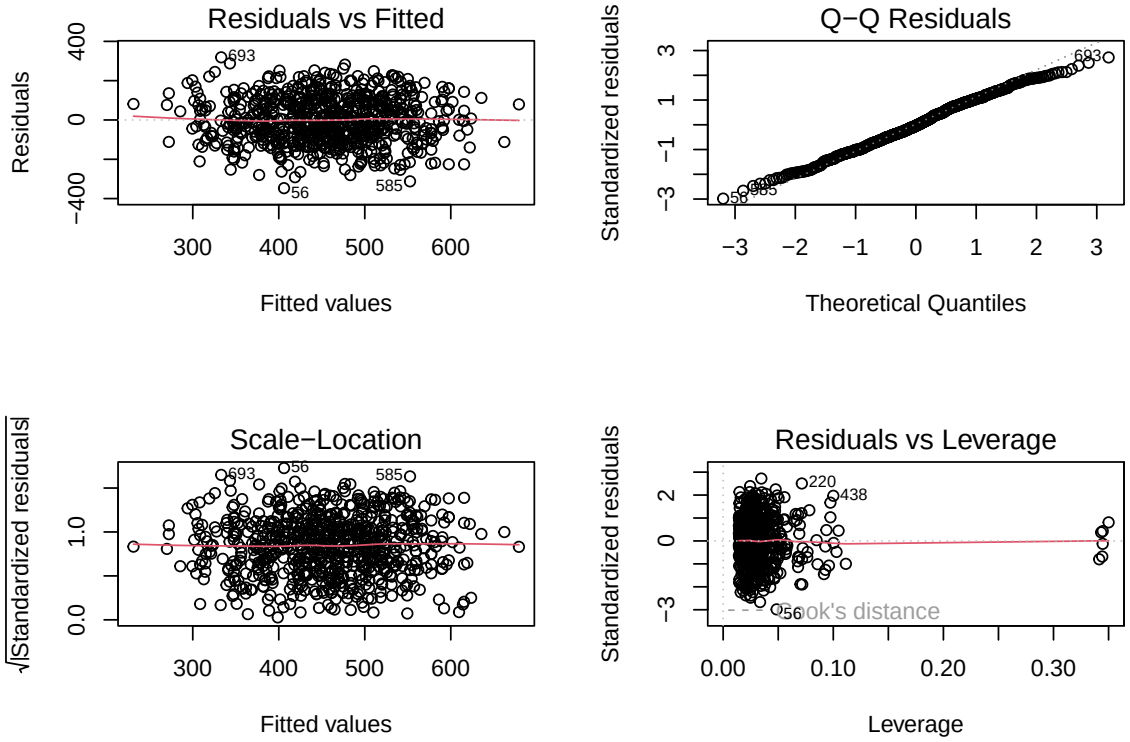


Table 19: Reading Score: Model 3

term	estimate	std.error	statistic	p.value
(Intercept)	525.600	72.134	7.286	9.08e-13
gendermale	-70.248	9.272	-7.576	1.20e-13
lunch_typefree/reduced	-77.050	9.691	-7.950	8.04e-15
is_first_childno:parent_educsome high school	-43.930	21.086	-2.083	3.76e-02
is_first_childyes:parent_educsome high school	-38.347	17.728	-2.163	3.09e-02
is_first_childno:parent_educassociate's degree	4.577	20.088	0.228	8.20e-01

term	estimate	std.error	statistic	p.value
is_first_childyes:parent_educassociate's degree	-2.973	17.210	-0.173	8.63e-01
is_first_childno:parent_educbachelor's degree	33.266	25.559	1.302	1.94e-01
is_first_childyes:parent_educbachelor's degree	17.545	20.166	0.870	3.85e-01
is_first_childno:parent_educhigh school	-77.601	21.657	-3.583	3.64e-04
is_first_childyes:parent_educhigh school	-39.030	16.996	-2.296	2.20e-02
is_first_childno:parent_educmaster's degree	40.272	36.783	1.095	2.74e-01
is_first_childyes:parent_educmaster's degree	43.490	24.380	1.784	7.49e-02
is_first_childno:parent_educsome college	-42.989	21.867	-1.966	4.97e-02
is_first_childyes:parent_educsome college	NA	NA	NA	NA
parent_marital_statussingle:wkly_study_hours< 5	-45.497	71.467	-0.637	5.25e-01
parent_marital_statusdivorced:wkly_study_hours< 5	7.992	73.110	0.109	9.13e-01
parent_marital_statusmarried:wkly_study_hours< 5	30.039	70.278	0.427	6.69e-01
parent_marital_statuswidowed:wkly_study_hours< 5	99.156	98.097	1.011	3.12e-01
parent_marital_statussingle:wkly_study_hours5-10	18.877	70.557	0.268	7.89e-01
parent_marital_statusdivorced:wkly_study_hours5-10	-3.342	70.862	-0.047	9.62e-01
parent_marital_statusmarried:wkly_study_hours5-10	41.672	69.965	0.596	5.52e-01
parent_marital_statuswidowed:wkly_study_hours5-10	41.945	77.611	0.540	5.89e-01
parent_marital_statussingle:wkly_study_hours> 10	51.112	73.132	0.699	4.85e-01

