Case Study 3: Filtering Noise From Audio

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***Abstract*—In this case study, a linear dynamical system model was implemented through vector-matrix operations in order to model a RC circuit's behavior over time. The effect of the sampling interval in time on the accuracy of the model was seen. In addition, two circuits were designed with sinusoidally varying input and the transfer functions of these two circuits were plotted. The behaviors, such as the shape and the delay of these circuits was observed, in order to distinguish between a high pass filter and a low pass filter. Further, a model for two cascaded RC circuits was created, and the voltage drops across the two cascaded circuits were observed. Finally, a circuit was designed to model the filtration of unnecessary noises from the audio itself in order to maximize the signal to noise ratio in an audio recording, and to get a clearer sound from the audio recording.**

I. Introduction

A RC circuits is a circuit with a combination of capacitors and resistors. RC circuits have many applications, and one of the main applications of such circuits is filtering, and in our case filtering sound. Circuits can be a low-pass, a band-pass or a high pass filter, depending on the design of the circuit (Legazpi). A low-pass filter is a circuit designed to block the high frequencies and let the low frequencies pass, and inversely, a high-pass filter is a circuit designed to block the low frequencies and let the high frequencies pass (Says). A band pass filter is a circuit designed to filter out frequencies that lie in a particular range (has a lower and higher bound). Therefore, by cascading two RC circuits, one that is low-pass and one that is high-pass, a circuit with band-pass filter can be designed (Band-Pass Filters). The frequencies that have to be filtered out from the sound are reached by setting a range. In this case study, RC circuits and cascaded RC circuits are designed and analyzed, and a cascaded RC circuit is used to filter sound from given audio.

II. Methods

A. Modeling an RC Circuit

From Ohm's Law and Kirchhoff's Voltage Law the equation, $v_{C,k+1} = \left(1 - \frac{h}{RC}\right) v_{C,k} + \left(\frac{h}{RC}\right) v_{in,k}$, was derived. $\left(1 - \frac{h}{RC}\right)$, and $\left(\frac{h}{RC}\right)$ were implemented in MATLAB as a 1x1 matrices A and B in order to fit a time model. A linear dynamical system was created by the built-in functions "lsim" and "ss". This system was simulated for two different time intervals, h and hNew, and for two different number of time steps (k). Then, the same system was simulated with the same two time intervals, h and hNew, using a theoretical curve. The change in voltage in the capacitor over time was plotted for every situation.

B. Comparing Two RC Circuits

Two different RC circuits were modelled using the same method stated in "Modelling and Designing a RC Circuit". However, this time the voltage of the resistor was taken as the output voltage instead of the voltage of the capacitor, and a sine function for the input voltage (v_{in}) was used. The output voltages of both the resistor and the capacitor were displayed with two different sinusoidal voltage inputs, one with a frequency of 50 Hz and the other with a frequency of 1000 Hz. These two circuits with different frequencies were plotted, and the behaviors of the output voltages were observed. Then, the transfer functions of each circuit were calculated, and the results were once again plotted on the same graph. The behaviors of the graphs were observed and compared.

C. Comparing Two Cascaded RC Circuits

In this part, models for two cascaded circuits were created. The first cascaded circuit had a capacitor (C_1) placed at the point of the voltage drop of v_1, and the second circuit had a resistor (R_1) at the point of the voltage drop v_1.

From applying Kirchhoff's Current Law and Ohm's Law to our first circuit, certain equations were given. These equations were put into matrix-vector operations, in the form of Ax = b, where x was given, and A and b were created by looking at the equations and the vector x, where matrix A was a 6x6 square matrix and b was a 6x1 vector. For the second circuit, equation 17 and equation 20 were updated to fit the new circuit. A new matrix A and a new vector b were created. The vector b was changed from b = (0,0,0,Vc1(i),Vc3(i),Vin(i)), to , b = (0,0,0,Vin(i),Vc2(i),Vc3(i)).

Each circuit had sinusoidal functions for the input voltage. For circuit C, the vectors Vc1, Vc3, and V_out were initialized to zero. The time interval, h, the time steps , k, and the values for the capacitors and the resistors were assigned. Having all the values, the system was simulated in a for loop from 1 to k, updating the vector b with each time step. The vector b was updated through equations, Vc1(i+1) = Vc1(i)+(h/C_1)*x(1), and Vc3(i+1) = Vc3(i)+(h/C_3)*x(3). The input function, and the output function of V_out(i) = x(6), were plotted on the same graph. The same was done for circuit D, and its values.

