

Average Number of Matches

Problem

The following are two versions of the matching problem:

(a) From a shuffled deck, cards are laid out on a table one at a time, face up from left to right, and then another deck is laid out so that each of its cards is beneath a card of the first deck. What is the average number of matches on the card above and the card below in repetitions of this experiment?

(b) A typist types letters and envelopes to n different persons. The letters are randomly put into the envelopes. On the average, how many letters are put into their own envelopes?

Part (a) Solution

We will apply the property of *Linearity of Expectation*. Let $X = X_1 + X_2 + \dots + X_{52}$ be the total number of card matches, where each X_i represents the number of matches (either 0 or 1) for the i^{th} card. We know $E[X_i] = \frac{1}{52}$. So, we can easily calculate the solution as follows:

$$\begin{aligned} E[X] &= E[X_1 + X_2 + \dots + X_{52}] \\ &= E[X_1] + E[X_2] + \dots + E[X_{52}] \\ &= 52 \left(\frac{1}{52} \right) \\ &= 1 \end{aligned}$$

In conclusion, we can expect 1 card to match on average.

Part (b) Solution

The same logic from Part (a) applies here, giving a solution of $n \left(\frac{1}{n} \right) = 1$ matching letters on average.