HW 3

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Question 1

A To find how many students have taken *at least one* of the two mentioned math classes, we can say that this is equal to (total number of students - number of student who have taken *neither*). We are given that the total number of students is 200, and that the number of students who have taken neither is 2. Therefore,
200 - 2 = **198 students have taken at least one of calculus and linear algebra**.

B Let us say that x represents the number of students that have not taken calculus in the past. Thus, based on the given information, we can say that $4x$ represents the number of students that have not taken linear algebra in the past. Further, based on the given information that 187 students have taken *both* a calculus and linear algebra class in the past and 2 students have taken *neither*, we see that 200 - 187 = 13 students haven't taken one or the other, or neither. Further, to account for over-counting, we must add the 2 students who have taken *neither* to get: 13 + 2 = 15 (number of classes not taken). Then, we can create the following equation

$$x + 4x = 15$$

$$5x = 15$$

$$x = 3$$

Now, we have that 3 students haven't taken calculus, and thus 4(3) = 12 students who haven't taken linear algebra. Since there are 200 total students, we can subtract the number of students who haven't taken linear algebra, 200 - 12 = **188 students who have taken linear algebra in the past**.

Question 2

A Using slot method, we want to calculate the probability that all four cards that we are choosing are the same suit. Thus, for the first card, we have probability = 1 because it can be any card; next, we have 12/51 because we have already chosen one card of a particular suit. Essentially, we subtract a card from the numerator (there are 13 cards in a suit) and denominator since we are looking for the probability of a card of the same suit. For the next two slots, the same logic applies, and we get the following fractional answer:

$$\mathbf{1 * \frac{12}{51} * \frac{11}{50} * \frac{10}{49}}$$

B Using slot method, we want to calculate the probability that all four cards that we are choosing are red. Thus, for the first card, we have probability = 1 because it can be any card; next, we have 25/51 because we have already chosen one card of a particular suit. Essentially, we subtract a card from the numerator (there are 26 cards of a color) and denominator since we are looking for the probability of a card of the same color. For the next two slots, the same logic applies, and we get the following fractional answer:

$$\mathbf{1 * \frac{25}{51} * \frac{24}{50} * \frac{23}{49}}$$

C Using slot method, we want to calculate the probability that all four cards that we are choosing have different suits. Thus, for the first card, we have probability = 1 because it can be any card; next, we have 13/51 because we have already chosen one card of a particular suit. Essentially, we decrease the numerator by 13 at each slot (there are 13 cards of a suit, and we want different suits, so we cannot include outcomes with suits we have already chosen) and subtract the denominator by one since we are picking up a card each time. For the next two slots, the same logic applies, and we get the following fractional answer:

$$\mathbf{1 * \frac{39}{51} * \frac{26}{50} * \frac{13}{49}}$$

Question 3

A For this question, we want to calculate two probabilities, one that player one will have 4 land cards, and the other that player two will have 4 land cards. We will then multiply these probabilities to get the probability of them both occurring ($A \cap B$).
For the first probability that player one has exactly 4 land cards in their deck, we do 30 choose 3, because 3 cards chosen must not be land cards (40 - 10 = 30 are not land cards). Then we multiply this probability with 10 choose 4, since 4 of the cards we choose must come from 10 land cards that are in the deck. We then divide this product by the sample space, 40 choose 7 (7 cards from 40).
For the second probability that player two has exactly 4 land cards in their deck, we do 20 choose 3, because 3 cards chosen must not be land cards (40 - 20 = 20 are not land cards). Then we multiply this probability with 20 choose 4, since 4 of the cards we choose must come from 20 land cards that are in the deck. We then divide this product by the sample space, 40 choose 7 (7 cards from 40).
We then multiply the probabilities of both events, resulting in:

$$\frac{(\binom{30}{3} * \binom{10}{4})}{\binom{40}{7}} * \frac{(\binom{20}{3} * \binom{20}{4})}{\binom{40}{7}}$$

