Università della Svizzera italiana
Institute of Computing CI

High-Performance Computing Lab **Institute of Computing**

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Solution for Project 6

1. Task: Install METIS 5.0.2, and the corresponding Matlab mex interface

For this part, the steps on the project document are followed and the installation is done accordingly.

2. Task: Construct adjacency matrices from connectivity data [10 points]

For all of the countries, the same steps are taken. First, the .csv files are read, then the adjacency matrix and the node coordinate list are constructed and saved in a new folder, and finally, the graph is visualized. The resulting visualization of the graphs for Norway and Vietnam can be found in Figure 1 (a) and (b), respectively. The adjacency matrix and the node coordinate list are constructed the following way for Norway as an example:

```
no_coord=table2array(no_coord);
save('Datasets/Countries_mat/no_coord.mat', 'no_coord');
N = graph(no_adj(:,1), no_adj(:,2) );
N = adjacency(N);
N = full(N);
no_adj = N;
save('Datasets/Countries_mat/no_adj.mat', 'no_adj');
```

(a) Norway (b) Vietnam

Figure 1

3. Task: Implement various graph partitioning algorithms [25 points]

In the specific section, the implementation of spectral graph bisection is carried out within the file `"bisection_spectral.m,"` utilizing the Fiedler eigenvector's entries. The process involves constructing the Laplacian, performing its eigen decomposition, labeling vertices based on the Fiedler vector components, and ultimately partitioning them around their median value (0).

Conversely, the inertial graph bisection is executed in the `"bisection_inertial.m"` file. Initially, the center of mass is computed, and matrix M is then constructed. Subsequently, the smallest eigenvector of M is determined. The line, denoted as L, containing the center of mass is identified, and points are partitioned around this line.

Table 1 provides the bisection edge-cut results for all toy meshes, generated or loaded in `"Bench_bisection.m."` An illustration of the outcomes for the dual of "airfoil" can be found in Figure 2 . The data in Table 1 indicates that the differences in results across all algorithms are not substantial, given the limited number of edges in the utilized meshes.

Table 1: Bisection results

Mesh	Coordinate	Metis 5.0.2	Spectral	Inertial
mesh1e1	18	17	18	19
mesh2e1	37	37	39	47
mesh3e1	19	19	26	19
mesh3em5	19	19	26	19
airfoil	94	77	132	94
netz4504_dual	25	23	23	30
stufe	16	16	16	16
3elt	172	124	117	209
barth4	206	97	127	194
ukerbe1	32	27	32	28
crack	353	201	233	377

