Assumption: $log(n)$ is assumed to be in base 2

1. $\lim_{n \to \infty} \frac{n^{cos(n)}}{n}$ is divergent. However, if we investigate the functions on our own we will

   see that since cosine function is in the interval $(-1, 1)$. We can write

   $$n^{cos(n)} \leq c * n \text{ , } whenever\ n \geq n_0$$

   for the values $c = 1$ and $n = 0$

   So, $n^{cos(n)} \in O(n)$

2. According to Stirling's approximation ($n! \sim \sqrt{2\pi n}(\frac{n}{e})^n$)

   $$\lim_{n \to \infty} \frac{n}{log(n!)} = \lim_{n \to \infty} \frac{n}{log(\sqrt{2\pi n}(\frac{n}{e})^n)} = 0$$

   So, $n \in o(log(n!))$

   Also notice,

   $$log(n!) = log(1) + log(2) + ... + log(n) \leq n * log(n)$$

   $$log(n!) \in O(n * log(n))$$

3. Using the logarithmic property,

   $$3^{log(n)} = n^{log(3)}$$

   $$\lim_{n \to \infty} \frac{n*log(n)}{n^{log(3)}} = \lim_{n \to \infty} \frac{log(n)}{n^{log(3)-1}}$$

   Apply L'Hopital's Rule

   $$= \lim_{n \to \infty} \frac{1/(n*ln(2))}{(-1+log(3))*x^{log(3)-2}} = \lim_{n \to \infty} \frac{1}{ln(2)*(-1+log(3))*n^{log(3)-1}}$$

   Expand $ln(2) * (1 - log(3)) =- ln(2) + ln(3)$

   $$= \frac{1}{-ln(2)+ln(3)} * \lim_{n \to \infty} \frac{1}{n^{log(3)-1}}$$

   $$= \frac{1}{-ln(2)+ln(3)} * \frac{1}{\infty} = 0$$

   So, $n * log(n) \in o(3^{log(n)})$

   So, $log(n!) \in o(3^{log(n)})$

4. $\lim_{n \to \infty} \frac{3^{log(n)}}{n^{log(n)}} = \lim_{n \to \infty} (\frac{3}{n})^{log(n)} = \lim_{n \to \infty} e^{log(n)*ln(\frac{3}{n})}$

Apply chain rule

$$= e^{\lim_{n \to \infty} (log(n) * ln(\frac{3}{n}))} = e^{\infty * (-\infty)} = e^{-\infty} = 0$$

So, $3^{log(n)} \in o(n^{log(n)})$

5. We know that $log(n) \in o(n)$. So, in this comparison we can say the denominator has a bigger growth rate since it has n on the power. We can easily say

   $$\lim_{n \to \infty} \frac{n^{log(n)}}{log(n)^n} = 0$$

   Thus, $n^{log(n)} \in log(n)^n$

Finally, we can say

$$n^{cos(n)} \leq n < log(n!) < 3^{log(n)} < n^{log(n)} < log(n)^n$$