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Gambler's Ruin

Problem

Player M has \$1, and Player N has \$2. Each play gives one of the players \$1 from the other. Player M is enough better than Player N that he wins $\frac{2}{3}$ of the plays. They play until one is bankrupt. What is the chance that Player M wins?

Solution

Let M and N represent victories by Player M and Player N, respectively. Then, consider the various sequences that result in Player M's victory:

$$\text{Case 0: } MM$$
$$\text{Case 1: } MNMM$$
$$\text{Case 2: } MNMNMM$$
$$\dots$$
$$\text{Case } n\text{: } (MN)^n MM$$

From the above, we can draw the following insights regarding sequences that result in Player M's victory:

- Player N must not win the 1^{st} play
- Player M winning 2 plays in a row results in a victory
- Player N must not win 2 players in a row

These insights lead to the following convergent geometric series representing Player M's probability of victory:

$$P(M\ Victory) = \sum_{i=0}^{\infty} \left[\left(\frac{2}{3}\right) \left(\frac{1}{3}\right) \right]^n \left(\frac{2}{3}\right)^2 = \frac{4}{7}$$