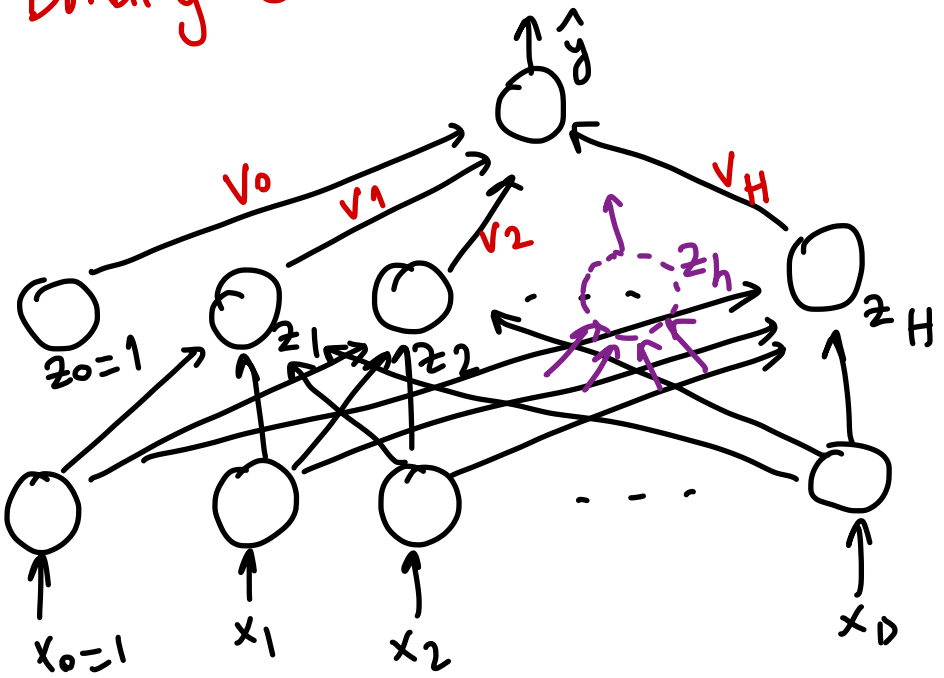
Multilayer Perceptrons

(A) Binary Classification



$$\hat{y}_i = \text{sigmoid}(v^T \cdot z_i)$$

sigmoid

$$z_{ih} = \text{sigmoid}(w_h^T \cdot x_i)$$

sigmoid

weights for all incoming edges to z_h

$$\text{Error}_i = -\left[y_i \log(\hat{y}_i) + (1-y_i) \log(1-\hat{y}_i) \right]$$ → binary cross-entropy

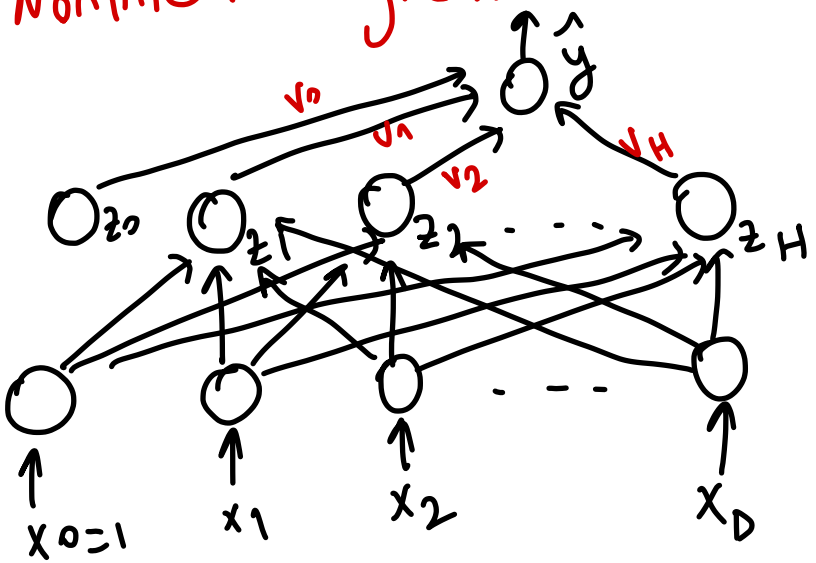
$$\Delta v_h = \eta \cdot (y_i - \hat{y}_i) \cdot z_{ih}$$

$$\Delta w_{hd} = \eta (y_i - \hat{y}_i)\, v_h \cdot z_{ih}(1 - z_{ih}) \cdot x_{id}$$

y_i's are either 0 or 1.