term	estimate	std.error	statistic	p.value
parent_marital_statusdivorced:wkly_study_hours> 10	-67.451	75.390	-0.895	3.71e-01
parent_marital_statusmarried:wkly_study_hours> 10	32.169	70.803	0.454	6.50e-01
parent_marital_statuswidowed:wkly_study_hours> 10	NA	NA	NA	NA
ethnic_groupgroup A:test_prepcompleted	31.199	29.474	1.059	2.90e-01
ethnic_groupgroup B:test_prepcompleted	14.231	23.496	0.606	5.45e-01
ethnic_groupgroup C:test_prepcompleted	2.532	21.272	0.119	9.05e-01
ethnic_groupgroup D:test_prepcompleted	36.452	22.970	1.587	1.13e-01
ethnic_groupgroup E:test_prepcompleted	61.828	24.623	2.511	1.23e-02
ethnic_groupgroup A:test_prepnone	-64.696	25.275	-2.560	1.07e-02
ethnic_groupgroup B:test_prepnone	-62.016	20.792	-2.983	2.96e-03
ethnic_groupgroup C:test_prepnone	-48.915	19.239	-2.543	1.12e-02
ethnic_groupgroup D:test_prepnone	-20.822	19.454	-1.070	2.85e-01
ethnic_groupgroup E:test_prepnone	NA	NA	NA	NA

