

# A comparative survey of SSVEP recognition algorithms based on template matching of training trials

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## Abstract

**Purpose** – Steady-state visual evoked potential (SSVEP) has been widely used in the application of electroencephalogram (EEG) based non-invasive brain computer interface (BCI) due to its characteristics of high accuracy and information transfer rate (ITR). To recognize the SSVEP components in collected EEG trials, a lot of recognition algorithms based on template matching of training trials have been proposed and applied in recent years. In this paper, a comparative survey of SSVEP recognition algorithms based on template matching of training trials has been done.

**Design/methodology/approach** – To survey and compare the recently proposed recognition algorithms for SSVEP, this paper regarded the conventional canonical correlated analysis (CCA) as the baseline, and selected individual template CCA (ITCCA), multi-set CCA (MsetCCA), task related component analysis (TRCA), latent common source extraction (LCSE) and a sum of squared correlation (SSCOR) for comparison.

**Findings** – For the horizontal comparative of the six surveyed recognition algorithms, this paper adopted the “Tsinghua JFPM-SSVEP” data set and compared the average recognition performance on such data set. The comparative contents including: recognition accuracy, ITR, correlated coefficient and R-square values under different time duration of the SSVEP stimulus presentation. Based on the optimal time duration of stimulus presentation, the author has also compared the efficiency of the six compared algorithms. To measure the influence of different parameters, the number of training trials, the number of electrodes and the usage of filter bank preprocessing were compared in the ablation study.

**Originality/value** – Based on the comparative results, this paper analyzed the advantages and disadvantages of the six compared SSVEP recognition algorithms by considering application scenes, real-time and computational complexity. Finally, the author gives the algorithms selection range for the recognition of real-world online SSVEP-BCI.

**Keywords** Non-invasive brain-computer interface, EEG signals, Template matching of training trials, Steady-state visual evoked potential, Optimal projection vector

**Paper type** Literature review

## 1. Introduction

Electroencephalograph (EEG) signal is one of the most commonly used brain signal types for non-invasive brain-computer interface researches and applications (Saha *et al.*, 2021). By constructing a communication pathway between the brain and the outside world by EEG signals, the participants of brain computer interface (BCI) can directly control various devices through the cognitive process of the brain, without relying on the peripheral nerves system and muscles system (Lebedev and Nicolelis, 2006). Three different categories of BCI patterns based on EEG signals are commonly used: event-related potential (ERP) (Ramirez-Quintana



*et al.*, 2021), sensorimotor rhythms (SMR) (Jin *et al.*, 2021) and steady-state visual evoked potential (SSVEP) (Liu *et al.*, 2021). Although the EEG based non-invasive BCI has a broad application prospect in many fields, the lower recognition accuracy and information transmission rate (ITR) of brain-controlled patterns are still the main reasons for restricting its applications (Yang *et al.*, 2021). Among the above three brain-controlled patterns, the SSVEP-BCI has the advantages of high recognition accuracy, high ITR and convenient to configure and utilize, so as to become a hot-spot for the research of BCI applications in recent years (Ming *et al.*, 2021; Yan *et al.*, 2021; Torres and Daly, 2021).

The component of SSVEP in EEG signals is expressed in the occipital lobe of the brain, and comes from the steady and continuous stimuli of different frequencies or phases received by the human's eye (Di Russo *et al.*, 2007). Based on such theory of evoking SSVEP component, the SSVEP-BCI designs different stimulus materials using flickers of different frequencies or phases. When a subject looks at a continuous fixed stimulus flicker for a long time, the SSVEP component of the corresponding frequency or phase can be analyzed and recognized in the occipital region from EEG signals (Chen *et al.*, 2015a). By recognizing the SSVEP component as brain patterns, the recognized brain patterns can be applied for the brain-controlled systems.

The earliest algorithm for recognizing SSVEP patterns was the canonical correlation analysis (CCA) algorithm (Lin *et al.*, 2006). Such algorithm constructed sine-cosine harmonic templates for different frequencies or phases, then computed the canonical relationship coefficients between each trial of EEG signals and each harmonic template, and selected the frequency or phase with the highest matching relationship as the recognition of the given trial of EEG signals. Although the computational process of the CCA algorithm is simple and efficient, such algorithm is highly sensitive to noises and does not have a good effect on frequency-intensive or phase-intensive stimuli. In addition, the biggest problem of CCA algorithm is that it is usually unable to distinguish closer stimuli. Therefore, a lot of research studies explored the improvements of conventional CCA algorithm, including filter bank CCA (FBCCA) (Chen *et al.*, 2015b), multi-way CCA (MCCA) (Zhang *et al.*, 2011), L1-regularized multi-way CCA (L1MCCA) (Zhang *et al.*, 2013), LASSO-regularized CCA (Zhang *et al.*, 2012) and multi-objective optimization based high-pass spatial filtering (Zhang *et al.*, 2022), and so on. These improvements are either to increase the signal-to-ratio of the EEG signals by filtering preprocess, or to make up for the insufficiency of the harmonic templates by constructing regularization constraints. The object of increasing signal-to-noise-ratio (SNR) and adding regularization is to improve the recognition accuracy of the conventional CCA algorithm for SSVEP-BCI.

Despite the aforementioned revised CCA algorithms achieve recognition performance improvements of SSVEP-BCI, but there is still a big bottleneck in the matching of harmonic templates. To break the bottleneck of directly using sine-cosine harmonic templates, researchers have proposed to regard the EEG trials that contained any SSVEP components as the training set, and then determine the specific frequency or phase of each testing trial to be recognized through the correlation coefficients between the training set and the testing trial (Stawicki *et al.*, 2021). Based on the correlation coefficients computational process (Edelmann *et al.*, 2021), different templates built by the existed training trials have been used for the recognition of SSVEP trials. The first recognition algorithm of using template matching of existed training trials was individual-template CCA (ITCCA) algorithm (Bin *et al.*, 2011). The algorithm directly used the average mean of existed training trials as a matching template for each stimulus category, and computed the correlation coefficients with the EEG trial that should be identified to recognize which stimulus category belonged to such EEG trial. In addition, the multiple-set CCA (MsetCCA) algorithm (Jiao *et al.*, 2018) learned the optimal matching template from multiple sets of existed training trials by a joint spatial filter, and then computed the correlation coefficients for the recognition of the following EEG trials. In recent years, a series of component analysis algorithms based on the SSVEP feature space

projections have been proposed, including task-related component analysis (TRCA) algorithm (Nakanishi *et al.*, 2017), latent common source extraction (LCSE) algorithm (Kumar and Reddy, 2019a) and sum of squared correlations (SSCOR) algorithm (Kumar and Reddy, 2019b). These algorithms all solved the optimal projection matrix from their respective perspectives, projected the existed EEG training trials to the new feature space that contained the SSVEP components, and then performed the optimal template matching for SSVEP categories recognition using the projected feature space.

For the researches among nearly a decade, SSVEP-BCI has produced a variety of different stimulus paradigms, and the recognition algorithms of SSVEP has also evolved from conventional sine-cosine harmonic templates matching to the optimal SSVEP components projected templates matching of existed training trials. However, current template matching of training trials algorithms were applied and tested to different stimulus paradigms and the experimental scenarios. To give an objective horizontal comparison, this paper selected six templates matching of training trials SSVEP recognition algorithms, and gave a comparative survey of selected recognition algorithms under the same stimulus paradigm and experimental scenario. This paper gave a detailed review of SSVEP-BCI recognition accuracy, ITR and running time under different time ranges of presenting stimuli. Based on the recognition results, the correlation coefficients and R-square values were also given for the comparative algorithms. In addition, ablation studies of all six surveyed template matching algorithms of training trials were given for different parameters settings, such as number of electrodes, number of trials and using filter bank or not. Finally, some suggestions on how to select the SSVEP-BCI recognition algorithms for different application scenarios were discussed.

The contribution of this paper is: Based on the “Tsinghua JFPM-SSVEP” data set, we compared and surveyed different indicators on recently proposed template matching of training trials algorithms, and provided advices for how to select the most suitable recognition algorithms in actual SSVEP-BCI scenarios. The rest of the paper is organized as follow: [Section 2](#) gives the survey of the conventional CCA algorithm and the five selected algorithms based on template matching of training trials. [Section 3](#) presents the comparative experiments and gave the results. [Section 4](#) deals with the ablation study and discussion on how to select suitable recognition algorithm. [Section 5](#) concludes the paper.

## 2. SSVEP recognition algorithm based on template matching of training trials

The earliest SSVEP recognition algorithm is the CCA algorithm, but it is limited to the matching ability of sine-cosine harmonic templates. In recent years, researchers have proposed SSVEP recognition based on template matching of training trials. We reviewed five representative algorithms, MsetCCA, ITCCA, TRCA, LCSE and SSCOR, for comparison and survey.

