Assignment: Sampling Demonstration

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1. Abstract
In the assignment, we focus on Phase-shift keying (PSK). We begin from the Binary-based Phase-shift keying (BPSK). Then, use the BPSK to improve the Quaternary Phase-shift keying (QPSK). PSK conveys data by changing the phase of a signal with constant frequency. Signals could be decomposed to be a series of sine or cosine waves by Fourier Transform. So, take a sine wave as an example that $s(t) = A\cos(2\pi f_0 t + \phi)$, where t denotes time; A denotes amplitude; f_0 denotes frequency; ϕ denotes phase. By doing multiplications with $s(t)$ and a sine or cosine wave, we change the phase of $s(t)$.
The results of the assignment show how probability error ($PrError$) and error rate change concerning the change of energy per bit over noise power spectrum density ($\frac{E_b}{N_0}$). And the standard deviation of the integrator output (σ_{out}) at each $\frac{E_b}{N_0}$ from -4 to 10 dB is unchanged. Moreover, the input signal information will be kept in the form of $\pm\frac{AT}{2}$.

2. Introduction
There are two kinds of PSK, the BPSK and the QPSK. The BPSK is the simpler form of PSK. It uses two separated phases "1" and "0" to reduce the effects of noise. Once we have a signal at phase "1" for a period T, the signal will be reversed at phase "0" for another period T. To test the ability to resist noise, we begin with a cosine signal $s(t) = \pm A\cos(2\pi f_0 t)$. As shown in Figure 1, we input the signal. Then, add noise $n(t)$ to the signal. The noise has power spectrum density (PSD) as N_0, standard deviation as σ_{in}, and mean as $\mu = 0$. We use the created signal to simulate what will happen in real life. Then, times the created signal with the local oscillator, another cosine wave ($\cos(2\pi f_0 t)$), in the Figure to do the BPSK process. Assume that the new cosine wave is coherent.

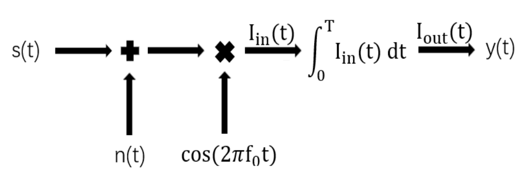


Figure 1: The process of BPSK.

Now, we can represent our signal as $I_{in}(t)$. We do integration of that to gain the total power among a period T and output the $I_{out}(t)$. When the output is too small, we could amplify it to get $y(t)$. However, under the condition of the assignment, the result can be observed clearly in plotted figures. So, let $I_{out}(t) = y(t)$. Meanwhile, based on Figure 1, we have an ideal formula for $I_{in}(t)$, $I_{out}(t)$, and the standard deviation σ_{out} of the result.

$$\begin{aligned}
I_{in}(t) &= [s(t) + n(t)] \cos(2\pi f_0 t) \\
&= [\pm A \cos(2\pi f_0 t) + n(t)] \cos(2\pi f_0 t) \\
&= \pm A \cos^2(2\pi f_0 t) + n(t) \cos(2\pi f_0 t) \\
&= \pm \frac{A}{2} + \frac{\cos(4\pi f_0 t)}{2} + n(t) \cos(2\pi f_0 t) \\
I_{out}(t) &= \int_0^T I_{in}(t) dt \\
&= \pm \frac{AT}{2} + \int_0^T n(t) \cos(2\pi f_0 t) dt \\
&= \pm \frac{AT}{2} + N \\
& for\ N = \int_0^T n(t) \cos(2\pi f_0 t) dt \\
\sigma_{out}^2 &= E(N^2) - E^2(N) \\
&= E\{[\int_0^T n(t) \cos(2\pi f_0 t) dt]^2\} \\
&= E[\int_0^T \int_0^T n(t) \cos(2\pi f_0 t) n(u) \cos(2\pi f_0 u) dt du] \\
&= \int_0^T \int_0^T E[n(t)n(t)] \cos(2\pi f_0 t) \cos(2\pi f_0 u) dt du \\
&= \int_0^T \int_0^T \frac{N_0}{2} \delta(tu) \cos(2\pi f_0 t) \cos(2\pi f_0 u) dt du \\
&= \int_0^T \frac{N_0}{2} \cos^2(2\pi f_0 t) dt \\
&= \int_0^T \frac{N_0}{2} [\frac{1}{2} + \cos(4\pi f_0 t)] dt \\
&= \frac{N_0 T}{4} \\
\sigma_{out} &= \frac{\sqrt{N_0 T}}{2}
\end{aligned}$$

The calculation results above are the ideal results for BPSK. We still need to measure the real values of the three parameters in the assignment.
QPSK is similar to BPSK. However, QPSK has four phases, “00”, “01”, “10”, and “11”. The input of QPSK is a combination of sine waves and cosine waves that $s(t) = \pm A \cos(2\pi f_0 t) \pm A \sin(2\pi f_0 t)$, which can depict a greater range of signals. The different kinds of combinations of positive and negative amplifiers of the sine and cosine waves correspond to the four phases. Since the four phases all have two bits, QPSK requires two paths to show each bit. And each path is similar to the BPSK, as shown in Figure 2. The upper path is completely the same as the BPSK. The

other one switches the local oscillator to $\sin(2\pi f_0 t)$ to maintain four different phases by multiplying with the input. The rest processes have the same idea as what is in BPSK. Also, we wanna know the three parameters in the QPSK case.

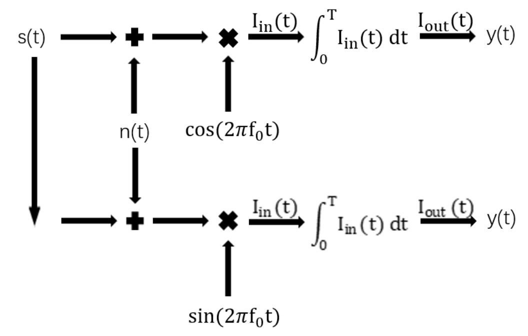


Figure 2: The process of QPSK.

