問題1

(1) $x^2-(7+\varepsilon)x+12+3\varepsilon=0$

$$x=\frac{7+\varepsilon\pm\sqrt{(7+\varepsilon)^2-4(12+3\varepsilon)}}{2}$$

$$=\frac{7+\varepsilon\pm\sqrt{49+14\varepsilon+\varepsilon^2-48-12\varepsilon}}{2}$$

$$=\frac{7+\varepsilon\pm\sqrt{\varepsilon^2+2\varepsilon+1}}{2}$$

$$=\frac{7+\varepsilon\pm(\varepsilon+1)}{2}=\varepsilon+4,\ 3$$

(2) $(x_0+\varepsilon x_1+\varepsilon^2 x_2+\cdots)^2-(7+\varepsilon)(x_0+\varepsilon x_1+\varepsilon^2 x_2+\cdots)+12+3\varepsilon=0$

$x_0^2+\varepsilon\cdot 2x_0x_1+\varepsilon^2(2x_0x_2+x_1^2)-7x_0-7\varepsilon x_1-7\varepsilon^2 x_2-\varepsilon x_0-\varepsilon^2 x_1$
$+12+3\varepsilon=0$

$x_0^2-7x_0+12+\varepsilon(2x_0x_1-7x_1-x_0+3)+\varepsilon^2(2x_0x_2-7x_2)+O(\varepsilon^3)$
$=0$

$O(1): x_0^2-7x_0+12=0 \longrightarrow x_0=3,4$

$O(\varepsilon): (2x_0-7)x_1=x_0-3 \longrightarrow x_1=\dfrac{x_0-3}{2x_0-7}$

$O(\varepsilon^2): (2x_0-7)x_2=0 \longrightarrow x_2=0$

$x_0=3$のとき、$x_1=0 \longrightarrow$ 解 $x=3$

$x_0=4$のとき、$x_1=1 \longrightarrow$ 解 $x=4+\varepsilon$

問題2

$J_\nu(x)$ の近似式の解を求める.

$$\sqrt{\frac{2}{\pi x}}\left\{\cos\left(x-\frac{\nu\pi}{2}-\frac{\pi}{4}\right)-\frac{4\nu^2-1}{8x}\sin\left(x-\frac{\nu\pi}{2}-\frac{\pi}{4}\right)\right\}=0$$

$$\frac{\cos\left(x-\frac{\nu\pi}{2}-\frac{\pi}{4}\right)}{\sin\left(x-\frac{\nu\pi}{2}-\frac{\pi}{4}\right)}=\frac{4\nu^2-1}{8x}$$

$$\cot\left(x-\frac{\nu\pi}{2}-\frac{\pi}{4}\right)=\frac{4\nu^2-1}{8x}\quad\cdots\ (2-0)$$

$x\gg1$ のときは $\frac{4\nu^2-1}{8x}\sim0$ と考え、

$$\cot\left(x-\frac{\nu\pi}{2}-\frac{\pi}{4}\right)=0$$

この式の解は

$$x-\frac{\nu\pi}{2}-\frac{\pi}{4}=n\pi+\frac{\pi}{2}$$

$$x=\left(n+\frac{\nu}{2}+\frac{3}{4}\right)\pi\quad\cdots\cdots\ (2-1)$$

(2-1) が0次近似解となる.

ここで $x=\left(n+\frac{\nu}{2}+\frac{3}{4}\right)\pi+\delta$ として (2-0) に代入すると

$$\cot\left(\left(n+\frac{\nu}{2}+\frac{3}{4}\right)\pi+\delta-\frac{\nu\pi}{2}-\frac{\pi}{4}\right)=\frac{4\nu^2-1}{8}\cdot\frac{1}{\left(n+\frac{\nu}{2}+\frac{3}{4}\right)\pi+\delta}$$

$$\cot\left(\left(n+\frac{1}{2}\right)\pi+\delta\right)=\frac{4\nu^2-1}{8}\cdot\frac{1}{\left(n+\frac{\nu}{2}+\frac{3}{4}\right)\pi}\cdot\frac{1}{1+\frac{\delta}{\left(n+\frac{\nu}{2}+\frac{3}{4}\right)\pi}}$$