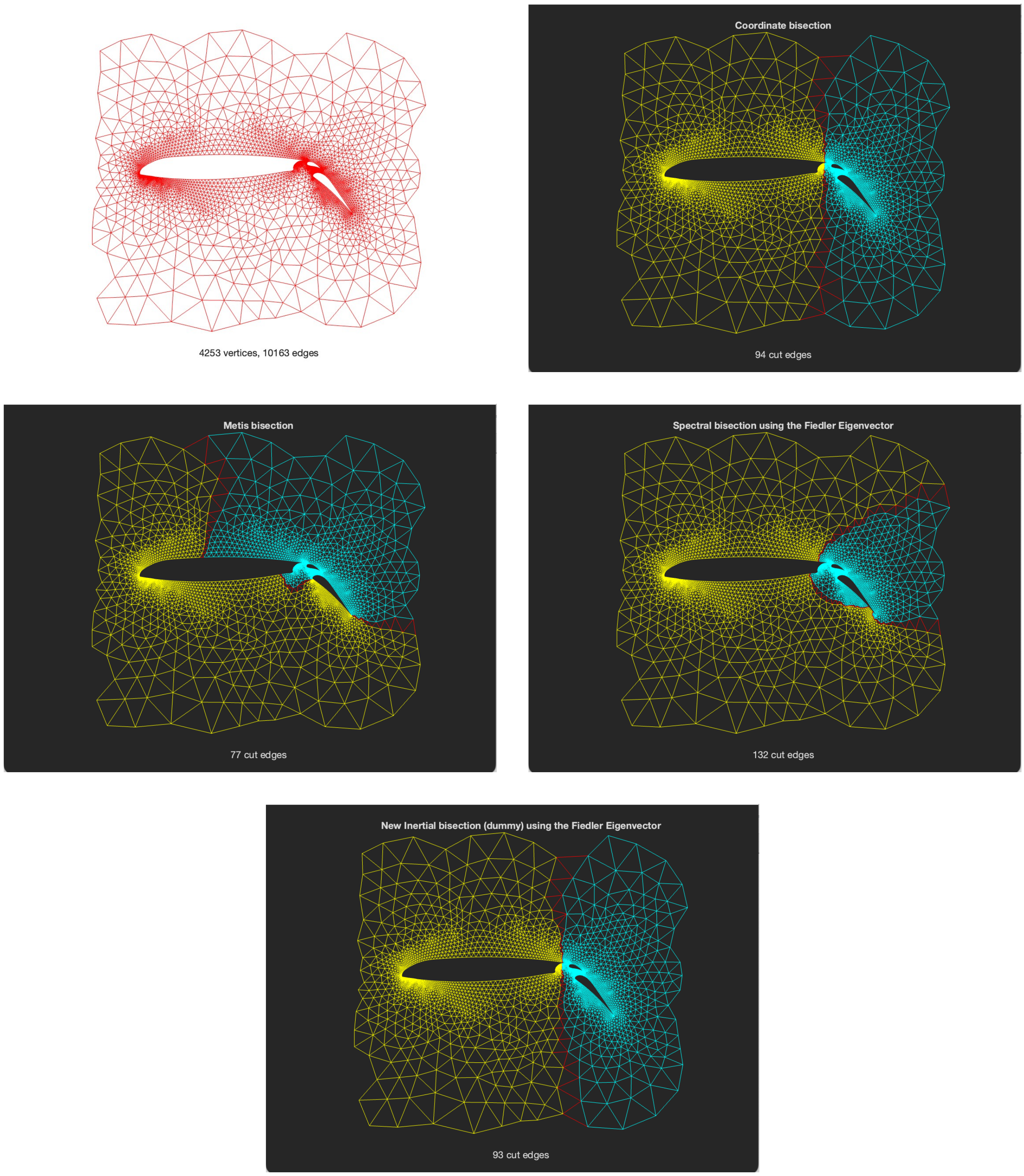
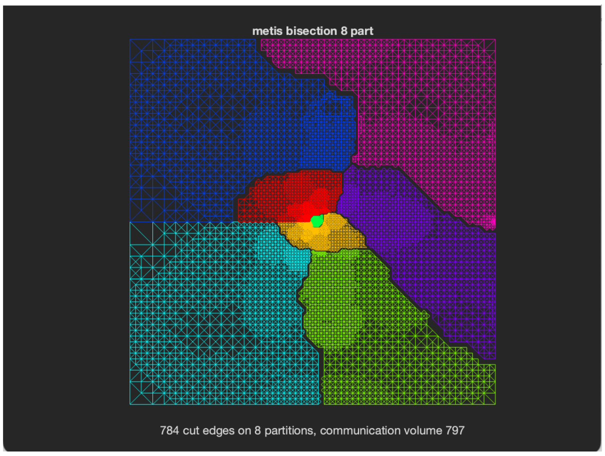


Figure 2: Visualizations for airfoil

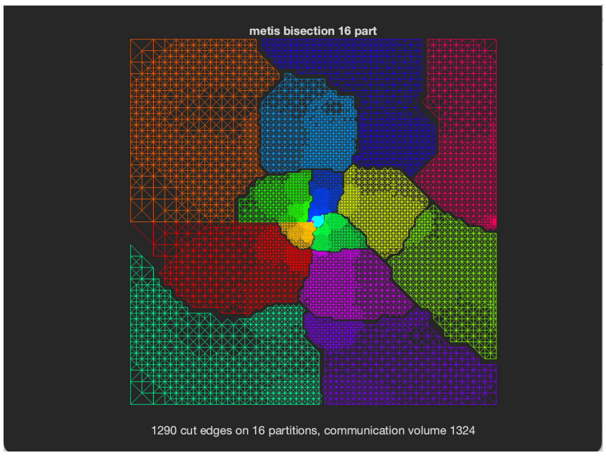
4. Task: Recursively bisecting meshes [15 points]

