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Birthday Pairings

Problem

What is the least number of persons required if the probability exceeds $\frac{1}{2}$ that two or more of them have the same birthday? (Year of birth need not match.)

Solution

We begin by calculating the probability that n people all have different birthdays, which is:

$$P(All\ Different\ Birthdays) = \frac{^{365}P_n}{365^n} = \frac{365!}{(365-n)!365^n}$$

In the above, the numerator represents the number of ways for n people to all have different birthdays, while the denominator represents the number of possible birthday configurations for n people.

The probability that at least 1 pair shares the same birthday is simply $1 - P(All\ Different\ Birthdays)$. So, we can solve the problem by graphing and choosing the smallest integer n that makes the probability exceed $\frac{1}{2}$. This gives the value $n = 23$.