$$\frac{\overset{0}{\cos}\left(\left(n+\frac{1}{2}\right)\pi\right)\cos(\delta)-\sin\left(\left(n+\frac{1}{2}\right)\pi\right)\sin\delta}{\sin\left(\left(n+\frac{1}{2}\right)\pi\right)\cos(\delta)+\underset{0}{\cos}\left(\left(n+\frac{1}{2}\right)\pi\right)\sin\delta}=\frac{4\nu^2-1}{8\left(n+\frac{\nu}{2}+\frac{3}{4}\right)\pi}\cdot\frac{1}{1+\frac{\delta}{\left(n+\frac{\nu}{2}+\frac{3}{4}\right)\pi}}$$

$$-\tan(\delta)=\frac{4\nu^2-1}{8\left(n+\frac{\nu}{2}+\frac{3}{4}\right)\pi}\cdot\frac{1}{1+\frac{\delta}{\left(n+\frac{\nu}{2}+\frac{3}{4}\right)\pi}}$$

$$f(x)=\frac{1}{1+x}\sim1-x$$

$$\frac{df}{dx}=-\frac{1}{(1+x)^2}\xrightarrow[x\to0]{}-1$$

$$-\tan(\delta)=\frac{4\nu^2-1}{8\left(n+\frac{\nu}{2}+\frac{3}{4}\right)\pi}\cdot\left(1-\frac{\delta}{\left(n+\frac{\nu}{2}+\frac{3}{4}\right)\pi}\right)$$

$$g(\delta)=\tan\delta\sim\delta$$

$$-\delta=\frac{4\nu^2-1}{8\left(n+\frac{\nu}{2}+\frac{3}{4}\right)\pi}-\frac{\delta}{8\left(n+\frac{\nu}{2}+\frac{3}{4}\right)^2\pi^2}$$

$$\left(\frac{1}{8\left(n+\frac{\nu}{2}\right)^2\pi^2}-1\right)\delta=\frac{4\nu^2-1}{8\left(n+\frac{\nu}{2}+\frac{3}{4}\right)\pi}$$

$$\frac{1-8\left(n+\frac{\nu}{2}+\frac{3}{4}\right)^2\pi^2}{8\left(n+\frac{\nu}{2}+\frac{3}{4}\right)^2\pi^2}$$

$$\delta=\frac{4\nu^2-1}{1-8\left(n+\frac{\nu}{2}+\frac{3}{4}\right)^2\pi^2}\cdot\left(n+\frac{\nu}{2}+\frac{3}{4}\right)^2\pi^2$$

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$$\delta = \frac{4\nu^2-1}{1-8\left(n+\frac{\nu}{2}+\frac{3}{4}\right)^2\pi^2}\cdot\left(n+\frac{\nu}{2}+\frac{3}{4}\right)^2\pi^2$$

$$x(\nu) = \left(n+\frac{\nu}{2}+\frac{3}{4}\right)\pi + (4\nu^2-1)\cdot\frac{\left(n+\frac{\nu}{2}+\frac{3}{4}\right)^2\pi^2}{1-8\left(n+\frac{\nu}{2}+\frac{3}{4}\right)^2\pi^2}$$

$$x(0) = \left(n+\frac{3}{4}\right)\pi + \frac{\left(n+\frac{3}{4}\right)^2\pi^2}{1-8\left(n+\frac{3}{4}\right)^2\pi^2}$$

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$$-\delta = \frac{4\nu^2-1}{8\left(n+\frac{\nu}{2}+\frac{3}{4}\right)\pi}, \quad \delta = \frac{1-4\nu^2}{8\left(n+\frac{\nu}{2}+\frac{3}{4}\right)\pi}$$

$$x(\nu) = \left(n+\frac{\nu}{2}+\frac{3}{4}\right)\pi + \frac{1-4\nu^2}{8\left(n+\frac{\nu}{2}\right)+\frac{3}{4})\pi}$$

$$= \left(n+\frac{\nu}{2}+\frac{3}{4}\right)\pi + \frac{1-4\nu^2}{2\pi(4n+2\nu+3)}$$