To recursively bisect the finite element meshes loaded within the script in 8 and 16 subgraphs, the recursive bisection algorithm in the file `rec_bisection.m` is utilized in `Bench_rec_bisection.m`. The inertial and spectral partitioning implementations, as well as the coordinate partitioning and the METIS bisection routine, are used. The results can be found in Table **??**. The visualization of the results for the case "crack" can be found in Figure 3.

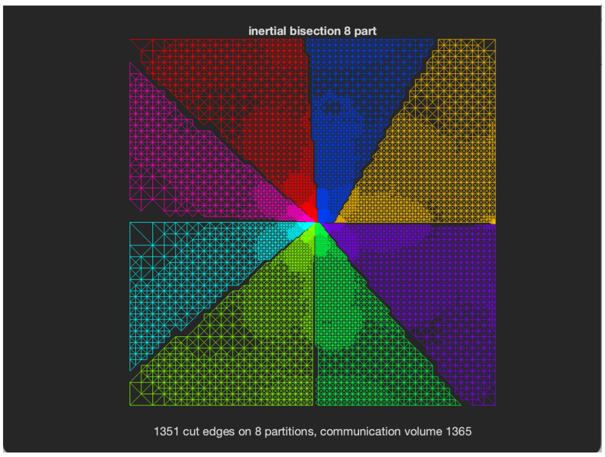
Case	Spectral		Metis 5.0.2		Coordinate		Inertial	
p	***8***	***16***	***8***	***16***	***8***	***16***	***8***	***16***
airfoil1	398	633	320	563	516	819	577	897
netz4504_dual	111	184	110	161	127	198	122	200
stufe	128	238	107	194	123	227	136	269
3elt	469	752	395	651	733	1168	880	1342
barth4	550	841	405	689	875	1306	891	1350
ukerbe1	398	695	128	224	225	374	280	468
crack	883	1419	784	1290	1343	1860	1061	1618



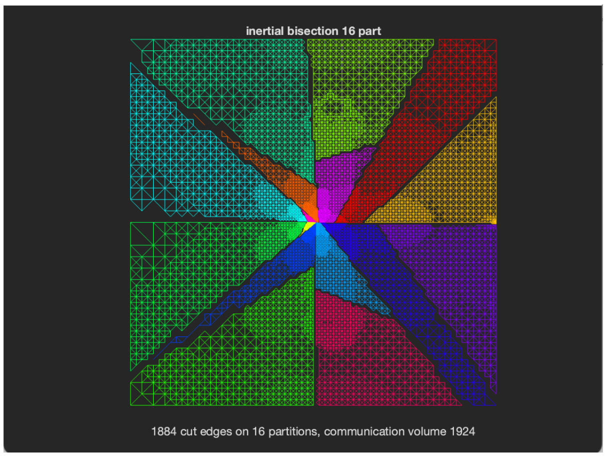
(a) Metis with p=8



(b) Metis with p=16



(c) Inertial with p=8



(d) Inertial with p=16

Figure 3

5. Task: Comparing recursive bisection to direct k-way partitioning [10 points]

For this part, the `Bench_metis.m` file is used. The cut obtained from Metis 5.0.2 after applying recursive bisection and direct multiway partitioning for the graphs is compared. The results can be found in Table ?? for 16 and 32 partitions. As can be observed, the recursive bisection produces more detailed results compared to multiway partitioning. This is expected since the recursive approach is not strictly limited in terms of the number of operations it does unlike k-way partitioning.

