Physics II Class Notes

Professor: Kunal Ghosh, Compiled: Skanda Kaashyap

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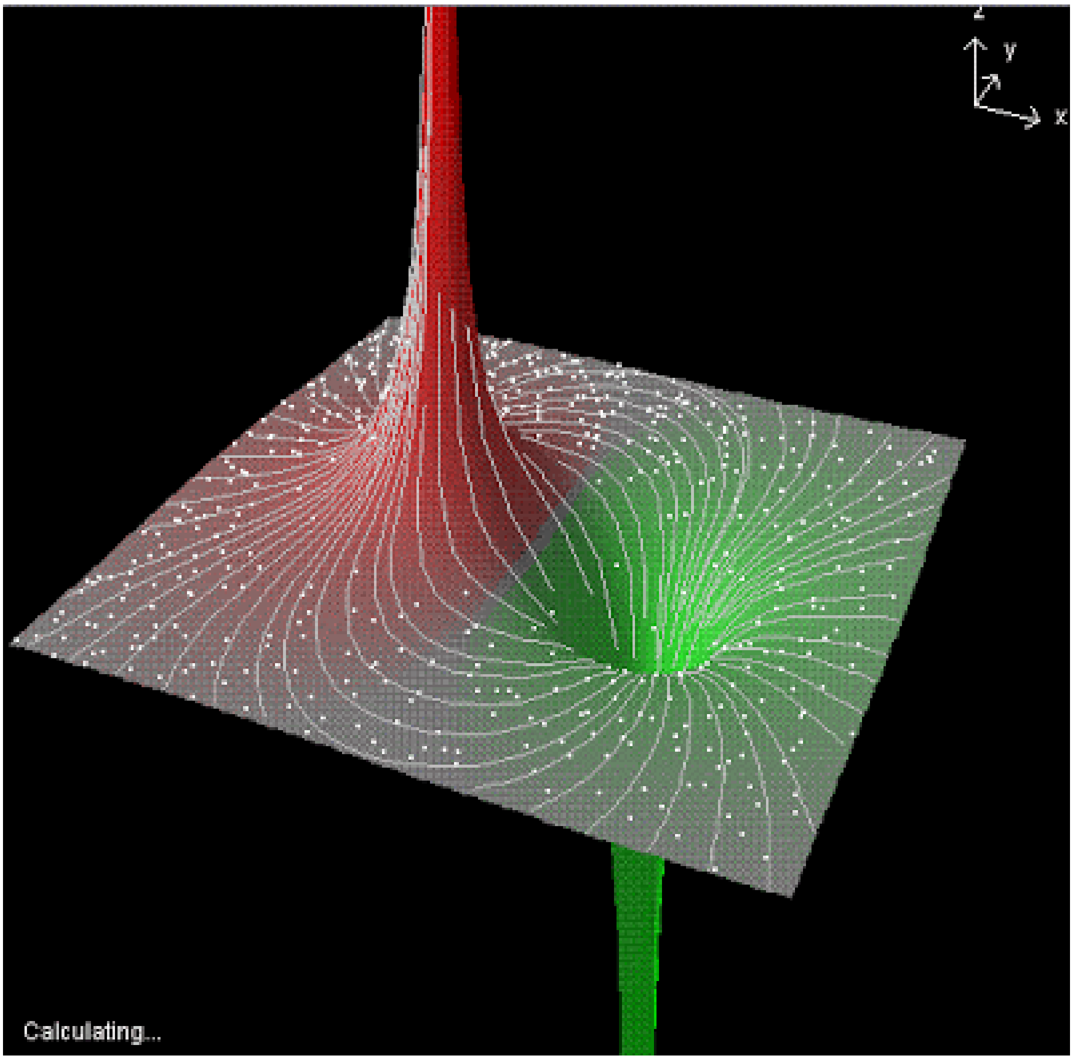
Chapter 1

Electric Field

1.1 What is the electric field?

On 5/21/19, we discussed the electric field. The electric field is a bit more abstract than some of the other concepts that we've dealt with so far; that is to say, it's not something that you can directly feel, like an electric force. It's often useful to think about the electric field the same way we often thinka about the gravitational field: like a trampoline. Imagine negative charges pushing down on the surface of the trampoline and positive charges pulling up. This is a helpful visual.