### 2.1 CCA algorithm

The CCA is a multivariate statistical computation algorithm, which was first proposed by [Hotelling \(1957\)](#), to compute the correlations between two different data sets. Assuming that a data set was composed of two random variables,  $X \in R^{I_1 \times J}$  and  $Y \in R^{I_2 \times J}$ , the purpose of CCA algorithm is to obtain the maximum correlation between the linear transformation of the data set by two liner transformations  $x = w^T X$  and  $y = v^T Y$ . This process can be expressed by the following equation:

$$\rho = \max_{w,v} \frac{E[xy^T]}{\sqrt{E[xx^T]E[yy^T]}} = \max_{w,v} \frac{w^T XY^T v}{\sqrt{w^T XX^T w v^T YY^T v}} \quad (1)$$

Where,  $\rho$  represents the maximum correlation coefficients between two data sets. The optimization problem of [equation \(1\)](#) can be solved by generalized eigenvalue decomposition.

Lin *et al.* (2006) first adopted the conventional CCA algorithms in the recognition application of SSVEP-BCI. Let us assume that there are  $M$  sparkle frequencies of stimuli in a SSVEP-BCI paradigm, we denote  $X \in R^{N_c \times N_t}$  to represent the recorded EEG signals from  $N_c$  electrodes, and there are  $N_t$  sampling points in each electrode.  $Y_m$  represents the corresponding reference signals ( $f_m (m = 1, 2, \dots, M)$ ) of  $m$  sparkle frequencies. The reference signals as a template are consisted by a series of sine-cosine harmonics:

$$Y_m = \begin{pmatrix} \sin(2\pi f_m t) \\ \cos(2\pi f_m t) \\ \dots \\ \sin(2\pi H f_m t) \\ \cos(2\pi H f_m t) \end{pmatrix}, t = \frac{1}{F}, \frac{2}{F}, \dots, \frac{J}{F} \quad (2)$$

Where  $H$  represents the number of sine-cosine harmonics to be used, and  $F$  represents the sampling rate. In the comparative experiments of CCA algorithms (Dmochowski *et al.*, 2014), the number of 2 or 3 harmonics was proved to be the best selection in the SSVEP recognition. By solving equation (1), the correlation coefficients between the EEG signals and each referenced signals were computed, and then the sparkle frequency with the largest correlation coefficient was deemed to the recognized result:

$$f_t = \max_{f_m} \rho_m, \quad m = 1, 2, \dots, M \quad (3)$$

### 2.2 ITCCA algorithm

The ITCCA algorithm was the earliest SSVEP recognition algorithm that used non-harmonic templates. This algorithm first computed the individual template of SSVEP by averaging the recorded trials of specific frequency, and then matched the EEG trial to be recognized to the corresponding frequency. Then, the recognized frequency can be encoded to the control system as the SSVEP-BCI (Bin *et al.*, 2011). For each target  $n$  of stimulus frequency, the individual template  $\bar{X}_n \in R^{N_c \times N_t}$  can be obtained by averaging all training trials:

$$\bar{X}_n = \frac{1}{N_t} \sum_{h=1}^{N_t} X_{nh} \quad (4)$$

Where  $X_{nh}$  represents the  $N_t$  training trials of the  $n$ -th stimulus frequency. For the matching process of using ITCCA algorithm, the conventional sine-cosine harmonic templates are replaced by the individual template computed by equation (4). Then, a similar method is used to compute the correlation coefficients of each stimulus frequency:

$$\rho_n = \max_{w_x, w_{\bar{x}}} \frac{E \left[ w_x^T X \bar{X}_n^T w_{\bar{x}} \right]}{\sqrt{E \left[ w_x^T X X^T w_x \right] E \left[ w_{\bar{x}}^T \bar{X}_n \bar{X}_n^T w_{\bar{x}} \right]}} \quad (5)$$

For the correlation coefficients computed from all stimulus frequencies, the maximum value can be obtained by equation (3), and the corresponding stimulus frequency is the matching result of the ITCCA algorithm.

### 2.3 MsetCCA algorithm

The MsetCCA algorithm (Jiao *et al.*, 2018) also no longer used a sine-cosine harmonic template as the reference template. From reference (Jiao *et al.*, 2018), researchers have found that for the

same subject, there were some common features for a certain stimulus frequency. Such common features can be extracted from a number of trials for the specific frequency, and can be suitable regarded as the reference template for the SSVEP recognition. To learn the optimal reference template from a multiple sets of training EEG trials by a joint spatial filter for different stimulus frequencies, the MsetCCA algorithm used the learned template for the recognition of SSVEP. The MsetCCA algorithm is a general form of CCA algorithm that extended from two sub-data sets. The purpose of such algorithm is to optimize the object function among canonical variables, which can be done by maximizing the correlation coefficients between canonical variables in the multiple sets. In the recognition of SSVEP by the MsetCCA algorithm, the strategy of maximizing eigenvalue of the co-correlation matrix is usually adopted in computation. It is assumed that  $X_{1,m}, X_{1,m}, \dots, X_{1,m} \in R^{N_c \times N_i}$  represent the  $N$  collected EEG trials from the  $m$ -th frequency, based on multivariate linear transformations  $w_{1,m}, w_{2,m}, \dots, w_{N,m}$ , the MsetCCA algorithm maintained the maximum correlation coefficients of canonical variables  $z_{1,m}, z_{2,m}, \dots, z_{N,m}$  by a joint spatial filter transform  $z_{i,m} = w_{i,m}^T X_{i,m} (i = 1, 2, \dots, N)$ . In general, the EEG trials to be processed are normalized to a criterion signal with a mean value of zero and a constant variance. The objective function to obtain the largest correlation between canonical variables can be defined as:

$$\begin{aligned} \max_{w_{1,m}, \dots, w_{N,m}} \quad & \rho = \sum_{i \neq j}^N w_{i,m}^T X_{i,m} X_{j,m}^T w_{j,m} \\ \text{s.t.} \quad & \frac{1}{N} \sum_{i=1}^N w_{i,m}^T X_{i,m} X_{i,m}^T w_{i,m} = 1 \end{aligned} \quad (6)$$

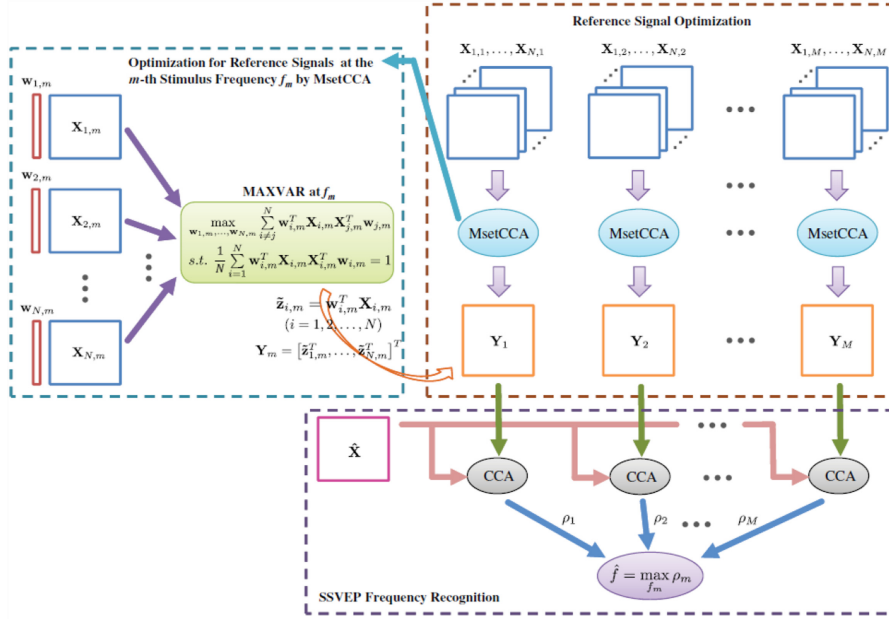
Where  $X_{i,m}, X_{j,m}$  represent the EEG trials from two different recording sessions.  $w_{i,m}, w_{j,m}$  represent the multivariate linear transformation, and satisfy  $z_{i,m} = w_{i,m}^T X_{i,m} (i = 1, 2, \dots, N)$ . The above quadratic optimization problem with restricted conditions can be solved by the Lagrangian multiplier method to extract the  $N$  eigenvectors that corresponding to the  $N$  largest eigenvalues:  $w_{1,m}, w_{2,m}, \dots, w_{N,m}$ . By performing the transformation of the eigenvectors to obtain the canonical variables  $z_{1,m}, z_{2,m}, \dots, z_{N,m}$ , and the template of the MsetCCA algorithm can be expressed as:

$$Y_m = [z_{1,m}^T, z_{2,m}^T, \dots, z_{N,m}^T]^T \quad (7)$$

After optimization, the correlation between the canonical variables  $Y_m$  and the stimulus frequency  $f_m$  is the largest, and the reference templates consisted by  $Y_m$  can be used for solving the CCA processing, and the average recognition accuracy is significantly higher than conventional sine-cosine harmonic templates. For the testing process, the tested EEG trials  $\hat{X}_m$  and the optimal reference templates  $Y_m$  are computed by [equation \(1\)](#) to give the recognition results. The training process of the MsetCCA algorithm needed to divide the EEG trials into a training set and a testing set, and a  $n * n$  cross-validation method is used for the actual recognition of SSVEP-BCI. [Figure 1](#) illustrated the flow chart of the MsetCCA algorithm.

