Case Study 2: Controlling Dynamic Networks

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***Abstract*—In this case study, a new population was created and the SIRD model in Section 9.3 of the book was implemented. This model described the evolution of the disease within the population in a set time. Then the SIRD model was generalized to three new populations to describe both within and across population dynamics. Complete immunization was assumed for these populations. In addition, the SIRD model given in section 9.3 of the book was extended to show the actions of immunization within a population. Finally, a control strategy to decrease and stop the virus from spreading was designed.**

Introduction

The goals of this case study were to analyze the spread of an epidemic within and across populations, to explore the effects of immunization, and to control the spread of infection in a network. The dynamics of an infection and the spread of an epidemic can be modeled using a linear dynamical system (Boyd 168). A linear dynamical system is a simple model for a system, in which the system changes in time (Colonius). In other words, a linear dynamical system is a model for a sequence, in which x_{t+1} is a linear function of t. (Boyd 163) The matrix used in the equation for the linear dynamical system is called a dynamics matrix. It's the "A" matrix in the equation, $x_{t+1} = Ax_t$. This matrix is used for a given model. In this case study, the SIRD model was used. The SIRD model, stands for susceptible, infected, recovered and deceased. Certain control strategies are implemented in order to regulate how the system is evolves, and control strategies deal with the regulation of dynamical systems operating within a time period. (Control Theory) While the efficiency of the control strategies was tried to be maximized the costs were tried to be minimized.

I. Methods

A. Creating Populations

First a population size was determined, and the "x_t" vector was initialized by multiplying the population size with the [1; 0; 0; 0] vector, meaning that everyone was susceptible at the start. The "A" matrix was a 4x4 matrix used to determine the evolution of the disease. For part 1 this matrix was given in Section 9.3 of the book, and for other parts the matrices were created by the user.

B. Creating A System for The Evolution of Disease Within Populations

After the population was created the time in which the system evolved was determined. For this part of the case study, the time of evolution was 50 days. The system was created using the "ss" function built in MATLAB. The "ss" function was used in the following way: ss(A,[],[],[],[]), where "A" was the matrix used the determined the evolution of the system, and the brackets were used to fill in unused parameters. Then this system was simulated by using "lsim", which is another built in MATLAB function. The function "lsim" was used in the following way: [~,~,x] = lsim(system, [], t, x_initial), where "system" was the system created, "t" was the time in which the system evolves and "x_initial" was the initial state of the population. Then, "x" was transposed in order to be plotted properly, and the evolution of the population in terms of susceptible, infected, recovered and deceased people were graphed.

C. Creating a System for The Evolution of Disease Across Populations

To model the spread of disease across populations, three new populations are created with the method explained in A. The matrices of these populations were designed with a simplifying assumption that people who got infected could not become susceptible again, meaning that once a person got infected that person became immune to the disease. The matrices of these 3 populations were concatenated using the "cat" function in MATLAB. The matrices created for each population were as follows: Pop1=cat(1,B,zeros(4),zeros(4)), Pop2=cat(1,zeros(4),C,zeros(4)), and Pop3=cat(1,zeros(4),zeros(4),D) where "B", "C", and "D" were 4x4 matrices that were used to determine the evolution of the disease in each population. Next a block matrix was created through combining these matrices: threePopulations = cat(2,Pop1,Pop2,Pop3). Then this matrix was edited in order to have people move from one population to another. The matrix was edited taking into consideration that recovered people after getting infected did not move across populations, and deceased people also did not move through populations, for obvious reasons. In addition, the assumption was made that people could not change state after moving across populations.

For this part of the case study assume population 1 has a population size four times smaller than the size of population 2 but two times the size of population 3. Population 1 is a "first world" country and has the best medical care and strongest economy out of the three countries. Population 2 is a more densely populated country, where the medical care is poor and the economy is in a recession. For this reason, people are leaving population 2 and mostly heading for population 1. Population 3 is a more rural developing country that does not have much infrastructure or medical care. For this reason, the infection is extremely dangerous in this country and death rates are very high. People are going from this country to population 1. Also a few people who are adventurous tourists from populations 1 and 2 travel to population 3. For this study, population 1 is represented by Canada, population 2 is represented by Bangladesh and population 3 is represented by Malawi.