$$x(0) = \left(n+\frac{3}{4}\right)\pi + \frac{1}{8\left(n+\frac{3}{4}\right)\pi} = \left(n+\frac{3}{4}\right)\pi + \frac{1}{(4n+3)2\pi}$$

$$x(1) = \left(n+\frac{5}{4}\right)\pi - \frac{3}{2\pi(4n+5)}$$

$$x(3) = \left(n+\frac{9}{4}\right)\pi - \frac{35}{2\pi(4n+9)}$$

$$x(5) = \left(n+\frac{13}{4}\right)\pi - \frac{99}{2\pi(4n+13)}$$

問題3

(1) 式(4)の特性方程式は、

$$\lambda^2+2\varepsilon\lambda+1=0.$$ これをといて

$$\lambda=-\varepsilon\pm i\sqrt{1-\varepsilon^2}$$

よって厳密解は

$$u(t)=Ae^{(-\varepsilon+i\sqrt{1-\varepsilon^2})t}+Be^{(-\varepsilon-i\sqrt{1-\varepsilon^2})t}$$

$$=e^{-\varepsilon t}\left(A'\cos(\sqrt{1-\varepsilon^2}\,t)+B'\sin(\sqrt{1-\varepsilon^2}\,t)\right)$$

$$\frac{du}{dt}=-\varepsilon e^{-\varepsilon t}\left(A'\cos(\sqrt{1-\varepsilon^2}\,t)+B'\sin(\sqrt{1-\varepsilon^2}\,t)\right)$$
$$+e^{-\varepsilon t}\left(-\sqrt{1-\varepsilon^2}A'\sin(\sqrt{1-\varepsilon^2}\,t)+\sqrt{1-\varepsilon^2}B'\cos(\sqrt{1-\varepsilon^2}\,t)\right)$$

$$=e^{-\varepsilon t}\left(-\varepsilon A'\cos(\alpha t)-\varepsilon B'\sin(\alpha t)-\sqrt{1-\varepsilon^2}A'\sin(\alpha t)+\sqrt{1-\varepsilon^2}B'\cos(\alpha t)\right)$$

$$=e^{-\varepsilon t}\left((-\varepsilon A'+\sqrt{1-\varepsilon^2}B')\cos(\alpha t)-(\sqrt{1-\varepsilon^2}A'+\varepsilon B')\sin(\alpha t)\right)$$

$$u(0)=A'=1,\quad \left.\frac{du}{dt}\right|_{t=0}=-\varepsilon A'+\sqrt{1-\varepsilon^2}B'=0$$

$$B'=\frac{\varepsilon}{\sqrt{1-\varepsilon^2}}$$

$$\therefore u(t)=e^{-\varepsilon t}\left(\cos(\sqrt{1-\varepsilon^2}\,t)+\frac{\varepsilon}{\sqrt{1-\varepsilon^2}}\sin(\sqrt{1-\varepsilon^2}\,t)\right)$$

$$f(x)=\cos(\sqrt{1-\varepsilon^2}\,x)\sim 1-\frac{1-\varepsilon^2}{2}x^2$$

$$\frac{df}{dx}=-\sqrt{1-\varepsilon^2}\sin(\sqrt{1-\varepsilon^2}\,x)\xrightarrow[x\to0]{}0$$

$$\frac{d^2f}{dx^2}=-(1-\varepsilon^2)\cos(\sqrt{1-\varepsilon^2}\,x)\xrightarrow[x\to0]{}-(1-\varepsilon^2)$$

$$g(x)=\sin(\sqrt{1-\varepsilon^2}\,x)\sim\sqrt{1-\varepsilon^2}\,x$$

$$\frac{dg}{dx}=\sqrt{1-\varepsilon^2}\cos(\sqrt{1-\varepsilon^2}\,x)\xrightarrow[x\to0]{}\sqrt{1-\varepsilon^2}$$

$$\frac{d^2g}{dx^2}=-(1-\varepsilon^2)\sin(\sqrt{1-\varepsilon^2}\,x)\xrightarrow[x\to0]{}0$$