The code for the voltage output was taken and a for loop was used to iterate through each frequency value (from 10 to 10000). Inside the for loop another for loop was used to iterate through the voltage output values (1 to k). For each frequency the maximum value of voltage output was taken and this maximum value was plotted against input frequencies using logarithmic axis, giving us the transfer functions.

D. Filtering Sound

Given the circuit diagram seen in Figure 1, the task was to design a circuit that acts as an audio filter. To do this Z_1 was set to be R_1 =510 Ω. Z_2 and Z_3 was set to C_2 and C_3 = 0.47μF. R_4 was given to be 16Ω. The model Ax=b was used once again, where b was the same b vector used for circuit "D", and A was the same matrix except for the changed resistor values. The circuit was designed and implemented using methods stated previously in the report. The circuit was simulated to filter audio and for each simulation the transfer function was plotted on a graph.

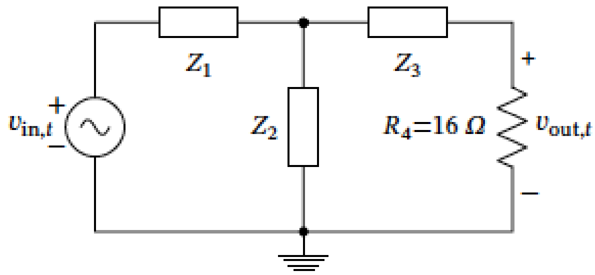

Figure 1: Circuit diagram for part 4

III. Results

A. Part 1: Model an RC Circuit

When modeling the behavior of the capacitor as a linear dynamical system the value of the sampling interval, h, was very important in producing an accurate charging curve for the capacitor. A smaller sampling interval meant there were more sample points recorded over a period of time compared to a larger sampling interval. The result of using different sized sampling intervals are seen in Figure 2 and Figure 3. In Figure 2 the h value was smaller than the h' value in Figure 3 so more samples were taken over a period of time and the charging curve was much smoother. In Figure 3, using a larger sampling interval resulted in a choppier graph that was less clear and accurate because less samples were taken.

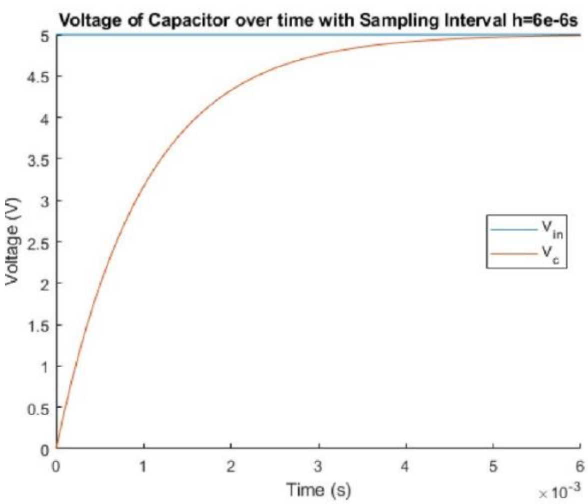

Figure 2: Voltage of capacitor with a constant input voltage using a time step of 6×10^{-6}s

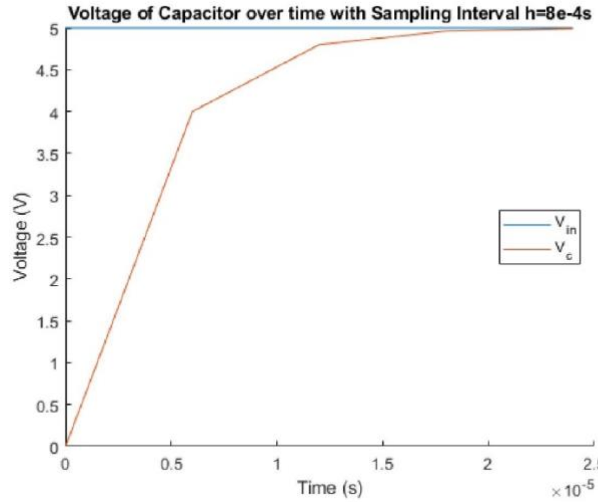

Figure 3: Voltage of capacitor with a constant input voltage using a time step of 8×10^{-4}s

