IE 202 INTRODUCTION TO MODELING AND OPTIMIZATION TERM PROJECT STAGE 2

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PART A

In Part A, we solved the mathematical model given to us while making proper assignments between the elements of the company by maximizing the monthly profit. Our objective function is to maximize profit, so we wrote an objective function by collecting the money from all projects, subtracting the penalty cost of undone projects from this all earned money, and then subtracting the salaries in the usual working hour for employees who are assigned to project for different positions and overworking hour given to the employees for different difficulty levels of projects. By subtracting the total expenses from the total income, we make our objective function in line with the maximize monthly profit that we want. Therefore, in mathematical model which is given to us, we have objective function and constraints and according to these to solve this problem, we used different solvers to figure out this problem which are Xpress, Cplex, and Gurobi. By using these solvers, we reached an outcome from solvers appropriate lists from our decision variables and maximum monthly profit. In our decision variables we have a project is assigned to a department, a employee is assigned to a project, a project is assigned to some department, a employee is assigned to some project, and total number of overworking hours of employee in a month for a project. With reference to these decision variables from solvers which we used, we examined our objective value is 905000. Here we also had chance to observe which decision variable take which value when optimized.

PART B

In Part B, we revised our model such that our objective function is maximize the number of completed projects and make an observation about how our monthly profit change in this case. At first, we revised our objective function in this report and then make comparison for our monthly profit in Part A and Part B. In new objective function, we try to maximize number of projects are assigned to some department. Again, same as with Part A, we used different solvers which were Xpress, Cplex, and Gurobi to obtain a outcome for this problem. By using these solvers, we reached an outcome from solvers appropriate lists from our decision variables and maximum number of completed projects.In our decision variables we have a project is assigned to a department, an employee is assigned to a project, a project is assigned to some department, an employee is assigned to some project, and total number of overworking hours of employee in a month for a project. According to these solvers, we obtained objective value is found 9 and monthly profit decreased to for Xpress is found 442500, for Cplex is found 506500, and for Gurobi is found 514500. Hence we observe that monthly profit is decreased due to new objective function. In addition, we understand that we have alternate optimal solutions for part b.

Comparison between Part A and Part B

While we compare Part A and Part B, we observed that for Xpress, Cplex, and Gurobi monthly profit is decreased as we mentioned in Part B. Then, we focus on why finding 3 different results in 3 different solvers is since even though the number of completed of projects is same which is 9, one of the reasons we found different monthly profits is that the projects to which the employee is assigned differ in the solvers. We observed this from the matrices that we got from all three solvers.Here we observe differences :if we look at [2,1] in all three matrices we encounter:1,0,0 for e_1,e_2 and e_3 respectively. Hence first solver made a different project assignment to employee 1 than others.

$$e_1 = \begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix} \quad e_2 = \begin{bmatrix} 0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 \end{bmatrix}$$

$$e_3 = \begin{bmatrix} 0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 \end{bmatrix}$$

Another reason is that the decision variable, which is the total number of overworking hours of employee in a month for a project, also varies in the solvers. As we can see below from the overworked matrices from all three solvers, they differ. For instance if we examine 4th worker, we see that only the second solver assigned 4th worker with an overwork on 4th project.

$$o_1 = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 40 & 0 & 40 & 0 & 0 & 0 & 0 & 0 \\
40 & 0 & 0 & 0 & 40 & 0 & 0 & 0 & 40 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
40 & 40 & 0 & 40 & 0 & 0 & 40 & 0 & 0 & 0 \\
0 & 40 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 40 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 40 & 0 & 40 & 40 & 40 \\
0 & 0 & 40 & 40 & 0 & 40 & 40 & 40 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}$$

$$
o_2 = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 40 & 40 & 0 & 30 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 40 & 40 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 40 & 0 & 0 & 0 \\
0 & 0 & 40 & 40 & 0 & 0 & 0 & 40 & 40 & 40 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
40 & 40 & 0 & 0 & 40 & 10 & 0 & 40 & 40 & 40 \\
40 & 40 & 0 & 0 & 0 & 0 & 40 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
$$

$$
o_3 \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
40 & 0 & 0 & 0 & 30 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 40 & 0 & 0 & 40 & 0 & 40 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 40 & 0 & 0 & 0 & 0 & 40 & 0 & 40 \\
0 & 40 & 40 & 0 & 0 & 40 & 0 & 0 & 0 & 40 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
40 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 40 & 0 & 40 & 10 & 40 & 40 & 0 & 40 & 0 \\
0 & 0 & 0 & 0 & 40 & 0 & 0 & 40 & 0 & 0
\end{bmatrix}
$$

In addition to this, we also noticed that in our solvers, they can't solve project 8 in all three of them. Therefore, when we can conclude that when we compare objective functions of Part A and Part B (for Part A maximize monthly profit and for Part B maximize the number of completed projects) we examine that 9 projects are assigned. However, monthly profit is decreased in Part B in all solvers.

