IE 202 INTRODUCTION TO MODELING AND OPTIMIZATION
TERM PROJECT
STAGE 1
CONSULTING FIRM PROBLEM

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CONSULTING FIRM PROBLEM

A&N is a consulting firm that operates in more than 30 countries. In this consulting firm, there are 6 different types of positions: associate, senior associate, manager, senior manager, director, and partner. In the Ankara office, there are K employees in total with different position levels.In addition, the AN consulting firm gives services in M different departments in this office. We assume that all employee has enough knowledge of all departments. Our aim is to maximize the profit by completing projects.

Assumptions:

1. Each person can be assigned to one department only

2. Maximum number of hours which a person can work a day is $H/30$ (including extra hours).

3. Number of required hours for a project and the workload of a person is integer

4. Similarity between fierce competition is assumed to be less than or equal to 2

5. Position of the employees and difficulties of each project are determined at the start and taken as parameter.

6. For the functions as min(), max() we assume that while having a real problem we will know these constants and easily choose the maximum of them

7. Assignment and completing a project are considered two different aspects. For instance, a person may be assigned to a level 3 project in order to start assigning level 2 projects to others without considering whether he or she can complete the assigned level 3 project.

English Description of Constraints:

1. Departments have to conduct at least one project.

2. Every project can be assigned to only one department.

3-4. Each personal have to assigned at least one project and also, have to be assigned at most five project.

5-6. A personal working in a position in a project must have an upper and lower limit, if this constraint is provided, the qualification of the department is also provided.

7. The time spent on a project (workload) equal or greater than the time spent by the employee working on a project. (Since there are two options either project is finished than they are equal, second project is not finished)

8. The usual salary an employee receives depends on the employee's assigned project and position.

9. When the time spent on a project is added up monthly for any employee, the maximum working time in the month given in the question cannot be over H hours.

10-11-12. Linerization every employee is assigned to only one department.

13-14. Since Department 1 and Department 2 are in a competition and the number of projects assigned to them is similar, the difference in the number of projects between them should not be too much.

15-16. If an employee is working more than the usual working time, the project they are working on should have a complexity level of 3.

17-18. If Department 1 is conducting Projects 1 and 2, then Project 3 cannot be conducted in Department 3.

19-20-21-22. The order in which projects are assigned is based on their level of complexity. If a project with difficulty level 3 is not assigned, a project with difficulty level 2 cannot be completed.

23-24. Linearization of whether employee worked extra hour or not.

25-26-27-28. Linearization of daily working hour and if employee has 3th level difficulty project's multiplication.

29-30. Positivity constraints.

Parameter s_{ps} meaning the position of the p-th person s being the position in the list [associate, senior associate, manager, senior manager, director, and partner] taking values from 1 to 6 respectively.

Why this choice of variables:

1. X variables are used to make sure that one project is assigned to only one department and every project is assigned to at least one department.

2. W variables are used to determine any employee's workload (in terms of hours) for each project and by using this variable it is possible to check whether project is finished or not.

3. Y decision variables are used to find which employees assigned to which projects. This helps the process while making sure any employee is assigned to maximum five projects, this variable also helps to meet the any project's employee requirements. For the first objective function this variable is also used to find employee costs since employees salaries are affected by the number of projects they are assigned to.

4. G variables are used to find overwork hour of any employee since overwork has additional cost and it is required for objective function one.

5. D variables are used to check whether an employee is assigned to third level project.

6. E variables are used to determine the number of p-th level projects assigned.

7. K variables are used to show which projects are finished since if the workload is not met for each project it can not be considered as finished and.

