Vehicle Suspension System: A Dance Between Comfort and Control

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Abstract— **Vehicle suspension systems are designed to respond in an ideal manner to all of the bumps and slopes that are ran over while driving. There are multiple answers to what is "ideal," and this case study will explore the options available to the designer of the system. We simulate the effect of a spring on the body of a car using differential equations, virtual terrain, and varying car speeds to create a comprehensive report on how to optimize this real-life problem.**

I. Background

A vehicle's suspension system is a complex problem that can be tackled using mathematical techniques such as frequency analysis, differential equations, and basic physics. Intuitively, an ideal suspension system is not so stiff so that you feel every bump and cannot drive over obstacles in your way, yet it is also not so forgiving so that you keep bouncing for some time on your springs after driving over a bump. Another negative side effect of your springs being too compressible would be scraping the nose of your car on the ground when faced with a sudden hill. The equation that models this system can be seen below.

$$\frac{d^2x(t)}{dt^2} + 2\zeta\omega_n\frac{dx(t)}{dt} + \omega_n^2 x(t) = \omega_n^2 u(t) \tag{1}$$

Equation (1) is a second order differential equation derived from Newton's second law containing the variables for the damping ratio ζ, the natural frequency ω_n, the input road surface $u(t)$, and the output of the system $x(t)$. The natural frequency is defined below in equation (2).

$$\omega_n = \sqrt{k/m} \tag{2}$$

In this equation, k is our spring constant and m is ¼ the mass of the vehicle, as we only model one wheel at a time. These equations and concepts are modeled in MATLAB to simulate the suspension system and find optimal settings for the various parameters.

II. Methods

A. Choosing a ζ

The damping ratio plays a critical role in how the car responds to various terrain. The system is considered underdamped if ζ is less than one, and oscillations will occur as the spring attempts to work its way back to its steady-state length, all the while overshooting that goal by a decreasing margin of error each time it reaches it. Because of this phenomenon, we knew to choose a value of ζ greater than or equal to one. However, the higher value we choose for ζ, the longer the rise-time is. This means that the shock absorber damping should be chosen to exactly equal one, the critically damped case.

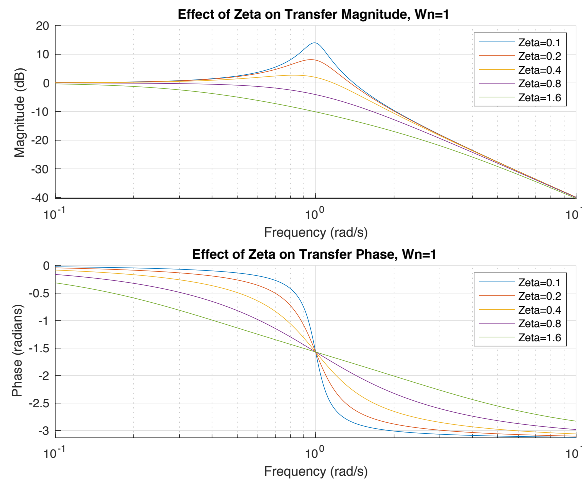

Fig. 1. Plot of the transfer magnitude and phase of our system with various ζ values. The peaks are undesirable and correspond to overshoot and ringing in the step response.

B. State-Space Model

In order to simulate this system in MATLAB, we used the function ss(A,B,C,D) to create a sys variable. This sys variable defines all of the laws governing the suspension system. This system is defined by the functions:

$$\dot{x} = Ax + Bu \text{ and } y = Cx + Du \tag{3}$$

We define the state matrix A as:

$$A = \begin{bmatrix} 0 & 1 \\ -\omega_n^2 & -2\zeta\omega_n \end{bmatrix} \tag{4}$$

And we continue to create the input-to-state matrix B, state-to-output matrix C, and the feedthrough matrix D in MATLAB with the following code (`wn` is ω_n):

```
B=[0;wn^2];
C=[1 0];
D=0;
```

This sys variable allows us to easily create our Pole-Zero map seen below. We have a multiplicity of two for the poles on the real axis. It is of note that if ζ was less than one the two poles would by symmetric on the real axis with each being the complex conjugate of the other. If ζ was more than one the poles would be spread out on the real axis. These two states of ζ correspond to the real-life conditions of the car either having more handling (ζ<1) or more comfort (ζ>1).