$\hat{y}_i$'s are between 0 and 1.

(B) Nonlinear Regression



linear $\quad \hat{y}_i = v^T . z_i \quad \Leftarrow \hat{y}_i \in \mathbb{R}$

sigmoid $\quad z_{ih} = \text{sigmoid}(w_h^T . x_i)$

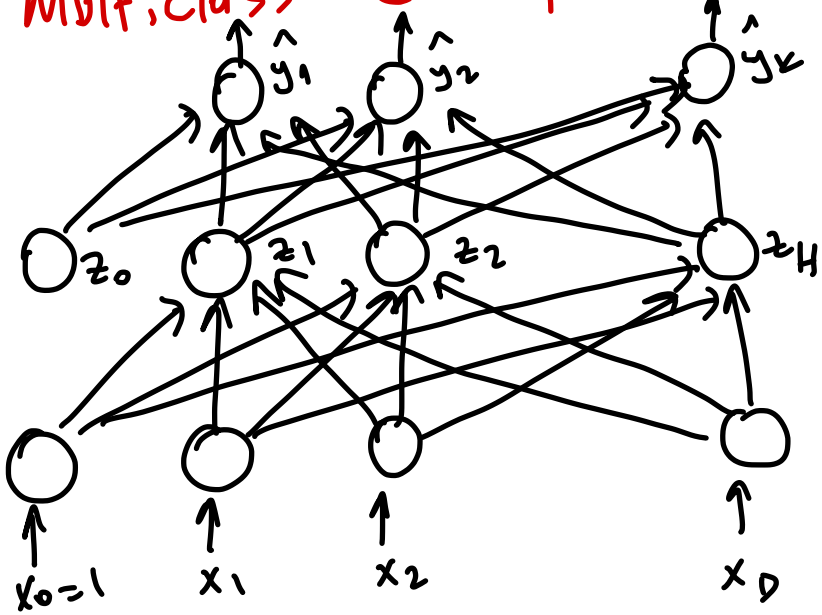
$$\text{Error}_i = \frac{1}{2}(y_i - \hat{y}_i)^2$$

$$\Delta v_h = \eta . (y_i - \hat{y}_i) z_{ih}$$

$$\Delta w_{hd} = \eta . (y_i - \hat{y}_i) . v_h . z_{ih} (1 - z_{ih}) . x_{id}$$

y_i's and $\hat{y}_i$'s are real numbers.

(c) Multiclass Classification



softmax

$$\hat{y}_{ic} = \frac{\exp\left[v_c^T \cdot z_i\right]}{\sum_{d=1}^{K} \exp\left[v_d^T \cdot z_i\right]}$$