1.2 How to Compute the Electric Field

The electric field is given by

$$\vec{E} = \frac{kQ}{|r|^2}\hat{r},$$

where $\hat{r}$ points from the charge inducing the field to the location at which you would like to measure the field. You will no doubt notice its striking resemblance to the Coulomb force, which is given by

$$\vec{F}_{12} = \frac{kQq}{|r_{21}|^2}\hat{r}_{21} \tag{1.1}$$

In fact, the E field is really $\vec{E} = \vec{F}/q$, which can be interpreted as "force per unit charge." One way to think about it is that in the same way that volume density is mass per unit volume, this is, in some sense, a "force density." It describes how the space around the object itself changes; it describes a *field.*

1.3 Finding $\vec{E}$ field Around Different Objects

In class, we discussed obtaining the electric field around various different objects. Try to see if you can find the electric field expression for the following objects with constant linear charge density.

- Straight line
- Ring
- Disk
- Sphere

Try to also see if you can obtain the near and far field limits for each of these objects.

1.3.1 Solving Technique

Here we will outline how to go about solving the E field for a straight line of charge with constant linear charge density. The problem solving method does not change for other geometries, only the mechanics. That being said, it's still useful to try those, since you might be expected to know how to compute different types of integrals, including ones with trig substitutions.

Solution. **What do we know?**

So far, we know that the E field expression for a point charge is given by $\vec{E} = (kq/r^2)\hat{r}$.

But we don't have a point charge! But since we don't have a point charge here, we can break up the line into an infinite number of point charges. We can compute the E field $d\vec{E}$ for a single charge differential dQ and sum them up to get our field for the line.

The integral becomes

$$d\vec{E} = \frac{kdQ}{|r|^2}\hat{r}$$
$$\Rightarrow \vec{E} = \int \frac{kdQ}{|r|^2}\hat{r}$$

Often times, dQ can be expressed as a density multiplied by a length differential, like $dQ = \lambda dx$ in this case. ■

1.4 Natural E-Field Calculations

In this section, I want to briefly discuss how calculating the $\vec{E}$ field expression for a uniform charge is a natural starting point to ultimately perform an integral to obtain the $\vec{E}$ field for a full, 3D object, assuming that we don't want to use Gauss's Law.

We calculated the $\vec{E}$ field for a line of charge by performing the following integral:

$$\vec{E} = \int \frac{\lambda k}{|r|^2}\hat{r}dx.$$

Now, if we imagine a singular "rod" and turning contorting it into a circle, then we have a ring. The geometry changes, but you are still, at the of the day, taking a line integral, so it will look of the form above. Add a bunch of rings of varying sizes and you get a disk; add a bunch of disks of the same size, you get a cylinder. Add a bunch of disks of *varying* sizes, you get a sphere.

My point here is that even though the geometries of the objects are changing, the mechanics of performing an integral stays the same and the fundamental property that performing an integral allows you to perceive higher dimensions also holds. This means that a ring of charge is just a line of charge, and adding a bunch of points (0 dimensions) gets you a 1 dimensional object (a line), adding a bunch of lines (1 dimension) gets you a plane, adding a bunch of rings gets you a disk, etc.

Chapter 2

Gauss's Law

2.1 Definition

Gauss's Law is defined by

$$\oint_A \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\epsilon_0} \tag{2.1}$$

2.2 Meaning

All Gauss's Law is saying is the following: "If we take the portion of the $\vec{E}$ field that is normal to the surface and we sum it up across the area of the surface, it will be proportional to the charge enclosed." Let's construct a surface to trap the $\vec{E}$ field vectors for the charge that *we're interested in* and sum them up. The number of $\vec{E}$ field vectors we trap should be proportional to the magnitude of the thing causing those vectors in the first place.

2.3 Application

This is a really simple and elegant statement because it reduces a lot of complexity in the system. Using Gauss's law, we only have to worry about 3 things:

1. Charge enclosed
2. Gaussian surface
3. Magnitude of charge, direction of $\vec{E}$ field vectors.

Once we have these 3 things, it's just the matter of computing a surface integral. I think a great example and practice problem for this was the special problem in homework 3. It forced you to think hard about what your Gaussian surface would look like. You had to draw the Gaussian surface not at the exact location of the shell, but a little bit above so you could encapsulate the information that the inner part of the shell was telling you. Otherwise, you would be given information about what was going on in the rod. Remember, Gauss's law only tells you about the *charge enclosed* within the Gaussian surface.