#### 2.4 TRCA algorithm

The TRCA algorithm ([Nakanishi et al., 2017](#)) is different from the conventional CCA algorithm. Instead of matching the collected EEG trials with the obtained templates by solving the maximum correlation coefficients, this algorithm used an optimized method to find the task-related parts from the stimulus frequency, which was more accurate for the



**Figure 1.**  
Flow chart of MsetCCA  
algorithm

**Source(s):** Derived from Reference (Jiao *et al.*, 2018)

SSVEP recognition. For the TRCA algorithm, the EEG trial  $X(t) \in R^{N_c \times N_t}$  was composed of the task-related signals  $s(t) \in R$  and the task-independent signals  $n(t) \in R$ :

$$X_i(t) = a_{i1}s(t) + a_{i2}n(t), \quad i = 1, 2, \dots, N_c \quad (8)$$

Where  $i$  represents the channel for collecting EEG signals, and  $N_c$  represents the number of channels.  $a_{i1}$  and  $a_{i2}$  represent the correlation coefficient of projecting the original collected EEG trial into task-related part and task-independent part. The purpose of the TRCA algorithm is to recover the task-related components from the collected EEG trial:

$$Y(t) = \sum_{i=1}^{N_c} w_i X_i(t) = \sum_{i=1}^{N_c} (w_i a_{i1} s(t) + w_i a_{i2} n(t)) \quad (9)$$

The above-mentioned signal recovery task can be achieved by maximizing the covariance matrix between experiments. The  $m$ -th EEG trial and the corresponding task-related components can be denoted as  $X^{(m)}(t)$  and  $Y^{(m)}(t)$ ,  $m = 1, 2, \dots, N_t$ , respectively. The duration of each experiment is  $T$ , and the time duration of collecting the EEG signals is fixed at  $t \in [t_m, t_m + T]$ . Then, the covariance matrices of two EEG trials,  $m_1$ -th and  $m_2$ -th, can be defined as:

$$\begin{aligned} C_{m_1, m_2} &= \text{Cov}(Y^{(m_1)}(t), Y^{(m_2)}(t)) \\ &= \sum_{i_1, i_2=1}^{N_c} w_{i_1} w_{i_2} \text{Cov}(X_{i_1}^{(m_1)}(t), X_{i_2}^{(m_2)}(t)) \end{aligned} \quad (10)$$



The combination of covariance matrices can be generalized to all EEG trials:

$$\sum_{m_1, m_2=1, m_1 \neq m_2}^{N_t} C_{m_1, m_2} = \sum_{m_1, m_2=1, m_1 \neq m_2}^{N_t} \sum_{i_1, i_2=1}^{N_c} w_{i_1} w_{i_2} \text{Cov}\left(X_{i_1}^{(m_1)}(t), X_{i_2}^{(m_2)}(t)\right) = w^T S w \quad (11)$$

In addition, the variance of event-related components has the following constraint condition:

$$\text{Var}(Y(t)) = \sum_{i_1, i_2=1}^{N_c} w_{i_1} w_{i_2} \text{Cov}(X_{i_1}(t), X_{i_2}(t)) = w^T Q w \quad (12)$$

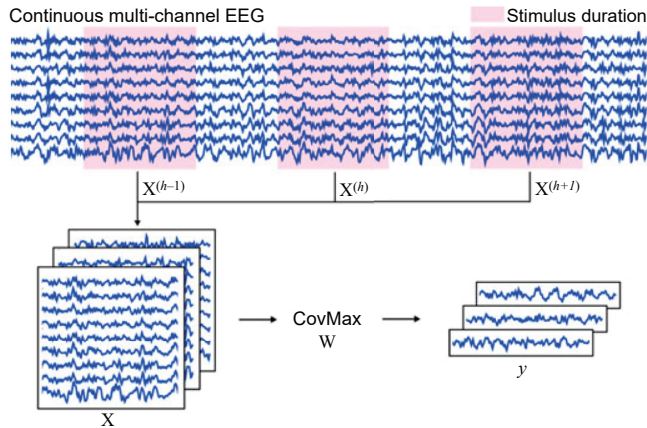
Therefore, the maximization of the covariance matrix with the restricted condition is given by:

$$\hat{w} = \underset{w}{\text{argmax}} \frac{w^T S w}{w^T Q w} \quad (13)$$

The above optimization problem can be solved by the eigenvalue decomposition of the given matrix. By sorting the eigenvalues of the matrix in a descend order, the eigenvectors corresponding to the eigenvalues will be selected as the solver of the optimization problem. In the recognition of SSVEP, the filter bank was first used to preprocess the EEG trials collected from each subject. Subsequently, for the stimulus frequency  $f_n$ , the optimal feature vector  $w_{i, \text{TRCA}} \in R^{N_c}$  was computed by the TRCA algorithm. For the input EEG trials of SSVEP, the projection matrix can be concatenated by the ensemble of projection vectors:  $w_{\text{TRCA}} = [w_1, w_2, \dots, w_N]$ . Finally, for the  $i$ -th single testing EEG trial  $X_{i, \text{test}} \in R^{N_c \times N_s}$  of each subject, the correlation coefficients between the testing EEG trial and the average individual-template  $\bar{X}$  of the EEG training trials (the same as ITCCA) can be computed as follow:

$$\rho_i = \rho\left(w_{i, \text{TRCA}}^T X_{i, \text{test}}, w_{i, \text{TRCA}}^T \bar{X}\right), \quad i = 1, 2, \dots, N_f \quad (14)$$

Where,  $\rho(\cdot)$  represents the Pearson correlation coefficients between two EEG trials. Similarly, the TRCA algorithm needs to divide the EEG trial into a training set and a testing set, and a  $n * n$  cross-validation strategy is used in the actual SSVEP recognition experiment. Figure 2 illustrated the flow chart of the TRCA algorithm.



**Figure 2.**  
Flow chart of TRCA  
algorithm

**Source(s):** Derived from Reference (Nakanishi *et al.*, 2017)

### 2.5 LCSE algorithm

The LCSE algorithm (Kumar and Reddy, 2019a) extracted a latent common source feature space from the multiple EEG trials, and the researchers called such process as “group configuration”. By such feature space expression, the recognition performance of SSVEP can be improved by the LCSE algorithm. For each sample  $X_{f_n}$  collected from the stimulus frequency  $f_n$ , the LCSE algorithm aims to solve a spatial filter on the latent common source feature space of all subjects. The learned filter can project any collected EEG trials on the same stimulus frequency onto a common source feature space, and such common source feature space is obtained by embedding multiple EEG training trials on the specific frequency. In fact, the LCSE algorithm is based on the generalized CCA algorithm, and extended to multiple models. In particular, the LCSE algorithm solved the optimal common source feature space by adopting the MAX-VAR equation. Therefore, the LCSE algorithm first projected the testing EEG trials onto the common source feature space, and then minimized the  $l_2$  norm between the common source feature space from the average template of the training trials (the same as ITCCA) and the common source feature space from the testing trial. The optimization object can be defined as:

$$\begin{aligned} \min_{\{w_i\}_{i=1}^{N_l}} \sum_{i=1}^{N_l-1} \|w_i^T X_i - C\|^2 \\ \text{s.t. } C^T C = I \end{aligned} \quad (15)$$

Where  $X_i \in R^{N_c \times N_l}$  represents the  $i$ -th EEG trial for the SSVEP response of the stimulus frequency.  $w_i \in R^{N_l \times K}$  represents the projected vector from the  $i$ -th trial and  $K$  represents the dimension of the projected vector. Assumed that the common source feature space of the SSVEP response can be obtained by the multiple collections of EEG trials, such common source feature space can be represented by  $C \in R^{l \times K}$ . In the vector of latent common source, the dimension  $K$  of feature cannot exceed the number of trials, namely  $K \leq \min(N, l)$ . By revising equation (15), we can obtain the optimal latent common source vector  $\hat{C}$  by the following equation:

$$C = \operatorname{argmax}_{C^T C = I} \operatorname{Tr} \left( C^T \left( \sum_{i=1, i \neq j}^N X_i X_j \right) C \right) \quad (16)$$

Equation (16) transforms the optimization problem given by equation (15) into the maximum covariance matrix of the active correlation matrix constructed by the combination of the EEG training trials. Based on the constructed covariance matrix  $E = \sum_{i=1, i \neq j}^N X_i X_j$ , the optimization object in equation (15) can be summarized as:  $\operatorname{argmax}_{C^T C = I} \operatorname{Tr}(C^T E C)$ . The solver

of this optimization object is the eigenvectors corresponding to the first  $K$  largest eigenvalues of the covariance matrix  $E$ . We call the matrix composed by  $K$  eigenvectors as the latent common source vector, and denote as projection vector  $w_i$ .