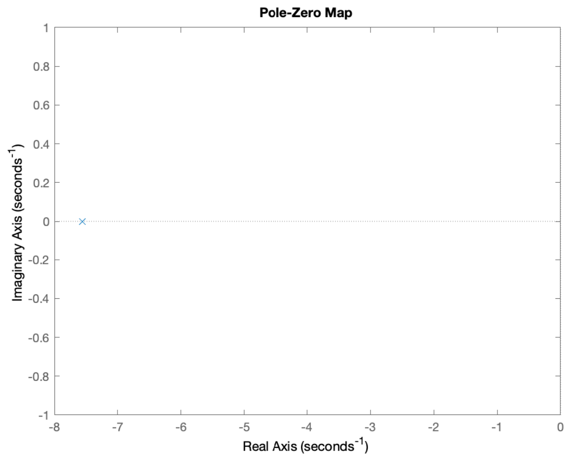

Fig. 2. Plot of the Pole-Zero map of our system. Two poles (mulitiplicity of 2) are present on the real axis between -8 and -7 seconds^{-1}.

C. *Frequency Response*

Once all of our variables and systems were formulated, we can find our frequency response. Using a load of 1200 kg, we finalize our relevant variables to be ζ=1, m = 400 kg, and k = 40,000 N/m to generate the plot below. The function `freqs` was used to calculate the Fourier coefficients.

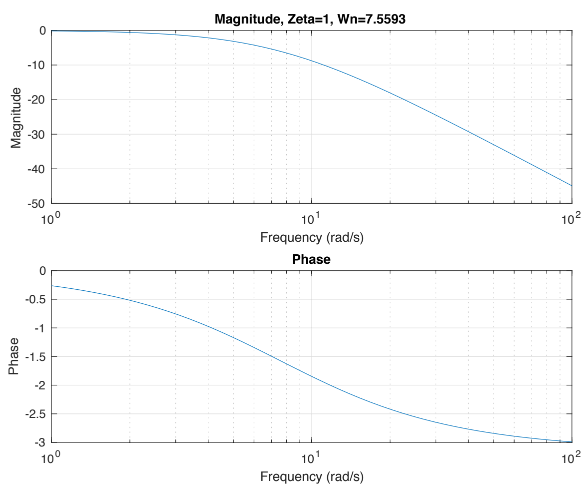

Fig. 3. Plot of the magnitude and phase of the frequency response of the suspension system.

D. *Additional Road Surface*

In addition to the sine speed bumps, trapezoidal bump, and potholes that were provided in our code, we were tasked with creating another custom surface. For this, we created 10 steps of varying heights and lengths. This gave us additional insight into how the car interacts with various surfaces. The plot of this surface can be seen in the bottom row of figures 6 and 7 in the next section.

III. Results

A. *Displacement factors*

We will first look into how each of the factors contributed to the overall displacement of the vehicle. As one might guess, the laws of inertia and the graph below show us that the higher the total mass of the car is, the less displacement the body of the car experiences. In this case study, we consider the mass of the vehicle to range from 1600 (fully unloaded) to 3000 (fully loaded) kg.

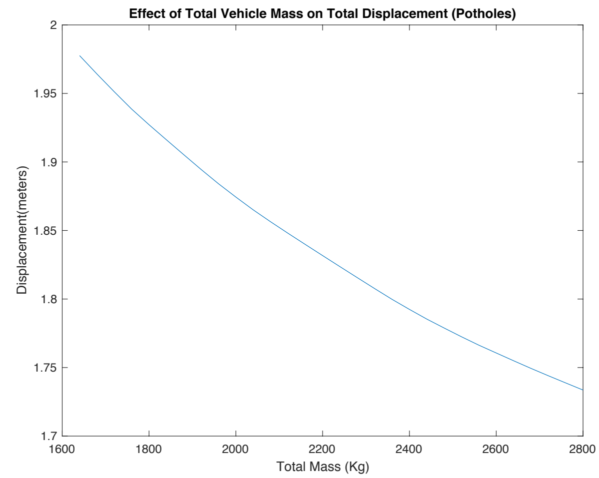

Fig. 4. Plot comparing the effect of total vehicle mass on total displacement. The vitrual car is sent over the same potholes with varying mass. The negative correlation proves our intuition that it is harder to move heavier objects so a pothole will move the heavier car less than the lighter one.

