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Proof of $\sqrt{2}$ Irrationality

Suppose to the contrary that $\sqrt{2}$ is rational. Then, there must exist two coprime integers p and q such that $\sqrt{2} = \frac{p}{q}$. We can rearrange this equation as follows:

$$\sqrt{2} = \frac{p}{q}$$
$$2q^2 = p^2$$

From the above it is clear that p^2 is even. It follows that p is also even, since the square of any odd number is odd. Thus, we can substitute p with $2k$, where k is any integer:

$$2q^2 = p^2$$
$$2q^2 = 4k^2$$
$$q^2 = 2k^2$$

By a similar argument, we can again see that both q^2 and q are even. The fact that both p and q are even contradicts the initial statement that the two are coprime. Thus, we have reached a contradiction, and $\sqrt{2}$ must be irrational.