For the input EEG trials of SSVEP, the projection matrix can be concatenated by the ensemble of projection vectors:  $w_{\text{LCSE}} = [w_1, w_2, \dots, w_N]$ . Assumed that the  $i$ -th EEG trial to be recognized is  $X_{i, \text{test}} \in R^{N_c \times N_c}$ , and  $\bar{X}$  represents the average individual-template of the EEG training trials (the same as ITCCA), we can compute the correlation coefficients by the projection matrix  $w_{i, \text{LCSE}}^T$  which is similar like the process of TRCA algorithm:

$$\rho_i = \rho \left( w_{i, \text{LCSE}}^T X_{\text{test}}, w_{i, \text{LCSE}}^T \bar{X} \right), \quad i = 1, 2, \dots, N_f \quad (17)$$

Similar, equation (3) can be used to obtain the largest correlation coefficients by LCSE algorithm as the matched stimulus frequency.



### 2.6 SSCOR algorithm

Based on the TRCA algorithm, researchers have proposed a faster SSVEP recognition algorithm of template matching of training trials based on a SSCOR (Kumar and Reddy, 2019b). The purpose of the SSCOR algorithm is to find a series of independent projection vectors  $w_{\text{SSCOR}}$ , and project the EEG trials to be recognized into a general SSVEP expressed feature space. The projection vectors are based on the sum of squares of the correlation coefficients, and are given by the generalized form of multi-view data using the conventional process of the CCA algorithm (Kettenring, 1971):

$$\max_{w_i \in \mathbb{R}^d} \sum_{i < j}^{N_t} \rho(w_i^T X_i, w_j^T X_j)^2 \quad (18)$$

The above equation gives the generalized form of the unrestricted SSCOR optimization objective. To meet the characteristics of the SSVEP recognition tasks, the objective function of the restricted SSCOR needs to be given and optimized. The restriction for the objective function of the SSCOR algorithm can be used to express the generic trial feature space of SSVEP. Therefore, in the SSCOR algorithm, the solving process of the hub-language problem was adopted to give the objective function under the restriction of  $i = 1$ . The objective function under restricted conditions allows optimizing the individual-template to obtain the optimal projection vectors, and the individual-specific EEG trials of SSVEP can be projected into the generic feature space of SSVEP through the optimal projection vector. The objective function with restricted conditions is given as follow:

$$\begin{aligned} \max_{w_i} \sum_{i=2}^N (w_1^T C_{(1,i)} w_i)^2 \\ \text{s.t. } w_i^T C_{(i,i)} w_i = 1, \forall i \end{aligned} \quad (19)$$

Where  $i = \{X||1, 2, \dots, N_t\}$  represents the concatenate vector with a length of  $N = (N_t + 1)$ .  $C_{(1,i)}$  represents the covariance matrix between the individual-template and the EEG trials to be recognized.  $C_{(i,i)}$  represents the covariance of each EEG trial to be recognized itself. The quadratic optimization problem with restricted conditions given in Equation (22) can be decomposed by Cholesky decomposition:  $C_{(1,i)} = K_i^T K_i$ . Using this decomposition, the covariance can be transformed to a simple form. By giving the definition of  $G_i = K_1^{-1} C_{(1,i)} K_i^{-1}$ , and defining  $v_i = K_i w_i$  to replace  $w_i$ , the quadratic optimization in Equation (19) can be transformed to the following form:

$$\begin{aligned} \max_{v_i} \sum_{i=2}^N (v_i^T G_i v_i)^2 \\ \text{s.t. } v_i^T v_i = 1, \forall i \end{aligned} \quad (20)$$

The above-mentioned quadratic optimization problem can be solved directly by the Lagrangian method, and the eigenvalue decomposition problem can be defined:

$$\left( \sum_{i=2}^N (G_i^T G_i) \right) v_1 + \lambda_1 v_1 = 0 \quad (21)$$

At this time, the eigenvector of the matrix  $\sum_{i=2}^N (G_i^T G_i)$  can provide the optimal solution  $v_1$  for expressing the generic trial feature space of SSVEP. The optimal projection vector  $w_{\text{SSCOR}}$  of the SSCOR algorithm can be obtained by the inverse transformation of the Cholesky decomposition for vector  $v_1$ :

$$w_{\text{SSCOR}} = K_1^{-1} v_1$$

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According to the aforementioned optimal projection vector solution process of the SSCOR algorithm, the maximum correlation coefficients between the individual-template and the EEG trials to be recognized can be computed like the same process of ITCCA. The ensemble projection matrix can be concatenated:  $w_{\text{LCSE}} = [w_1, w_2, \dots, w_N]$ . Assumed that the  $i$ -th EEG trial to be recognized is  $X_{i,\text{test}} \in R^{N_c \times N_s}$ , and  $\bar{X}$  represents the average individual-template of the EEG training trials (the same as ITCCA), we can compute the correlation coefficients by the projection matrix  $w_{i,\text{SSCOR}}^T$  which is similar like the process of TRCA and LCSE algorithm:

$$\rho_i = \rho(w_{i,\text{SSCOR}}^T X_{i,\text{test}}, w_{i,\text{SSCOR}}^T \bar{X}), i = 1, 2, \dots, N_f \quad (23)$$

Similar, equation (3) can be used to obtain the largest correlation coefficients by SSCOR algorithm as the matched stimulus frequency.

### 3. Comparative experiments and results analysis

To compare the SSVEP recognition algorithms based on the template matching of training trials, this paper used the CCA algorithm as the baseline, and compared the five template matching algorithms based on the training trials on a benchmark data set of “Tsinghua JFPM-SSVEP” (Wang *et al.*, 2016). For the comparison, the average recognition accuracy, average efficiency and ITR are used for the measurement of performance.

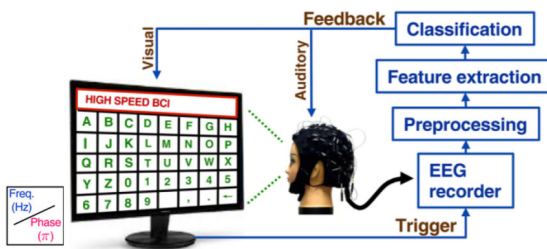
#### 3.1 Data set description

The “Tsinghua JFPM-SSVEP” data set adopts the joint frequency-phase modulation (JFPM) for designing the sparkle stimulus (Chen *et al.*, 2015a), and the presented stimulus includes not only different frequencies but also different phases. The phase difference ensures that the frequencies with litter differences can also be distinguished and used for design SSVEP. As shown in Figure 3(a), the data set contains a total of 40 sparkle stimuli. The blue words are the stimulus frequency, and the red words are the stimulus phase. The experimental data set contains an entity of 35 subjects. The stimulus frequency ranges from 8.0 Hz to 15.8 Hz, with an increment of 0.2 Hz. The stimulus phase ranges from  $0.0\pi$  to  $0.6\pi$ , with an increment of  $0.1\pi$ . The phase increased  $0.35\pi$  for each base frequency, and there are 6 stimuli of phases under each base frequency. The square of each sparkle stimulus contains  $32 \times 32$  pixels.