The next relation we will look at is the effect of velocity on the displacement of the vehicle. The figure below shows a similar negative correlation, although slightly more sloped. In this case study, we consider the speed of the vehicle to range from 0 to 40 m/s.

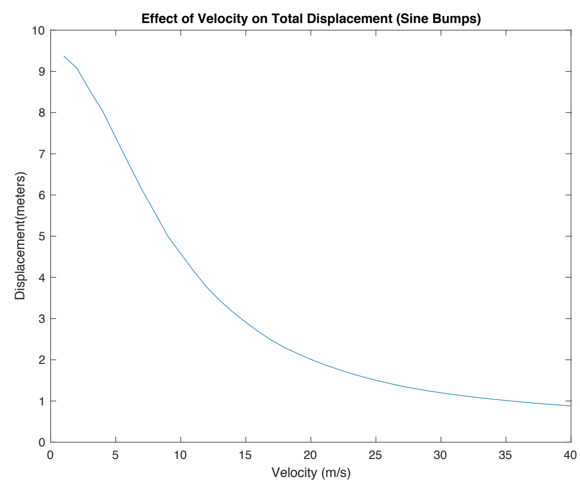

Fig. 5. Plot comparing the effect of velocity on total displacement. The faster the car went, the less displacement occurred.

There was an option to put the peak of this curve at a value greater than 0 m/s but we opted to keep the natural frequency ω_n at 10 to get the most comfort at low speeds. The relation seen in the above figure can also be seen on all four terrains when the car simulation is run at speeds of 5 m/s and 40 m/s.

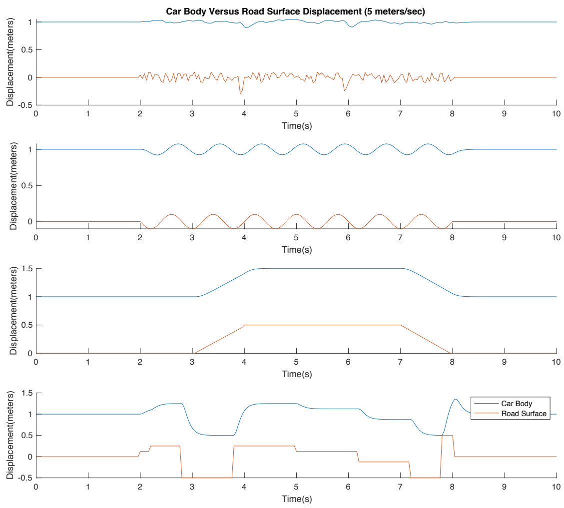

Fig. 6. Plot showing the car body interacting with the road surface beneath it. The fast response time of the car body at 5 m/s creates large amounts of displacement when tasked with driving over bumps.

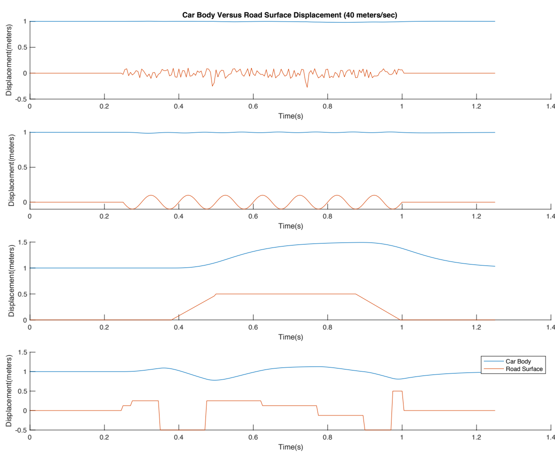

Fig. 7. Plot showing the car body interacting with the road surface beneath it. The slower response time of the car body at 40 m/s creates smaller amounts of displacement when tasked with driving over bumps, allowing for a smoother driving experience.

Another factor on the total displacement is the natural frequency ω_n. The plot shows us that there is a positive relationship between ω_n and displacement. Since the natural frequency is proportional to the spring constant k, we opted to use the smallest spring constant allowed in this case study, which is 40,000 N/m. Our frequency of $\omega_n/2\pi$ equals about 1.2 Hz, which is in the recommended range for optimal comfort.

