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Theater Row

Problem

Eight eligible bachelors and seven beautiful models happen randomly to have purchased single seats in the same 15-seat row of a theater. On the average, how many pairs of adjacent seats are ticketed for marriageable couples*?

Solution

Let X be a random variable denoting the number of pairs that are marriageable. We can rewrite X as $X = X_1+X_2+\ldots+X_{14}$, where each X_i is a random variable representing the number of marriageable couples in the i^{th} pair of adjacent seats. Note that each X_i can take on either 0 or 1 only.

We can easily calculate $E[X_i]$ by considering the two arrangements that result in a marriageable pair, which are *Male-Female* and *Female-Male*:

$$E[X_i] = \left(\frac{8}{15}\right)\left(\frac{7}{14}\right) + \left(\frac{7}{15}\right)\left(\frac{8}{14}\right) = \frac{8}{15}$$

Then, applying the *Linearity of Expectation* property, we can calculate $E[X]$ as follows:

$$\begin{aligned} E[X] &= E[X_1 + X_2 + \ldots + X_{14}] \\ &= E[X_1] + E[X_2] + \ldots + E[X_{14}] \\ &= 14\left(\frac{8}{15}\right) \end{aligned}$$

Thus, the expected number of marriageable pairs in the row of 15 seats is $E[X] \approx 7.46$.

*According to outdated cultural norms from the time of this problem's publication.