sigmoid

$$z_{ih} = \text{sigmoid}\left(w_h^T \cdot x_i\right)$$

$$\text{Error}_i = -\sum_{c=1}^{K} y_{ic} \log\left(\hat{y}_{ic}\right)$$

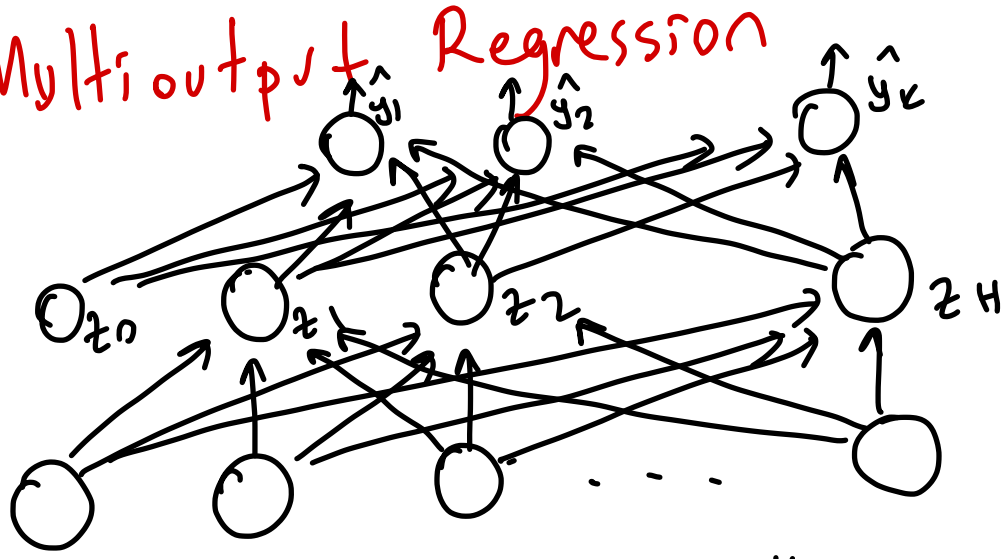
$$\Delta v_h = \eta \left(y_{ic} - \hat{y}_{ic}\right) \cdot z_{ih}$$

$$\Delta w_{hd} = \eta \left[\sum_{c=1}^{K} \left(y_{ic} - \hat{y}_{ic}\right) \cdot v_{ch}\right] \cdot z_{ih}\left[1 - z_{ih}\right] \cdot x_{id}$$

y_{ic}'s are either 0 or 1.

$\hat{y}_{ic}$'s are between 0 and 1.

(D) Multioutput Regression



linear

$$\hat{y}_{ic} = v_c^T \cdot z_i$$

sigmoid

$$z_{ih} = \text{sigmoid}(w_h^T x_i)$$

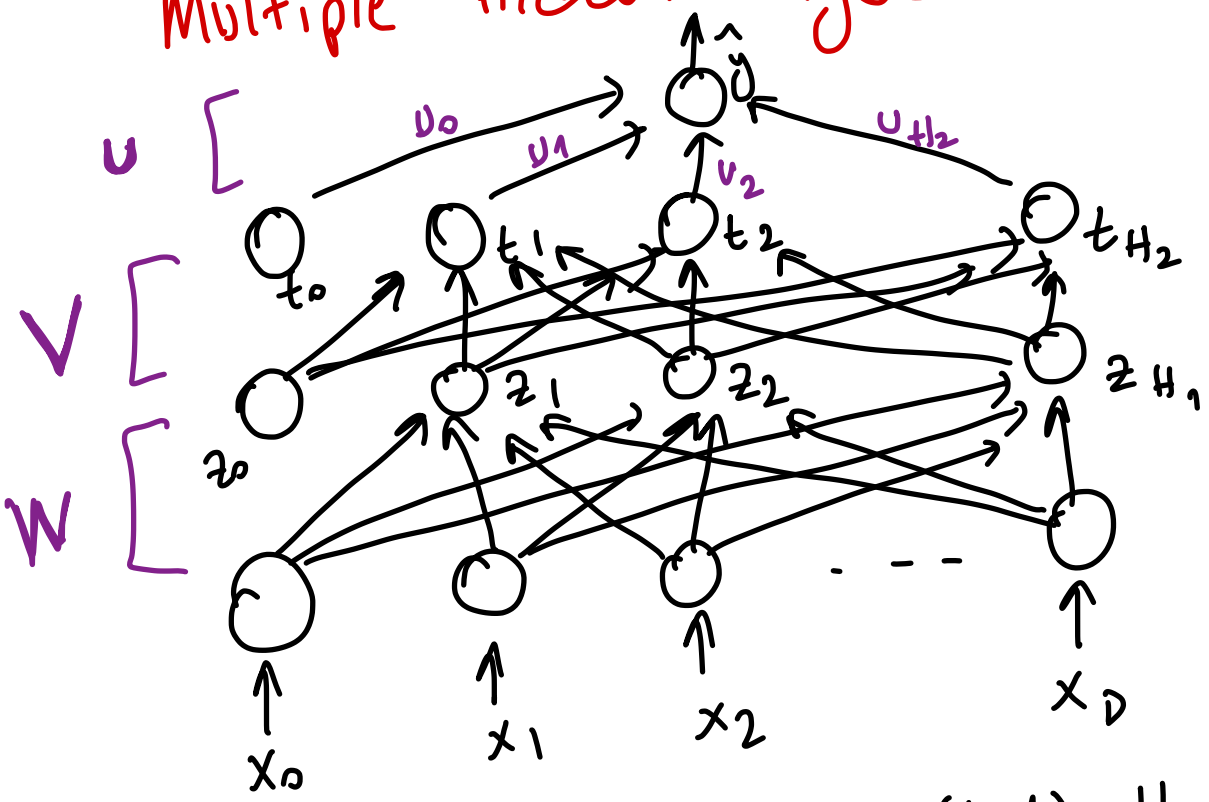
$$\text{Error}_i = \frac{1}{2} \sum_{c=1}^{K} (y_{ic} - \hat{y}_{ic})^2$$