$$
\begin{aligned}
I_{in,upper}(t) &= [s(t) + n(t)]\cos(2\pi f_0 t) \\
&= [\pm A\cos(2\pi f_0 t) \pm A\sin(2\pi f_0 t) + n(t)]\cos(2\pi f_0 t) \\
&= \pm\frac{A}{2} + \frac{\cos(4\pi f_0 t)}{2} \pm A\frac{\cos(4\pi f_0 t)}{2} + n(t)\cos(2\pi f_0 t) \\
I_{out,upper}(t) &= \int_0^T I_{in}(t)dt \\
&= \int_0^T [\pm\frac{A}{2} + \frac{\cos(4\pi f_0 t)}{2} \pm A\frac{\cos(4\pi f_0 t)}{2} + n(t)\cos(2\pi f_0 t)]dt \\
&= \pm\frac{AT}{2} + N \\
&\quad for\ N = \int_0^T n(t)\cos(2\pi f_0 t)dt \\
I_{in,lower}(t) &= [s(t) + n(t)]\sin(2\pi f_0 t) \\
&= [\pm A\cos(2\pi f_0 t) \pm A\sin(2\pi f_0 t) + n(t)]\sin(2\pi f_0 t) \\
&= \pm\frac{A}{2} - \frac{\cos(4\pi f_0 t)}{2} \pm A\frac{\sin(4\pi f_0 t)}{2} + n(t)\sin(2\pi f_0 t) \\
I_{out,lower}(t) &= \int_0^T I_{in}(t)dt \\
&= \int_0^T [\pm\frac{A}{2} - \frac{\cos(4\pi f_0 t)}{2} \pm A\frac{\sin(4\pi f_0 t)}{2} + n(t)\sin(2\pi f_0 t)]dt \\
&= \pm\frac{AT}{2} + N
\end{aligned}
$$

Because the value of N is fixed, the standard deviation's formula is the same as the one in the BPSK case. The three parameters in QPSK are also ideal.

Another thing we need to consider is the PrError. Due to the $\pm$ sign in the inputs, we will gain two outputs centered at $+\frac{AT}{2}$ and $-\frac{AT}{2}$, as shown in Figure 3. Due to the effect from the noise, the values of $y(t)$ may vary. But the results of largest probability are at $+\frac{AT}{2}$ and $-\frac{AT}{2}$. The yellow and blue range represents the $+A$ and $-A$ cases. However, we can see that some yellow ones go to the negative part and some blue ones go to the positive. These phenomena are the errors. We should use the number of errors over the total number of calculations to find out PrError.

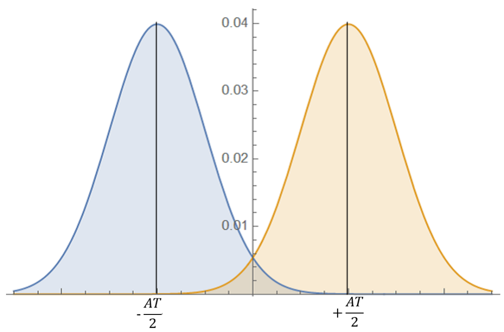


Figure 3: The result of $+A$ and $-A$ cases. When it is $+A$, the output plot centers at $+\frac{AT}{2}$, as the yellow one; when it is $-A$, the output plot centers at $-\frac{AT}{2}$, as the blue one.

Ideally, PrError can be calculated by the input amplitude and standard deviation.

$$\begin{aligned}
PrError &= Pr(+A) \cdot Pr(-A/+A) + Pr(-A) \cdot Pr(+A/-A) \\
&= \frac{1}{2}[Pr(-A/+A) + Pr(+A/-A)] \\
&= Pr(-A/+A) = Pr(+A/-A) \quad by\ symmetry \\
&= \frac{1}{2} erfc(\frac{A/2}{\sigma_{out}\sqrt{2}}) \\
&= \frac{1}{2} erfc(\sqrt{\frac{A^2T}{2N_0}}) \\
&= \frac{1}{2} erfc(\sqrt{\frac{E_b}{N_0}})
\end{aligned}$$

3. Design
There are two kinds of PSK. Thus, we create two LabVIEW files for the BPSK and QPSK cases. And to check what will happen to the PrError and standard deviation if we change $\frac{E_b}{N_0}$, we build a new file for each of them to call the cases files at different values of $\frac{E_b}{N_0}$.
We begin with BPSK, in Figure 4. The variable is the $\frac{E_b}{N_0}$ (ebn0 in the Figure). The input standard deviation ($sigma\_in\ 2 = 1$) of setting the noise, the number of samples (s and $\#s = 1000$), and the sampling frequency ($F_s = 100\ Hz$) are fixed for the whole process. In the green region, we use the calculated amplitude (A) and the

previous quantities to compute the ideal standard deviation ($Sigma\_out$ $(Ideal)$) and probability error ($Pr(error)$ $Ideal$). Meanwhile, the quantities and A set the icons in the for loop. The formula in the math script is given by the following steps.

$$
\begin{aligned}
power &= \int PSDdf = N_0 \frac{F_s}{2} = \sigma_{in}^2 \\
N_0 &= \frac{2\sigma_{in}^2}{F_s} \\
\frac{E_b}{N_0} &= \frac{A^2 T}{2N_0} \\
&= \frac{A^2 T}{2} \frac{F_s}{2\sigma_{in}^2} \\
&= \frac{A^2 T F_s}{4\sigma_{in}^2} \\
A &= \sqrt{4 \frac{E_b}{N_0} \frac{\sigma_{in}^2}{s}} \\
for\ T &= \frac{s}{F_s}
\end{aligned}
$$

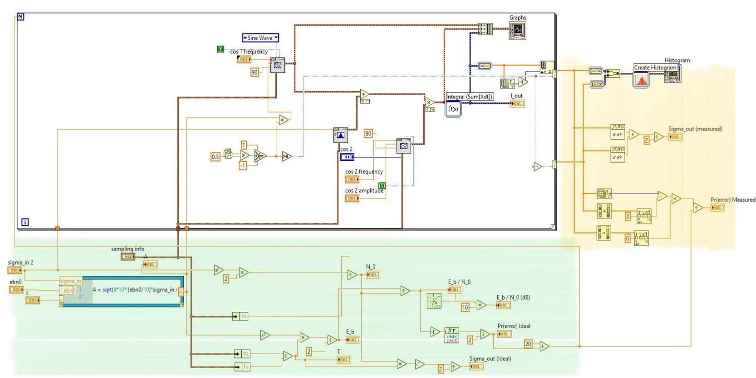


Figure 4: The block diagram of BPSK. In the for loop is the main process of doing BPSK, and we plot out the outputs of each step by the $Graphs$. The yellow region is how we show the measured PrError, σ_{out}, and the final value for both $\pm A$ cases. The ideal PrError and σ_{out} are calculated in the green region. Since we define $\frac{E_b}{N_0}$ as the variable in the design, we use the processes in the green region to initially set up the for loop and the waveforms in the loop.

