Kaan Aksoy — April 28, 2020

Consecutive Dice Throws

Problem

What is the expected number of times one must roll a fair dice before getting 2 consecutive 6's?

Solution

According to the geometric distribution, we can expect 6 rolls before landing the first 6. At that point, there is a $\frac{1}{6}$ chance that the next roll is a 6, which would result in two consecutive 6's. On the other hand, there is a $\frac{5}{6}$ chance of landing a number that is not 6, at which point we must reset our count. This formulation can be represented by the following recurrence, where X is a random variable representing the number of rolls for 2 consecutive 6's:

$$E[X] = 6 + \frac{1}{6}(1) + \frac{5}{6}(1 + E[X])$$
$$E[X] = 42$$

Thus, the expected number of rolls to land 2 consecutive 6's is 42.

This approach can be generalized to an arbitrary number of consecutive 6's. For example, suppose Y represents the number of rolls for 3 consecutive 6's. Knowing that $E[X] = 42$, we can write a recurrence for $E[Y]$ as follows:

$$E[Y] = E[X] + \frac{1}{6}(1) + \frac{5}{6}(1 + E[Y])$$
$$E[Y] = 258$$