$$h(x)=e^{-\varepsilon x}\sim 1-\varepsilon x+\frac{\varepsilon^2}{2}x^2$$

$$\frac{dh}{dx}=-\varepsilon e^{-\varepsilon x}$$

$$\frac{d^2h}{dx^2}=\varepsilon^2e^{-\varepsilon x}$$

$$u(t)\sim\left(1-\varepsilon t+\frac{\varepsilon^2}{2}t^2\right)\left(1-\frac{1-\varepsilon^2}{2}t^2+\frac{\varepsilon}{\sqrt{1-\varepsilon^2}}\times\sqrt{1-\varepsilon^2}\,t\right)$$

$$=\left(1-\varepsilon t+\frac{\varepsilon^2}{2}t^2\right)\left(1-\frac{1}{2}t^2+\varepsilon t+\frac{\varepsilon^2}{2}t^2\right)$$

$$=1-\frac{1}{2}t^2-\varepsilon t\left(1-\frac{1}{2}t^2\right)+\frac{\varepsilon^2}{2}t^2\left(1-\frac{1}{2}t^2\right)+\varepsilon t-\varepsilon t(\varepsilon t)+\frac{\varepsilon^2}{2}t^2$$

$$=1-\frac{1}{2}t^2-\cancel{\varepsilon t}+\frac{\varepsilon}{2}t^3+\cancel{\frac{\varepsilon^2}{2}t^2}-\frac{\varepsilon^2}{4}t^4+\cancel{\varepsilon t}-\cancel{\varepsilon^2t^2}+\cancel{\frac{\varepsilon^2}{2}t^2}$$

$$=1-\frac{1}{2}t^2+\frac{\varepsilon}{2}t^3-\frac{\varepsilon^2}{4}t^4$$

(2)

関数$f(t)$に対し、$\dfrac{d^k f}{dt^k} := f^{\overbrace{\prime\cdots\prime}^{k}}(t)$と表す。

$u(t) = u_0(t) + \varepsilon u_1(t) + \varepsilon^2 u_2(t) + \cdots$ を (4)式に代入すると、

$$\left(u_0''(t) + \varepsilon u_1''(t) + \varepsilon^2 u_2''(t) + \cdots\right) + 2\varepsilon\left(u_0'(t) + \varepsilon u_1'(t) + \varepsilon^2 u_2'(t)\right)$$

$$+\left(u_0(t) + \varepsilon u_1(t) + \varepsilon^2 u_2(t)\right) + O(\varepsilon^3) = 0$$

$$\left(u_0''(t) + u_0(t)\right) + \left(u_1''(t) + 2u_0'(t) + u_1(t)\right)\varepsilon + \left(u_2''(t) + 2u_1'(t) + u_2(t)\right)\varepsilon^2 + O(\varepsilon^3) = 0$$

$$O(1): \left.\begin{aligned} &u_0'' + u_0 = 0 \\ &u_0(0) = 1, \ \left.\frac{du_0}{dt}\right|_{t=0} = 0 \end{aligned}\right\} ①$$

$$O(\varepsilon): \left.\begin{aligned} &u_1'' + u_1 = -2u_0' \\ &u_1(0) = 0, \ \left.\frac{du_1}{dt}\right|_{t=0} = 0 \end{aligned}\right\} ②$$

$$O(\varepsilon^2): \left.\begin{aligned} &u_2'' + u_2 = -2u_1' \\ &u_2(0) = 0, \ \left.\frac{du_2}{dt}\right|_{t=0} = 0 \end{aligned}\right\} ③$$

①をとく。

$$u_0(t) = a\cos(t) + b\sin(t)$$

$$\frac{du_0}{dt} = -a\sin(t) + b\cos(t)$$

$$u_0(0) = a = 1, \quad \left.\frac{du_0}{dt}\right|_{t=0} = b = 0$$

$$\therefore u_0(t) = \cos(t), \quad \frac{du_0}{dt} = -\sin(t)$$

②をとく。 $u_1'' + u_1 = 2\sin(t)$

$-\alpha\sin(t)$ t

同次形の解 $: u_1(t) = a\cos(t) + b\sin(t)$

特解 $: u_1(t) = \alpha t\cos(t)$

$$\frac{du_1}{dt} = \alpha\cos(t) - \alpha t\sin(t)$$

$$\frac{d^2u_1}{dt^2} = -\alpha\sin(t) - \alpha\sin(t) - \alpha t\cos(t) = -2\alpha\sin(t) - \alpha t\cos(t)$$