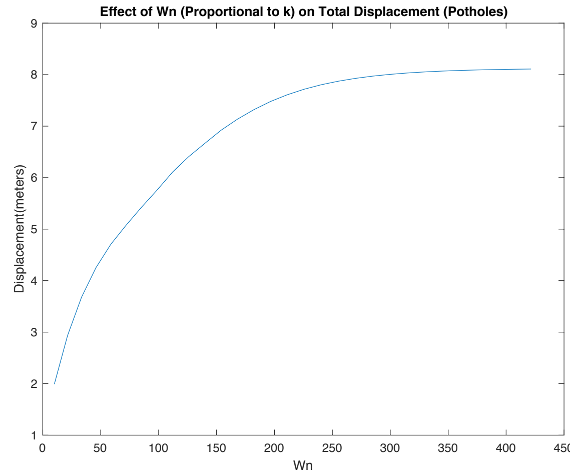

Fig. 8. This plot of the effect of ω_n shows that a smaller oscillation period allows for lower amounts of displacement. These various factors must be balanced and considered to create an ideal suspension system.

B. System Characteristics

After finalizing our initial values and our system configuration, we are able to calculate the step response and impulse response as seen in the below figures.

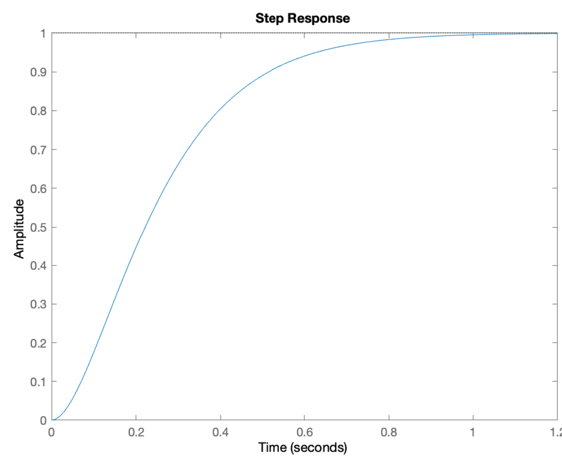

Fig. 9. Plot of the step response. One second was decided to be an approximately ideal amount of time for the body of the car to move up after running over a single step. One second is not so fast that the bump hurts the passengers or the car but not so slow that it oscillates one climbing the step.

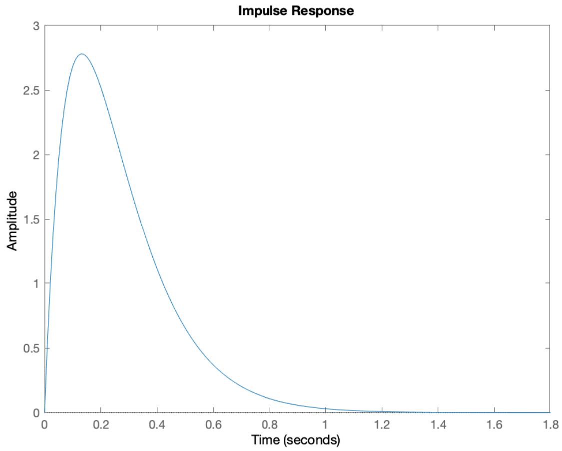

Fig. 10. Plot of the impulse response. Once again, we deemed one second to be an appropriate length of time for the vehicle to completely react to any bumps in the road.

IV. Conclusion

In conclusion, we have simulated the response of a suspension system when faced with various factors such as load weight, velocity, and terrain, as well as varying internal factors such as the damping coefficient and the spring constant. Our research into this case study allowed us to find optimal settings for these variables. We also created an additional road surface for the car to “drive” over. Some limitations of this model is that it does not take into account all of the factors of a real car and we don’t look at all of the effects of bumps. These factors could include shock tire type and weight distribution and the effects could include damage on the car and fuel efficiency. All in all, this was a very informative case study for understanding the applications of our mathematical and coding education.

References

[1] A. W. Openheim, A. S. Willsky, and S. H. Nawab, “Signals and Systems” Second Edition, Prentice-Hall, Boston, MA, USA, 1996