D. Implementing Immunization to a Population

The "A" matrix from section 9.3 of the textbook was used for this section of the case study. In this model the assumption that people did not go back to susceptible after being infected did not apply. It was decided that immunization was given only to the infected people, so people who went from infected to susceptible did not get the cure. Susceptible people also did not get the cure. This meant that the immunization made infected people go to recovered. So, a new matrix for immunization was created named "immunization" which had the value of "-0.3" in the second row, second column and the value "0.3" in the third row, second column. This meant that people who got the vaccine went from infected to recovered. In addition, there was limited amount of the vaccine, so not everyone who was infected could receive the vaccine. Then this immunization matrix was added to the matrix "A" and a new matrix named "newA" was created. Then, a new system was created and simulated using the methods previously described.

E. *Designing a Control Strategy for Virus Spread*

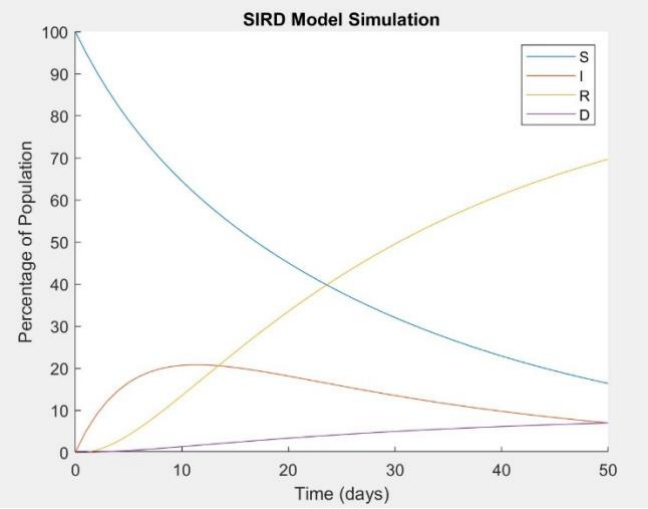
In order to implement a strategy to control the spread of an epidemic in a small world topology network, first, the lattice network needed to be converted into a small world topology network. This was done by adding edges between different nodes in the network to provide both short and long range connections. Creating a small world topology network allowed the network to be much more connected, which caused disease to spread through the network quicker. After small world topology was implemented, the size of the control vector was set with the following line of MATLAB code: control_vector = zeros([length(x_out),1]). "x_out" was the matrix which showed the evolution of each population by the given time. So, the length of x_out was 100, as it had 100 populations (rows) and 90 days (columns). Then the number of columns were checked. If the number of columns was only 1, nothing was done to the control vector and its entries remained as all zeros. Otherwise, a "for" loop was used to check the difference between each row of the last column (each row of "cols") with each row of the previous column ("cols-1"). If the difference between the row in index i of the last column (cols) and the previous column was greater than 0.001 then the row in index i in the last column (cols) of the x_out matrix was multiplied by -1 in order to stop the spread and eradicate the disease.

II. Results

A. *Part 1: Implementing SIR Model in Section 9.3*

Implementing the SIR model from the book produced the results shown in Figure 1. In this model 95% of the susceptible population remains susceptible and the other 5% of the susceptible people become infected. Also 85% of the infected population remains infected, 10% recover and become immune, 4% recover and become susceptible again, and 1% die.

Figure 1:



B. *Part 2: Within and Across Population Dynamics*

Three populations were modeled for both the situation where they were isolated and the situation where people travel across populations. Figures 2 show the SIR models for the populations 1, 2, and 3 (Canada, Bangladesh, and Malawi) when they were isolated populations and people were not leaving or entering the population. Here the assumption that infected people did not return to the susceptible state applies. In Canada 95% of the susceptible people remained susceptible and 5% of the susceptible people became infected. Also 75% of infected people remained infected, 24% of infected people recovered, and 1% of infected people died. In Bangladesh 70% of the susceptible people remained susceptible and 30% of the susceptible people became infected. 75% of infected people remained infected, 10% of infected people recovered, and 15% of infected people died. Finally, in Malawi 50% of the susceptible people remained susceptible the other 50% of the susceptible people became infected. 15% of infected people remained infected, while only 5% of infected people recovered, and 80% of infected people died. Due to the poor medical care available in Malawi the disease was extremely deadly and barely anyone survived. In Bangladesh, the epidemic caused a lot of deaths as well, but a little under half of the Bangladesh population was able to recover and survive. In Canada, the disease did little damage and a very small percentage of the population died. This was due to Canada having the best medical care out of the three countries which allowed many of the infected people to be cured before they died. Also, it should be noted that the percentages of the populations in the graphs are relative to the population of Canada. So, for Bangladesh (which is assumed to be four times the population size of Canada) 200% represents 200% of the Canadian population, which is 50% of the Bangladesh population. Likewise, for Malawi (which is assumed to be half the population size of Canada) 50% represents 50% of the Canadian population which is 100% of the Malawi population.

