

DT049A: Probability Theory and Stochastic Processes

Lab Exam

Contact: Sarder Fakhrul Abedin (010-142 83 60)

Instructions

The lab consists of three parts, and it is graded with pass/fail. The requirement for a pass is submitting a report with at least 50% of the tasks completed. You are required to work individually, which means that any textbooks and course notes are allowed, but it is not allowed to use code found on the Internet or developed by other course participants. **The pseudocode for each problem is provided to guide you through the implementation and understanding of the problems.** However, your knowledge of stochastic processes is required.

Submission

The submission, in the form of a .zip file named `firstname_secondname_DT049_HT25_lab.zip`, must include the following:

1. A short report is a .pdf file discussing the assigned tasks, including the MATLAB/Python-generated figures. Use a word processor of your choice to write the report and properly reference the figures in the discussion.
2. The .m files/.py file should be used for generating the figures. The files should include a top comment with your name and a brief description of the code's structure.
3. Upload the report and code to the lab submission portal in Canvas.

Submission Deadline: Friday, November 28, 2025, h 23:59.

Good Luck!

1. Basic Probability

- (a) Suppose X_i are all uniformly distributed over $(0, 1)$. Plot in MATLAB/Python and show the results of estimating the CDF using n IID random variables for $n = 10, n = 100$, and $n = 1000$. Explain what happens if n gets larger.
- (b) A fair coin is tossed 10 times. What is the probability of a run of exactly 5 heads in a row? Do not count runs of 6 or more heads in a row. Write the solution in MATLAB/Python and show the mathematical reasoning in the report.

Algorithm 1 Pseudocode for Basic Probability Problems 1(a)

```
1: n_values ← [10, 100, 1000]           ▷ Define sample sizes to be used
2: for each  $n$  in n_values do             ▷ Loop over each sample size
3:   Generate  $n$  IID random variables  $X_i \sim U(0, 1)$    ▷ Generate  $n$  uniform random
   variables
4:   Sort  $X_i$  in ascending order           ▷ Order the variables to create an empirical CDF
5:   Plot empirical CDF using  $X_i$  and  $n$            ▷ Plot the empirical CDF
6:   Label and show the plot               ▷ Add labels and display the plot
7: end for
```

Algorithm 2 Pseudocode for Basic Probability Problems 1(b)

```
1: p_head ← 0.5           ▷ Probability of getting a head
2: p_tail ← 0.5           ▷ Probability of getting a tail
   ▷ Calculating probability for different scenarios
3: P_5_heads_middle ← p_head5 × p_tail ▷ Probability of getting 5 heads followed by a
   tail, not at the start
4: P_5_heads_start ← p_head5 × p_tail  ▷ Probability of getting 5 heads starting from
   the first toss
5: P_5_heads_end ← p_tail × p_head5 ▷ Probability of getting 5 heads at the end of the
   sequence
   ▷ Summing up all the probabilities
6: Total_P ← P_5_heads_middle + P_5_heads_start + P_5_heads_end
7: Display Total_P        ▷ Displaying the total probability
```

2. Poisson Processes

Let X be an exponential random variable of parameter λ , modeling the inter-arrival times of a Poisson arrival process. Then:

- (a) Plot the PDF f_X for $\lambda = 0.3$, $\lambda = 1$, $\lambda = 3$ using MATLAB. What do you observe? Explain the physical meaning of the parameter λ .

Algorithm 3 Pseudocode for Poisson Processes Problem 2(a)

```
1: lambda_values ← [0.3, 1, 3]           ▷ Define different  $\lambda$  values to be used
2: Define  $x$  as a range of values to evaluate the PDF           ▷ Specify the range for  $x$ 
3: for each  $\lambda$  in lambda_values do           ▷ Loop over each  $\lambda$  value
4:    $f_X \leftarrow \lambda \times \exp(-\lambda \times x)$            ▷ Calculate the PDF of the exponential distribution
5:   Plot  $x$  vs  $f_X$            ▷ Plot the PDF
6:   Label and show the plot           ▷ Add labels and display the plot
7: end for
```

3. Markov process

A common example of Markov chains (and Markov processes in general) is that of queueing systems. Consider the following scenario,

A taxi stand at a busy airport. A line of taxis, which for all practical purposes can be taken to be infinitely long, is available to serve travelers. Customers wanting a taxi enter a queue and are given a taxi on a first-come, first-serve basis. Suppose it takes one unit of time (say, a minute) for the customer at the head of the queue to load himself and his luggage into a taxi. Hence, during each unit of time, one customer in the queue receives service and leaves the queue while some random number of new customers enter the end of the queue. Suppose at each time instant, the number of new customers arriving for service is described by a discrete distribution (p_0, p_1, p_2, \dots) , where p_k is the probability of new customers. For such a system, the transition probability matrix of the Markov chain would look like $\mathbf{P} = \begin{bmatrix} p_0 & p_1 & p_2 & \cdots \\ p_0 & p_1 & & p_2 & \cdots \\ \dots & \dots & & \dots & \dots \end{bmatrix}$

The manager of the taxi stand might be interested in knowing the probability distribution of the queue length. If customers have to wait too long, they may get dissatisfied and seek other forms of transportation.

- (a) Write the MATLAB/Python code to simulate the distribution of the queue length of the taxi stand described in the example described above. For this example, take the number of arrivals per time unit, X , to be a Poisson random variable whose PMF is $P_X(k) = \frac{\lambda^k \exp(-\lambda)}{k!}$ and an average arrival rate of 0.85 arrivals per time unit. For the code, initialize the length of the simulation, $N = 10000$, $k = [0 : 10]$.

Algorithm 4 Pseudocode for Markov process Problem 3(a)

```

1: lambda ← 0.85                                ▷ Define the average arrival rate
2: N ← 10000                                     ▷ Define the length of the simulation
3: k ← [0 : 10]                                 ▷ Define a range for the number of arrivals
4: queue_length ← 0                             ▷ Initialize the queue length
5: queue_length_distribution ← [ ]               ▷ Initialize a list to store queue lengths
6: for i = 1 to N do                             ▷ Loop through each time unit in the simulation
7:     Generate a random number of arrivals using Poisson PMF
        ▷  $P_X(k) = (\text{lambda}^k \cdot \exp(-\text{lambda}))/k!$ 
8:     Update queue_length based on arrivals and service rate    ▷ Modify the queue length
9:     Record queue_length in queue_length_distribution          ▷ Store the current queue length
10: end for
11: Plot histogram of queue_length_distribution                ▷ Visualize the distribution
12: Label and show the plot                                    ▷ Add labels and display the plot

```
