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Lazy Caterer's Sequence

The Lazy Caterer's Sequence describes the maximum number of slices that can be made on a pizza using a given number of straight cuts. The sequence begins as follows:

Cuts	0	1	2	3	4	...
Slices	1	2	4	7	11	...

Intuitively, to maximize slices, each straight cut should pass through all prior cuts. Furthermore, that cut should not pass through the intersection of two prior cuts. In this way, the n^{th} cut will pass through $n-1$ previous cuts and be divided into n segments. Each of those n segments will split an existing slice in 2, resulting in n new slices.

Formally, this can be expressed by the following recurrence relation, with base case $f(0) = 1$. $f(n)$ represents the maximum number of slices with n cuts:

$$f(n) = n + f(n-1)$$

The closed form of this recurrence can be developed using the triangle numbers:

$$\begin{aligned} f(n) &= n + f(n-1) \\ &= n + (n-1) + f(n-2) \\ &= n + (n-1) + (n-2) + (n-3) + \ldots + 1 + f(0) \\ &= \frac{n(n+1)}{2} + 1 \\ &= \frac{n^2 + n + 2}{2} \end{aligned}$$