$$\Delta v_{ch} = \eta \cdot (y_{ic} - \hat{y}_{ic}) \cdot z_{ih}$$

$$\Delta w_{hd} = \eta \cdot \left[\sum_{c=1}^{K} (y_{ic} - \hat{y}_{ic}) \cdot v_{ch} \right] \cdot z_{ih} \cdot (1 - z_{ih}) \cdot x_{id}$$

y_{ic}'s and $\hat{y}_{ic}$'s are real numbers.

Multiple Hidden Layers



sigmoid $\hat{y}_i = \text{sigmoid}(U^T \cdot t_i)$

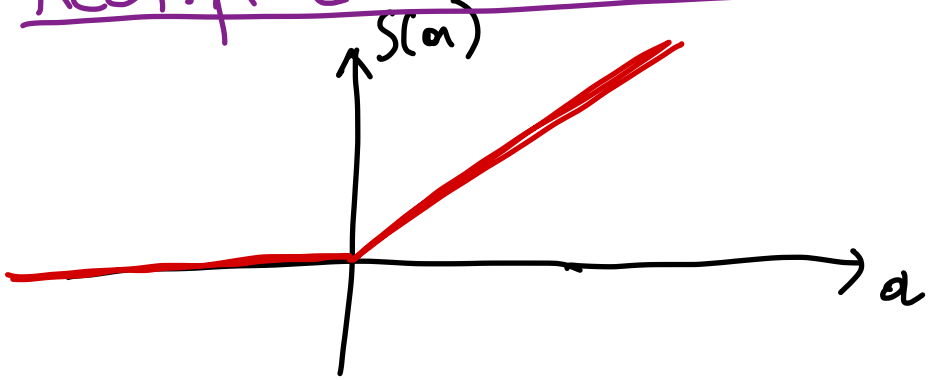
sigmoid $t_{ih} = \text{sigmoid}(V_h^T \cdot z_i)$

sigmoid $z_{ih} = \text{sigmoid}(w_h^T \cdot x_i)$

$$\#\text{ of parameters} = \underbrace{(D+1)\cdot H_1}_{W} + \underbrace{(H_1+1)\cdot H_2}_{V} + \underbrace{(H_2+1)}_{U}$$

"Vanishing gradients" → approaching to 0.