>> HIGH SPEED BCI															
8.0	9.0	10.0	11.0	12.0	13.0	14.0	15.0								
0.00	1.75	1.50	1.25	1.00	0.75	0.50	0.25								
8.2	9.2	10.2	11.2	12.2	13.2	14.2	15.2								
0.35	0.10	1.85	1.60	1.35	1.10	0.85	0.60								
8.4	9.4	10.4	11.4	12.4	13.4	14.4	15.4								
0.70	0.45	0.20	1.95	1.70	1.45	1.20	0.95								
8.6	9.6	10.6	11.6	12.6	13.6	14.6	15.6								
1.05	0.80	0.55	0.30	0.05	1.80	1.55	1.30								
8.8	9.8	10.8	11.8	12.8	13.8	14.8	15.8								
1.40	1.15	0.90	0.65	0.40	0.15	1.90	1.65								

Presentation of Stimuli

(a)



EEG signals processing procedure

(b)

Figure 3.  
Tsinghua JFPM-  
SSVEP data set

Source(s): Derived from Reference (Chen *et al.*, 2015a)

A 23.6-inch LCD display with a refresh rate of 60 Hz and a resolution of  $1,920 \times 1,080$  was used for stimulus presentation. As shown in Figure 3(b), the stimulus object is composed of  $5 \times 8$  squares, with a total of 40 stimulus frequencies. The sparkle materials contained 26 letters, 10 numbers and 4 punctuation marks. The color of the sparkle materials was green and the color of background was white. The device for EEG signals collection was NeuroScan SynAmp 2 with a sensitivity of  $0.024 \mu\text{V/bit}$ . The sampling rate is 1,000 Hz, and 9 electrodes ( $P_z, PO_5, PO_3, PO_z, PO_4, PO_6, O_1, O_z$  and  $O_2$ ) among the occipital brain region were used for EEG signals collection. During the offline experiment, each subject completed 40 stimuli in a given order of all 40 sparkle stimuli. In each session, the subject observed each sparkle stimulus for 5 s to evoke the SSVEP, and each subject repeated six sessions.

According to the process of EEG signals collection, the EEG trials of SSVEP for each subject were composed of a 4D matrix:  $X \in R^{N_e \times N_t \times N_f \times N_p}$ , where  $N_e = 9$  represents the number of electrodes,  $N_t = 1000$  represents the sampling points in time domain,  $N_f = 40$  represents the number of stimulus frequencies and  $N_p = 6$  represents the number of trials for each stimulus frequency. For the EEG signals collection, there was a delay of 0.14 s to ensure that an effective SSVEP was evoked at the occipital region of the subject. Therefore, the recognition of SSVEP, the effective sampling points of each stimulus needed to be delayed for 140. In addition, the effective EEG signals of brain are generally concentrated within 100 Hz. A band-pass filter of 7–90 Hz and a notch filter of 48–52 Hz were used for filtering the raw EEG signals.

### 3.2 Evaluation criterion of performance

The performance evaluation of SSVEP recognition includes objective evaluation and characteristics evaluation. The objective evaluation includes average recognition accuracy and ITR. The recognition performance can be cross-validated using the “leave one rest” strategy. For each subject, five trials are selected as training, and the remaining one is used for testing, and there are five rounds of cross-validation. In addition, the ITR index is another index for measuring the control performance of the SSVEP-BCI:

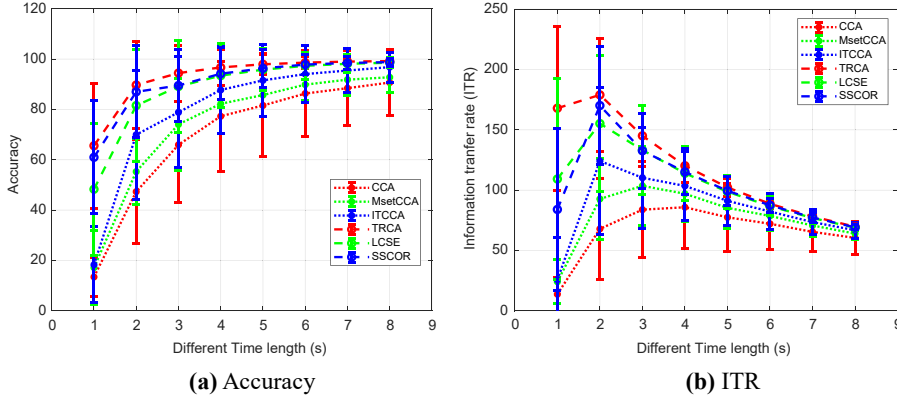
$$\text{ITR} = \left( \log_2 N_f + P \log_2 P + (1 - P) \log_2 \left[ \frac{1 - P}{N_f - 1} \right] \right) \times \left( \frac{60}{T} \right) \quad (24)$$

Where  $P$  represents the average recognition accuracy, and  $T$  represents the average time duration of producing each control signal. Under different average time duration, the average recognition accuracy will be varied differently, which will affect the performance results of ITR. Therefore, we selected different average time durations to evaluate the ITR performance for the SSVEP-BCI. The average time durations range from 0.5 to 4 s, with an increment of 0.5 s.

For the characteristics evaluation, the ratio of subject intention (SSVEP) to the variance of non-related attention (no task) is adopted. The r-square value of is obtained by the ratio of the correlation coefficients of the intention for the target stimulus  $\rho_\tau$  and the maximum correlation coefficients  $\rho_{n \neq \tau}$  of the intention for the non-target stimulus. To verify the feasibility of the SSVEP recognition algorithms, the average time complexity of all algorithms is also compared, including preprocessing time, training time and testing time. Different number of training trials, electrodes and the usage of filter bank are also evaluated.

### 3.3 Recognition results of SSVEP-BCI

Figure 4(a) showed the average recognition accuracy of all subjects under different durations of stimulus presentation. Among them, the number of training trials is five, and all nine electrodes are used for recognition. It can be seen from the results in Figure 4(a) that all recognition algorithms based on template matching of training trials significantly outperformed than the conventional CCA algorithm. The average recognition accuracy of



**Figure 4.**  
Average recognition  
accuracy and ITR for  
all subjects under eight  
time duration ranges

the ITCCA algorithm surpassed the MsetCCA algorithm, which was consistent with the results in the reference (Nakanishi *et al.*, 2015), but there is no significant difference among the 35 subjects ( $p = 0.837$ ). The recent proposed template matching of training trails algorithms (TRCA/LCSE/SSCOR) significantly surpassed the ITCC algorithm in terms of the average recognition accuracy ( $p < 0.05$ ). In addition, the average recognition accuracy of six compared algorithms has a greater difference with a shorter stimulus presentation time duration. To verify the difference of six compared algorithms in different presentation time durations, the ANOVA analysis was used to compare the eight time durations of six compared algorithms. Table 1 illustrated the ANOVA results with the significant level.

From the results in Table 2, it can be seen that the six compared algorithms have a significant improvement ( $p < 0.05$ ) in average recognition accuracy for all presentation time durations, and all F-values satisfy the critical values of the testing. In a control system of the

Durations of presentation	F-value	p-value
$L = 0.5$ s	$F(5, 204) = 68.58$	$p < 0.05$
$L = 1.0$ s	$F(5, 204) = 23.58$	$p < 0.05$
$L = 1.5$ s	$F(5, 204) = 9.76$	$p < 0.05$
$L = 2.0$ s	$F(5, 204) = 5.83$	$p < 0.05$
$L = 2.5$ s	$F(5, 204) = 4.69$	$p < 0.05$
$L = 3.0$ s	$F(5, 204) = 3.87$	$p < 0.05$
$L = 3.5$ s	$F(5, 204) = 3.59$	$p < 0.05$
$L = 4.0$ s	$F(5, 204) = 3.21$	$p < 0.05$

**Table 1.**  
Average recognition  
accuracy under  
different time duration  
ranges and their  
ANOVA results

Compared algorithms	ITR	Recognition accuracy	Durations of presentation
CCA	$85.64 \pm 34.27$	$77.29 \pm 21.86$	$L = 2.0$ s
MsetCCA	$103.58 \pm 32.89$	$74.07 \pm 18.31$	$L = 1.5$ s
ITCCA	$124.13 \pm 61.09$	$69.95 \pm 25.64$	$L = 1.0$ s
TRCA	$178.97 \pm 46.86$	$89.70 \pm 17.36$	$L = 1.0$ s
LCSE	$155.22 \pm 56.39$	$81.54 \pm 22.22$	$L = 1.0$ s
SSCOR	$170.12 \pm 48.65$	$86.99 \pm 18.26$	$L = 1.0$ s