この式を、②に代入すると

$$-2\alpha\sin(t) - \alpha t\cos(t) + \alpha t\cos(t) = 2\sin(t) \longrightarrow \alpha = -1$$

$$\therefore u_1(t) = -t\cos(t)$$

一般解：$u_1(t) = a\cos(t) + b\sin(t) - \quad t\cos(t)$

$$\frac{du_1}{dt} = -a\sin(t) + b\cos(t) - \quad \cos(t) + \quad t\sin(t)$$

$$= -a\sin(t) + (b - 1)\cos(t) + \quad t\sin(t)$$

$$u_1(0) = a = 0, \quad \left.\frac{du_1}{dt}\right|_{t=0} = b - 1 = 0 \rightarrow b = 1$$

$$\therefore u_1(t) = \quad \sin(t) - \quad t\cos(t)$$

③をとく. $u_2'' + u_2 = 2t\cos(t) - 2\sin(t)$

同次形の解：$u_2(t) = a\cos(t) + b\sin(t)$

特解：$u_2(t) = \alpha t^2\sin(t) + \beta t\cos(t)$

$$\frac{du_2}{dt} = 2\alpha t\sin(t) + \alpha t^2\cos(t) + \beta\cos(t) - \beta t\sin(t)$$

$$= (2\alpha - \beta)t\sin(t) + \alpha t^2\cos(t) + \beta\cos(t)$$

$$\frac{d^2u_2}{dt^2} = (2\alpha - \beta)(\sin(t) + t\cos(t)) + \alpha(2t\cos(t) - t^2\sin(t)) - \beta\sin(t)$$

$$= (2\alpha - 2\beta)\sin(t) + (4\alpha - \beta)t\cos(t) - \alpha t^2\sin(t)$$

$$(2\alpha - 2\beta)\sin(t) + (4\alpha - \beta)t\cos(t) - \cancel{\alpha t^2\sin(t)} + \cancel{\alpha t^2\sin(t)} + \beta t\cos(t)$$

$$= 2t\cos(t) - 2\sin(t)$$

$$\begin{cases} 2\alpha - 2\beta = -2 \\ 4\alpha = 2 \end{cases} \Rightarrow \begin{cases} \alpha = \frac{1}{2} \\ \beta = \frac{3}{2} \end{cases} \quad \therefore u_2(t) = \frac{1}{2}t^2\sin(t) + \frac{3}{2}t\cos(t)$$

一般解：$u_2(t) = a\cos(t) + b\sin(t) + \frac{1}{2}t^2\sin(t) + \frac{3}{2}t\cos(t)$

$$\frac{du_2}{dt} = -a\sin(t) + b\cos(t) + t\sin(t) + \frac{1}{2}t^2\cos(t) + \frac{3}{2}\cos(t) - \frac{3}{2}t\sin(t)$$

$$u_2(0) = a = 0, \quad \left.\frac{du_2}{dt}\right|_{t=0} = b + \frac{3}{2} = 0 \rightarrow b = -\frac{3}{2}$$

$$\therefore u_2(t) = -\frac{3}{2}\sin(t) + \frac{1}{2}t^2\sin(t) + \frac{3}{2}t\cos(t)$$

$$\therefore u(t) = \cos(t) + (\sin(t) - t\cos(t))\varepsilon + \left(-\frac{3}{2}\sin(t) + \frac{1}{2}t^2\sin(t) + \frac{3}{2}t\cos(t)\right)\varepsilon^2 + O(\varepsilon^3)$$

(3) u を ε^2 のオーダーまで求める。

$$\frac{d}{dt}=\frac{\partial}{\partial t_0}\frac{dt_0}{dt}+\frac{\partial}{\partial t_1}\frac{dt_1}{dt}+\frac{\partial}{\partial t_2}\frac{dt_2}{dt}=\frac{\partial}{\partial t_0}+\varepsilon\frac{\partial}{\partial t_1}+\varepsilon^2\frac{\partial}{\partial t_2}$$