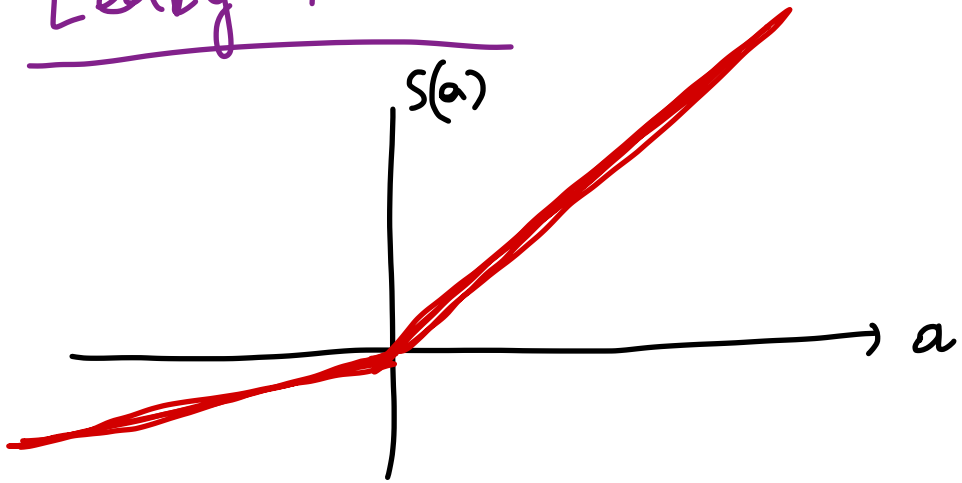
Rectified Linear Unit (ReLU)



$$s(a) = \begin{cases} a & \text{if } a > 0 \\ 0 & \text{otherwise} \end{cases}$$

$$s'(a) = \begin{cases} 1 & \text{if } a > 0 \\ 0 & \text{otherwise} \end{cases}$$

Leaky ReLU



$$s(a) = \begin{cases} a & \text{if } a > 0 \\ \alpha a & \text{otherwise} \end{cases}$$

$$s'(a) = \begin{cases} 1 & \text{if } a > 0 \\ \alpha & \text{otherwise} \end{cases}$$

usually $\alpha = 0.01$

TRAINING PROCEDURES

Momentum:

(state) variable storage of the previous step

gradient of the current step

$$s_h^{(t)} = \underbrace{\alpha s_h^{(t-1)}}_{\text{memory}} + \underbrace{(1-\alpha)\frac{\partial Error^{(t)}}{\partial w_h}}_{\text{Current opinion}}$$

$$\Delta w_h^{(t)} = -\eta s_h^{(t)}$$

Adaptive learning Rate:

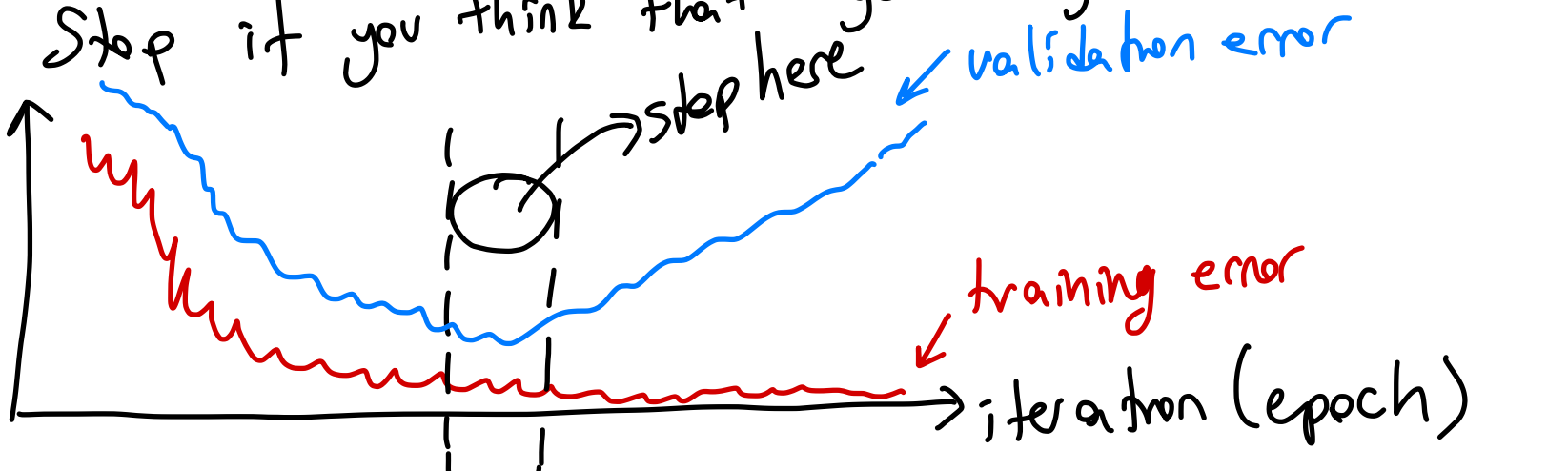
$$\eta^{(t)} = \eta^{(t-1)} 0.99$$

$\eta = ?$

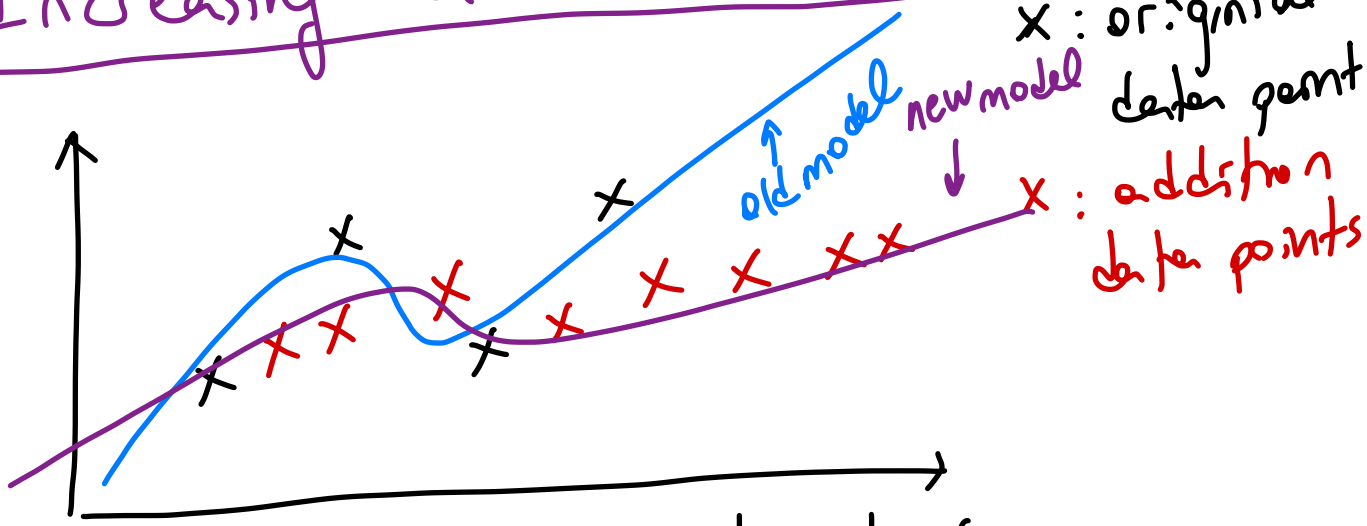
- starts with a large value
- decrease if error increases