The visualization of partitioning results for both graphs for 32 partitions can be found in Figure 5.

Table 2: Comparing the number of cut edges for recursive bisection and direct multiway partitioning in Metis 5.0.2.

	Luxemburg		**usroads-48**		**Greece**		**Switzerland**		**Vietnam**		**Norway**		**Russia**	
Partitions	***16***	***32***	***16***	***32***	***16***	***32***	***16***	***32***	***16***	***32***	***16***	***32***	***16***	***32***
kway	170	279	579	961	278	471	673	1042	245	411	255	439	551	933
recursive	197	322	607	988	297	509	730	1089	245	445	284	470	616	1006

6. Task: Utilizing graph eigenvectors [25 points]

Section a

The plot for the entries of the eigenvectors associated with the first (λ_1) and second (λ_2) smallest eigenvalues of the graph Laplacian matrix $\mathbf{L}$ for the graph "airfoil1" can be found in Figure 4. It can be observed that the second eigenvector is a straight line, whereas the first one takes different values, which is expected since the second eigenvector is to be used for partitioning.

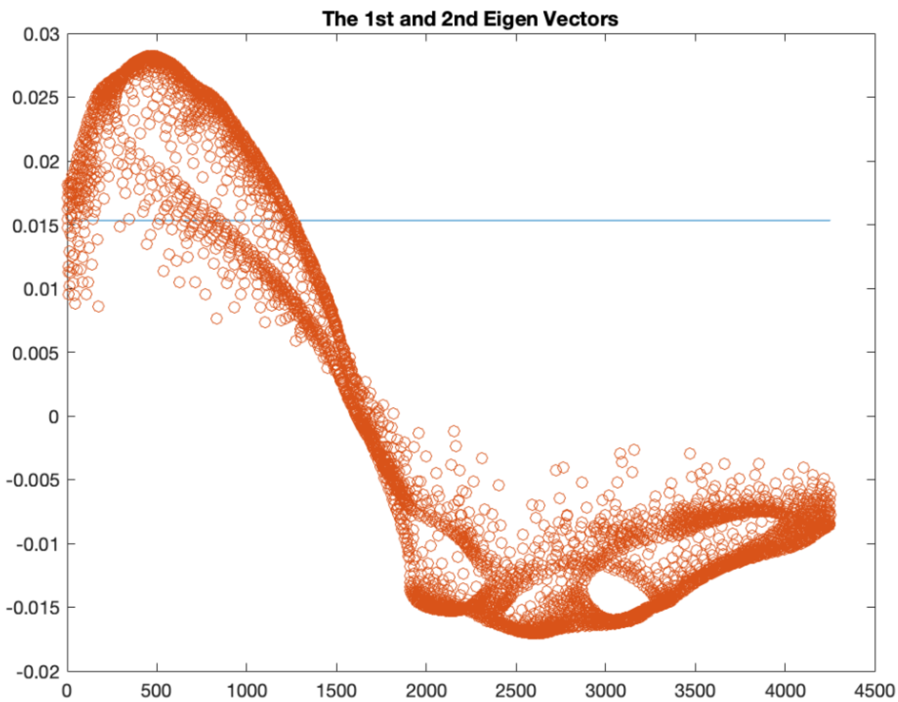


Figure 4: Plot of the entries of the eigenvectors associated with the first (λ_1) and second (λ_2) smallest eigenvalues of the graph Laplacian matrix $\mathbf{L}$ for the graph "airfoil1."