8. Z variables are used to linearize multiplication of G and P random variable.

Parameters: c_i = penalty cost for i-th project, $i = 1, 2, \ldots, N$

l_{is} = max limit on i-th project from s-th position, $i = 1, 2, \ldots, N, s = 1, 2, \ldots, 6$

t_{is} = min limit on i-th project from s-th position, $i = 1, 2, \ldots, N, s = 1, 2, \ldots, 6$

t_i = workload of i-th project, $i = 1, 2, \ldots, N$

b_{ps} = salary of p-th personnel from s-th position per project, $p = 1, 2, \ldots, K$, $s = 1, 2, \ldots, 6$

m_p = usual salary of p-th personnel

$s_{ps} = 1$ if p-th personnel is from s-th position, $s = 1, 2, \ldots, 6, p = 1, 2, \ldots, K$

r_i = number of i-th level projects, $i = 1, 2, \ldots, N$

p_i = profit from i-th project, $i = 1, 2, \ldots, N$

Decision Variables:

$$x_{ij} = \begin{cases} 1, & \text{if project } i \text{ is assigned to department } j \\ 0, & \text{otherwise} \end{cases} \quad i \in \{1, 2, \ldots, N\} \; j \in \{1, 2, \ldots, M\}$$

w_{pi} : the workload of the p-th personnel on the i-th project. $i \in \{1, 2, \ldots, N\}$

$$y_{pi} = \begin{cases} 1, & \text{if personnel } p \text{ is assigned to project } i \\ 0, & \text{otherwise} \end{cases} \quad p \in \{1, 2, \ldots, K\} \; j \in \{1, 2, \ldots, M\}$$

g_{pq} : the number of hours worked by the p-th personnel on q-th day.
$p \in \{1, 2, \ldots, K\} \; q \in \{1, 2, \ldots, 30\}$

$$d_p = \begin{cases} 1, & \text{if personnel } p \text{ is assigned to a level 3 project} \\ 0, & \text{otherwise} \end{cases} \quad p \in \{1, 2, \ldots, K\}$$

e_p : the number of assigned p-th level projects $p \in \{1, 2, 3\}$

$$k_i = \begin{cases} 1, & \text{if project i is completed} \\ 0, & \text{otherwise} \end{cases} \quad i \in \{1, 2, \ldots, N\}$$

$z_{pi} = g_{pq} \times d_p \quad i \in \{1, 2, \ldots, N\} \; p \in \{1, 2, 3\} \; q \in \{1, 2, \ldots, 30\}$

Model (Equivalent Linear Form forcing z variables to take value 0):

A) $\max \left(-\sum_{i=1}^{N} \sum_{p=1}^{K} m_p w_{pi} + \sum_{i=1}^{N} p_i k_i - \sum_{i=1}^{N} c_i(1 - k_i) - 50 \sum_{i=1}^{N} \sum_{p=1}^{K} z_{pi} - 8d_p \right)$

B) $\max \left(\sum_{i=1}^{N} k_i \right)$

subject to

1. $1 \leq \sum_{i=1}^{N} x_{ij} \quad j = 1, 2, \ldots, M$

2. $\sum_{j=1}^{M} x_{ij} \leq 1 \quad i = 1, 2, \ldots, N$

3. $1 \leq \sum_{i=1}^{M} y_{pi} \quad p = 1, 2, \ldots, K$

4. $\sum_{i=1}^{M} y_{pi} \leq 5 \quad p = 1, 2, \ldots, K$

5. $f_{is} \leq \sum_{p=1}^{K} y_{pi} \times s_{ps} \quad i = 1, 2, \ldots, N, \quad s = 1, 2, \ldots, 6$

6. $\sum_{p=1}^{K} y_{pi} \times s_{ps} \leq l_{is} \quad i = 1, 2, \ldots, N, \quad s = 1, 2, \ldots, 6$

7. $t_i \geq \sum_{p=1}^{K} w_{pi} \quad i = 1, 2, \ldots, N$

8. $m_p = \sum_{t=1}^{N} \sum_{s=1}^{6} b_{ps} \times y_{pi} \quad p = 1, 2, \ldots, K$

9. $\sum_{q=1}^{30} g_{pq} \leq H \quad p = 1, 2, \ldots, K$

10. $z_1 \leq x_{ij} \quad i = 1, 2, \ldots, N, \quad j = 1, 2, \ldots, M \quad z_1 \in \{0, 1\}$