**Table 2.**  
The highest  
recognition accuracy  
and ITR for six  
compared algorithms

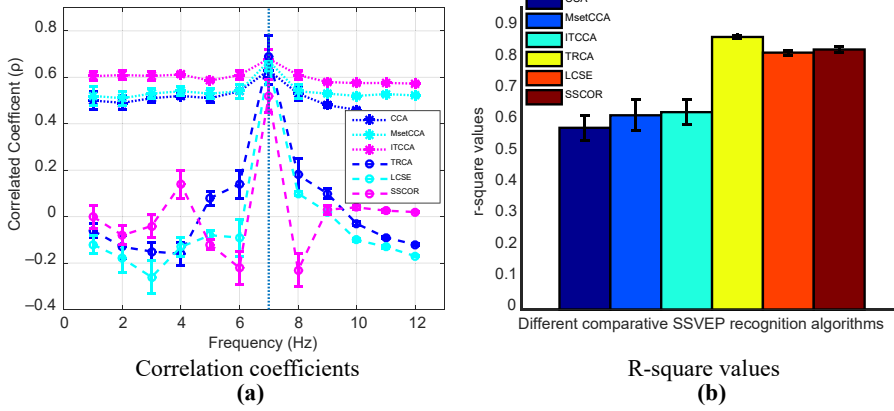
SSVEP-BCI, smaller time duration of stimulus presentation will bring a higher control performance. For the stimulus presentation time duration of  $L = 0.5$  s, the six compared algorithms average recognition accuracy on 35 subjects are: CCA:  $13.44 \pm 7.77$ , MsetCCA:  $17.14 \pm 14.59$ , ITCCA:  $18.29 \pm 15.04$ , TRCA:  $65.58 \pm 24.88$ , LCSE:  $48.29 \pm 26.17$  and SSCOR:  $60.98 \pm 22.43$  (unit: %). For the stimulus presentation time duration of  $L = 1.0$  s, the six compared algorithms average recognition accuracy on 35 subjects are: CCA:  $47.32 \pm 20.73$ , MsetCCA:  $55.29 \pm 13.02$ , ITCCA:  $69.95 \pm 25.64$ , TRCA:  $89.70 \pm 17.36$ , LCSE:  $81.54 \pm 22.22$  and SSCOR:  $86.99 \pm 18.26$  (unit: %). Whether the presentation time duration is  $L = 0.5$  or  $L = 1.0$  s, the algorithms based on optimal projection vectors achieved significant higher average recognition accuracy than conventional CCA/MsetCCA/ITCCA algorithms ( $p < 0.05$ ). From the comparative results, we have suggested that whether it was the optimal projection of sample differences between individuals or the projection of individual differences, the signal-to-noise of SSVEP in EEG trials can be significantly improved, and the average recognition accuracy can be improved for the SSVEP-BCI.

Figure 4(b) showed the average ITR of all subjects under different stimulus presentation time durations. Among them, the ITR was computed by Equation (24), and all parameters were kept the same as in Figure 4(a). The results in Figure 4(b) have shown that the changes in ITR indexes produced by six compared algorithms were mainly the same as the changes in average recognition accuracy. When the time duration of stimulus presentation was  $L = 1.0$  s, the average ITR of the optimal projection algorithms (TRCA/LCSE/SSCOR) was significantly higher than the conventional algorithms (CCA/MsetCCA/ITCCA) ( $p < 0.05$ ). Table 2 illustrated the highest ITR of the six compared algorithms and their corresponding time durations of stimulus presentation. From the results in Table 2, we have found that the template matching of training trials based algorithms (ITCCA/TRCA/LCSE/SSCOR) achieved the highest ITR at  $L = 1.0$  s, while the MestCCA algorithm achieved the highest ITR at  $L = 1.5$  s. For the conventional CCA algorithm, the highest ITR was obtained at  $L = 2.0$  s. In fact, for the algorithms obtained the highest ITR at an earlier time, they owned a higher efficiency for iterations. Therefore, such algorithms were more suitable to be applied to the online SSVEP-BCI control system. In addition, the performance of the SSVEP-BCI control system can also be improved by optimizing various parameters, including the number of stimuli and the conversion time of different stimuli.

#### 4. Ablation study and results discussion

##### 4.1 Characteristics evaluation for template matching of training trials algorithms

For the conventional CCA algorithm and its variants, due to the sine-cosine harmonics template was hard to cope with diversity of SSVEP, the ITCCA algorithm introduced individual-template to fix the diversity of template, which achieved a significant improvement of recognition performance. The experimental results were also shown that using the subject's training trials as the matching template cannot only retain the frequency parts of the sine-cosine harmonic template, but also simulate the corresponding phase parts. Therefore, the MsetCCA/ITCCA algorithm obtained a significant recognition performance improvement under the JFPM paradigm of SSVEP. To validate that the TRCA/LCSE/SSCOR algorithms also have a good ability to simulate the frequency parts and the phase parts, the sparkle stimulus of 12.0 Hz/1.0 phase of all subjects was selected, and correlation coefficients and  $R$ -square value were given on the selected sparkle stimulus by using the statistical analysis method. Figure 5 showed the average correlation coefficients and  $R$ -square value of the six compared algorithms on the sparkle stimulus of 12.0 Hz/1.0 phase. It can be seen from Figure 5(a) that the conventional CCA/MsetCCA/ITCCA algorithm used CCA and produced correlation coefficients in a range of  $[0, 1]$ . However, the TRCA/LCSE/SSCOR algorithm used projection vectors and analyzed the correlation in the projected



**Figure 5.**  
Characteristics  
evaluation results on a  
sparkle stimulus of  
12.0 Hz/1.0 phase

feature space, and produced correlation coefficients in a range of  $[-1, 1]$ . Therefore, such projection vector based template matching algorithms has a wider negative correlation coefficients space, so it has higher distinguishability between the target stimulus frequency and the non-target stimulus frequency.

This paper used the “Tsinghua JFPM-SSVEP” data set, and the stimulus frequencies of this data set were designed by the joint encoding paradigm of frequency and phase, so that the SSVEP of adjacent frequencies can be strongly and negatively correlated with the target frequency. From the recognition results of JFPM-SSVEP, we have found that the JFPM paradigm can ensure that the correlation coefficients between the target stimulus frequency and adjacent frequencies have a significant improvement in resolution. The decent resolution is suitable for the template matching of training trials based algorithms, and the results of six compared algorithms are consistent in reference (Kettenring, 1971). No matter for the conventional CCA algorithm or for the MsetCCA/ITCCA algorithm, the stimulus closest to the target stimulus frequency has higher correlation coefficients than other non-target frequencies, resulting in incorrect recognition results for the nearest neighbor stimulus. However, by computing the optimal projection vector, for the projected feature space, the TRCA/LCSE/SSCOR algorithm can significantly reduce the correlation coefficients of the nearest neighbor stimulus, thereby ensuring the correct recognition results of SSVEP.

Taking 12.0 Hz as an example in Figure 5(a), it can be seen that 11.8 Hz and 12.2 Hz, which are the closest to such frequency, have significantly reduced the correlation coefficients for TRCA/LCSE/SSCOR algorithm, but the correlation coefficients of the conventional CCA/MsetCCA/ITCCA algorithm is not significantly reduced. Figure 5(b) showed the R-square value of the 12.0 Hz/1.0 phase stimulus within the presentation time duration of  $L = 1.0$  s. From the results in Figure 5(b), we have seen that the R-square values produced by the six compared algorithms were consistent with the results of the average recognition accuracy and ITR (CCA:  $0.58 \pm 0.04$ , MsetCCA:  $0.62 \pm 0.05$ , ITCCA:  $0.63 \pm 0.04$ , TRCA:  $0.87 \pm 0.006$ , LCSE:  $0.82 \pm 0.008$ , SSCOR:  $0.83 \pm 0.01$ ). At the same time, the ANOVA statistical analysis between the six compared algorithms also exhibited significant differences ( $F(5, 204) = 8.47, p < 0.05$ ).

The above experimental results further proved that the construction of an online SSVEP-BCI control system needed to address the differences of different individual EEG trials. The most commonly used algorithm to eliminate individual differences is transfer learning, which mainly includes trial-level transformation, feature-level domain adaptation or domain generalization and the transfer of recognition models (He and Wu, 2019). However, most of the



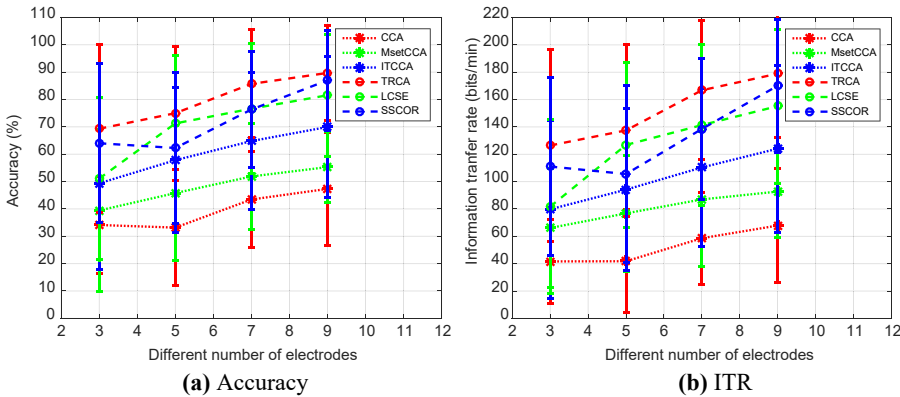
existing transfer learning algorithms were used in motor imagery and ERPs (He and Wu, 2020). How to apply transfer learning algorithms to the recognition of SSVEP will be the key points to the future researches for constructing online SSVEP-BCI control systems.