$$\frac{d^2}{dt^2}=\left(\frac{\partial}{\partial t_0}+\varepsilon\frac{\partial}{\partial t_1}+\varepsilon^2\frac{\partial}{\partial t_2}\right)\left(\frac{\partial}{\partial t_0}+\varepsilon\frac{\partial}{\partial t_1}+\varepsilon^2\frac{\partial}{\partial t_2}\right)$$

$$=\frac{\partial^2}{\partial t_0^2}+\varepsilon\frac{\partial^2}{\partial t_0\partial t_1}+\varepsilon^2\frac{\partial^2}{\partial t_0\partial t_2}+\varepsilon\frac{\partial^2}{\partial t_1\partial t_0}+\varepsilon^2\frac{\partial^2}{\partial t_1^2}+\varepsilon^3\frac{\partial^2}{\partial t_1\partial t_2}$$

$$+\varepsilon^2\frac{\partial^2}{\partial t_2\partial t_0}+\varepsilon^3\frac{\partial^2}{\partial t_2\partial t_1}+\varepsilon^4\frac{\partial^2}{\partial t_2^2}$$

$$=\frac{\partial^2}{\partial t_0^2}+\varepsilon\, 2\frac{\partial^2}{\partial t_0\partial t_1}+\varepsilon^2\left(\frac{\partial^2}{\partial t_1^2}+2\frac{\partial^2}{\partial t_0\partial t_2}\right)+\varepsilon^3\cdot 2\frac{\partial^2}{\partial t_1\partial t_2}+\varepsilon^4\frac{\partial^2}{\partial t_2^2}$$

上式を(4)式に代入

$$\frac{d^2}{dt^2}(u_0+\varepsilon u_1+\varepsilon^2u_2+\cdots)+2\varepsilon\frac{d}{dt}(u_0+\varepsilon u_1+\varepsilon^2u_2+\cdots)+(u_0+\varepsilon u_1+\varepsilon^2u_2+\cdots)=0$$

$$\Big\{\left(\frac{\partial^2u_0}{\partial t_0^2}+2\varepsilon\frac{\partial^2u_0}{\partial t_0\partial t_1}+\varepsilon^2\left(\frac{\partial^2u_0}{\partial t_1^2}+2\frac{\partial^2u_0}{\partial t_0\partial t_2}\right)+2\varepsilon^3\frac{\partial^2u_0}{\partial t_1\partial t_2}+\varepsilon^4\frac{\partial^2}{\partial t_2^2}\right)$$

$$+\varepsilon\left(\frac{\partial^2u_1}{\partial t_0^2}+2\varepsilon\frac{\partial^2u_1}{\partial t_0\partial t_1}+\varepsilon^2\left(\frac{\partial^2u_1}{\partial t_1^2}+2\frac{\partial^2u_0}{\partial t_0\partial t_2}\right)+2\varepsilon^3\frac{\partial^2u_1}{\partial t_1\partial t_2}+\varepsilon^4\frac{\partial^2u_1}{\partial t_2^2}\right)$$

$$+\varepsilon^2\left(\frac{\partial^2u_2}{\partial t_0^2}+2\varepsilon\frac{\partial^2u_2}{\partial t_0\partial t_1}+\varepsilon^2\left(\frac{\partial^2u_2}{\partial t_1^2}+2\frac{\partial^2u_0}{\partial t_0\partial t_2}\right)+2\varepsilon^3\frac{\partial^2u_2}{\partial t_1\partial t_2}+\varepsilon^4\frac{\partial^2u_2}{\partial t_2^2}\right)\Big\}$$

$$+2\varepsilon\Big\{\left(\frac{\partial u_0}{\partial t_0}+\varepsilon\frac{\partial u_0}{\partial t_1}+\varepsilon^2\frac{\partial u_0}{\partial t_2}\right)+\varepsilon\left(\frac{\partial u_1}{\partial t_0}+\varepsilon\frac{\partial u_1}{\partial t_1}+\varepsilon^2\frac{\partial u_1}{\partial t_2}\right)+\varepsilon^2\left(\frac{\partial u_2}{\partial t_0}+\varepsilon\frac{\partial u_2}{\partial t_1}+\varepsilon^2\frac{\partial u_2}{\partial t_2}\right)\Big\}$$