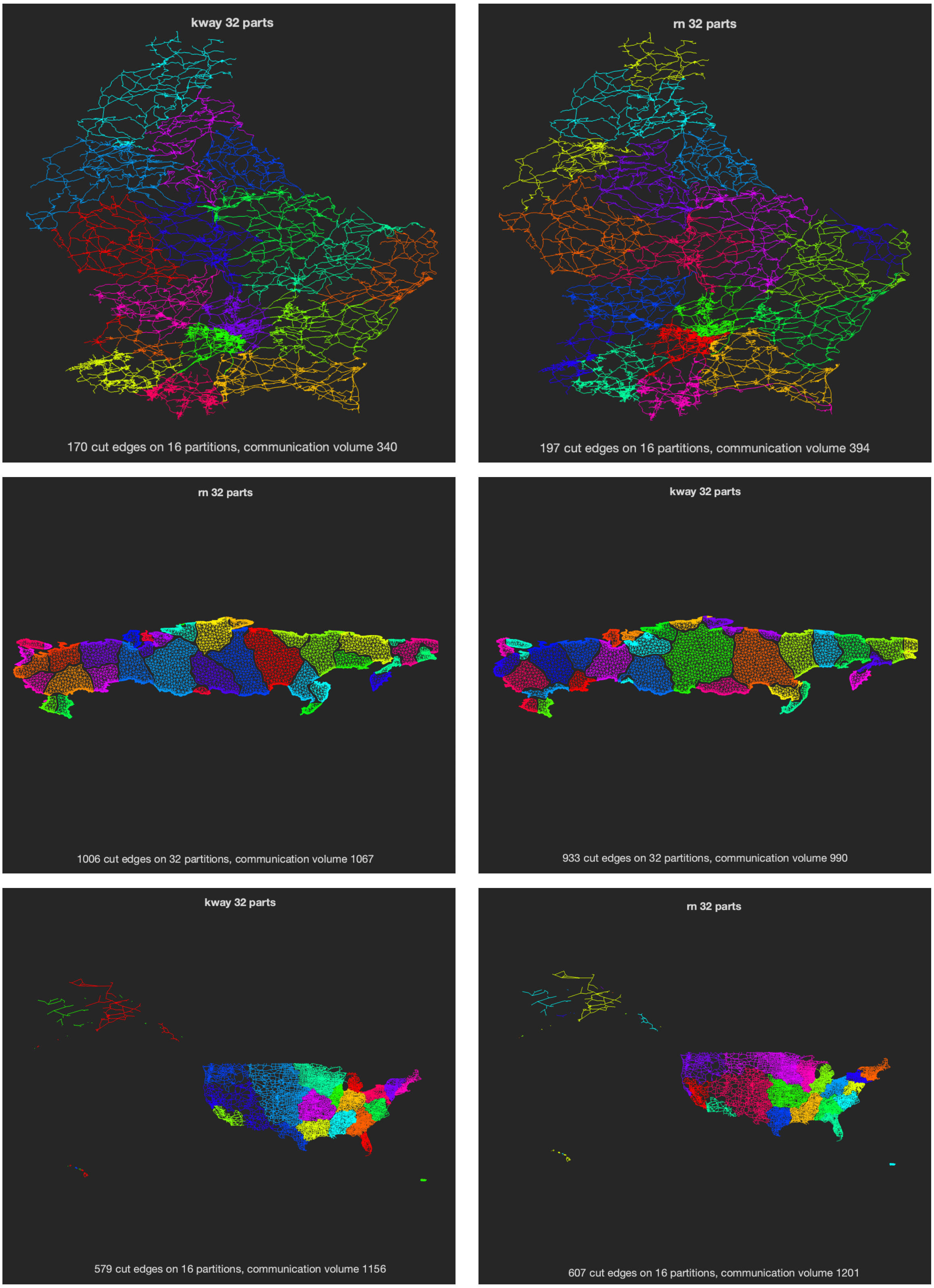


Figure 5: Visualization of partitioning results for both graphs for 32 partitions.

Section b

The plots for the entries of the eigenvector associated with the second smallest eigenvalue λ_2 of the Graph Laplacian matrix $\mathbf{L}$ are given in Figure 6.

