Strategic Commitment to a Production Schedule with Uncertain Supply and Demand: Renewable Energy in Day-Ahead Electricity Markets

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We consider a day-ahead electricity market that consists of multiple competing renewable firms (e.g., wind generators) and conventional firms (e.g., coal-fired power plants) in a discrete-time setting. The market is run in every period, and all firms submit their price-contingent production schedules in every day-ahead market. Following the clearance of a day-ahead market, in the next period, each (renewable) firm chooses its production quantity (after observing its available supply). If a firm produces less than its cleared day-ahead commitment, the firm pays an undersupply penalty in proportion to its underproduction. We explicitly characterize firms' equilibrium strategies by introducing and analyzing a supply function competition model. The purpose of an undersupply penalty is to improve reliability by motivating each firm to commit to quantities it can produce in the following day. We prove that in equilibrium, imposing or increasing a market-based undersupply penalty rate in a period can result in a strictly larger renewable energy commitment at all prices in the associated day-ahead market, and can lead to lower equilibrium reliability in all periods with probability 1. We also show in an extension that firms with diversified technologies result in lower equilibrium reliability than single-technology firms in all periods with probability 1.

Key words: renewable energy, supply uncertainty, supply function equilibrium, day-ahead electricity market, reliability, demand uncertainty, production schedule, production quantity, penalty, subsidy

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1. Introduction

The penetration of variable renewable energy (such as wind and solar) in the electricity generation mix has significantly increased in the world (REN21 2015). In the E.U., wind and solar power accounted for 43.7% and 29.7% of the new power capacity in 2014, respectively (EWEA 2015). Many countries have implemented policies that set targets for renewable energy generation, which boosted the capacity and production of wind and solar energy. For example, Denmark targets to meet all of its electricity demand with renewable power by 2050, and it produced almost 33% of its electricity with wind energy in 2013 (Vittrup 2014, REN21 2015). In fact, during 2015, there have been days in which Denmark met 100% of its electricity demand with wind power (Guardian 2015). Similarly, in the U.S., with the renewable portfolio standard and other renewable energy policies, variable renewable energy capacity and generation have substantially increased. In 2014, the U.S. ranked first for wind energy generation and second for total wind capacity in the world (REN21 2015).

With increasing penetration of wind and solar in the generation mix, the participation of variable renewable power producers in U.S. day-ahead electricity markets has significantly increased, mainly because of various rules implemented in regional electricity markets. For example, recently, Midcontinent Independent System Operator, Inc. (MISO) has created a new category of wind energy resource called "Dispatchable Intermittent Resource (DIR)" (MISO 2014*a*). DIRs are treated as any conventional firm in MISO's day-ahead electricity markets (MISO 2013), which means that wind producers' day-ahead committed production schedule must be taken into account in MISO's day-ahead market clearance (MISO 2016*b*). This initiative enabled scheduling a large amount of wind power in MISO's day-ahead markets: After the addition of DIRs, in 2014, around

35 million MWh of wind power was cleared in MISO's day-ahead markets (MISO 2015*b*). Other regional electricity markets (such as PJM Interconnection (PJM) and ISO New England (ISO-NE)) also implemented rules that allowed for more wind energy participation and scheduling in day-ahead ahead electricity markets (UWIG 2011). The caveat is that in many electricity markets, wind producers are subject to penalties if there is a mismatch between their actual production quantity and day-ahead production commitment (UWIG 2011).

The fundamental challenge with variable renewable energy is supply uncertainty. Despite increasing production commitments by variable renewable energy producers in day-ahead electricity markets and the high penetration of wind and solar in the electricity generation mix, the day-ahead production schedule decision of a strategic firm that faces supply uncertainty and penalties has not yet been studied. Our paper analyzes this issue by considering a day-ahead market with competing conventional firms (such as coal-fired or nuclear power plants) and variable renewable firms that can commit to their production schedules in day-ahead markets. In this context, our paper studies four main research questions. First, (i) what are the day-ahead production commitment and actual production quantity of each firm in equilibrium? Our second research question is related to "reliability" that is a key performance metric for an independent system operator (ISO) (MISO 2014*a*). In fact, FERC Order No. 888 states that "an ISO should have the primary responsibility in ensuring short-term reliability of grid operations" (FERC 1996). In a day-ahead electricity market, reliability can be measured by the probability of the actual aggregate production in the operating day to be larger than the total day-ahead production commitment. (The subsequent paragraph will explain how a typical day-ahead market operates; the formal definition of the reliability will be introduced in Section 2.) To improve the reliability in the presence of supply uncertainty, independent system operators impose penalties on variable renewable energy imbalances, as mentioned above. However, these penalty rules have been subject to many changes in the U.S. (UWIG 2009, 2011). In light of this, our second research question is (ii) what are the implications of penalty rules for reliability, renewable firms' committed production schedules and actual production strategies in equilibrium? Third, in day-ahead electricity markets, there can be firms that generate electricity from both variable renewable resources and conventional resources. Based on this, (iii) what are the implications of firms that can generate electricity from different technologies for day-ahead commitments, actual production quantity and the reliability in equilibrium? Finally, (iv) how does subsidizing renewable firms for their production affect their committed production schedules and actual production strategies in equilibrium?