In order to model the theoretical charging curve of the capacitor, shown in Figure 4, the equation $vC(t)$=5[1−exp(−t/RC)] was used. The theoretical charging curve was very similar to the curve produced in Figure 2 where h = 6×10^{-6}s. An important feature of the graph to note is the meaning of τ=RC, called the RC time constant. The RC time constant is a measure that helps to figure out how long it will take a capacitor to charge to a certain voltage level and the at this time, τ, the capacitor is charged to about 63% of the input voltage (RC Time Constant). For this circuit it was given that R=1kΩ and C=1μF, so τ=RC=0.001s. This value is larger than the h' value and would be too large of an h value to use for an accurate, smooth graph. A value smaller than h'=8×10^{-4}s was needed to produce such a graph.

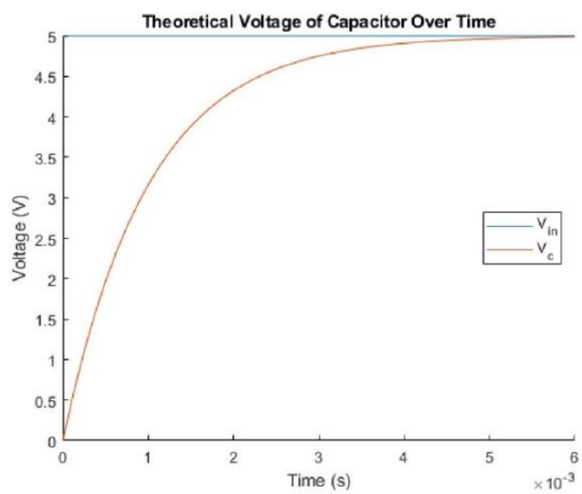

Figure 4: Theoretical voltage of capacitor with a constant input voltage

B. Part 2: Compare Two RC Circuits

The two RC circuits being compared were Circuit "A" which was used in part 1 and Circuit "B" which was the same circuit but with the locations of the capacitor and resistor switched. In Figure 5 and Figure 6 below the output of Circuit "A" is V_C and the output of Circuit "B" is V_R. When the input function was at a frequency of 50 Hz, the input voltage (V_{in}) had an amplitude of 5V, the voltage stored in the capacitor (V_C) had an amplitude a little less than 5V, and the resistor voltage (V_R) had an amplitude close to 1. When the input function was at a frequency of 1 kHz, V_{in} again had an amplitude of 5V, but V_C had an amplitude of about 1V and V_R had an amplitude of around 5V. In at both these frequencies the capacitor was behind the input function and the resistor was ahead. It was observed that at the lower frequency of 50Hz, the first circuit, Circuit "A" was much better at reproducing the 50Hz wave without delay. However, at 1kHz, Circuit "B" was much better at reproducing the sinusoidal input wave.