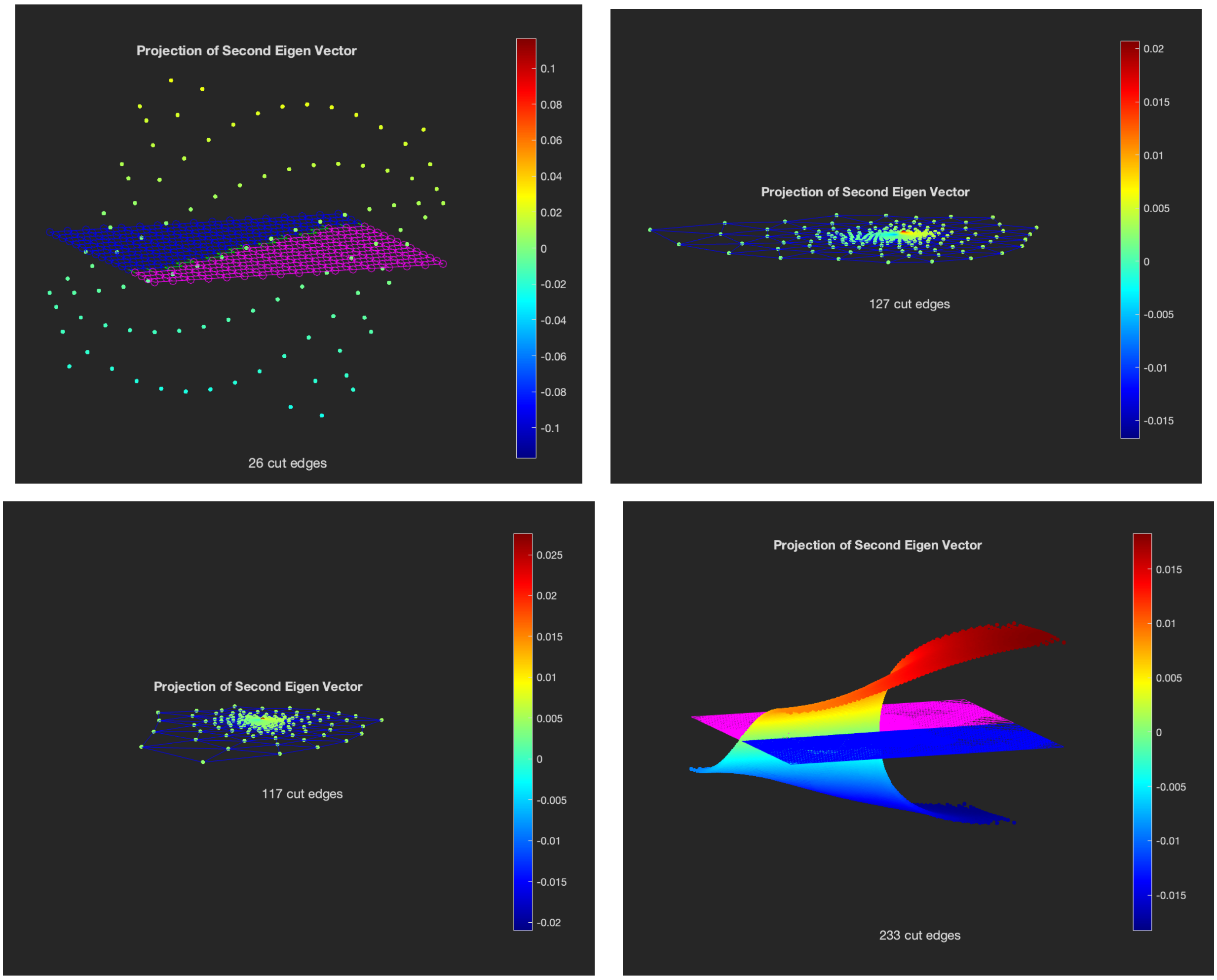


Figure 6: The plots for the entries of the eigenvector associated with the second smallest eigenvalue λ_2 of the Graph Laplacian matrix $\mathbf{L}$.

Section C

The plots for the graphs mesh3e1, barth4, 3elt, crack, and their **spectral bi-partitioning** results using the eigenvectors to supply coordinates can be found in Figure 7.

