Kaan Aksoy — Feb 10, 2020

The Sock Drawer

Part A

We would like to find the fewest number of socks (which are either red or black) such that the probability of two drawn socks both being red is $\frac{1}{2}$. Let r be the number of red socks and b be the number of black socks.

The probability that the two drawn socks are both red is:

$$\frac{r}{r+b} \cdot \frac{r-1}{r+b-1} = \frac{1}{2}$$

Note that, for $b > 0$, $\frac{r}{r+b} > \frac{r-1}{r+b-1}$. Using this, we can say that:

$$\left(\frac{r}{r+b}\right)^2 > \frac{1}{2} > \left(\frac{r-1}{r+b-1}\right)^2$$

Taking the square root and rearranging gives:

$$\frac{1-\sqrt{2}-b}{1-\sqrt{2}} > r > \frac{b}{\sqrt{2}-1}$$

Trying $b = 1$, we get $3.414 > r > 2.414$, which leaves only $r = 3$. Thus, the fewest number of socks for a $\frac{1}{2}$ probability of drawing two reds is 4 (with 3 red socks, and 1 black sock).

Part B

Now, we would like to solve the same question with the added constraint that the number of black socks is even. To do so, we will try increasing even values for b along with the inequality from *Part A*.

b	Inequality	Potential r	Probability of two reds
2	$4.828 < r < 5.828$	5	0.476
4	$9.656 < r < 10.656$	10	0.494
6	$14.485 < r < 15.485$	15	0.50

Thus, The fewest number of socks with an even number of black socks is 21 (with 15 reds and 6 blacks).