Figure 2:

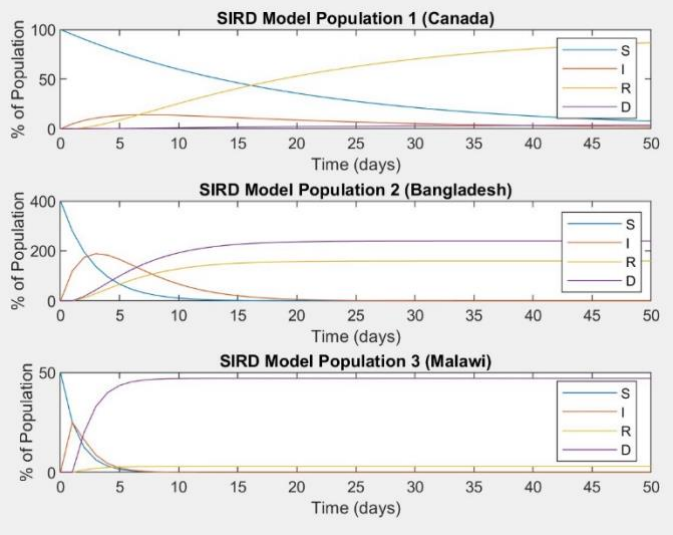
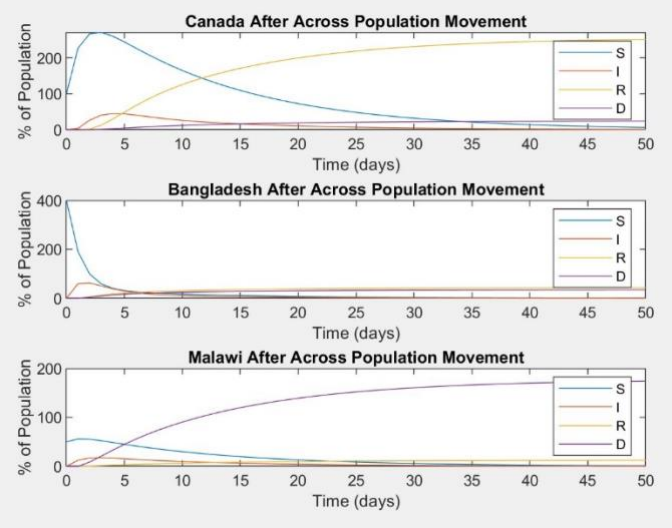


Figure 3 shows these same populations after people traveled across populations. In Canada 33% of the susceptible people from Bangladesh and 20% of susceptible people from Malawi came into the country. Also 23% of the infected population in Bangladesh and 13% of the infected population in Malawi traveled to Canada. People from Bangladesh and Malawi wanted to come to Canada because they had the best medical care and people had a better chance of recovering in Canada. This caused the percentage of death in Canada to be bigger than it was when Canada was isolated, but the disease still did little damage to Canada as most people we able to recover. In Bangladesh only 4% of susceptible people from Canada and 5% of susceptible people form Malawi entered the country. Only 3% of infected Canadians and 2% of the infected from Malawi went to Bangladesh. The result for Bangladesh compared to when it was isolated was that the percentage of deaths dropped significantly, but this was not caused by advances in medicine in Bangladesh, but rather by a large amount of the population fleeing from Bangladesh and heading to Canada where the epidemic is controlled better. Finally, in Malawi 7% of susceptible Canadians and 6% of susceptible people in Bangladesh entered the country. Also 1% of

infected people from Canada and 3% of infected people from Bangladesh went to Malawi. The results for Malawi after the population movement are more tragic than when each country was isolated because when all the people from Canada and Bangladesh come in most of them end up dying which increases the percentage of deaths in the population. Again, it should be noted in this figure that the percentages of the populations in the graphs are relative to the population of Canada.