B For this question, we want to calculate two probabilities, one that player one will have 2 land cards, and the other that player two will have 3 land cards. We will then multiply these probabilities to get the probability of them both occurring ($A \cap B$).
For the first probability that player one has exactly 2 land cards in their deck, we do 30 choose 5, because 5 cards chosen must not be land cards (40 - 10 = 30 are not land cards). Then we multiply this probability with 10 choose 2, since 2 of the cards we choose must come from 10 land cards that are in the deck. We then divide this product by the sample space, 40 choose 7 (7 cards from 40).
For the second probability that player two has exactly 3 land cards in their deck, we do 20 choose 4, because 4 cards chosen must not be land cards (40 - 20 = 20 are not land cards). Then we multiply this probability with 20 choose 3, since 3 of the cards we choose must come from 20 land cards that are in the deck. We then divide this product by the sample space, 40 choose 7 (7 cards from 40).
We then multiply the probabilities of both events, resulting in:

$$\frac{(\binom{30}{5} * \binom{10}{2})}{\binom{40}{7}} * \frac{(\binom{20}{4} * \binom{20}{3})}{\binom{40}{7}}$$

C Looking at figure 1 below, we can see that we used two summations to describe the probability of player two (20 land cards in deck) having more land cards than player one (10 land cards in deck). Essentially, what I did here was to represent all situations where player two would have more land cards than player one for my event outcomes. For example, if player one has 1 land card, then to be in the outcome, player two can have 2 - 7 land cards; next, we could say if player 1 had 2 land cards, then player two can have 3 - 7 land cards. Thus we can represent this in the Python code for every scenario, using combinations to select the cards for each player, exactly like parts A and B. After this, I divided all of these by the sample space (40 choose 7) to get the probability. Then, I summed these probabilities to get the total probability.

Figure 1: This is the Python code I used to calculate and represent my summations.

```
probability = 0

for index in range(0, 8):
    for num in range(index + 1, 8):
        total_combinations = nCk(40, 7) * nCk(40, 7)
        event_outcomes = (nCk(30, 7 - index) * nCk(10, index)) * (nCk(20, 7 - num) * nCk(20, num))

        current_probability = event_outcomes / total_combinations

        probability = probability + current_probability
print(probability)
```

```
0.7843337197977621
```

Question 4

A Ω = **((AL1, P1, ADD), (AL1, P1, ADC), (AL1, P1, ADB), (AL1, P1, ADA), (AL1, S1, ADE), (AL1, S1, ADD), (AL1, S1, ADC), (AL1, S1, ADB), (AL1, S1, ADA))**
The above outcomes define the total sample space based on the schedule and timings provided for the classes.

B The equation for independence is given by: P$(A \cap B) = P(A) * P(B)$, given that events A and B are indepedent.
i. Applying the formula, we see that in this case,

$$\frac{4}{9} = \frac{4}{9} * \frac{9}{9}$$

$$\frac{4}{9} = \frac{4}{9}$$

Therefore, the statement is true and the events are independent.

ii. Applying the formula, we see that in this case,

$$0 = \frac{4}{9} * \frac{1}{9}$$

$$0 \neq \frac{4}{81}$$

Therefore, the statement is false and the events are dependent.

iii. Applying the formula, we see that in this case,

$$\frac{1}{9} = \frac{5}{9} * \frac{2}{9}$$

$$\frac{1}{9} \neq \frac{10}{81}$$

Therefore, the statement is false and the events are dependent.

Question 5

A Based on the given information, the probability that the student knows the answer is $\mathbf{P(K) = 0.7}$.

B Based on the given information, the probability that the student gets the answer correctly given that they know the answer is clearly $\mathbf{P(K|R) = 1}$.

C For this question, we can use the definition of conditional probability:

$$P(K|R) = \frac{P(R|K) * P(K)}{P(R)}$$

Further, we know (based on conditional probability) that

$$P(R) = P(R|K)P(K) * P(K) + P(R|K^c) * P(K^c)$$

The probability of $P(R|K^c) = \frac{1}{N}$ because with every answer choice N added to the question, we will always have a $\frac{1}{N}$ chance when choosing randomly. Lastly, we know that $P(K^c) = 1 - 0.7 = 0.3$ based on the given information and part A. Plugging values into this equation, we get:

$$P(R) = (1)(0.7) + \frac{1}{N} * (0.3)$$

Now, using the equation for P(R) in the equation for $P(K|R)$, we get:

$$\mathbf{P(K|R)} = \frac{\mathbf{(0.7)(1)}}{\mathbf{.7 + \frac{.3}{N}}}$$

D To find where P(K—R) ¿ .99, we plug in 0.99 as our probability. We get:

$$\frac{(0.7)(1)}{.7 + \frac{.3}{N}} = 0.99$$

Solving this equation, we get:

$$N = 42.43$$

answer choices will ensure a probability of 0.99.
We then round up to ensure that this is a whole number and would be a probability greater than .99, to get $\mathbf{N = 43}$