$$f(n) = f(\sqrt{n}) + 1$$

We will to **disprove** that $f(n) \in \theta(log(n))$

Let $n = 2^m$

$$f(2^m) = f(2^{m/2}) + 1$$

Let $f(2^m) = S(m)$ such that S is a function of m.

$$S(m) = S(m/2) + 1$$

Apply Master Theorem, which is convenient for eventually non-decreasing functions which satisfy below conditions.

$$x(n) = a * x(n/b) + g(n) \text{ such that } x(1) = c,\ n = b^k,\ b > 1,\ a \geq 1,\ k \geq 1$$

So,

$$a = 1,\ b = 2,\ k = 1,\ g(n) = 1$$

Master Theorem states that if $g(n) \in \theta(n^p),\ p \geq 0$, then

$$x(n) \in \begin{cases} \theta(n^p), & a < b^p \\ \theta(n^p * logn), & a = b^p \\ \theta(n^{log_b a}), & a > b^p \end{cases} \quad , for\ n \in \mathbb{N}$$

Since we have $p = 0$,

$$x(n) = S(m) \in \theta(n^p * logn) = \theta(logn)$$

Also $n = 2^m \Rightarrow m = log_2 n$

$$f(n) = S(m) = S(logn)$$

So,

$$f(n) \in log(logn)$$

Hence, disproved