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# The Prisoner's Dilemma

## Problem

Three prisoners, $A$, $B$ and $C$, with apparently equally good records have applied for parole. The parole board has decided to release two of the three, and the prisoners know this but not which two. A warder friend of prisoner $A$ knows who are to be released. Prisoner A realizes that it would be unethical to ask the warder if he, $A$, is to be released, but thinks of asking for the name of one prisoner other than himself who is to be released. He thinks that before he asks, his chances of release are $\frac{2}{3}$. He thinks that if the warder says "$B$ will be released", his own chances have now gone down to $1/2$, because either $A$ and $B$ or $B$ and $C$ are to be released. And so $A$ decides not to reduce his chances by asking. However, $A$ is mistaken in his calculations. Explain.

## Solution

Prisoner $A$'s mistake is not considering the entire sample space of pairs of prisoners that could be released, which is $\Omega = \{AB, AC, BC\}$. In the case of $AB$, the warden will say that $B$ will be released, and $A$ has a 100% chance of release. Similarly, in the case of $AC$, the warden will say that $C$ will be released, and $A$ has a 100% chance of release. In the case of $BC$, the warden will say that either $B$ or $C$ will be released, and $A$ has a 0% chance of release. Each outcome has a $\frac{1}{3}$ chance of occurring. When combined, this gives the overall chance of release with the warden's statement as:

$$P(A\ Released) = \left(\frac{1}{3}\right)1 + \left(\frac{1}{3}\right)1 + \left(\frac{1}{3}\right)0 = \frac{2}{3}$$

Thus, asking the warden has no effect on prisoner $A$'s chances of being released.