Chapter 3

Class Notes

3.1 Notes 5/22/19

QABS = Question Asked By Student

- Recall, the force of particle 2 on 1 is given by $F_{1,2}$.
- Question: what are the units of the electric field? Answer: Well, what would happen if you had multiple charges?
- How do we solve the uniformly charged rod?
- QABS: How do we visualize the tiny charge differentials?
- QABS: What is dr?
- QABS: How do we find charge density? Ans:

  $$dQ = \frac{dy}{L} Q_0,$$

  where Q_0 is the total charge.

  Solution. The rule is always the same. We want to find the sum

  $$\vec{F} = \sum_i \vec{E}_i q_i.$$

  Recall, the electric field is a *vector.*

  That being said, let's either guess or predict the direction of the electric field/force.

  So we can split things up into the positive and negative directions.

  We found the E field differential. After we find the E field differential, we want to integrate over the whole thing. We want to find the total E field.

  Important things to keep an eye out basically all the time:

  - Look at limits.
  - Look for symmetries.
  - You can always pull out constants.

Basically, when we have these squared looking integrals, we want to see if we can rewrite these in terms of trigonometric identities.

Ask yourself: What trig substitutions have you seen in class?

Things to check after solving a problem:

1. Does it make sense? Are the vectors pointing in the right directions? Do we have scalars when we should have vectors? Is it in roughly the same order of magnitude?
2. Are the dimensions correct?
3. Check close field and far field limits.

■

3.2 Notes 5/24/19

- Solve the electric field for a hollowed out disk. Solution: just find the E field for big disk, small disk, then subtract. Again, this uses the *superposition principle.*
- QABS:
- It is very important to recall the **Taylor expansion relation** $(1+\alpha)^\beta = 1 + \alpha\beta$ as long as $\alpha << 1$.
- In order to find close and far field limits, if we have denominators like $(a+b)^n$, we can always reduce to obtain an expression of the form $(1+\alpha)^\beta$.
- Question to consider: In order to show that the $\vec{E}$ field inside a spherical shell is zero, can we imagine a ring, rotate it around its axis, and then integrate?
- Summary of what we did today:
  - We solved the disk with hole.
  - We solved the near and far field limits of the disk. In so doing, we introduced the binomial theorem/Taylor expansion of $(1+\alpha)^\beta$. How can we prove/derive $(1+\alpha)^\beta$ with the binomial theorem?
  - We discussed the electric field inside a hollow sphere and a outside a hollow sphere.
  - Discussed the motion and electric field inside a uniformly charged, nonconducting sphere. This is basically the motion inside of the earth.
  - What if we had an electric force that varied as $\frac{1}{r^3}$? More generally, what if it varied as $1/r^n$?
  - Introduced Gauss's Law.
  - If we construct a "Gaussian surface." If we perform the following calculation,

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\epsilon_0}.$$

3.3 Notes 5/28/19

- Starting/looking at Gauss's law: $\oint \vec{E} \cdot d\vec{A} = q_{enc}/\epsilon_0$.

- What would the electric field be around a rod that is not infinitely long?
- We also introduced the cylindrical coordinate system. The spherical coordinate system is characterized by ρ, ϕ, z.
- The conversion rate is given by

$$
\begin{aligned}
x &= \rho \cos \phi \\
y &= \rho \sin \phi \\
z &= z
\end{aligned}
$$

- The basis vectors are given by:

$$
\begin{aligned}
\hat{\rho} &= \cos \phi \hat{\imath} + \sin \phi \hat{\jmath} \\
\hat{\phi} &= -\sin \phi \hat{\imath} + \cos \phi \hat{\jmath} \\
\hat{z} &= \hat{z}
\end{aligned}
$$

- The generalized expression for the electric field in cylindrical coordinates is going to be:

$$\hat{E}_p = \hat{\rho} E_\rho(\rho, \phi, z) + \hat{\phi} E_\phi(\rho, \phi, z) + \hat{z} E_z(\rho, \phi, z).$$

- Use Coulomb's law, basis of electric attraction and repulision to reduce $\hat{A}, \hat{B}, \hat{C}$ as much as possible.
- Consider an infinitely long, positively charged rod. In such a situation, a point charge some distance ρ away will feel a force necessarily in the $\hat{\rho}$ direction.
- The rod looks the same anywhere in $\hat{\phi}$ direction and the $\hat{z}$ direction.
- Problem solving algorithm:
  1. Look for symmetries. This is useful in not just E&M, but basically everywhere.
  2. Find a coordinate system that also has the same symmetries. For example, with a rod, a cylindrical Gaussian surface is suitable.
  3. Remember, the generalized $\vec{E}$ field is given by

$$
\begin{aligned}
\vec{E}_p = {} & \hat{A} E_A(A, B, C) \\
& + \hat{B} E_B(A, B, C) \\
& + \hat{C} E_C(A, B, C)
\end{aligned}
$$

