SVD Exercise 1:

This NASA photo (TarantulaNebula.jpg) comes from the Hubble telescope and presents a dramatic picture of this extra-galactic formation. Download it.

Matlab provides various image processing utilities. In this case, read the image in using the following command.

```
nasacolor=imread('TarantulaNebula.jpg');
```

The variable nasacolor will be a 567x630x3 matrix of integer values between 0 and 255.

Display the color plot using the command

```
image(nasacolor)
```

The third subscript of the array nasa refers to the red, green, and blue color components. To simplify this exercise, turn it into a greyscale with ordinary double precision values 0-255 using the following commands:

```
nasa=sum(nasacolor,3,'double');  %sum up red+green+blue

m=max(max(nasa));                %find the max value

nasa=nasa*255/m;                 %make this be bright white
```

The result from these commands is that nasa is an ordinary $ 567\times630$ array of double precision numbers.

Remark: This is a cheap but dirty way to create a grayscale image from an rgb image. It is good enough for our purpose here.

Matlab has the notion of a ``colormap'' that determines how the values in the matrix will be colored. Use the command

```
colormap(gray(256));
```

to set a grayscale colormap. Now you can show the picture with the command

```
image(nasa)

title('Grayscale NASA photo');
```

Use the command [U S V]=svd(nasa); to perform the singular value decomposition.

Plot the singular values on a semilog scale: semilogy(diag(S)). You should observe that the values drop off very rapidly to less than 2% of the maximum in fewer than 50 values. Please send me this plot with your summary.

Construct the three new matrices

nasa100=U(:,1:100)*S(1:100,1:100)*V(:,1:100)';

nasa50=U(:,1:50)*S(1:50,1:50)*V(:,1:50)';

nasa25=U(:,1:25)*S(1:25,1:25)*V(:,1:25)';

These matrices are of lower rank than the nasa matrix, and can be stored in a more efficient manner. (The nasa matrix is 567x630 and requires 357,210 numbers to store it. In contrast, nasa50 requires subsets of U and V that are 567x50 and the diagonal of S, for a total of 56,750. This is better than four to one savings in space.) Do not include these matrices with your summary!

Plot the three matrices nasa100, nasa50 and nasa25 as images, including descriptive titles. You should only very slight differences between the original and the one with 100 singular values, some noticeable differences with 50 singular values while you should see serious degradation of the image in the case of 25 singular values. Comment on the results.