RULE:

A function f is a member of a $O(g(n))$ if and only if there exist positive constants c and n_0 such that

$$f(n) \le c * g(n), \text{ whenever } n \ge n_0$$

PROOF:

We are given

$f_1(n) \in O(g_1(n))$ and $f_2(n) \in O(g_2(n))$ which implies

$f_1(n) \le c_1 * g_1(n)$ and $f_2(n) \le c_2 * g_2(n)$, c_1, c_2 are positive constants

When we sum these inequalities, we get

$$f_1(n) + f_2(n) \le c_1 * g_1(n) + c_2 * g_2(n)$$

Assuming $max(g_1(n), g_2(n))$ is $g_1(n)$ and $c_1 + c_2 = c_3$ we may write

$f_1(n) + f_2(n) \le c_3 * g_1(n)$ such that c_3 is a positive constant

From this equation, we conclude

$$f_1(n) + f_2(n) \in O(g_1(n))$$

Which is equivalent to

$$f_1(n) + f_2(n) \in O(max(g_1(n), g_2(n))$$