$$+(u_0+\varepsilon u_1+\varepsilon^2u_2+\cdots)+O(\varepsilon^4)=0$$

$$\Big\{\left(\frac{\partial^2u_0}{\partial t_0^2}+2\varepsilon\frac{\partial^2u_0}{\partial t_0\partial t_1}+\varepsilon^2\left(\frac{\partial^2u_0}{\partial t_1^2}+2\frac{\partial^2u_0}{\partial t_0\partial t_2}\right)+\cancel{2\varepsilon^3\frac{\partial^2u_0}{\partial t_1\partial t_2}}\right)+\left(\varepsilon\frac{\partial^2u_1}{\partial t_0^2}+2\varepsilon^2\frac{\partial^2u_1}{\partial t_0\partial t_1}\right.$$

$$\left.\left.+\varepsilon^3\cancel{\left(\frac{\partial^2u_1}{\partial t_1^2}+2\frac{\partial^2u_0}{\partial t_0\partial t_2}\right)}\right)\right)+\left(\varepsilon^2\frac{\partial^2u_2}{\partial t_0^2}+\cancel{2\varepsilon^3\frac{\partial^2u_2}{\partial t_0\partial t_1}}\right)\Big\}+\Big\{\left(2\varepsilon\frac{\partial u_0}{\partial t_0}+2\varepsilon^2\frac{\partial u_0}{\partial t_1}+\cancel{2\varepsilon^3\frac{\partial u_0}{\partial t_2}}\right)$$

$$+\left(2\varepsilon^2\frac{\partial u_1}{\partial t_0}+\cancel{2\varepsilon^3\frac{\partial u_1}{\partial t_1}}\right)+\cancel{2\varepsilon^3\frac{\partial u_2}{\partial t_0}}\Big\}+(u_0+\varepsilon u_1+\varepsilon^2u_2+\cdots)+O(\varepsilon^3)=0$$

$$\left.\frac{du}{dt}\right|_{t=0}=\left(\frac{\partial}{\partial t_0}+\varepsilon\frac{\partial}{\partial t_1}+\varepsilon^2\frac{\partial}{\partial t_2}\right)(u_0+\varepsilon u_1+\varepsilon^2u_2+\cdots)\Big|_{(t_0,t_1,t_2)=(0,0,0)}$$

$$=\frac{\partial u_0}{\partial t_0}+\varepsilon\left(\frac{\partial u_1}{\partial t_0}+\frac{\partial u_0}{\partial t_1}\right)+\varepsilon^2\left(\frac{\partial u_0}{\partial t_2}+\frac{\partial u_1}{\partial t_1}+\frac{\partial u_2}{\partial t_0}\right)+O(\varepsilon^3)\Big|_{(0,0,0)}=0$$

$$O(1):\ \left.\begin{aligned}&\frac{\partial^2u_0}{\partial t_0^2}+u_0=0\\&u_0(0,0,0)=1,\ \left.\frac{\partial u_0}{\partial t_0}\right|_{(0,0,0)}=0\end{aligned}\right)\ (4)$$

$$O(\varepsilon):\ \left.\begin{aligned}&\frac{\partial^2u_1}{\partial t_0^2}+u_1=-2\frac{\partial^2u_0}{\partial t_0\partial t_1}-2\frac{\partial u_0}{\partial t_0}\\&u_1(0,0,0)=0,\ \left.\frac{\partial u_1}{\partial t_0}+\frac{\partial u_0}{\partial t_1}\right|_{(0,0,0)}=0\end{aligned}\right)\ (5)$$

$$O(\varepsilon^2):\ \left.\begin{aligned}&\frac{\partial^2u_2}{\partial t_0^2}+u_2=-\frac{\partial^2u_0}{\partial t_1^2}-2\frac{\partial^2u_0}{\partial t_0\partial t_2}-2\frac{\partial u_0}{\partial t_1}-2\frac{\partial u_1}{\partial t_0}-2\frac{\partial^2u_1}{\partial t_0\partial t_1}\\&u_2(0,0,0)=0,\ \left.\frac{\partial u_0}{\partial t_2}+\frac{\partial u_1}{\partial t_1}+\frac{\partial u_2}{\partial t_0}\right|_{(0,0,0)}=0\end{aligned}\right)\ (6)$$

④をとく.