Part C

At part C, there is a completely new problem. New problem is stated below:

YigitBora Partners is a consulting firm that wants to assign its four senior members; Yigit, Bora, Gozde, and Michelle to four companies, namely; Amazor, ProctorGambler, Pegasos, Sielens. Each company has 3 available projects. It means there are 12 available projects in total. The hourly works must be done to complete each project is given below.

Assumptions:

1. An employee can do all the projects.
2. More than 1 employee could assign to a single project of any companies.
3. The given hourly works of companies and specific projects is equal to the required minimum total working hours of employees.

English Description of Constraints:

1. Bora can't work more than 80 hours.
2. Michelle can't work more than 80 hours.
3. Any employee can not work less than 60 hours.
4. Any employee can not work more than 1000 hours.
5. Gözde needs to work more than 150 hours on project 1.
6. Gözde can not work at project 3.
7. Michelle does not work at project 1 of any company.
8. Yiğit needs to work at least 30 hours with company ProctorGambler.
9. Number of hours worked on a project must be bigger than required working hours for each project. (Stated in parameters)
10. Total working hour for any employee is bigger than 0. (This constraint is redundant for now but it can help us to avoid problems for newly introduced specıal cases.

Why this choice of variables:

1. X_{ijk} variables are used to determine how many hours i-th employee works on the j-th company's k-th project.

Parameters:

c_i = I-th employee's cost per working hour. $i = 1, 2, \ldots, 4$

h_{ij} = Required number of hours for i-th company's j-th project.
$i = 1, 2, \ldots, 4$, $j = 1, 2, 3$

Decision Variables:

X_{ijk} = i-th employee works on the j-th company's k-th project. $i = 1, 2, \ldots, 4$, $j = 1, 2, \ldots, 4$, $i = 1, 2, 3$

Model:

$$\max \sum_{i=1}^{4} \sum_{j=1}^{4} \sum_{k=1}^{3} c_i X_{ijk}$$
$$Subjectto$$
$$\begin{array}{ll}
1. & \sum_{j}^{4} \sum_{k=1}^{3} X_{2jk} \leq 80 \\
2. & \sum_{j}^{4} \sum_{k=1}^{3} X_{4jk} \leq 80 \\
3. & \sum_{j}^{4} \sum_{k=1}^{3} X_{ijk} \geq 60 \quad i = 1, 2, \ldots, 4 \\
4. & \sum_{j}^{4} \sum_{k=1}^{3} X_{ijk} \leq 1000 \quad i = 1, 2, \ldots, 4 \\
5. & \sum_{k=1}^{3} X_{31k} \leq 150 \\
6. & \sum_{k}^{3} X_{33k} = 0 \\
7. & \sum_{k}^{3} X_{41k} = 0 \\
8. & \sum_{j}^{4} X_{1j2} \geq 30 \\
9. & \sum_{k}^{3} X_{ijk} \geq H_{jk} \quad k = 1, 2, \ldots, 4 \quad j = 1, 2, 3 \\
10. & X_{ijk} \geq 0 \quad i = 1, 2, \ldots, 4 \quad j = 1, 2, 3 \quad k = 1, 2, \ldots, 4
\end{array} \tag{1}$$

Data:

h_{jk} *Data Matrix*

$$\begin{bmatrix} 13 & 70 & 45 & 60 \\ 45 & 66 & 33 & 80 \\ 60 & 55 & 98 & 40 \end{bmatrix}$$

c_i *Data Matrix*

$$\begin{bmatrix} 100 \\ 85 \\ 90 \\ 80 \end{bmatrix} \tag{2}$$

Sensitivity Analysis Analysis:

First Table: Decision Variables
From the decision variables table we can see how many hours a person worked at each project from the "Final Value" column. Reduced cost column shows us the coefficients of row zero on optimal table for any variable (for our example it is X which states how many hours a person worked at a company's any job) this means it indicates how much our objective function value will change for unit change in that variable. "Objective value" column is hourly pay for our example and we can see how we can change hourly pay without changing our objective function value from the "Allowable increase" and "Allowable decrease" columns. To be more specific, for example for the M3 row from the first table we can see Yiğit's hourly payment is 100 dollars from objective coefficient value column and by looking allowable increase and allowable decrease columns we can find the range [100,110] for objective coefficient value of M3 row. Similarly, here are intervals for objective coefficient value ranges for non zero final value cells:

M3 - [100,110]	L5 - [0,100]
R3 - [90, 100]	N5 - [0,100]
J4 - [85, 85]	O5 - [0,100]
P4 - [85, 85]	Q5 - [0,100]
S4 - [-15, 85]	R5 - [0,100]
H5 - [0, 100]	J6 - [80,80]
I5 - [0, 100]	M6 - [70,80]
K5 - [0, 100]	

For zero final value cells interval logic is the same. However we also see infinity values there, reason for that is if someone is not working at a certain project, it is not surprising that that person will not work if their salary per hour goes to infinity since this is a cost minimization problem but if we decrease their salary more than the given interval they MAY work on that specific project which can change the solution.
From the reduced cost column we can see how much change is needed for a non basic variable to become a basic variable. For example for the constraint "Yiğit Amazor1" that value is 10.

Second Table: Constraints
By looking the Final Value columns we can see the value of our constraints at the optimal solution we found. For example Gözde Project1 constraint's final value is 208, this shows us our constraint about Gözde's total working hour on any project 1 is 208. Furhtermore,

from the allowable increase or from allowable decrease columns we can see what are the limits if we want to change the right hand side of that specific constraint. Returning to our Gözde Project 1 example we can see since on our solution that constraint's value is 208 we can only increase the current Right hand side value (150) at most 58. After that point our current solution will not satisfy the new requirements. We need the check the optimality of our solution. Since we are changing the RHS we may disturb the primal optimality and as a result of that we may need to use dual simplex to return to optimality. Even though we are in the ranges of the allowable increase and decrease thus our solution is still optimal, our optimal solution value will change according to our constraint's shadow price. To visualize it, "Hours Worked Pegasos1" constraint has 90 shadow price, this means if we increase this contraint by 5 units our optimal solution value (which is not wanted since this is a minimization pronblem) will also increase 5 times our constraint's shadow price which is 450. For futher analysis if RHS value and Final values are equal to each other we can say that it is a binding constraint. To give an example, "Hours Worked P&G2" constraint is a binding constraint.

Also there is another report that is called "Feasibility Report" from this report we can see our total cost which is 59580 and this is our optimal solution value. Any action that makes this value to increase is an unwanted outcome since this problem is a minimization problem and we want our total cost low as possible.