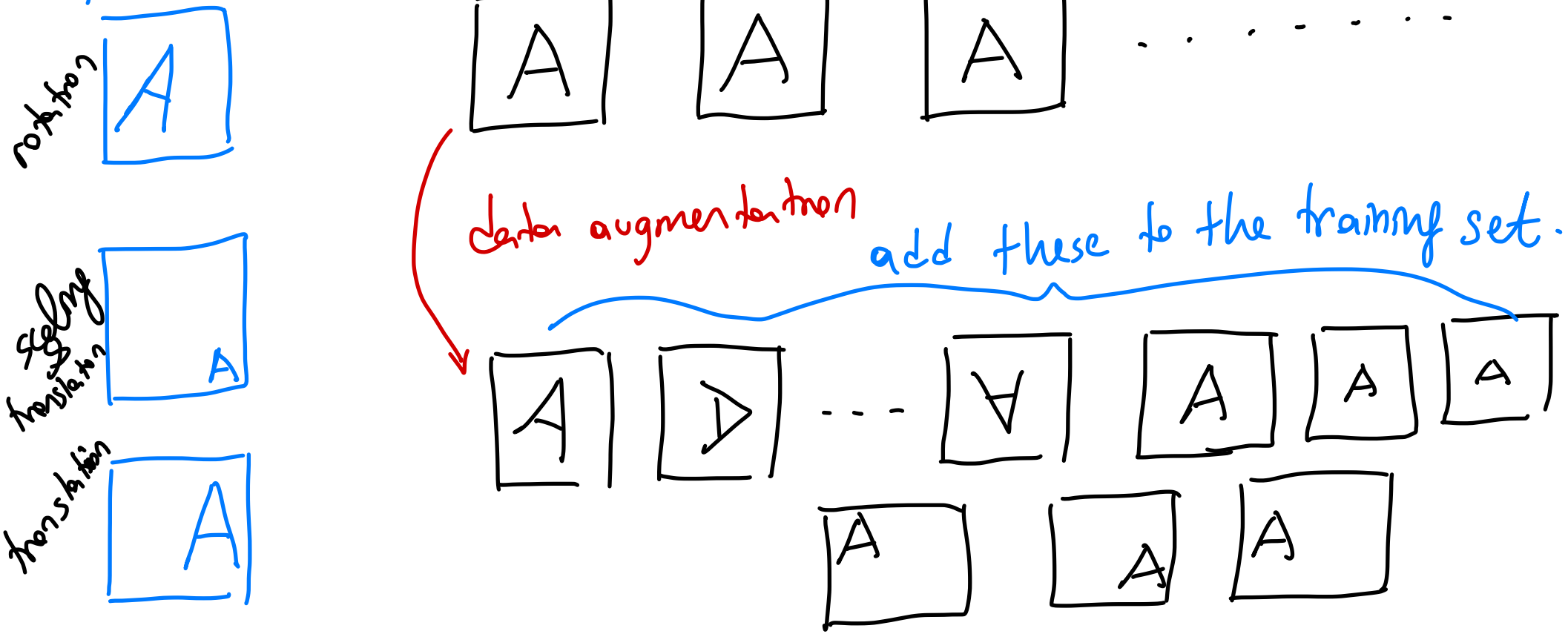
Early Stopping:

Stop if you think that your algorith is overfitting.



Increasing Data Set Size





Weight Decay:

$$Error' = Error + \underbrace{\frac{\lambda}{2} \cdot \sum_{h=1}^{H} w_h^2}_{\text{weight decay}} \qquad \lambda > 0$$

weight decay

l_2-norm regularization

$$\frac{\partial Error'}{\partial w_h} = \frac{\partial Error}{\partial w_h} + \frac{\lambda}{2} \cdot 2 \cdot w_h$$

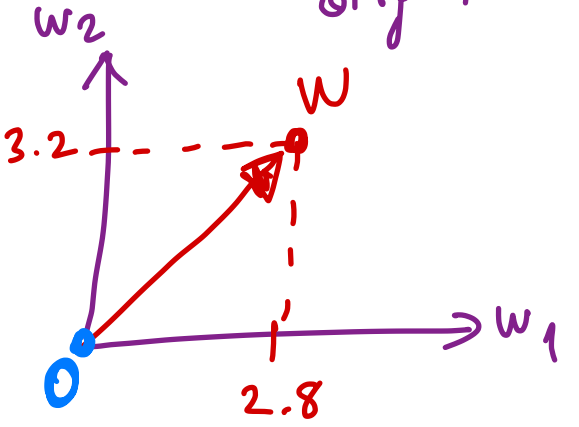
$$= \frac{\partial Error}{\partial w_h} + \lambda \cdot w_h$$

$$\Delta w_h = -\eta \left[\frac{\partial Error}{\partial w_h} + \lambda w_h \right]$$

$$\sum_{h=1}^{H} w_h^2 = \|w\|_2^2$$

↓ distance to the origin.

$$w = \begin{bmatrix} 2.8 \\ 3.2 \end{bmatrix}$$



$$w_1^2 + w_2^2 = \left(\sqrt{(w_1-0)^2+(w_2-0)^2}\right)^2$$

Euclidean distance between w and 0.