In the U.S., day-ahead markets consist of multiple competing firms (U.S. EIA 2014*b*). These markets typically operate as follows. A day-ahead market is run one day before every operating day (i.e., the actual production day), and a day is divided into 24 time blocks. Each firm submits its production schedule (i.e., supply function or offer curve) for each of the 24 time blocks, without observing the day-ahead demand for electricity or other firms' submitted production schedules in any time block. A firm's submitted *production schedule* in a time block represents how much the firm commits to produce in that time block of the following day for any potential market clearing price. After firms submit their production schedules for 24 time blocks and day-ahead

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demands for all 24 time blocks are realized, later in the same day, the independent system operator determines a day-ahead market clearing price for each time block by matching the day-ahead demand and day-ahead commitment for every time block separately. At any given time block of the following day, each firm must produce its committed production quantity at the aforementioned market clearing price of that time block. A firm is subject to penalties if for any time block, there is a mismatch between its cleared commitment in the day-ahead market and its production in the operating day, which is the day following the day-ahead market clearance. A common feature of these penalties in many electricity markets is that penalty rates are market-based, that is, the penalty rates depend on the realized price in either the day-ahead electricity market or another market that is related to that, such as balancing markets (MISO 2016*b*, PJM 2015, UWIG 2011).[1]In line with practice, this paper considers market-based penalty rates.

Variable renewable energy firms (such as DIRs in the MISO system), in short *renewable firms*, have the flexibility to commit to a different production schedule in every day-ahead electricity market; they can even submit a different production schedule for different time blocks of the same day-ahead market. However, nuclear or coal-fired power generators do not have such flexibility because they are used as base-load generators that must run continuously and at a *constant rate* to achieve operational efficiency (U.S. EIA 2016). Therefore, these conventional firms do not change their production quantities or commitments for a long time (i.e., for months). Based on this, hereafter, firms with this type of electricity generation will be called *inflexible firms*. In fact, inflexible firms and renewable firms together meet a major portion of electricity demand in the U.S. markets. For instance, in 2014, nearly 80% of the electricity demand was met by inflexible firms and wind power producers in MISO (MISO 2015*b*). Apart from their differences in flexibility, inflexible firms and renewable firms differ in the following two main aspects. First, inflexible firms have positive cost of production, whereas renewable firms incur no (or negligibly small) cost for production. Second, the renewable energy potential in the following day is uncertain (i.e., a firm does not exactly know how much wind is going to blow in the following day); on the other hand, an inflexible firm is not exposed to such uncertainty in its available supply.

Considering these practical issues, to answer our research questions, we study a discrete time setting in which a day-ahead market is run in every period for a finite time horizon. Each period in our formulation can be interpreted as a day. To avoid unnecessary notational complexity, in our model, there is one time block in each period.[2] The market consists of multiple competing inflexible firms and renewable firms. In every period, each renewable firm's strategy is to choose (a) a committed production schedule, which is a *function* that maps any possible day-ahead market clearing price to a committed production quantity (that must be delivered in the following period) and (b) an actual production quantity (that is related to the cleared day-ahead commitment in the previous period); the strategy of each inflexible firm consists of (c) a constant production commitment

[1] An electricity market is called "related to the day-ahead electricity market" or a "related electricity market" in short, if there is a dependency between the realized price in that market and the realized day-ahead market clearing price.

[2] This is a common modeling feature in the literature (see, for instance, Al-Gwaiz et al. (2017), Anderson and Philpott (2002) and references therein, among others). Our analysis trivially extends to the scenario in which a period consists of 24 time blocks by adding another index for the time block in the analysis. With this additional index, all of our results hold as stated, and such notational complexity does not generate any additional insights.