What's in the for loop is similar to Figure 1. We begin with a cosine signal with $\cos\ 1\ frequency = 1\ Hz$, the amplitude from the multiplication between A and a select function to make the $\pm$ sign, and the sampling information (F_s and $\#s$). Then, add the noise determined by $sigma\_in\ 2 = 1$ and sampling information. Later on, times the cosine wave with $\cos\ 2\ frequency = 1\ Hz$, $\cos\ 2\ amplitude = 1\ V$, and sampling information. Then, integrate the current signal and send the result out of

the loop to the *Histogram*. The outputs of each step are collected to the *Graphs*. In the yellow region, the result is plotted and the real σ_{out} ($Sigma\_out$ $(measured)$) and PrError ($Pr(error)$ $Measured$) are computed based on the returned data from the for loop.
Figure 5 is calling Figure 4 at different values of $\frac{E_b}{N_0}$. Based on Figure 4, distinct $\frac{E_b}{N_0}$ gives different A. Then, the inputs and outputs of BPSK are varied. In Figure 5, we assemble the outputs including $y(t)$, PrError, and σ_{out} from $\frac{E_b}{N_0} = -4\ dB$ to $10\ dB$. Then, we analyze how these three outputs change with the change of $\frac{E_b}{N_0}$.

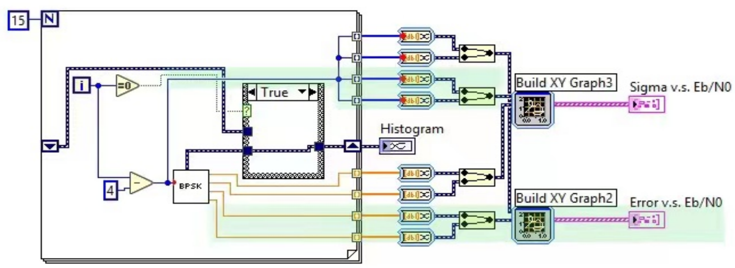


Figure 5: The block diagram of plotting $y(t)$ and calculating PrError and σ_{out} of BPSK. The green region refers to the steps of finding PrError v.s. $\frac{E_b}{N_0}$ in $Error\ v.s.\ Eb/N0$; another plot is showing σ_{out} v.s. $\frac{E_b}{N_0}$ in $Sigma\ v.s.\ Eb/N0$; The *Histogram* collects all the outputs $y(t)$ under different values of $\frac{E_b}{N_0}$.

The QPSK follows the same idea as BPSK. Looking at Figure 1 and Figure 2, the QPSK has one more path than the BPSK. Therefore, in Figure 6, we add the path in the for loop. The designing thoughts in green and yellow regions are similar to the thoughts in the green and yellow regions in Figure 4. The only change happens in the for loop. The input $s(t)$ is contributed by a sine wave and a cosine wave at the top left. Then, add noise to $s(t)$. Based on Figure 2, the signal is separated into two paths, the one in the blue region and another hasn't been highlighted. The blue one is the path with a cosine wave as the oscillator; the not highlighted one contains a sine wave as the oscillator. The others, including the physical quantities, are the same as what in the BPSK case. Thus, we'll achieve two plots, *Graphs* (cos) and *Graphs* (sin), of each step's result in the QPSK process. And two $y(t)$s histograms, *Histogram* (cos) and *Histogram* (sin).

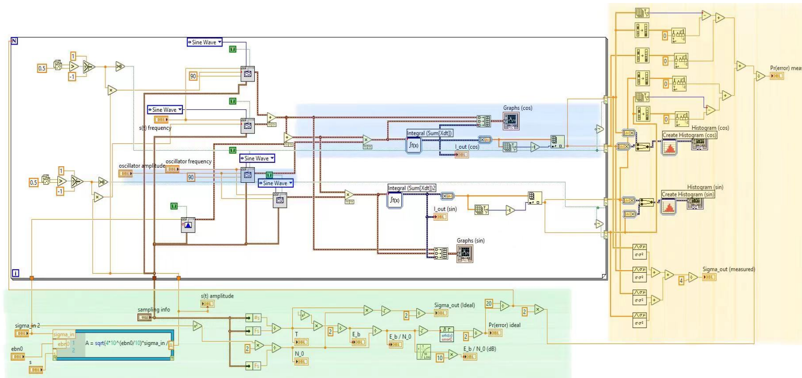


Figure 6: The block diagram of BPSK. In the for loop is the main process of doing BPSK, and we plot out the outputs of each step by the *Graphs*. The yellow region is how we show the measured PrError, σ_{out}, and the final value for both $\pm A$ cases. The ideal PrError and σ_{out} are calculated in the green region. Since we define $\frac{E_b}{N_0}$ as the variable in the design, we use the processes in the green region to initially set up the for loop and the waveforms in the loop.

To view the change of PrError, σ_{out}, and $y(t)$ with respect to $\frac{E_b}{N_0}$, in Figure 7, we use the identical idea that we showed in Figure 5. Since there is only one exact PrError and σ_{out} for each $\frac{E_b}{N_0}$, we don't make any difference with the BPSK one. For $y(t)$, we combine the outputs from the two paths and show all of them under the 15 values of $\frac{E_b}{N_0}$ together in *Histogram*.

