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# Finding Your Birthmate

## Problem

You want to find someone whose birthday matches yours. What is the least number of strangers whose birthday you need to ask about to have a 50-50 chance?

## Solution

Let $X$ be a random variable representing the number of people you need to ask before finding someone whose birthday matches yours. Then, applying the geometric distribution, we know $P(X = n)$ is:

$$P(X = n) = \left(\frac{364}{365}\right)^n \left(\frac{1}{365}\right)$$

Extending the above, the probability that $X$ is no more than some value $n$ is simply:

$$P(X \le n) = \sum_{i=0}^{n} \left(\frac{364}{365}\right)^n \left(\frac{1}{365}\right)$$

To finish this problem, we assign $P(X \le n) = \frac{1}{2}$ and solve for $n$:

$$\frac{1}{2} = \sum_{i=0}^{n} \left(\frac{364}{365}\right)^n \left(\frac{1}{365}\right) = \frac{\frac{1}{365}\left[1 - \left(\frac{364}{365}\right)^n\right]}{1 - \frac{364}{365}}$$

$$\frac{1}{2} = \left(\frac{364}{365}\right)^n$$

The first line above comes from applying the formula for the sum of a finite geometric series, which is $s_n = \frac{a_1(1-r^n)}{1-r}$. The final answer is $\log_{\frac{364}{365}} \frac{1}{2} \approx 253$.