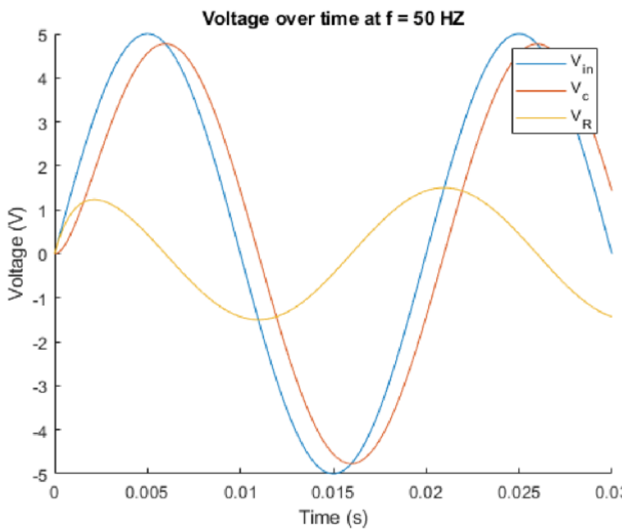

Figure 5: Outputs of two RC Circuits with a sinusoidal input at f=50Hz

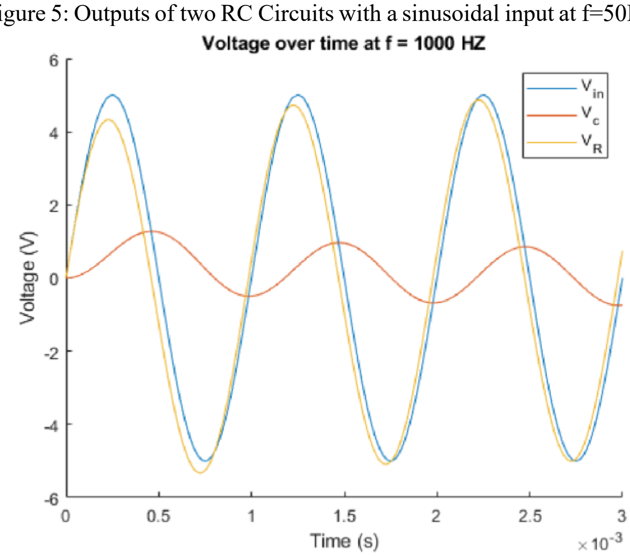

Figure 6: Outputs of two RC Circuits with a sinusoidal input at f=1kHz

As seen in Figure 7 below, one of the two circuits being compared was a low pass filter and the other was a high pass filter. The first circuit, Circuit "A", which has the transfer function H1 in the graph is a low pass filter. The second circuit, Circuit "B", which has the transfer function H2 in the graph is a high pass filter.

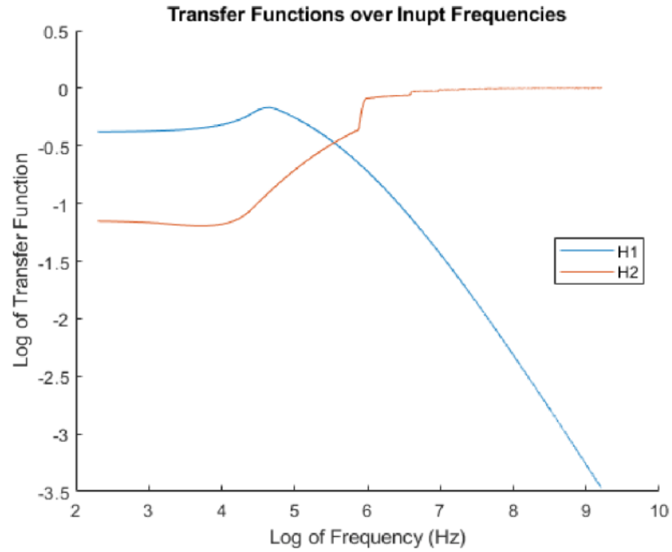

Figure 7: Transfer Functions for Circuit "A" (H1) and Circuit "B" (H2)

C. Part 3: Compare Two Cascaded RC Circuits

The two circuits being compared were Circuit "C" and Circuit "D", which were both cascaded RC circuits. For this part of the case study the input voltage was given by V_{in}=5sin($2\pi f_1 t$) + sin($2\pi f_2 t$). This function is the sum of two sinusoidal functions each containing different frequencies (f_1 = 440 Hz, f_2 = 3 kHz). When the output of each circuit was plotted it was observed that their outputs were also the sum of two sinusoids. The output for Circuit "C" was had a greater amplitude and frequency than the output for Circuit "D". Looking at the transfer functions for these circuits in Figure 10 it was determined that Circuit "C" was a high pass filter and that Circuit "D" was a band pass filter.

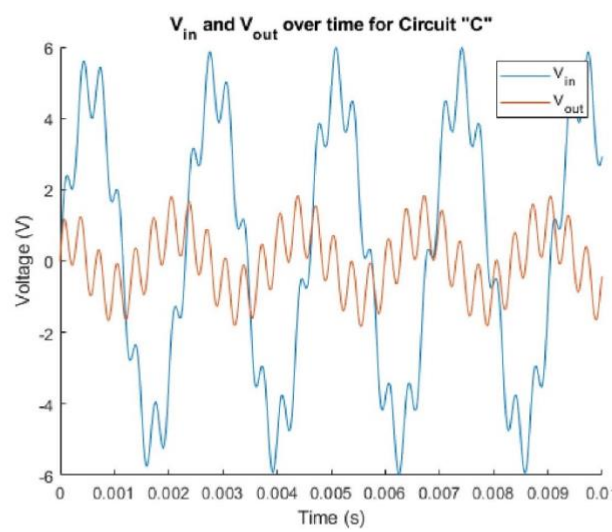

Figure 8: Input and output for Circuit "C" given an input signal that is the sum of two sinusoidal functions with different frequencies