$$\frac{\partial^2 u_0}{\partial t_0^2}+u_0=0$$

$$u_0(t_0,t_1,t_2)=A_{02}(t_2)A_{01}(t_1)e^{it_0}+B_{02}(t_2)B_{01}(t_1)e^{-it_0}$$

$$\Rightarrow\left\{\begin{aligned}\frac{\partial u_0}{\partial t_0}&=iA_{02}(t_2)A_{01}(t_1)e^{it_0}-iB_{02}(t_2)B_{01}(t_1)e^{-it_0}\\ \frac{\partial^2 u_0}{\partial t_0\partial t_1}&=iA_{02}(t_2)\frac{dA_{01}}{dt_1}e^{it_0}-iB_{02}(t_2)\frac{dB_{01}}{dt_1}e^{-it_0}\end{aligned}\right.$$

上式を⑤に代入.

$$\begin{aligned}\frac{\partial^2 u_1}{\partial t_0^2}+u_1&=-2iA_{02}(t_2)\frac{dA_{01}}{dt_1}e^{it_0}+2iB_{02}(t_2)\frac{dB_{01}}{dt_1}e^{-it_0}\\ &\quad-2iA_{02}A_{01}e^{it_0}+2iB_{02}(t_2)B_{01}(t_1)e^{-it_0}\\ &=-2iA_{02}\left(A_{01}+\frac{dA_{01}}{dt_1}\right)e^{it_0}+2iB_{02}\left(B_{01}+\frac{dB_{01}}{dt_1}\right)e^{-it_0}\end{aligned}$$

$A_{02}=0$ または $A_{01}=A_{01}(0)e^{-t_1}$ かつ $B_{02}=0$ または $B_{01}=B_{01}(0)e^{-t_1}$

$$\frac{\partial^2 u_1}{\partial t_0^2}+u_1=0 \quad ⑤$$

$$u_1(t_0,t_1,t_2)=A_{12}(t_2)A_{11}(t_1)e^{it_0}+B_{12}(t_2)B_{11}(t_1)e^{-it_0}$$

$$\frac{\partial^2 u_0}{\partial t_0^2}=-A_{02}A_{01}e^{it_0}-B_{02}B_{01}e^{-it_0},\quad \frac{\partial^2 u_0}{\partial t_0\partial t_2}=i\frac{dA_{02}}{dt_2}A_{01}e^{it_0}-i\frac{dB_{02}}{dt_2}B_{01}e^{-it_0}$$

$$\frac{\partial u_0}{\partial t_1}=A_{02}\frac{dA_{01}}{dt_1}e^{it_0}+B_{02}\frac{dB_{01}}{dt_1}e^{it_0},\quad \frac{\partial u_1}{\partial t_0}=iA_{12}A_{11}e^{it_0}-iB_{12}B_{11}e^{-it_0}$$

$$\frac{\partial^2 u_1}{\partial t_0\partial t_1}=iA_{12}\frac{dA_{11}}{dt_1}e^{it_0}-iB_{12}\frac{dB_{11}}{dt_1}e^{-it_0}$$

上式を⑥に代入

$$\frac{\partial^2 u_2}{\partial t_0^2}+u_2=$$

係数決定ができません(?)

$\lambda+1$

$$u_0(0,0,0)=A_{02}(0)A_{01}(0)+B_{02}(0)B_{01}(0)=1$$

$$\left.\frac{\partial u_0}{\partial t_0}\right|_{(0,0,0)}=A_{02}(0)\left.\frac{dA_{01}}{dt_1}\right|_{t_1=0}-B_{02}(0)\left.\frac{dB_{01}}{dt_1}\right|_{t_1=0}=0$$

$$\frac{B_{02}(0)\left.\frac{dB_{01}}{dt_1}\right|_{t_1=0}}{\left.\frac{dA_{01}}{dt_1}\right|_{t_1=0}}A_{01}(0)+B_{02}(0)B_{01}(0)=1$$

$$B_{02}(0)\left(\frac{A_{01}(0)}{\left.\frac{dA_{01}}{dt_1}\right|_{t_1=0}}\left.\frac{dB_{01}}{dt_1}\right|_{t_1=0}+B_{01}(0)\right)=1$$