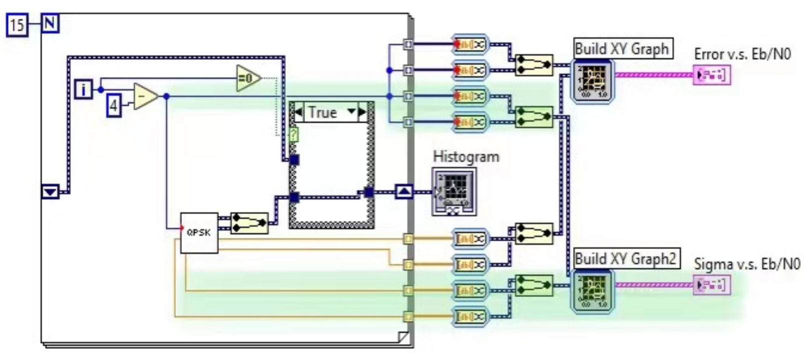
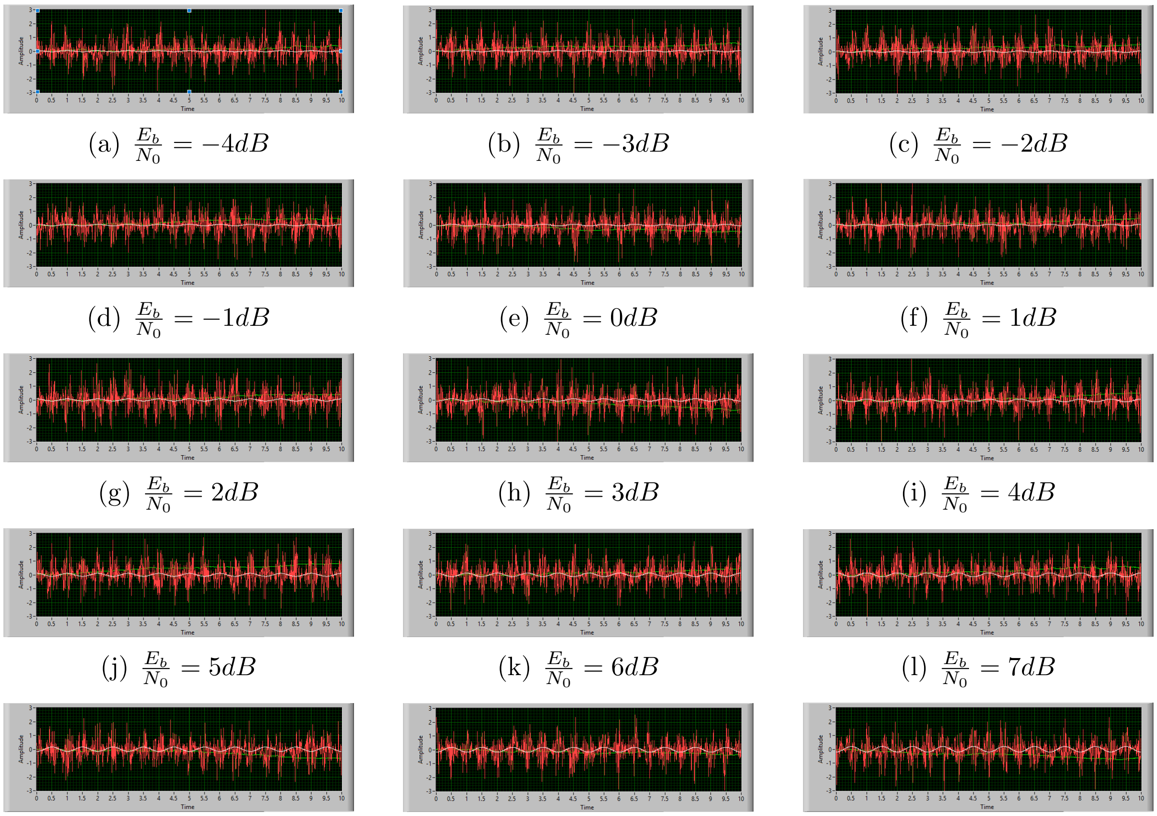


Figure 7: The block diagram of plotting $y(t)$ and calculating PrError and σ_{out} of QPSK. The green region refers to the steps of finding PrError v.s. $\frac{E_b}{N_0}$ in *Error v.s. Eb/N0*; another plot is showing σ_{out} v.s. $\frac{E_b}{N_0}$ in *Sigma v.s. Eb/N0*; The *Histogram* collects all the outputs $y(t)$ under different values of $\frac{E_b}{N_0}$.

4. Operation/Testing

   (a) BPSK

   In Figure 8, we care most about the green line. The rightmost point of the green line is the value of $y(t)$. Ideally, the line should end at $\pm\frac{AT}{2}$. However, by considering the effect from the noise, $y(t) = \pm\frac{AT}{2} + N$. So, the green line varies each time we run the file. And, if we collect all the values of measured $y(t)$ after calling the for loop a large number of times, we will get the plots shown in Figure 9.



(a) $\frac{E_b}{N_0} = -4dB$ (b) $\frac{E_b}{N_0} = -3dB$ (c) $\frac{E_b}{N_0} = -2dB$

(d) $\frac{E_b}{N_0} = -1dB$ (e) $\frac{E_b}{N_0} = 0dB$ (f) $\frac{E_b}{N_0} = 1dB$

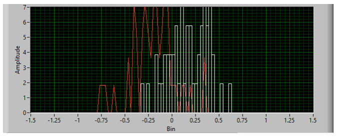
(g) $\frac{E_b}{N_0} = 2dB$ (h) $\frac{E_b}{N_0} = 3dB$ (i) $\frac{E_b}{N_0} = 4dB$

(j) $\frac{E_b}{N_0} = 5dB$ (k) $\frac{E_b}{N_0} = 6dB$ (l) $\frac{E_b}{N_0} = 7dB$

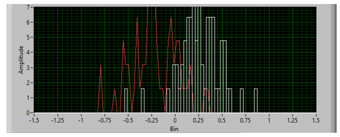
(m) $\frac{E_b}{N_0} = 8dB$ (n) $\frac{E_b}{N_0} = 9dB$ (o) $\frac{E_b}{N_0} = 10dB$

Figure 8: The outputs of each steps in BPSK. The white line is the $s(t)$; the red line denotes the I_{in}; the green line shows the $y(t)$.

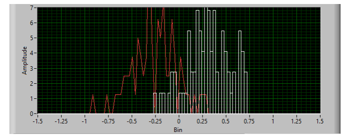
Due to the design in Figure 4, the number of times of calling the for loop is related to the value of $\frac{E_b}{N_0}$. So, we don't get many values of $y(t)$ for small $\frac{E_b}{N_0}$. Then, the top plots are not smooth as the bottom plots in Figure 9. The white and red curves in the plots are almost symmetric to each other and peaks locate at corresponding $\pm\frac{AT}{2}$. Since the noise is normally distributed, $y(t) = \pm\frac{AT}{2}$ has the highest probability to be attained.



