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Curing the Compulsive Gambler

Problem

Mr. Brown always bets a dollar on the number 13 at roulette against the advice of Kind Friend. To help cure Mr. Brown of playing roulette, Kind Friend always bets Brown $20 at even money that Brown will be behind at the end of 36 plays. How is the cure working?

(Most American roulette wheels have 38 equally likely numbers. If the player's numbers comes up, he is paid 35 times his stake and gets his original stake back; otherwise he loses his stake.)

Solution

First, we must calculate the number of plays out of 36 that Mr. Brown must win in order to not be behind at the end of 36 plays. To do so, we solve the following equation, where w represents the number of wins ($0 \leq w \leq 36$):

$$\begin{aligned} 35w - (36 - w) &= 0 \\ w &= 1 \end{aligned}$$

From the above, we can see that Mr. Brown will break even at $w = 1$ win. Thus, for $w \geq 1$, Mr. Brown will not be behind at the end of 36 plays.

Next, we will calculate the expected value of 36 plays with the $20 bet incorporated. Let Y be the random variable representing the earnings from the game. Then, by applying the binomial distribution, we can calculate $E[Y]$ as follows:

$$\begin{aligned} E[Y] &= \sum_{i=1}^{36} \binom{36}{i} \left(\frac{37}{38}\right)^{36-i} \left(\frac{1}{38}\right)^{i} (35i - (36 - i) + 20) + \left(\frac{37}{38}\right)^{36} (-36 - 20) \\ &\approx 0.617(-16) - 21.440 + 36 \sum_{i=1}^{36} i \binom{36}{i} \left(\frac{37}{38}\right)^{36-i} \left(\frac{1}{38}\right)^{i} \\ &\approx -31.312 + 36(36)\left(\frac{1}{38}\right) \\ &\approx \$2.79 \end{aligned}$$

In the above equation, the summation represents the expected value for the case with $w \geq 1$, while the expression to its right represents the expected value for the case with $w = 0$. In the 3^{rd} line, the formula for the expected value of the binomial distribution ($E[X] = np$) is applied to replace the summation.

Thus, when incorporating the \$20 bet, Mr. Brown can expect to win on average \$2.79 for every set of 36 plays. In conclusion, Kind Friend's cure is not working.