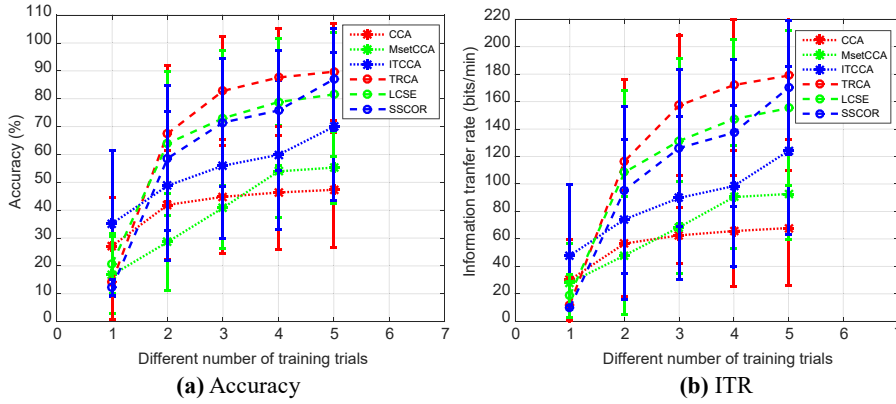
#### 4.2 Discussion on the influence of different parameters for six compared algorithms

During the SSVEP recognition, three parameters, electors, training trials, and usage of filter bank, influenced the performance of six compared algorithms. In this section, we have done three ablation studies to measure the influence of three different parameters. In the common setting of SSVEP recognition, we adopted all nine electrodes in the experiment. First, to verify the influence brought by the number of electrodes, similar in the presentation time duration of  $L = 1.0$  s, we also selected three different situations: three electrodes ( $O1$ ,  $Oz$  and  $O2$ ), five electrodes ( $PO5$ ,  $PO3$ ,  $POz$ ,  $PO4$  and  $PO6$ ) and seven electrodes ( $PO5$ ,  $PO3$ ,  $PO4$ ,  $PO6$ ,  $O1$ ,  $Oz$  and  $O2$ ). Figure 6 exhibited the SSVEP recognition results of six compared algorithms under different number of electrodes. From the results in Figure 6, we have found that the average recognition accuracy and ITR of SSVEP for the six compared algorithms are basically proportional to the number of the electrodes. A larger number of electrodes will bring higher average recognition accuracy and ITR, but will also accordingly increase the time complexity of computations. During reducing the number of electrodes, the performance reduction of two indexes for TRCA/SSCOR algorithm is not as obvious as other compared algorithms. In consequence of a restricted SSVEP-BCI environment and only a limited number of electrodes can be applied, the TRCA algorithm is preferred to be selected as the recognition algorithm. In addition, the time complexity of SSCOR algorithm is lower than the TRCA algorithm. If there is a demand for the efficiency of the algorithm in a restricted environment, the SSCOR algorithm is the best selection.

Second, the original SSVEP recognition experiment used all five trials for training, and the rest one for testing. To verify the influence of different number of trials for training, we also adopted the presentation time duration of  $L = 0.1$ , and used different trials number of 4, 3, 2, 1 for training. For more than one testing trials, we randomly selected one for testing. Figure 7 exhibited the SSVEP recognition results of six compared algorithms under different number of training trials. Among them, the CCA algorithm exhibited the average recognition accuracy of different number of trials. It can be seen from the results in Figure 7 that the six compared algorithms can maintain a high average recognition accuracy and ITR with enough number of training trials. Among them, the recognition performance of conventional

**Figure 6.**  
Average recognition  
accuracy and ITR  
comparison with  
different number of  
electrodes





**Figure 7.**  
Average recognition  
accuracy and ITR  
comparison with  
different number of  
training trials

CCA/MsetCCA/ITCCA algorithm decreased linearly as the number of training trials decreased.

However, with the number of training trials decreased, the recognition performance of TRCA/LCSE/SSCOR algorithm did not significantly reduced. But, when the number of training trials was less than two, there will be a significant decrease for the recognition performance of TRCA/LCSE/SSCOR algorithm. When the training trials decreased to one, the recognition performance of such three algorithms was much lower than the conventional CCA algorithm, which meant that only one training trial cannot fully compute the optimal projection vector for such three algorithms. It is impossible to project the EEG trials into the separable feature space for only one trial, resulting in a low average recognition accuracy and ITR. When the number of training trials is limited, the recognition performance of the optimal projection vector based algorithms is lower than conventional average training trials template or sine-cosine harmonic template directly matched algorithms. Future research studies will focus on the specific reasons for this phenomenon and the solution, since the lack of specific EEG trials in actual online BCI system is extremely common (Zhang *et al.*, 2020). Therefore, the novel optimal projection vector computation algorithms should be researched for limited number of training trials, to solve the limitation of trials in online BCI scenarios.

Finally, for the algorithms based on template matching of training trials, the filtering pre-processing before template computation is also important. Researchers (Islam *et al.*, 2017) have shown that a pre-processing of filter bank will greatly improve the separability of filtered EEG trials. The filter bank pre-processing is also suitable for SSVEP recognition (Chen *et al.*, 2015b). In the above experiments, a filter bank of five filters was adopted during recognition for all template matching of training trials algorithms. To measure the significance of filter bank, we also adopted the presentation time duration of  $L = 0.1$ , and compared the usage of filter bank or not for six compared algorithms. Table 3 illustrated the influence of performance for using filter bank in SSVEP recognition or not.

From the results in Table 3, we have suggested that the filter bank preprocessing has a great influence on the recognition of SSVEP. Among the six compared algorithms, the average recognition accuracy and ITR with filter bank preprocessing significantly outperformed than the situation of without filter bank preprocessing. In fact, the collected EEG trial of SSVEP belongs to the classical multi-dimensional time series data. Most of the current existed recognition algorithms adopted the form of covariance matrix, which only considered the time domain and spatial domain information of the raw EEG trials, but did not consider the frequency domain information (Zhang *et al.*, 2020). After a filter bank

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preprocessing, the SSVEP non-related frequency components were filtered out, and the constructed covariance matrix had higher distinguishability for different frequencies and phases, so algorithms based on the template matching of training trials with filter bank preprocessing achieved a higher average recognition accuracy and ITR.

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4.3 Discussion on the efficiency of six compared algorithms

The efficiency is another index for selecting the recognition algorithm for SSVEP-BCI. We have done the experiments to compare the time complexity of six algorithms. Table 4 showed the average time complexity of six compared algorithms under different presentation time duration of stimulus. The hardware environment is: AMD Ryzen 3700U CPU, 8 GB memory, Windows 10 operating system and the algorithms running platform adopted Matlab R2018a. For the training process, the time complexity of computing the optimal projection vector was given. For the testing process, the time complexity of projecting EEG trial and computing correlation coefficients was given.

From the comparison results in Table 4, it is suggested that the time complexity of the conventional CCA/MsetCCA/ITCCA algorithm was smaller. Compared with the computation of optimal projection vector, the time complexity of computing average template of training trials or computing sine-cosine harmonic template was lower. Compared with the ITCCA algorithm, the time complexity of TRCA/LCSE/SSCOR algorithm includes the training time of the optimal projection vector, and the testing time of projecting and the correlation coefficient on the optimal projected space. Judging from the gaps between the training and testing, the time consumption of training the optimal projection vector computation was not large, but the optimal projected space conversion and matching in during the testing process was relatively large.