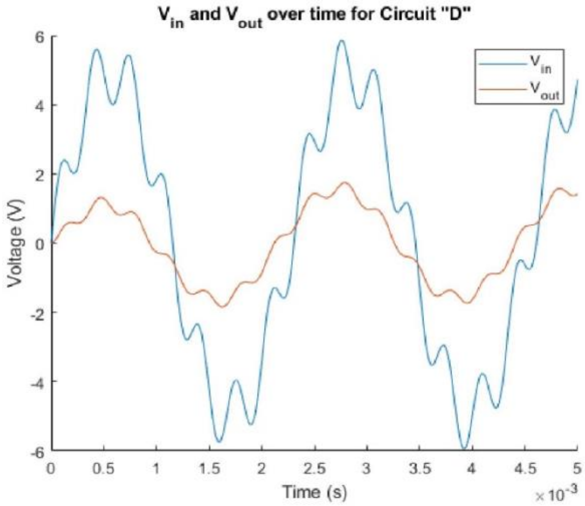

Figure 9: Input and output for Circuit "C" given an input signal that is the sum of two sinusoidal functions with different frequencies

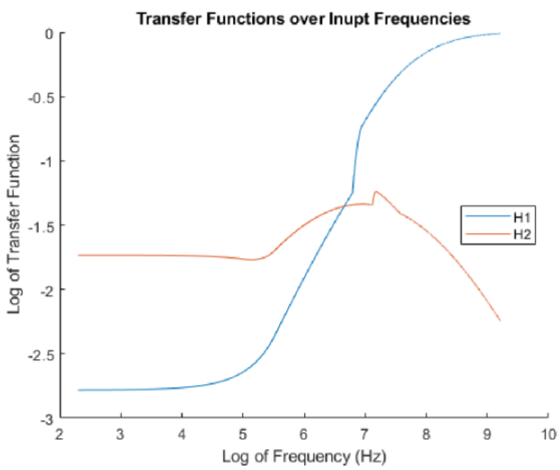

Figure 10: Transfer Functions for Circuit "C" (H1) and Circuit "D" (H2)

D. *Part 4: Competition*

The designed RC filter was designed to be a band pass filter like Circuit "D" in an attempt to eliminate both high and low frequencies. In both the Handel and Apollo 11 audio files there was a large amount of high frequency noise and the designed filter did a good job at cleaning up these audio files. When played with and without the filter both files sounded much less noisy when the filter was applied. Looking at Figure 12 and Figure 13, it was seen that the range of high frequency power/frequency was reduced with the use of a filter. Without the filter, the range of high frequency noise at 10^4 Hz was about -100 db/Hz to -225 db/HZ as seen in Figure 12. However, when the filter was used in Figure 13 the range of high frequency noise at 10^4 Hz was about -125 db/Hz to -250 db/HZ. After listening to the original sounds, and then the filtered sounds it was observed that our circuit was able to filter the sound to get rid of some of the hissing noise, even though it didn't filter all the static noises and let some of the noises pass.

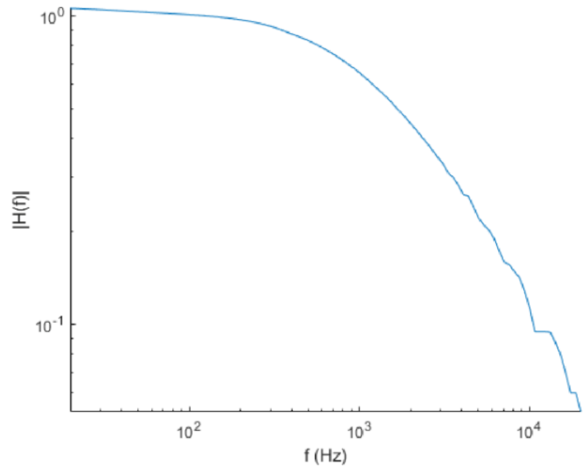

Figure 11: Transfer function of the designed circuit when applied to filter an audio file of the Apollo 11 landing

