CSE 6140 / CX 4140 Assignment 1
due Sep 12, 2019 at 6pm on Canvas

Please type your answers (LaTeX highly recommended) and upload a single PDF named `assignment.pdf`. If you want to draw a graph by hand you can take a picture of your drawing and insert it to your PDF file. Please make sure that your inserted picture can be clearly read.

1 Simple Complexity (8pt)

a) For each pair of functions f and g, write whether f is in $\mathbb{O}(g)$, $\Omega(g)$, or $\Theta(g)$. (5pt)

   (a) $f = (n + 1000)^4$, $g = n^4 - 3n^3$

   (b) $f = \log_{1000} n$, $g = \log_2 n$

   (c) $f = n^{1000}$, $g = n^2$

   (d) $f = 2^n$, $g = n!$

   (e) $f = n^n$, $g = n!$

b) Prove the additivity of big O notation (3pt):
f, g and h are functions of input size n. Prove that if $f \in \mathbb{O}(h)$ and $g \in \mathbb{O}(h)$, then $f + g \in \mathbb{O}(h)$.

2 Greedy 1 (12pt)

You are given n distinct points and one line l on the plane and some constant $r > 0$. Each of the n points is within distance at most r of line l (as measured along the perpendicular). You are to place disks of radius r centered along line l such that every one of the n points lies within at least one disk. Devise a greedy algorithm that runs in $O(n \log n)$ time and uses a minimum number of disks to cover all n points; prove its optimality.

3 Greedy 2 (12pt)

You are on a hike along the Appalachian Trail when you happen upon an abandoned mine. Against your better judgment, you venture inside and find a large deposit of various precious minerals. Luckily, you have a bag and a pickaxe with you, but you can't carry everything. Therefore, you must figure out how much of each mineral to take to maximize the value of your bag.

Formally: Given a bag with weight limit L, and a list of n minerals, where mineral i has value v_i and weight w_i, design a greedy algorithm to maximize the value of items you pack in your bag. (Note: v_i is the value of the entierty of item i, so if you decide you only want to take half of item i you only get value $\frac{v_i}{2}$). Prove your algorithm gives the optimal result using an **exchange argument**.

4 Shortest Path (10pt)

Let $G = (V, E)$ be a directed graph with positive edge length. Let $t \in V$. Give an algorithm that runs in $O(|V|^2)$ time for finding shortest paths between all pairs of nodes, such that these paths pass through t.

5 Minimum Spanning Tree (8pt)

Give a counter-example or prove the following statement: Let $G = (V, E)$ be a weighted undirected graph. Let C be one cycle in G and let e be an edge in C. If the weight of e is strictly larger than any other edge in C, then e is not in any minimum spanning tree of G.