Table 1: Feasibility Report

Cell	**Name**	**First Value**	**Last Value**
H14	Total Cost	59580	59580

Table 2: Decision Variables

Cells	Name	Final Value	Reduced Costs	Objective Value	Allowable Increase	Allowable Decrease
H3	Yiğit Amazor1	0	10	100	1E+100	10
I3	Yiğit Amazor2	0	10	100	1E+100	10
J3	Yiğit Amazor3	0	0	100	1E+100	0
K3	Yiğit P&G 1	0	10	100	1E+100	10
L3	Yiğit P&G 2	0	10	100	1E+100	10
M3	Yiğit P&G 3	30	0	100	10,0000001	0
N3	Yiğit Pegasos1	0	10	100	1E+100	10
O3	Yiğit Pegasos 2	0	10	100	1E+100	10
P3	Yiğit Pegasos 3	63	0	100	0	10,0000001
Q3	Yiğit Sielens 1	0	10	100	1E+100	10
R3	Yiğit Sielens 2	0	10	100	1E+100	10
S3	Yiğit Sielens 3	0	0	100	1E+100	0
H4	Bora Amazor1	0	10	85	1E+100	10
I4	Bora Amazor2	0	10	85	1E+100	10
J4	Bora Amazor3	5	0	85	0	0
K4	Bora P&G 1	0	10	85	1E+100	10
L4	Bora P&G 2	0	10	85	1E+100	10
M4	Bora P&G 3	0	0	85	1E+100	0
N4	Bora Pegasos1	0	10	85	1E+100	10
O4	Bora Pegasos 2	0	10	85	1E+100	10
P4	Bora Pegasos 3	35	0	85	0	0
Q4	Bora Sielens 1	0	10	85	1E+100	10
R4	Bora Sielens 2	0	10	85	1E+100	10
S4	Bora Sielens 3	40	0	85	0	100,0000001
H5	Gözde Amazor1	13	0	90	10,0000001	90,0000001
I5	Gözde Amazor2	45	0	90	10,0000001	90,0000001
J5	Gözde Amazor3	0	0	90	1E+100	1E+100
K5	Gözde P&G 1	70	0	90	10,0000001	90,0000001
L5	Gözde P&G 2	66	0	90	10,0000001	90,0000001
M5	Gözde P&G 3	0	0	90	1E+100	1E+100
N5	Gözde Pegasos1	45	0	90	10,0000001	90,0000001
O5	Gözde Pegasos 2	33	0	90	10,0000001	90,0000001
P5	Gözde Pegasos 3	0	0	90	1E+100	1E+100
Q5	Gözde Sielens 1	80	0	90	10,0000001	90,0000001
R5	Gözde Sielens 2	60	0	90	10,0000001	90,0000001
S5	Gözde Sielens 3	0	0	90	1E+100	1E+100
H6	Michelle Amazor1	0	10	80	1E+100	10
I6	Michelle Amazor2	0	10	80	1E+100	10
J6	Michelle Amazor3	55	0	80	0	0
K6	Michelle P&G 1	0	10	80	1E+100	10
L6	Michelle P&G 2	0	10	80	1E+100	10
M6	Michelle P&G 3	25	0	80	0	10,0000001
N6	Michelle Pegasos1	0	10	80	1E+100	10
O6	Michelle Pegasos 2	0	10	80	1E+100	10
P6	Michelle Pegasos 3	0	0	80	1E+100	0
Q6	Michelle Sielens 1	0	10	80	1E+100	10
R6	Michelle Sielens 2	0	10	80	1E+100	10
S6	Michelle Sielens 3	0	0	80	1E+100	0

Table 3: Constraints

Cells	Name	Final Value	Shadow Price	RHS Value	Allowable Increase	Allowable Decrease
B19>=150	Gözde Project1	208	0	150	58	1E+100
B20>=30	Yiğit P&G	30	0	30	5	30
H6=0	Michelle Amazor1	0	0	0	0	0
H7>=H8	Hours Worked Amazor1	13	90	13	588	13
I7>=I8	Hours Worked Amazor2	45	90	45	588	45
J5=0	Gözde Amazor3	0	-10	0	5	0
J7>=J8	Hours Worked Amazor3	60	100	60	35	5
K6=0	Michelle P&G 1	0	0	0	0	0
K7>=K8	Hours Worked P&G 1	70	90	70	588	58
L7>=L8	Hours Worked P&G 2	66	90	66	588	66
M5=0	Gözde P&G 3	0	-10	0	5	0
M7>=M8	Hours Worked P&G 3	55	100	55	35	5
N6=0	Michelle Pegasos1	0	0	0	0	0
N7>=N8	Hours Worked Pegasos1	45	90	45	588	45
O7>=O8	Hours Worked Pegasos 2	33	90	33	588	33
P5=0	Gözde Pegasos 3	0	-10	0	33	0
P7>=P8	Hours Worked Pegasos 3	98	100	98	907	33
Q6=0	Michelle Sielens 1	0	0	0	0	0
Q7>=Q8	Hours Worked Sielens 1	80	90	80	588	58
R7>=R8	Hours Worked Sielens 2	60	90	60	588	60
S5=0	Gözde Sielens 3	0	-10	0	33	0
S7>=S8	Hours Worked Sielens 3	40	100	40	35	33
T3<=1000	Yiğit Total Working Hour	93	0	1000	1E+100	907
T4<=1000	Bora Total Working Hour	80	0	1000	1E+100	920
T5<=1000	Gözde Total Working Hour	412	0	1000	1E+100	588
T6<=1000	Michelle Total Working Hour	80	0	1000	1E+100	920
T3>=60	Yiğit Total Working Hour	93	0	60	33	1E+100
T4>=60	Bora Total Working Hour	80	0	60	20	1E+100
T5>=60	Gözde Total Working Hour	412	0	60	352	1E+100
T6>=60	Michelle Total Working Hour	80	0	60	20	1E+100
T4<=80	Bora Total Working Hour	80	-15	80	33	20
T6<=80	Michelle Total Working Hour	80	-20	80	5	20