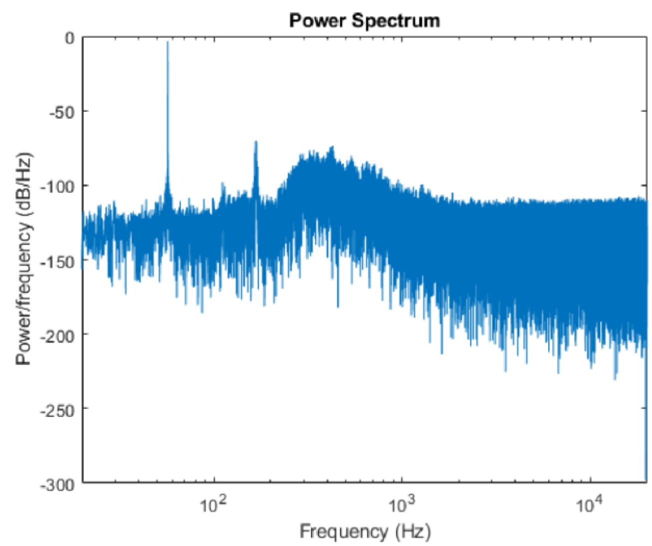

Figure 12: Unfiltered power spectrum of Apollo 11 landing audio file

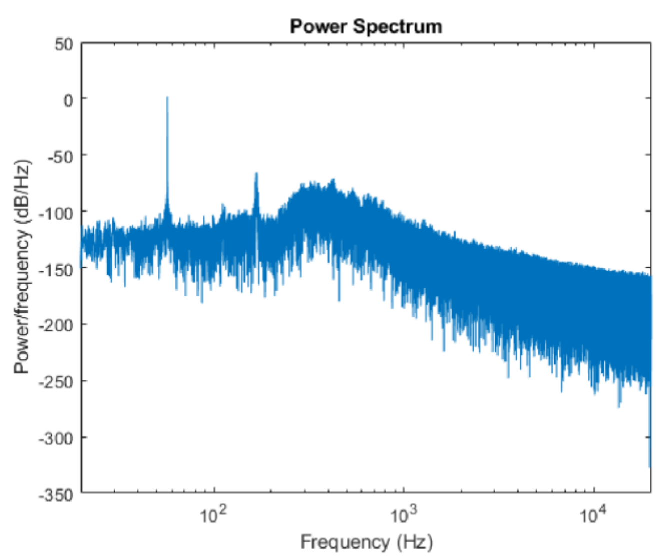

Figure 13: Filtered power spectrum of Apollo 11 landing audio file using designed RC filter

IV. Conclusions

Multiple conclusions were reached from this case study. First, it was determined that without appropriate values for the time interval and the time steps the graph of voltage vs time doesn't give clear and accurate results. It was also concluded that the RC time constant, τ, should be equal to the time it takes for the capacitor to be charged to about 63% of the input voltage. Additionally, it was determined that a value of h smaller than 8×10^{-4}, produced an accurate voltage vs time graph.

Moreover, after comparing the plots of the transfer functions of the two different circuits in Part 2, it was determined that the circuit with the capacitor as the output voltage (Circuit A) was a low-pass filter, and the circuit with the resistor as the output voltage (Circuit B) was the high-pass filter. In addition, it was concluded from the graphs that Circuit A, the low-pass filter, was much better at producing the plot of the 50 Hz wave without delay, and Circuit B, the high-pass filter, was much better of producing the graph of the wave with frequency of 1000 Hz.

Further, after looking at the graphs of the cascaded circuits it was determined that when two sine functions are added up to give the input voltage the resulting output voltage is also the sum of two sine functions. After looking at the graph of the transfer functions of Circuits C and D, it was concluded that the cascaded Circuit C was a high-pass filter, and the cascaded Circuit D was a band-pass filter.

After playing the unfiltered audio and the filtered audio it was concluded that the circuit deigned in Part 4 was able to filter some of the unwanted noises in the audio, even though our circuit wasn't able to filter all the "hissing" sounds. Therefore, the circuit that was designed wasn't a great filter. Finally, the conclusion that the designed circuit was a band-pass filter was reached, as it filtered values of frequencies in a certain range.

References

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