

Hurried Duelers

Problem

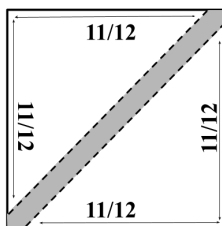
Duels in the town of Discretion are rarely fatal. There, each contestant comes at a random moment between 5 AM and 6 AM on the appointed day and leaves exactly 5 minutes later, honor served, unless his opponent arrives within the time interval and then they fight. What fraction of duels lead to violence?

Solution

Let X be the arrival time of the first contestant, and Y that of the second. For simplicity, we will scale the duration between 5 AM and 6 AM to the value 1, so that $\frac{1}{12}$ corresponds to 5 minutes. Using this notation, the fraction of duels that will to violence can be stated as:

$$P\left(|X - Y| < \frac{1}{12}\right) = P\left(-\frac{1}{12} < X - Y < \frac{1}{12}\right)$$

Graphing this inequality gives the following shaded region representing duels that lead to violence:



The relevant region is a unit square (since $0 \leq X \leq 1$ and $0 \leq Y \leq 1$), and X and Y are both uniformly distributed, so we can calculate the probability of violence as the total area of the square, minus the areas that do not lead to violence, which gives $1 - \left(\frac{11}{12}\right)^2 = \frac{23}{144} \approx \frac{1}{6}$.