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and (d) actual production quantity. In each day-ahead market, all firms simultaneously submit their production commitments before the realization of the day-ahead demand, which is time-dependent and subject to a random shock in every period. At the time of the day-ahead commitment, a renewable firm cannot observe its available supply in the following period; hence, it is a random variable for the firm. The length of the finite horizon represents the time frame during which inflexible firms cannot change their commitments; thus, an inflexible firm chooses its commitment in the initial day-ahead market, and that commitment remains the same throughout the finite horizon. In contrast, renewable firms can change their day-ahead committed production schedules in every period. Firms with the same generation technology (i.e., inflexible or renewable) are identical and consider the same strategies. If a firm produces less (respectively, more) than its cleared production commitment determined in the previous period's day-ahead market, the firm has to pay an *undersupply penalty rate* for each unit of its underproduction (respectively, an *oversupply penalty rate* for each unit of its overproduction). Our paper also analyzes various variants of this base model (see Sections 4 and 5, and Sections EC.1.1 through EC.1.4 in the Electronic Companion). For instance, Section EC.1.4 analyzes a setting where renewable firms gain revenue for their overproduction instead of paying an oversupply penalty.

1.1. Overview of Main Results

Using the setting explained above, Proposition 2 characterizes firms' equilibrium day-ahead committed production schedules and actual production strategies in every period, which have not been studied in the literature (see Section 1.2 for a detailed discussion). By Proposition 2, a renewable firm's day-ahead committed production schedule in every period is characterized by an ordinary differential equation (ODE) subject to a monotonicity constraint and an initial condition.

Our paper establishes three unexpected results related to the implementation of the undersupply penalty rates. First, one might expect that if the undersupply penalty rate increases in a period, each renewable firm would decrease its associated day-ahead commitment in equilibrium to reduce its underproduction and hence its undersupply penalty. In contrast, Proposition 4 shows that increasing or imposing a market-based undersupply penalty rate can lead to a strictly larger day-ahead commitment by each renewable firm for any price in equilibrium. Second, intuition suggests that imposing or increasing a penalty rate on underproduction would improve the reliability by motivating firms to submit day-ahead commitments that they can deliver in the following period. However, Propositions 5 and 6 show that increasing or imposing a market-based undersupply penalty rate in a period can lead to strictly lower reliability *with probability 1* (i.e., for any realization of random day-ahead demand shock) in the associated day-ahead market. Third, Proposition 6 proves that increasing or imposing a market-based undersupply penalty rate in one period can indeed result in strictly lower equilibrium reliability in *all* periods with *probability 1*. The intuition behind Propositions 4 and 5 relies on renewable firms' manipulation power on the undersupply penalty rate. If the penalty rate is linked to the day-ahead market price or to the price in a market that is related to the day-ahead electricity market (e.g., as in MISO (MISO 2016*b*) and PJM (PJM 2015)), renewable firms can manipulate the undersupply penalty rate through their day-ahead commitments. When said manipulation power is large, as a response to an increase in the penalty rate,

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each renewable firm can profitably inflate its production commitment to mitigate the increase in the undersupply penalty rate by reducing the day-ahead market clearing price. Proposition 6 is due to the aforementioned commitment inflation by renewable firms and inflexible firms' strategic reaction to that inflation.

Sections 4 and 5, and Sections EC.1.1 through EC.1.4 in the Electronic Companion show that the explained unexpected results hold in various variants of the base model, and hence they are robust.

Section 4 analyzes a variant of the base model where each firm generates electricity from both inflexible and renewable resources. Proposition 9 shows that in a day-ahead market, firms with diversified technologies result in lower equilibrium reliability than single-technology firms in *all* periods with *probability 1*.

Section 5 extends the base model to consider production subsidies for renewable firms. In this setting, Proposition 11 demonstrates an interesting theoretical property of a renewable firm's committed production schedule in equilibrium. Specifically, it shows that with a large initial subsidy rate, there exists a critical quantity that divides up the quantity space into two intervals, and in each of these intervals, said committed production schedule is a solution of a different ODE.

Proposition 7 shows that even if the manipulation power of renewable firms is limited due to intense competition, a larger market-based undersupply penalty rate can still cause renewable firms to inflate their day-ahead commitments for some price range. Our numerical example based on MISO's day-ahead electricity market in Section 6 demonstrates that even this partial commitment inflation by renewable firms can strictly reduce expected reliability in the associated day-ahead market. Section 6 further demonstrates that the penalty rule has a large impact on firms' day-ahead commitments and reliability.

