

(1) 以下の論理式を \forall と \exists を使わない論理式にせよ.

$$S = \{a, b, c\}, s \in S, T = \{d, e\}, t \in T$$

$$(i) \forall s (P(s) \rightarrow Q(s))$$

$$\equiv (P(a) \rightarrow Q(a)) \wedge (P(b) \rightarrow Q(b)) \wedge (P(c) \rightarrow Q(c))$$

$$(ii) \exists s (P(s) \wedge Q(s))$$

$$\equiv (P(a) \wedge Q(a)) \vee (P(b) \wedge Q(b)) \vee (P(c) \wedge Q(c))$$

$$(iii) \forall s \exists t R(s, t)$$

$$\equiv \forall s (R(s, d) \vee R(s, e))$$

$$\equiv (R(a, d) \vee R(a, e)) \wedge (R(b, d) \vee R(b, e)) \wedge (R(c, d) \vee R(c, e))$$

2. 以下を形式的に証明せよ.

$$(i) \forall x (P(x) \rightarrow Q(x)) \wedge \neg \exists x Q(x) \rightarrow \forall x \neg P(x)$$

1. $\forall x (P(x) \rightarrow Q(x))$	P
2. $\neg \exists x Q(x)$	P
3. $P(x) \rightarrow Q(x)$	1, VI
4. $\forall x \neg Q(x)$	2, T (ト"エルカ"ン)
5. $\neg Q(x)$	4, VI
6. $\neg P(x)$	3, 5, MT
7. $\forall x \neg P(x)$	6, VG
QED	1-7, CP

$$(ii) \neg Q(c) \wedge (\exists x P(x) \rightarrow \forall y Q(y)) \rightarrow \forall x \neg P(x)$$

1. $\neg Q(c)$	P
2. $\exists x P(x) \rightarrow \forall y Q(y)$	P
3. $\exists x P(x)$	P [$\exists x P(x) \rightarrow \forall y Q(y)$ 仮定]
4. $\forall y Q(y)$	2, 3, MP
5. $Q(c)$	4, VI
6. False	1, 5, Contr
7. $\neg \exists x P(x)$	3-6, IP
8. $\forall x \neg P(x)$	7, T (ト"エルカ"ン)
QED	1-8, CP

$$(1) \neg \exists x \exists y (R(x, y) \wedge \neg P(x, y)) \wedge \forall x \forall y (\neg R(x, y) \rightarrow P(x, y)) \\ \longrightarrow \exists x \exists y P(x, y)$$

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|----------------------------------------------------------------|---------------|
| 1. $\neg \exists x \exists y (R(x, y) \wedge \neg P(x, y))$ | P |
| 2. $\forall x \forall y (\neg R(x, y) \rightarrow P(x, y))$ | P |
| 3. $\forall x \neg \exists y (R(x, y) \wedge \neg P(x, y))$ | 1, T(ドモルガソ) |
| 4. $\forall x \forall y \neg (R(x, y) \wedge \neg P(x, y))$ | 3, T(ドモルガソ) |
| 5. $\forall x \forall y (\neg R(x, y) \vee \neg \neg P(x, y))$ | 4, T(ドモルガソ) |
| 6. $\forall x \forall y (\neg R(x, y) \vee P(x, y))$ | 5, DN |
| 7. $\forall y (\neg R(x, y) \vee P(x, y))$ | 6, UI |
| 8. $\neg R(x, y) \vee P(x, y)$ | 7, UI |
| 9. $\forall y (\neg R(x, y) \rightarrow P(x, y))$ | 2, UI |
| 10. $\neg R(x, y) \rightarrow P(x, y)$ | 9, UI |
| 11. $\neg \exists x \exists y P(x, y)$ | P |
| 12. $\forall x \neg \exists y P(x, y)$ | 11, T(ドモルガソ) |
| 13. $\forall x \forall y \neg P(x, y)$ | 12, T(ドモルガソ) |
| 14. $\forall y \neg P(x, y)$ | 13, UI |
| 15. $\neg P(x, y)$ | 14, UI |
| 16. $\neg \neg R(x, y)$ | 10, 15, MT |
| 17. $P(x, y)$ | 8, 16, DS |
| 18. $\exists y P(x, y)$ | 17, EG |
| 19. $\exists x \exists y P(x, y)$ | 18, EG |
| 20. False | 16, 19, Contr |
| 21. $\exists x \exists y P(x, y)$ | 11-20, IP |
| QED | 1-21, CP |

3. 以下の論理式の階数を求めよ.

$$(i) S(x) \wedge T(S, x) \rightarrow U(T, S, x)$$

3階 //

$$(ii) \forall x \exists S (S(x) \wedge T(S, x) \rightarrow U(T, S, x))$$

5階 //

$$4. (\forall x \neg P(x, x)) \wedge (\forall x \forall y \forall z (P(x, y) \wedge P(y, z) \rightarrow P(x, z))) \\ \rightarrow \forall x \forall y (P(x, y) \rightarrow \neg P(y, x))$$

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|---------------------------------------------------------------------------------|------------------------------------------------|
| 1. $\forall x \neg P(x, x)$ | P |
| 2. $\forall x \forall y \forall z (P(x, y) \wedge P(y, z) \rightarrow P(x, z))$ | P |
| 3. $\neg P(x, x)$ | 1, UI |
| 4. $P(x, y) \wedge P(y, x) \rightarrow P(x, x)$ | 2, UI |
| 5. $\neg (P(x, y) \wedge P(y, x))$ | 3, 4, MT |
| 6. $\neg P(x, y) \vee \neg P(y, x)$ | 5, T ("E/LP") |
| 7. $P(x, y)$ | $P[P(x, y) \rightarrow \neg P(y, x)]$
["I"] |
| 8. $\neg \neg P(x, y)$ | 7, DN |
| 9. $\neg P(y, x)$ | 6, 8, DS |
| 10. $P(x, y) \rightarrow \neg P(y, x)$ | 7-9, CP |
| 11. $\forall x \forall y (P(x, y) \rightarrow \neg P(y, x))$ | 10, VG |
| 12. QED | 1-11, CP |