

## Faulty Coin

### Problem

You have 1000 coins, one of which is faulty: it has a head on both sides. You randomly draw a coin, and, without examining it, toss it 10 times. As it happens, you get 10 heads in a row. What's the probability that it's the faulty one?

### Solution

Bayes theorem is useful for solving this problem. Let  $P(F)$  represent the prior probability that the drawn coin is faulty. Let  $P(H)$  represent the prior probability of the coin landing on heads 10 times in a row.

We can calculate  $P(H)$  by considering the two possible cases for the drawn coin—it is either faulty or not:

$$P(H) = \frac{1}{1000}(1) + \frac{999}{1000} \left( \frac{1}{2^{10}} \right) = \frac{2023}{1024000}$$

We trivially know that  $P(F) = \frac{1}{1000}$ , since only one coin is faulty. Additionally, we know  $P(H|F) = 1$ , since the faulty coin always lands on heads.

Inserting these values into Bayes' theorem gives the following solution:

$$\begin{aligned} P(F|H) &= \frac{P(F)P(H|F)}{P(H)} \\ &= \left( \frac{1}{1000} \right) \left( \frac{1024000}{2023} \right) \\ &\approx 0.506 \end{aligned}$$