1.2. Literature Review

Our paper belongs to the growing literature on renewable energy/resources operations. Aflaki and Netessine (2017) show that the intermittency of renewable energy significantly impacts the effectiveness of the environmental policies, and find that increasing the emissions cost can decrease the share of renewable capacity investments in an energy investment portfolio. Kök et al. (2016) show that for a utility firm, flat pricing results in a higher solar energy investment than peak pricing, and the impact of pricing scheme on wind energy investment depends on the output pattern. Murali et al. (2015) characterize the optimal groundwater allocation and control policies when the water can be transferred. Hu et al. (2015) establish that the granularity of the demand and renewable output data plays a crucial role in renewable capacity investments. Zhou et al. (2011) study the performance of various heuristic policies and show that the energy storage adds significant value to renewable generators. Wu and Kapuscinski (2013) show that reducing renewable energy output can have a higher economic value with energy storage. Lobel and Perakis (2011) study welfare-maximizing subsidy design for customers' solar technology adoption, and conclude that current subsidy rate in Germany is very low. Alizamir et al. (2016) analyze the socially-optimal design of feed-in-tariffs for renewable technology, and show that a constant profitability policy is mostly suboptimal. To the best of our knowledge, our paper is the first that analyzes strategic renewable energy producers' equilibrium production schedule commitments in a day-ahead market with various penalty rules and subsidies in effect.

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Our paper also contributes to the supply function equilibrium (SFE) literature by introducing and analyzing a SFE with both supply and demand uncertainty. A comprehensive literature review on SFE models can be found in Holmberg and Newbery (2010); here we only include papers that are most relevant to ours. Klemperer and Meyer (1989) introduce and analyze a supply function competition model with demand uncertainty and convex production cost, and show the existence of a symmetric SFE. Green and Newbery (1992) and Green (1996) are among the first that apply SFE models in an electricity market. Green and Newbery (1992) analyze a linear supply function competition model; Green (1996) calibrates a supply competition model for British spot electricity market. It is well-established that the explicit analysis of an asymmetric SFE in a general setting is usually infeasible (see Johari and Tsitsiklis (2011) for a discussion). Thus, most of the SFE literature focuses on the analysis of a symmetric SFE. Rudkevich et al. (1998) analyze a symmetric SFE when the demand is inelastic and the cost of production is a piecewise constant and convex function. Anderson and Philpott (2002) solve a symmetric SFE with an inelastic demand and convex production cost. Holmberg (2008) identify the conditions under which there exists a unique symmetric SFE when the demand is inelastic. Focusing primarily on deterministic demand and the linear supply functions, Vives (2011) studies a symmetric SFE model with private information about production cost, and establishes that the supply function can be decreasing in price. Unlike these papers, our paper introduces and analyzes a supply function competition model with both supply and demand uncertainty under various penalty/credit rules, which has not been studied in the SFE literature. Considering the supply uncertainty together with the penalty rules in day-ahead markets give rise to results (summarized in Section 1.1), which have not been identified in the literature. Furthermore, different from the literature that analyzes linear supply functions, in our paper, each renewable firm's committed production schedule is allowed to be any *function* in equilibrium.

Our paper complements the recent "priority dispatch" literature that studies settings where renewable energy is prioritized in the electricity dispatch. Some of the leading papers in that literature are Buygi et al. (2012) and Al-Gwaiz et al. (2017). The work by Al-Gwaiz et al. (2017) is relevant to our paper, but its focus is very different than ours. Al-Gwaiz et al. (2017) analyze an electricity market where there are multiple competing conventional firms, and renewable energy is prioritized. They show that the implementation of economic curtailment policy for renewable energy results in intensified market competition. In their setting, there is no decision to be made by renewable firms in the market, meaning that renewable firms are not allowed to submit their production schedules in the market and cannot choose their production quantities; a fixed price is paid for unit renewable energy output. In contrast, our paper analyzes renewable firms' equilibrium decisions. Specifically, motivated by the recent developments in the U.S. electricity markets, our paper studies how each renewable firm dynamically commits to its production schedule and sets its production strategy in equilibrium when there are multiple competing renewable firms and inflexible firms.

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2. Model

Consider a day-ahead electricity market consisting of finitely many competing firms in a discrete-time setting. There are two types of firms in the market: $N_r \geq 2$ renewable firms indexed by $n = 1, 2, \ldots, N_r$, and $N_i \geq 2$ inflexible (conventional) firms (such as coal-fired power plants) indexed by $k = N_r + 1, \ldots, N_r + N_i$. There are three important differences between a renewable firm and an inflexible firm. First, different from an inflexible firm, the available supply of a renewable firm is uncertain in the future. Second, compared to an inflexible firm, a renewable firm incurs no (or negligibly small) production cost. Third, a renewable firm can change its committed production schedule in every day-ahead market. However, an inflexible firm does not change its production schedule that frequently. In fact, an inflexible firm continuously produces electricity at a *constant* rate for a long time (U.S. EIA 2016) because changing the production quantity leads to excessive operational inefficiencies for the firm. All of these elements will be included in our formulation.