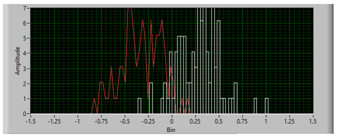
(a) $\frac{E_b}{N_0} = -4dB$



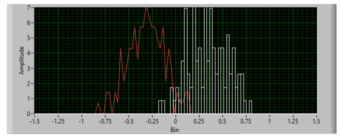
(b) $\frac{E_b}{N_0} = -3dB$



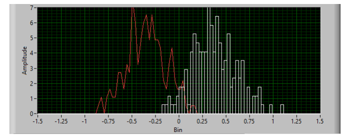
(c) $\frac{E_b}{N_0} = -2dB$



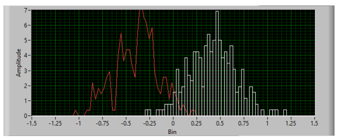
(d) $\frac{E_b}{N_0} = -1dB$



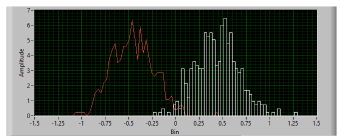
(e) $\frac{E_b}{N_0} = 0dB$



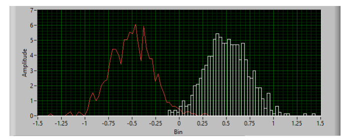
(f) $\frac{E_b}{N_0} = 1dB$



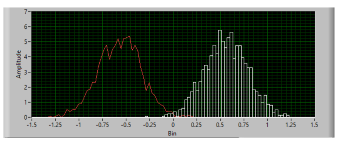
(g) $\frac{E_b}{N_0} = 2dB$



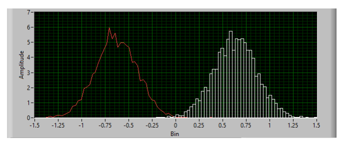
(h) $\frac{E_b}{N_0} = 3dB$



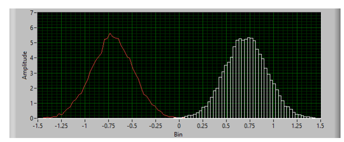
(i) $\frac{E_b}{N_0} = 4dB$



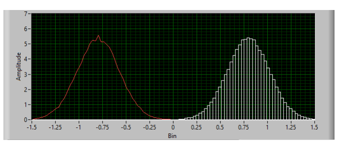
(j) $\frac{E_b}{N_0} = 5dB$



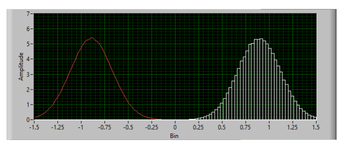
(k) $\frac{E_b}{N_0} = 6dB$



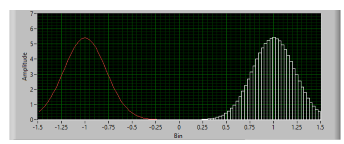
(l) $\frac{E_b}{N_0} = 7dB$



(m) $\frac{E_b}{N_0} = 8dB$



(n) $\frac{E_b}{N_0} = 9dB$



(o) $\frac{E_b}{N_0} = 10dB$

Figure 9: The histograms of all $y(t)$ in BPSK. The white line is for the $+A$ case; the red line is for the $-A$ case. The $x-$axis represents the value of $y(t)$, and the $y-$axis represents the probability of getting the value of $y(t)$.

By Figures 8 and 9, under different values of $\frac{E_b}{N_0}$, some quantities from BPSK may change. So, we use the design in Figure 5 to find the change of PrError (Figure 10), σ_{out} (Figure 11), and the histogram of $y(t)$ (Figure 12). It is clear to view in the plots of Figure 9 that with the increase of $\frac{E_b}{N_0}$, the overlapping of $+A$ and $-A$ curves decreases. By the definition of PrError, PrError is inversely proportional to $\frac{E_b}{N_0}$. This phenomenon is shown in Figure 10. Because the quantities for calculating σ_{out} are fixed, σ_{out} is also fixed. This is proved in Figure 11. As explained in Figure 9, the peaks for the curves are at $\pm\frac{AT}{2}$ and A is proportional to $\sqrt{\frac{E_b}{N_0}}$. So, in Figure 12, the $+A$ and $-A$ curves move apart from each other.

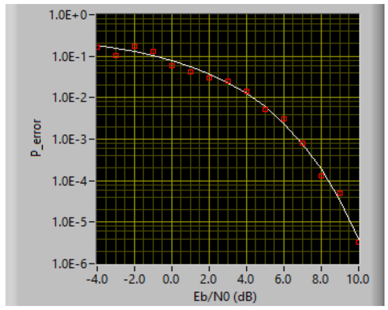


Figure 10: The PrError of BPSK. The white line is the ideal curve, and the red points imply the measured PrError at corresponding $\frac{E_b}{N_0}$.

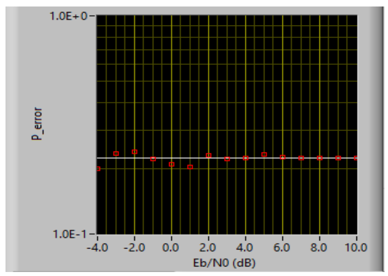


Figure 11: The σ_{out} of BPSK. The white line is the ideal value, and the red points imply the measured σ_{out} at corresponding $\frac{E_b}{N_0}$.

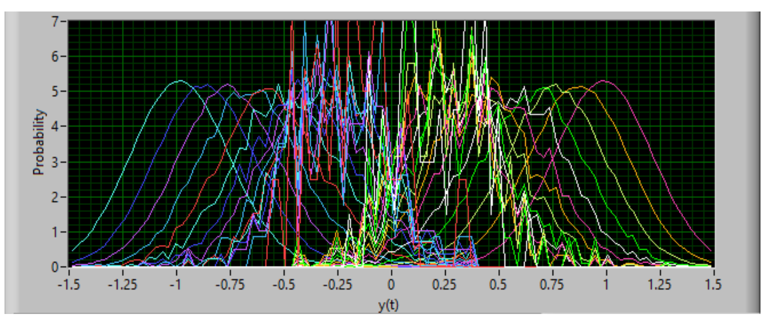
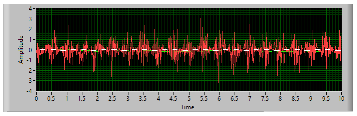
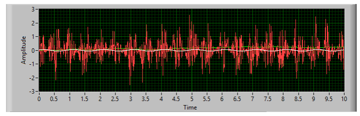


Figure 12: The combination of all $y(t)$ histogram of BPSK at different values of $\frac{E_b}{N_0}$.