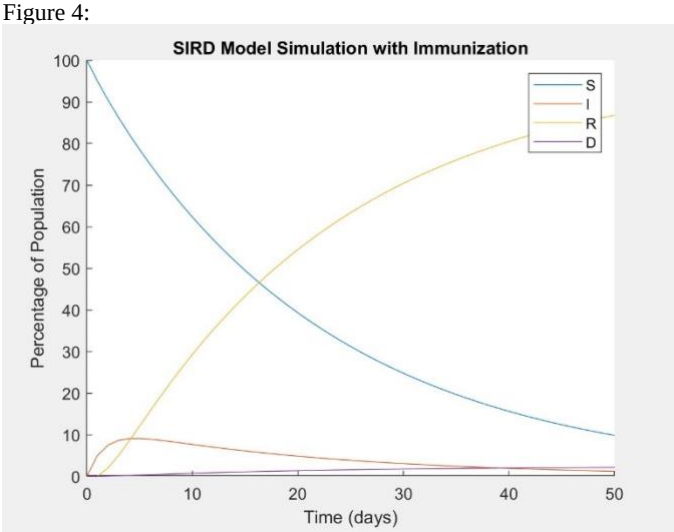
Figure 3:



C. *Part 3: Immunization Within a Population*

This part of the case study compared used the same SIR model from the textbook that was used in part 1 except this simulation allowed for immunization. As expected, when immunization was used the percentage of infected and dead people in the population in Figure 4 was less than in Figure 1, where there was no immunization. Also, the number of recovered people increased and the number of infected people decreased in a shorter period of time in Figure 4 than in Figure 1.

Figure 4:



D. *Part 4: Using a Control Strategy to Mitigate Virus Spread*

Originally the lattice structure of the network produced the network displayed in Figure 5, but when small world topology was integrated the network shown in figure 6 was produced.

Figure 5:

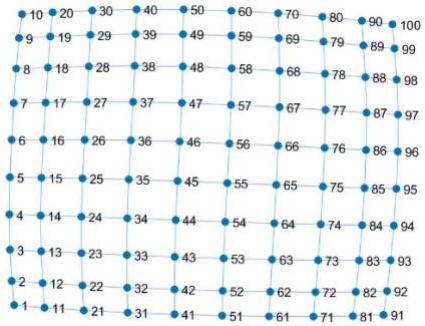
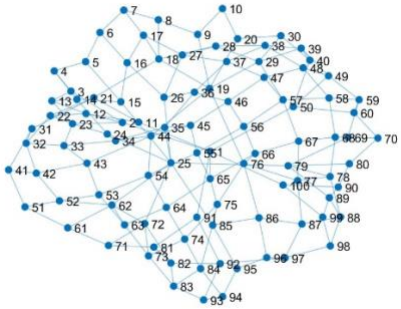


Figure 6:



With the more connected network, the virus spreads faster and when the simulation was run without a control strategy the results in Figure 7 and Figure 8 were produced. Figure 7 shows the activation of the virus over time and when the activation reached 1000, this means the virus was fully activated and spread across the whole network. Figure 8 shows the control magnitude versus time which was the cost of the control strategy. If there is no control strategy, the control magnitude is zero as seen in Figure 8.

Figure 7:

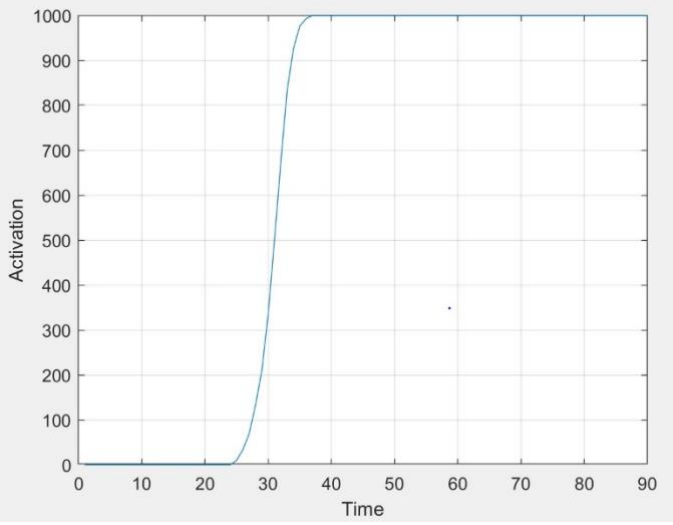
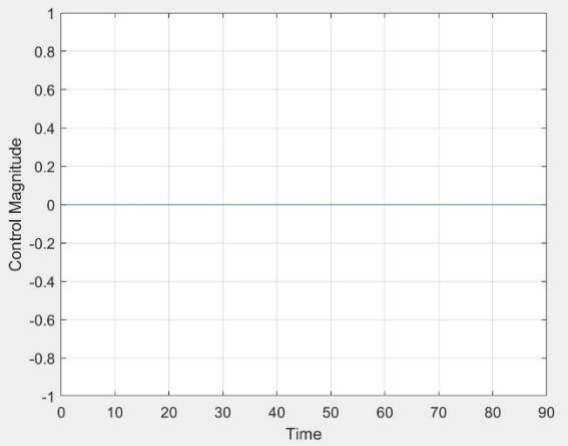


Figure 8:



With a connected network, the virus spreads fast so a control strategy was necessary in order to mitigate the virus. When the control vector was implemented to introduce vaccines into the network the result in Figure 9 and Figure 10 were produced. Figure 9 shows the activation of the virus in the network which was only about 24 compared to 1000 when no control strategy was used. Figure 10 shows the cost of using the control vector which reaches a maximum magnitude of about 24 but is effective in eradicating the virus from the network.