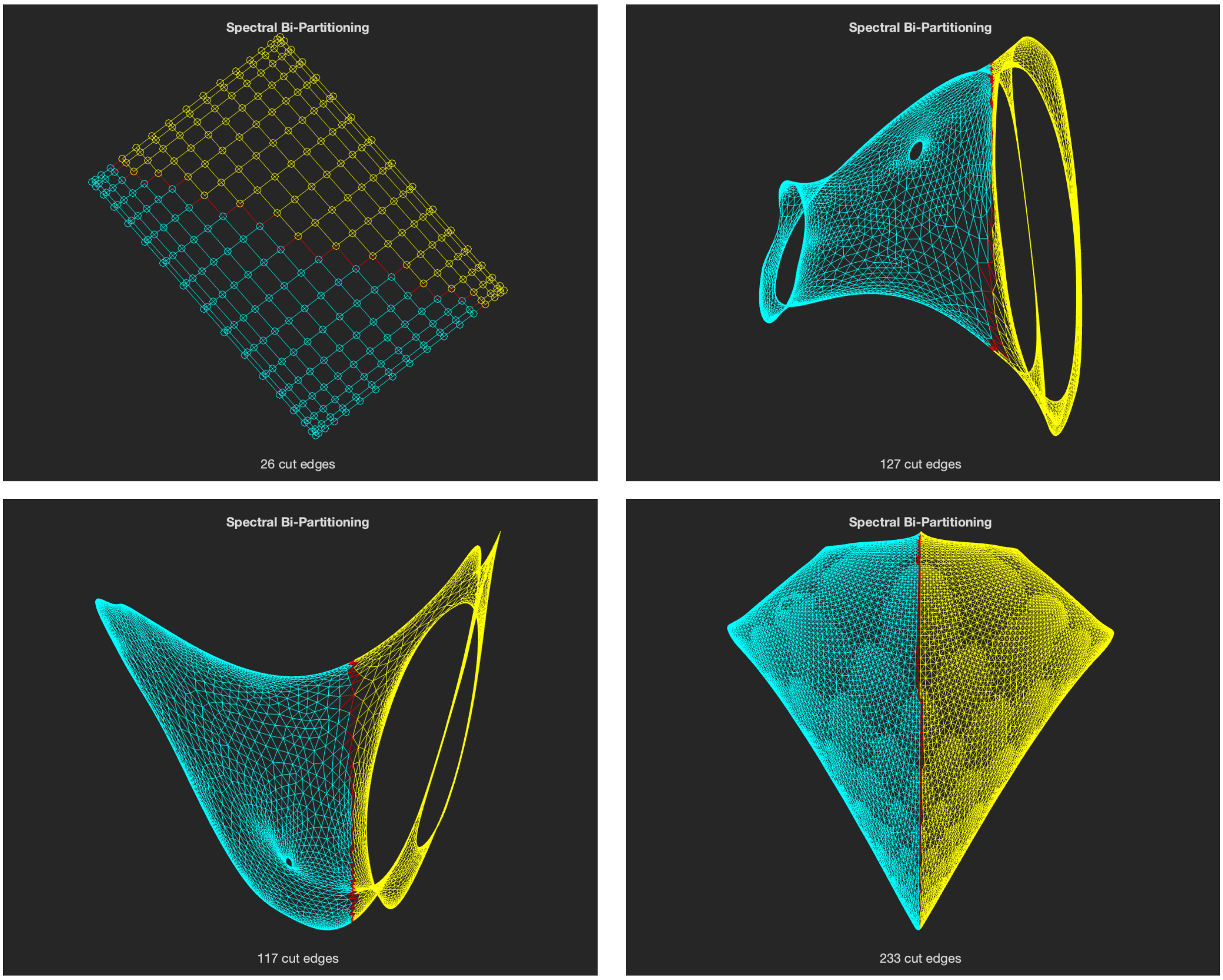


Figure 7: Plots for the graphs mesh3e1, barth4, 3elt, crack, and their **spectral bi-partitioning** results.