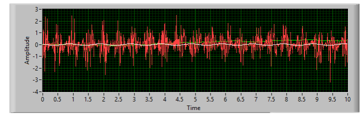
(b) QPSK
Similar to the BPSK case, in Figures 13 and 14, we care most about the green line. The rightmost point of the green lines is the values of $y(t)$. Ideally, the line also should end at $\pm\frac{AT}{2}$. This statement stands for both the cosine and sine paths and is been proved by the formulas in the introduction section.



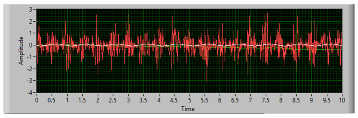
(a) $\frac{E_b}{N_0} = -4dB$



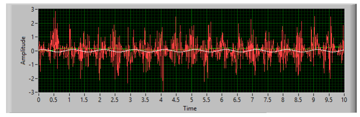
(b) $\frac{E_b}{N_0} = -3dB$



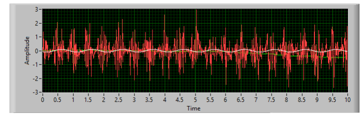
(c) $\frac{E_b}{N_0} = -2dB$



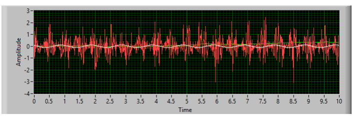
(d) $\frac{E_b}{N_0} = -1dB$



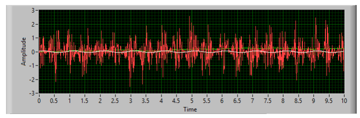
(e) $\frac{E_b}{N_0} = 0dB$



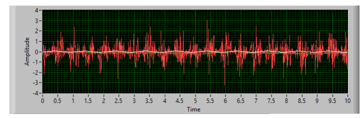
(f) $\frac{E_b}{N_0} = 1dB$



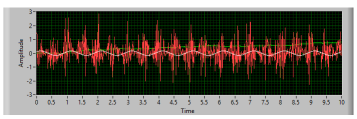
(g) $\frac{E_b}{N_0} = 2dB$



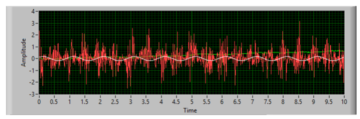
(h) $\frac{E_b}{N_0} = 3dB$



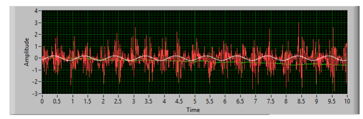
(i) $\frac{E_b}{N_0} = 4dB$



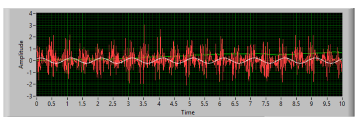
(j) $\frac{E_b}{N_0} = 5dB$



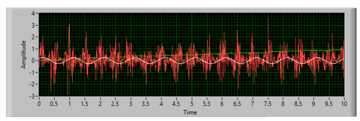
(k) $\frac{E_b}{N_0} = 6dB$



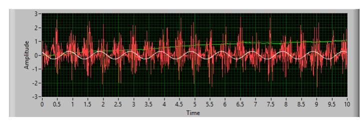
(l) $\frac{E_b}{N_0} = 7dB$



(m) $\frac{E_b}{N_0} = 8dB$

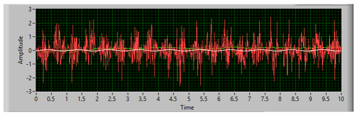


(n) $\frac{E_b}{N_0} = 9dB$

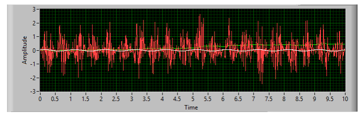


(o) $\frac{E_b}{N_0} = 10dB$

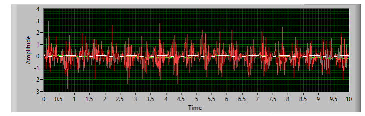
Figure 13: The outputs of each steps in QPSK for the path that the oscillator is a cosine wave. The white line is the $s(t)$; the red line denotes the I_{in}; the green line shows the $y(t)$.



