HW 5

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Question 1

A Using a binomial distribution's probability function, we can solve this problem. We can use a binomial distribution here because our scenario revolves around repeated Bernoulli distribution examples. Next, the probability of a binomial distribution is

$$\binom{N}{k} * p^k * (1-p)^{N-k}$$

In our scenario (fair coin, p = 0 .5), the event that there is an odd person out only occurs when there is 1 Tail and 2 Heads or 1 Head and 2 Tails. Using our formula, we obtain:

$$\binom{3}{1} * p^1 * (1-p)^2 = \frac{3}{8}$$

N represents the 3 flips and k represents the number of heads. However, we will multiply our probability by 2 to account for both aforementioned scenarios (their probabilities will be the same since it's a fair coin and p = 0.5 and both heads and tails will be 2 or 1). So, our total probability is

$$\frac{3}{8} * 2 = \mathbf{\frac{3}{4}}$$

B Using a binomial distribution's probability function, we can solve this problem. We can use a binomial distribution here because our scenario revolves around repeated Bernoulli distribution examples. Next, the probability of a binomial distribution is

$$\binom{N}{k} * p^k * (1-p)^{N-k}$$

In our scenario (fair coin, p = 0 .5), the event that there is an odd person out only occurs when there is 1 Tail and 3 Heads or 1 Head and 3 Tails. Using our formula, we obtain:

$$\binom{4}{1} * p^1 * (1-p)^3 = \frac{1}{4}$$

N represents the 3 flips and k represents the number of heads. However, we will multiply our probability by 2 to account for both aforementioned scenarios (their probabilities will be the same since it's a fair coin and p = 0.5 and both heads and tails will be 3 or 1). So, our total probability is

$$\frac{1}{4} * 2 = \mathbf{\frac{1}{2}}$$

C In this scenario, we can use a geometric distribution since we are looking for the expected value until there is an odd person out. The expected value for a geometric distribution is , $E[X] = \frac{1}{p}$. Thus, we need to find the probability. There are 2^5 total possible events since there are 5 people playing. There is an odd person out when there is 1 tail and 4 heads or 1 head and 4 tails. We can represent each one of these scenarios as $\binom{5}{1}$; we multiply this by 2 to account for both scenarios.

$$2 * \binom{5}{1} = 10$$

Thus, our probability is

$$\frac{10}{32}$$

Thus,

$$E[X] = \frac{1}{\frac{10}{32}} = \mathbf{3.2}$$

Question 2

A In total, there are 4 possible outcomes from the 2 flips with the given probability p for the coin being heads. The scenarios are: HH, TT, HT, TH. However, we are only concerned with the last two scenarios because the result of the first flip is only reported when the two coins are opposite coins. Looking at these two scenarios, HT, TH, we see that the first coin is only head in one out of the two scenarios where we actually report the first coin's value. Thus, our probability is one out of two, or $\frac{1}{2}$.

B For this scenario, since we are looking for the expected number of flips **until** we report a result, we can use a geometric distribution. The expected value for a geometric distribution is

$$E[X] = \frac{1}{p}$$

The probability that we report a result is again when we have HT, TH. For the first scenario, the probability of this occurring is $P(H) * P(T) = p * (1-p)$. For the second scenario, the probability of this occurring is $P(T) * P(H) = (1-p) * p$. We can add the probabilities of the two scenarios (law of total probability) to obtain $2p(1-p)$. Now, plugging the probability into the aforementioned expected value formula, we get

$$E[X] = \mathbf{\frac{1}{2p(1-p)}}$$

where X represents the number of flips

Question 3

A The probability that the pilot flies only happens when there is at least one female passenger. We can represent this as:
P(at least one female) = 1 - P(0 females).
For this probability, we can use binomial distributions since we are choosing between two values (male and female) several times. The formula for probability in a binomial distribution is:

$$\binom{N}{k} * p^k * (1-p)^{N-k}$$

Plugging this in (N is number of seats, k is number of males, and p is 0.5 for probability of gender) we get the following:

$$\binom{6}{6} * 0.5^6 * (0.5)^0 = 1 - \frac{1}{64} = \mathbf{\frac{63}{64}}$$

B For this question, we can use the same formula for binomial distribution since we are choosing between passengers showing up (p) or not (1 - p). The formula for probability in a binomial distribution is:

$$\binom{N}{k} * p^k * (1-p)^{N-k}$$

However, this time, we can use a summation with this probability to represent all scenarios with one or more empty seats. We will create this by choosing i to represent filled seats (hence why we can have up to s - 1 filled seats, as shown in the summation) out of the t tickets sold, and p will be our probability of the passenger turning up for departure. We also choose between the minimum of s-1 seats and t in case t is smaller than the number of available seats, s. Together, this will be the following:

$$\sum_{\mathbf{i=0}}^{\min(\mathbf{s-1,t})} \binom{\mathbf{t}}{\mathbf{i}} * \mathbf{p^i} * \mathbf{(1-p)^{t-i}}$$

Question 4

A Here, we can apply integration definition the normal distribution for all parts. It will be like the following:

$$P(h \in [495000, 505000]) = \int_{505000}^{495000} \frac{1}{500\sqrt{2\pi}} exp(\frac{-(x-500000)^2}{2 * 250000})dx$$

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B Here, we can apply integration definition the normal distribution for all parts. It will be like the following:

$$P(h \in [9000, \infty]) = \int_{9000}^{\infty} \frac{1}{500\sqrt{2\pi}} exp(\frac{-(x-5000)^2}{5000})dx$$

C Here, we can apply integration definition the normal distribution for all parts. It will be like the following:

$$P(h < 40, h > 60]) = 1 - \int_{40}^{60} \frac{1}{500\sqrt{2\pi}} exp(\frac{-(x-5000)^2}{5000})dx$$

Question 5

A The rate, or $\lambda = 0.1$ posts per 1 hour. Using the definition of the Poisson distribution, we obtain the probability function as follows:

$$P(X = 0) = \frac{e^{-.1} * .1^0}{0!}$$

$$P(X = 0) = e^{-.1}$$

B The rate, or $\lambda = 0.1$ posts per 1 hour. Using the definition of the Poisson distribution, we obtain the probability function as follows:

$$P(X = 1) = \frac{e^{-.1} * .1^1}{1!}$$

$$P(X = 1) = .1 * e^{-.1}$$

C The rate, or $\lambda = 0.1$ posts per 1 hour. Using the definition of the Poisson distribution, we obtain the probability function as follows:

$$P(X = 2) = \frac{e^{-.1} * .1^2}{2!}$$

$$P(X = 2) = .005 * e^{-.1}$$

D The rate in this case will be $\lambda = 24 * 0.1 = 2.4$ posts per 24 hours due to the time period we are looking at. Using the definition of the Poisson distribution, we obtain the probability function as follows:

$$P(X = 0) = \frac{e^{-2.4} * 2.4^0}{0!}$$

$$P(X = 0) = e^{-2.4}$$