11. $z_1 \leq y_{pi} \quad i = 1, 2, \ldots, N, \quad p = 1, 2, \ldots, K \quad z_1 \in \{0, 1\}$

12. $z_1 \geq x_{ij} + y_{pi} - 1 \quad i = 1, 2, \ldots, N \quad j = 1, 2, \ldots, M \quad p = 1, 2, \ldots, K \quad z_1 \in \{0, 1\}$

13. $\sum_{i=1}^{N} (x_{i1} - x_{i2}) \leq 2 \quad i = 1, 2, \ldots, N$

14. $-2 \leq \sum_{i=1}^{N} (x_{i1} - x_{i2}) \quad i = 1, 2, \ldots, N$

15. $g_{pq} - 8 \leq (\frac{H}{30} - 8) \times z_2 \quad z_2 \in \{0, 1\} \quad p = 1, 2, \ldots, K \quad q = 1, 2, \ldots, 30$

16. $1 - d_p \leq (\frac{H}{30} - 8)(1 - z_2) \quad z_2 \in \{0, 1\} \quad p = 1, 2, \ldots, K$

17. $x_{11} + x_{21} \leq z_4 \quad z_4 \in \{0, 1\}$

18. $x_{33} \leq 1 - z_5 \quad z_5 \in \{0, 1\}$

19. $e_2 \leq max(r_3, r_2)z_6 \quad z_6 \in \{0, 1\}$

20. $r_3 - e_3 \leq max(r_3, r_2)(1 - z_6) \quad z_6 \in \{0, 1\}$

21. $e_1 \leq max(r_1, r_2)z_7 \quad z_7 \in \{0, 1\}$

22. $r_2 - e_2 \leq max(r_1, r_2)(1 - z_7) \quad z_7 \in \{0, 1\}$

23. $\sum_{p=1}^{k} w_{pi} - t_i + 1 \leq z_8 \quad p = 1, 2, \ldots, K \quad i = 1, 2, \ldots, N \quad z_8 \in \{0, 1\}$

24. $1 - k_i \leq (1 - z_8) \quad z_8 \in \{0, 1\} \quad i = 1, 2, \ldots, N$

Model (Equivalent Linear Form forcing z variables to take value 0):(cont)

25. $z_{pi} \leq g_{pq}$ $p = 1, 2, \ldots, K$ $i = 1, 2, \ldots, N$ $q = 1, 2, \ldots, 30$
26. $z_{pi} \leq \left(\frac{H}{30}\right)$ $z_{pi} \in \{0, 1\}$ $i = 1, 2, \ldots, N$ $p = 1, 2, \ldots, K$
27. $w_{pi} \geq 0$ $i = 1, 2, \ldots, N$ $p = 1, 2, \ldots, K$
28. $g_{pq} \geq 0$ $p = 1, 2, \ldots, K$ $q = 1, 2, \ldots, 30$
29. $e_p \geq 0$ $p = 1, 2, \ldots, K$
30. $z_{pi} \geq 0$ $i = 1, 2, \ldots, N$ $p = 1, 2, \ldots, K$

B) $\max \left(\sum_{i=1}^{N} k_i \right)$ (*a better looking function in the model!*)

Scenario A: In the question, it says, " Due to this, if a project with difficulty level 3 is left unassigned, then no project with difficulty level 2 can be completed." because of that, in order to complete many projects, we can satisfy the requirement by assigning level three and level two projects, respectively but completing the easier ones first. This can cause our profit to decrease or even money loss, but since our objective function aims to increase the number of completed projects, we would achieve our goal.

Scenario B: Since we do not have information about the values of the penalty costs of level three projects and the profit amount from finishing level one project, both objective functions can lead to the same output. To clarify the situation, completing level one projects may give us such profit that penalty costs of unfinished but assigned second and third-level projects can be negligible; in this case, we maximize profit and the number of completed projects at the same time.