  4. Use Coulomb's law to figure out electric attractions and repulsions. This will allow you to determine which components of the $\vec{E}$ field should remain.
  5. Use symmetry arguments to reduce the number of variables.
  6. Then simply solve for $|\vec{E}|$.
  7. Basically, when choosing our Gaussian surface, we want to find something whose $\vec{E} \cdot d\vec{A}$ has no angular component.
  8. Find *representative points.* These are points that "represent" each unique surface. If our surface is a cylinder, there are 3 main representative points: the body of the cylinder, and the two bases. Since this is not continuous, you will need 3 integrals.
  9. Solve all these integrals.

3.4 Notes 5/29/19

- So what are some of the benefits of Gauss's Law? Well consider finding the E field of a solid charged sphere. If you had to use the tools you learned in chapter 21, this would be a much more difficult calculus problem. You will have to decompose the sphere into a series of disks and integrate the E field through the entire sphere since you already know the E field for a disk. But now, we can instead use Gauss's Law.
- We introduce spherical coordinates: The E field in spherical coordinates is described by:
$$\begin{aligned}\vec{E}_p &= \hat{r}E_r(r,\theta,\phi)\\ &+\hat{\theta}E_\theta(r,\theta,\phi)\\ &+\hat{\phi}E_\phi(r,\theta,\phi)\end{aligned}$$
- The conversion rate for spherical coordinates is given by:
$$\begin{aligned}x &= r\cos\phi\sin\theta\hat{x}\\ y &= r\sin\phi\sin\theta\hat{y}\\ z &= r\cos\theta\hat{z}\end{aligned}$$
Therefore, the spherical basis vectors are given by:
$$\begin{aligned}\hat{r} &= \cos\phi\sin\theta\hat{x}+\sin\phi\sin\theta\hat{y}+\cos\theta\hat{z}\\ \hat{\phi} &= \frac{d\hat{r}/d\phi}{|d\hat{r}/d\phi|} = \sin\phi\hat{x}+\cos\phi\hat{y}+0\hat{z}\\ \hat{\theta} &= \frac{d\hat{r}/d\theta}{|d\hat{r}/d\theta|} = \cos\phi\cos\theta\hat{x}+\sin\phi\cos\theta\hat{y}-\sin\theta\hat{z}\end{aligned}$$
- Problem solving steps:
  1. Expect $\vec{E}$ to go in the $\hat{r}$ direction.
  2. We're probably going to be using spherical polar coordinates.
  3. Think about what would happen empirically/experimentally.
  4. Make as many symmetry arguments as possible to reduce your calculations.
  5. Choose a fish net such that the fish net contains all the "fish." What this means is that we want to choose a Gaussian surface such that the surface encloses all the points.
  6. Choose your "representative points."
  7. Always remember, laziness is crucial to solving Gauss's law problems. Without it, you will most definitely fail.
  8. In all seriousness, try to make your own life as easy as possible. Reduce as much as possible, pick the most convenient Gaussian surface as possible, and ensure that your chosen Gaussian surface encapsulates all the information you want. That is, you are interested in what is going on WITHIN the Gaussian sphere, but are not interested in ANY less or ANY more.
- We are saying that the E field must only be in the z direction and that the E field relies only on $\vec{E}_p = \hat{z}E(z)$.
- Ask yourself: What are the 4 requirements of determining a Gaussian surface?

Chapter 4

Fun Problems

4.1 Dipoles and Charged Rods: From Lectures 5/20 - 5/22

1. Consider a uniformly charged rod of linear density:

$$\rho = nx,$$

or parabolic density:

$$\rho = nx^2,$$

general case for polynomial density, where

$$\rho = nx^p.$$

Find the electric field a distance d from the center of the rod.

2. What if we made a dipole out of two rods? What would the electric field around that look like?

3. What is the electric field of a dipole with two rods of varying charge density?

4. Consider a rod, where half of it is positive and the other half is negative. At which point is it justifiable to say that the rod becomes a dipole? Is it justifiable at all? This question is motivated by the fact that a rod at a far field limit begins looking like a point.

5. Consider an infinitely long and wide charged plane. The density is given by γ, given in units of μC/m^2. What is the electric field due to *only a single side of the sheet?*