Under the stimulus presentation time duration of  $L = 1.0$  s, the computational time of the optimal projection vector during training process was TRCA (1,055 ms), LCSE (2,021 ms),

Compared algorithms	Indexes for comparison			
	Average		ITR	
	With filter bank	Without filter bank	With filter bank	Without filter bank
CCA	47.32 ± 20.73	65.13 ± 21.86	67.91 ± 41.68	108.89 ± 51.50
MsetCCA	49.59 ± 22.67	68.29 ± 13.02	67.35 ± 45.53	92.75 ± 33.31
ITCCA	44.95 ± 29.54	69.95 ± 25.64	69.11 ± 59.28	124.13 ± 61.09
TRCA	44.75 ± 22.24	89.70 ± 17.36	63.21 ± 44.80	178.97 ± 46.86
LCSE	41.98 ± 25.02	81.54 ± 22.22	59.44 ± 52.88	155.22 ± 56.39
SSCOR	50.88 ± 22.78	86.99 ± 18.26	75.87 ± 49.51	170.12 ± 48.65

Compared algorithms	Duration of presentation							
	$L = 1.0$ s		$L = 2.0$ s		$L = 3.0$ s		$L = 4.0$ s	
	Training	Testing	Training	Testing	Training	Testing	Training	Testing
FBCCA	–	1.643	–	1.986	–	2.717	–	3.573
MsetCCA	0.492	1.721	0.682	2.176	0.982	2.372	1.271	2.897
ITCCA	0.979	1.670	1.188	2.007	1.329	2.283	1.595	3.549
TRCA	1.055	4.030	1.273	4.928	1.438	5.995	1.693	6.120
LCSE	2.021	1.950	5.672	3.044	12.418	3.420	25.178	4.412
SSCOR	0.913	3.263	1.148	4.095	1.237	5.122	1.410	6.705

SSCOR (913 ms) and the computational time of projection space conversion and matching during testing process was respectively TRCA (4,030 ms), LCSE (1,950 ms) and SSCOR (3,263 ms). For the optimal projection vector based algorithms, they relied on a time complexity of <2,000 ms during training process to achieve a significant improvement of recognition performance. However, in the actual SSVEP-BCI, we also required to set a delay of 2,000–3,000 ms for stimulus target conversion in the actual SSVEP-BCI. Therefore, for the time complexity matching, the TRCA/LCSE/SSCOR algorithm was suitable for constructing the online SSVEP-BCI control system. However, for the application scenarios with extremely high efficiency and no time reserved for stimulus target conversion, the CCA algorithm with filter bank preprocessing (FBCCA (Chen *et al.*, 2015b)) was the most suitable one.

Moreover, compared with the TRCA/LCSE algorithm, the SSCOR algorithm directly solved the optimal projection vector by using the Lagrangian optimization, and its computational time complexity was lower. However, the operations of SSCOR algorithm will reduce its adaptation of EEG trials, and cannot have a high average recognition accuracy and ITR for all EEG trials. The TRCA algorithm consumed the highest time, but it also obtained the highest recognition performance of SSVEP. For a complex SSVEP-BCI control system that required less time complexity and reserved time duration for stimulus target conversion, the TRCA algorithm was the most suitable selection.

#### 4.4 Discussion on algorithms selection for online SSVEP-BCI

To select the appropriate recognition algorithm for constructing the online SSVEP-BCI control system, Table 5 illustrated the characteristics of the six compared algorithms. Among them, the minimum requirements for the number of trials and the number of electrodes were subject to an average recognition accuracy of 70%. From the comparison in Table 5, we have suggested that the conventional CCA algorithm was simple to compute and contained a high efficiency. However, its average recognition performance was relatively low, and it was suited for the application scenarios without training. The efficiency of the ITCCA algorithm was medium, but the average recognition performance of SSVEP has been greatly improved, and it was suitable for application scenarios with certain data calibration and high efficiency requirements. The template matching algorithms based on the optimal projection vector achieved the lowest efficiency and required the support of sufficient number of training trials.

Compared algorithms	Need training	Trials required	Comparison of characteristics			Time complexity	References
			Electrodes required	Recognition method	Accuracy		
CCA	No need	No required	$\geq 9$	Canonical correlation	Low	Very fast	Lin <i>et al.</i> (2006)
MsetCCA	Need	$\geq 5$	$\geq 9$	Canonical correlation	Medium	Fast	Jiao <i>et al.</i> (2018)
ITCCA	Need	$\geq 5$	$\geq 9$	Correlation coefficients	Medium	Fast	Bin <i>et al.</i> (2011)
TRCA	Need	$\geq 2$	$\geq 3$	Correlation coefficients	Very high	Medium	Nakanishi <i>et al.</i> (2017)
LCSE	Need	$\geq 2$	$\geq 5$	Correlation coefficients	High	Medium	Kumar and Reddy (2019a)
SSCOR	Need	$\geq 3$	$\geq 7$	Correlation coefficients	Very high	Fast	Kumar and Reddy (2019b)

**Table 5.**  
Characteristics for six compared algorithms

Such algorithms were suitable for application scenarios with requirements of high recognition accuracy and stimulus target conversion time duration.

Furthermore, according to the results in Table 4, we have also suggested that as the length of the time duration for the stimulus presentation increased, the time complexity gap between the TRCA/LCSE/SSCOR algorithms was not big, while the gap between the CCA/MsetCCA/ITCCA algorithms was getting bigger and bigger. From the results in Figure 4, a larger length of the time duration for the stimulus presentation will bring higher average recognition accuracy, but a lower average ITR. Therefore, the construction of an online SSVEP-BCI control system should try to select the presentation time duration within  $L = 2.0$  s for recognition to ensure the recognition performance.

#### *4.5 Discussion on the SSVEP-BCI recognition algorithms development*

Recently, the template matching of training trials-based algorithms gradually replaced the classical CCA algorithm and its variants in the SSVEP-BCI recognition. Adopting training trials to replace the sine-cosine harmonic templates has a strong robustness in complex and variable EEG signals. Actually, due to the nonlinear and non-stationary characteristics of EEG signals, EEG signals recorded from different subjects for the same task (such as SSVEP) contain different distributions of feature space. Meanwhile, the EEG signals for the same subjects in different recording sessions also have different distributions of feature space, which called the covariate shift problem (Li *et al.*, 2010). Therefore, most of the existed templates matching of training trials-based algorithms only construct individual and session-specific templates to combat the characteristics of EEG signals. However, such individual and session-specific templates will cause a lot of computational consume, which leads to a low practical efficiency for applications. To solve the influence of covariate shift problem, a lot of researches based on transfer learning have been successfully applied to motor imagery-based BCI and achieved excellent results (Wei and Lin, 2020; Li, 2020). In recent years, some researchers have also carried out the exploration research studies for cross-subject SSVEP-BCI with transfer learning (Wang *et al.*, 2021). Nevertheless, there are few current research studies on cross-subject SSVEP-BCI, further development is pressing, required to be able to construct more efficient and robust online SSVEP-BCI recognition algorithms.

Moreover, recent convolutional neural networks (CNN) have achieved great progress on pattern recognition, and CNN have also been applied to the EEG based BCI applications. Different from motor imagery-based BCI or P300-based BCI, SSVEP-BCI has only a few EEG trials in a single session, which cannot directly train an effective CNN model, although there are currently some researchers exploring the use of CNN model to construct the SSVEP recognition (Ravi *et al.*, 2020). However, if the SSVEP recognition task is under the introduced cross-subject and cross-session condition, the distribution differences of cross-domain EEG trials can be prevented by the transfer learning algorithms at first, and then a large amount of EEG trials consistent with the identical independent distribution can be used together. Based on these EEG trials, new breakthroughs will be made in the construct of deep neural network models. In addition, filter bank can play an important role in the recognition of SSVEP, but existed template matching of training trials-based algorithms all directly matched the filtered EEG trials, which inevitably introduced noises. On the basis of CNN model, the template matching process can be performed on the features extracted by the CNN model. These extracted features not only have cross-domain invariance, but also are discriminative for recognition, which have more excellent recognition performance efficiency in template matching. In the future, online SSVEP-BCI will become more stable and efficient under the fusion of multiple algorithms among transfer learning, deep learning and template matching.

## 5. Conclusion

In this paper, to recognize the SSVEP in the BCI, the commonly used CCA algorithm was applied as the baseline, and another five recognition algorithms based on template matching of training trials were surveyed and compared for experiments. For the six compared recognition algorithms, this paper used the “Tsinghua JFPM-SSVEP” data set, and compared the average recognition accuracy and ITR under different time duration of stimulus presentation. Based on the optimal presentation time duration, we also gave the efficiency comparison for the six algorithms. In view of computing the correlation for the six compared algorithms, we also gave the correlation coefficients and R-square values with the difference significance detection through a detailed statistical analysis. In addition, we also compared different parameters settings of the six compared algorithms, including different number of training trials, different number of electrodes and usage of filter bank preprocessing. In all, we surveyed and compared the advantages and disadvantages of the mainstream template matching of training trials based algorithms, and verified the algorithms selection for different application scenarios. Future works include exploring template matching algorithms for the scenarios of lacking training trials, researching the transfer learning for the SSVEP trials from different subjects, and using deep neural network for the generalized SSVEP recognition. In actual application scenarios, it is important to flexibly select recognition algorithm for constructing a real-world online SSVEP-BCI.

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### Further reading

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