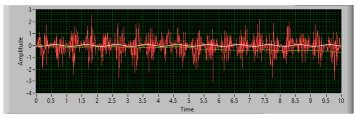
(a) $\frac{E_b}{N_0} = -4dB$



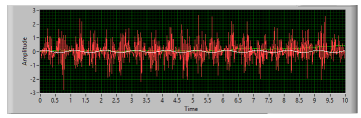
(b) $\frac{E_b}{N_0} = -3dB$



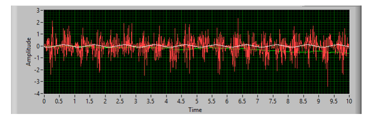
(c) $\frac{E_b}{N_0} = -2dB$



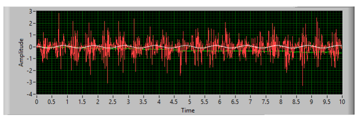
(d) $\frac{E_b}{N_0} = -1dB$



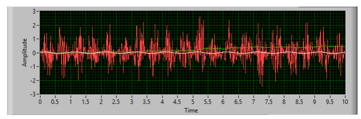
(e) $\frac{E_b}{N_0} = 0dB$



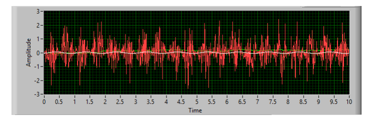
(f) $\frac{E_b}{N_0} = 1dB$



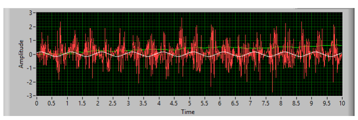
(g) $\frac{E_b}{N_0} = 2dB$



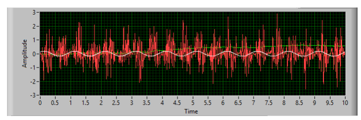
(h) $\frac{E_b}{N_0} = 3dB$



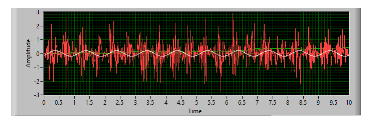
(i) $\frac{E_b}{N_0} = 4dB$



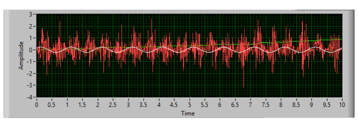
(j) $\frac{E_b}{N_0} = 5dB$



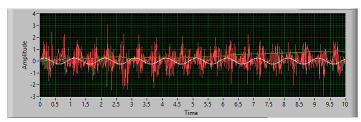
(k) $\frac{E_b}{N_0} = 6dB$



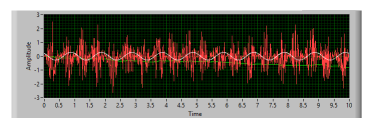
(l) $\frac{E_b}{N_0} = 7dB$



(m) $\frac{E_b}{N_0} = 8dB$



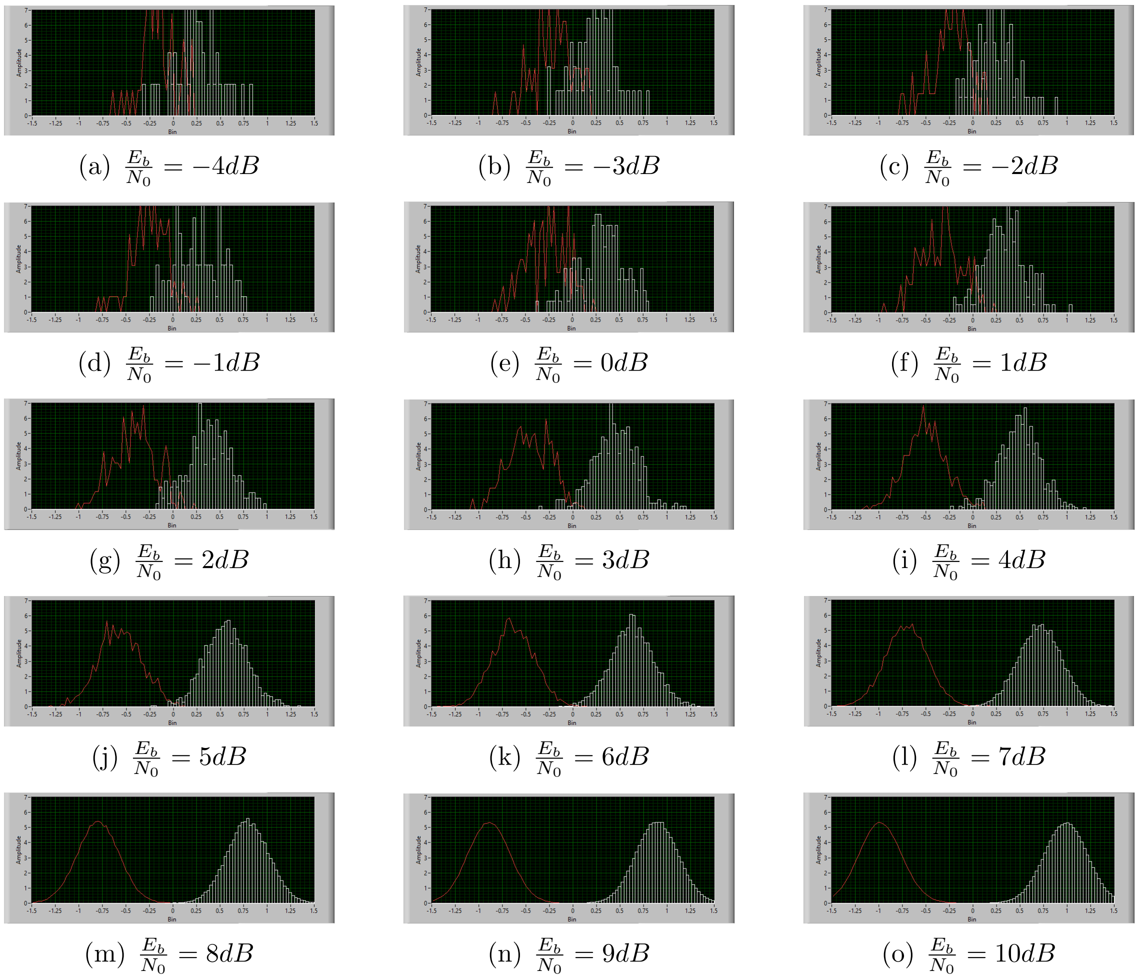
(n) $\frac{E_b}{N_0} = 9dB$



(o) $\frac{E_b}{N_0} = 10dB$

Figure 14: The outputs of each steps in QPSK for the path that the oscillator is a sine wave. The white line is the $s(t)$; the red line denotes the I_{in}; the green line shows the $y(t)$.

Holding the same idea with Figure 9, the number of times of calling the for loop is related to the value of $\frac{E_b}{N_0}$. So, the top plots are not smooth as the bottom plots in Figures 15 and 16. The white and red curves in the plots are almost symmetric to each other and peaks are at corresponding $\pm\frac{AT}{2}$. Moreover, $y(t) = \pm\frac{AT}{2}$ has the highest probability to be achieved.



(a) $\frac{E_b}{N_0} = -4dB$ (b) $\frac{E_b}{N_0} = -3dB$ (c) $\frac{E_b}{N_0} = -2dB$

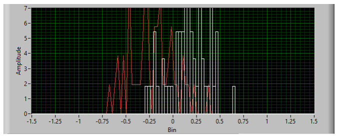
(d) $\frac{E_b}{N_0} = -1dB$ (e) $\frac{E_b}{N_0} = 0dB$ (f) $\frac{E_b}{N_0} = 1dB$

(g) $\frac{E_b}{N_0} = 2dB$ (h) $\frac{E_b}{N_0} = 3dB$ (i) $\frac{E_b}{N_0} = 4dB$

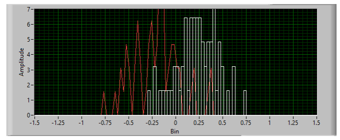
(j) $\frac{E_b}{N_0} = 5dB$ (k) $\frac{E_b}{N_0} = 6dB$ (l) $\frac{E_b}{N_0} = 7dB$

(m) $\frac{E_b}{N_0} = 8dB$ (n) $\frac{E_b}{N_0} = 9dB$ (o) $\frac{E_b}{N_0} = 10dB$

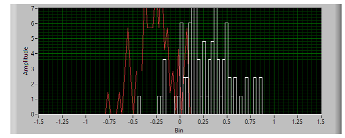
Figure 15: The histograms of all cosine path's $y(t)$ in QPSK. The white line is for the $+A$ case; the red line is for the $-A$ case. The $x-$axis represents the value of $y(t)$, and the $y-$axis represents the probability of getting the value of $y(t)$.