Figure 9:

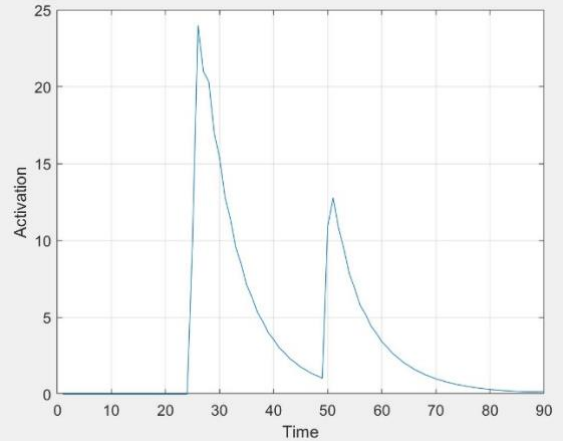
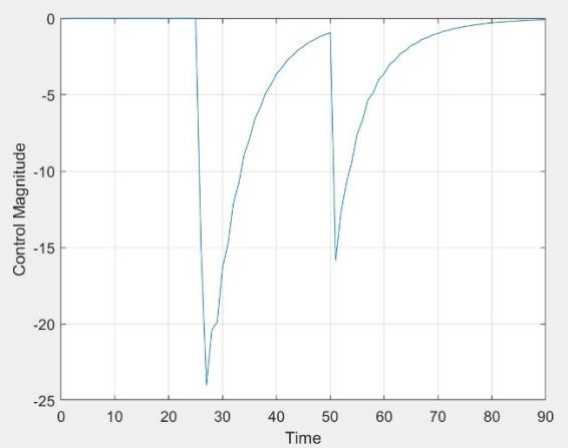


Figure 10:



III. Conclusions

In this case study it was shown that vector- matrix operations could be used to create and simulate dynamical systems that affect the evolution of populations with different models and characteristics. In Part 1 the effect of a given SIRD model on the evolution of a population was observed. By creating more populations and observing the interactions between populations in Part 2 it was observed that the interaction of different populations can greatly influence the states of other populations. It was determined that depending on the state of the travelling people across population movement can increase or decrease the percentage of the infected and susceptible people in that population. However, the model implemented in this part included some unrealistic assumptions, like people that have moved to different populations cannot change states, and like recovered people don't move across populations. These unrealistic assumptions of our model limited the how realistic our model was. The effect immunization has was seen by comparing the results from Part 1 and Part 3, and it was determined that immunization increases the number of recovered people in a population by the same amount it decreases the number of infected. In Part 4, the effect of a control strategy to mitigate a virus spread was examined, and the costs were tried to be minimized while the activation was tried to kept constant. Even though the strategy can be improved to stop virus spread in even more severe situations, it was observed that the control vector was sufficient to mitigate the spread of the virus, while taking cost and activation into consideration. Therefore, it was determined that control strategies can be used to regulate and change the evolution of a dynamic system, using matrix vector multiplication. In this case study the SIRD model on dynamic systems of populations with the use of vector-matrix operations was observed

References

1. Boyd, Stephen P., and Lieven Vandenberghe. *Introduction to Applied Linear Algebra: Vectors, Matrices, and Least Squares*. Cambridge University Press, 2018.

2. "Part 1: Matrices and Linear Dynamical Systems." *Dynamical Systems and Linear Algebra*, by Fritz Colonius and Wolfgang Kliemann, A.M.S., 2014, pp. 3–95. "Outlier." Outlier - an Overview | ScienceDirect Topics, www.sciencedirect.com/topics/mathematics/outlier.

3. "Control Theory." *Engineering*, engineering.fandom.com/wiki/Control_theory

4. "A,B,C,D,E." *Create State-Space Model, Convert to State-Space Model - MATLAB*, www.mathworks.com/help/control/ref/ss.html?searchHighlight=ss&s_tid=doc_srchtitle.

5. "Linear System Analyzer." *Simulate Time Response of Dynamic System to Arbitrary Inputs - MATLAB*, www.mathworks.com/help/control/ref/lsim.html?s_tid=doc_ta.