Figure 17: Reading Score Model 3

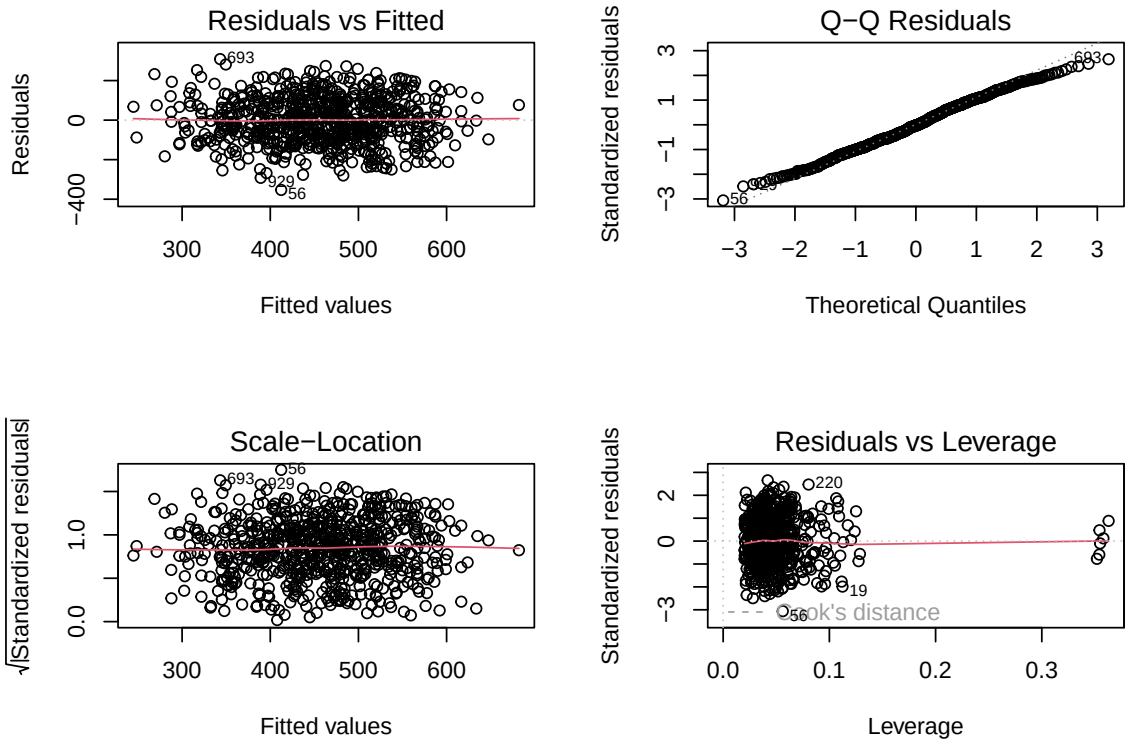


Figure 18: Correlation of Observed and Predicted Math Score

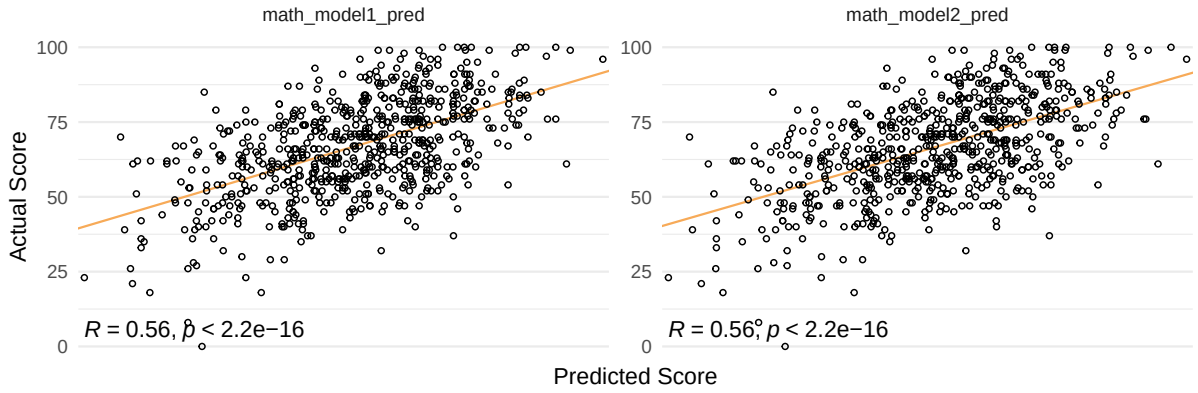


Table 20: Performance matrices of the 2 Models in Predicting (Math Score + 1)ˆ1.3

model_id	RMSE	Rsquared	MAE	RMSESD	RsquaredSD	MAESD
w/o no. of sibling	62.188	0.283	51.084	3.817	0.060	3.556
w/ no. of sibling	62.479	0.277	51.456	4.739	0.085	3.737

Figure 19: Correlation of Observed and Predicted Writing Score

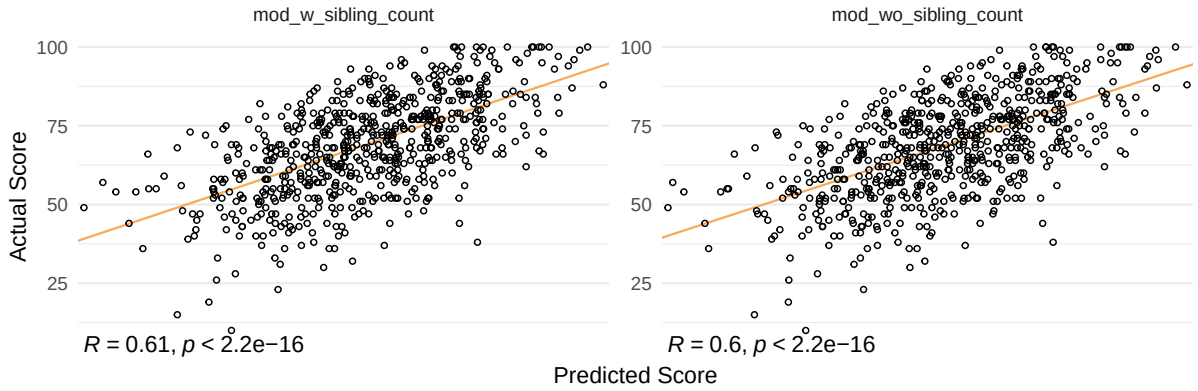


Table 21: Performance matrices of the 2 Models in Predicting (Writing Score)ˆ1.5

model_id	RMSE	Rsquared	MAE	RMSESD	RsquaredSD	MAESD
w/o no. of sibling	157.769	0.330	128.940	10.013	0.097	8.118
w/ no. of sibling	158.340	0.325	129.863	10.698	0.063	9.883

Table 22: Performance matrices of the 3 Multiple Linear Regression Models

model_id	intercept	RMSE	Rsquared	MAE	RMSESD	RsquaredSD	MAESD
1	TRUE	126.482	0.163	103.415	8.198	0.060	6.670
2	TRUE	120.738	0.242	99.581	7.413	0.067	6.934
3	TRUE	121.910	0.229	100.444	7.614	0.056	6.756