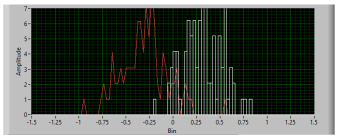
(a) $\frac{E_b}{N_0} = -4dB$



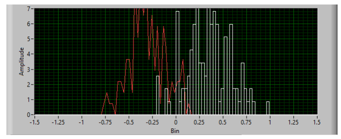
(b) $\frac{E_b}{N_0} = -3dB$



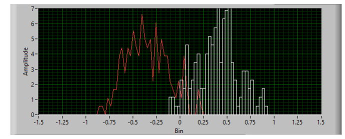
(c) $\frac{E_b}{N_0} = -2dB$



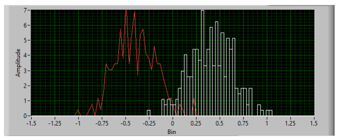
(d) $\frac{E_b}{N_0} = -1dB$



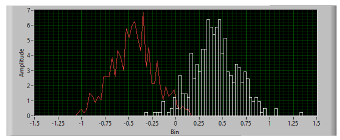
(e) $\frac{E_b}{N_0} = 0dB$



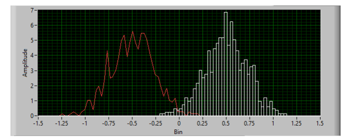
(f) $\frac{E_b}{N_0} = 1dB$



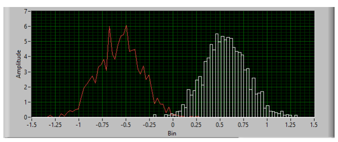
(g) $\frac{E_b}{N_0} = 2dB$



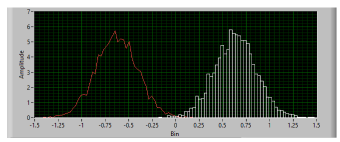
(h) $\frac{E_b}{N_0} = 3dB$



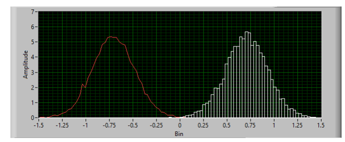
(i) $\frac{E_b}{N_0} = 4dB$



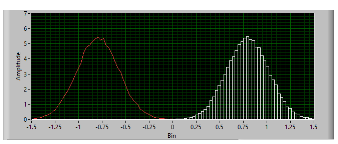
(j) $\frac{E_b}{N_0} = 5dB$



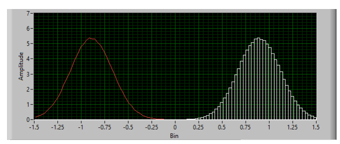
(k) $\frac{E_b}{N_0} = 6dB$



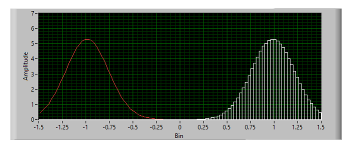
(l) $\frac{E_b}{N_0} = 7dB$



(m) $\frac{E_b}{N_0} = 8dB$



(n) $\frac{E_b}{N_0} = 9dB$



(o) $\frac{E_b}{N_0} = 10dB$

Figure 16: The histograms of all cosine path's $y(t)$ in QPSK. The white line is for the $+A$ case; the red line is for the $-A$ case. The $x-$axis represents the value of $y(t)$, and the $y-$axis represents the probability of getting the value of $y(t)$.

Except for the signals, there is nothing different between the BPSK and QPSK now. Thus, we come to analyze the change of PrError (Figure 17), σ_{out} (Figure 18), and the histogram of $y(t)$ (Figure 19) with respect to $\frac{E_b}{N_0}$. With the decline of $+A$ and $-A$ curves in Figures 15 and 16, PrError decreases in Figure 17. And in Figure 18, σ_{out} keeps the same. Lastly, the raise of $\sqrt{\frac{E_b}{N_0}}$ causes $+A$ and $-A$ curves to move away from each other in Figure 19.

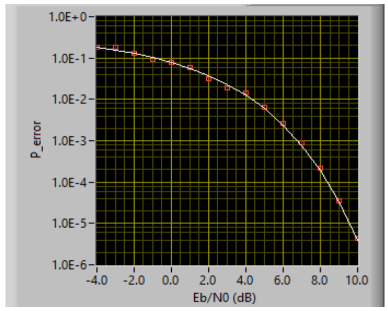


Figure 17: The PrError of QPSK. The white line is the ideal curve, and the red points imply the measured PrError at corresponding $\frac{E_b}{N_0}$.

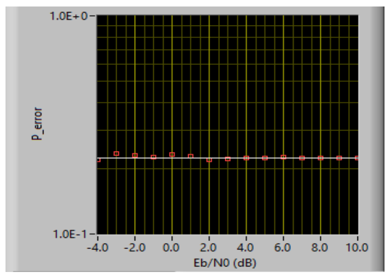


Figure 18: The σ_{out} of QPSK. The white line is the ideal value, and the red points imply the measured σ_{out} at corresponding $\frac{E_b}{N_0}$.

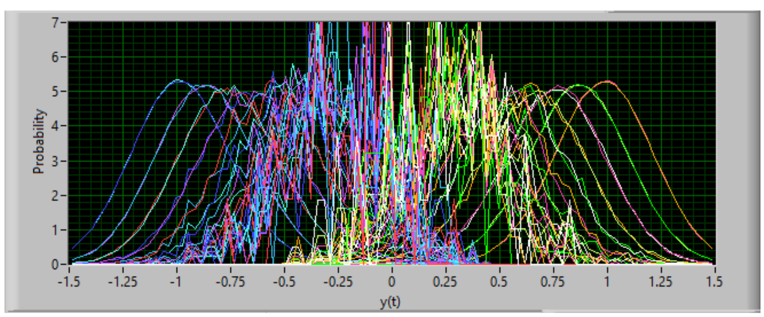


Figure 19: The combination of all $y(t)$ histogram of QPSK at different values of $\frac{E_b}{N_0}$.

5. Discussion/Conclusion
   We analyze how well the BPSK and QPSK maintain signal information under noise's interruption. We find that the amplitudes of the input signals will be kept as shown in Figures 9, 12, 15, 16, and 19. Also, the greater the $\frac{E_b}{N_0}$ is, the smaller PrError we'll meet. Lastly, the σ_{out} remains the same under the change of $\frac{E_b}{N_0}$.