There are τ commitment periods indexed by $t = 1, \ldots, \tau$. In our formulation, $\tau \geq 2$ represents the number of periods during which each inflexible firm commits to the same production quantity. In practice, inflexible firms do not change their production commitments for months, sometimes even for a year (Clark 2012, IRENA 2015). Thus, τ is in the order of months in practice. A day-ahead market is run in each commitment period t. Because a day-ahead market is run every day in U.S. electricity markets, each period t can be interpreted as a day.[3]

Day-ahead electricity demand in period t is $D_t(p, \epsilon_t)$, which is a function of random shock ϵ_t and price $p \in \mathbb{R}$; $D_t(p, \epsilon_t)$ strictly increases in ϵ_t, and strictly decreases and is concave in p. The day-ahead demand shock ϵ_t is a random variable with a differentiable cumulative distribution function $\Phi_t(\cdot)$ and a continuous density function $\phi_t(\cdot) > 0$ on support $[\tilde{\epsilon}_t(0, p_\ell), \infty)$. Here, $\tilde{\epsilon}_t(y, p)$ is the realization of ϵ_t that results in demand y at price p in the period-t day-ahead market and p_ℓ is the minimum possible market clearing price over τ periods. (Because $D_t(p, \epsilon_t)$ strictly increases in ϵ_t for any p, we can take the inverse of $D_t(p, \epsilon_t)$ with respect to ϵ_t to obtain the function $\tilde{\epsilon}_t : \mathbb{R}_+ \times \mathbb{R} \to \mathbb{R}$.)

For $t = 1, \ldots, \tau$, the period-t day-ahead market is run as follows. Before the uncertainty in the period-t day-ahead demand is resolved, all firms simultaneously submit their production schedules which represent the amount of electricity they commit to generate in the following period as a function of price. Specifically, each renewable firm $n \leq N_r$ chooses its twice continuously differentiable *production schedule*, i.e., *supply function*

$$S_{n,t} : \mathbb{R} \to \mathbb{R}_+.$$

The production schedule $S_{n,t}(\cdot)$ gives the quantity renewable firm n commits to produce in period t+1 as a function of period-t day-ahead market clearing price. If $t = 1$, each inflexible firm k chooses its day-ahead production commitment $\bar{S}_k \leq K$ for $p \geq p_\ell$, where K is the capacity of firm k; for $t > 1$, inflexible firm k's day-ahead production commitment remains the same and is equal to $\bar{S}_k$. After the day-ahead demand uncertainty

[3] Recall that, in our formulation, each period consists of one time block for readability. This is a common modeling feature in the literature (e.g., Al-Gwaiz et al. (2017)). All our proofs trivially extend to the case where there are 24 time blocks in each period by adding another index for the time block in the analysis (to represent non-stationary effects within a day). With this additional index, all our findings hold as stated.

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is resolved, that is, after ϵ_t is realized, in the same period, the independent system operator (ISO) determines the day-ahead market clearing price p_t^* that matches the day-ahead demand and total production commitment in period t:

$$D_t(p_t^*, \epsilon_t) = \sum_{n=1}^{N_r} S_{n,t}(p_t^*) + \sum_{k=N_r+1}^{N_r+N_i} \bar{S}_k. \tag{1}$$

This market clearing procedure is well established in the literature; see, for instance, Anderson and Philpott (2002), Rudkevich (1999), Bolle (1992) and Klemperer and Meyer (1989), among others. With (1), the ISO meets the period-t day-ahead demand at minimum cost because day-ahead demand decreases in price and we will establish later in Section 3 that in equilibrium, total production commitment increases in price.[4] Outcomes of the period-t day-ahead market determine how much electricity each firm must produce in period t+1. Here, $S_{n,t}(p_t^*)$ represents renewable firm n's cleared production commitment in period t, which is the amount of electricity firm n commits to deliver in period t+1.

In period t, there is uncertainty about the renewable firms' available supply in period t+1 (e.g., because firms do not exactly know how much wind is going to blow in the following period). Therefore, at the time of the commitment in period t, the available supply of renewable firm n in period t+1 is a random variable $Q_{n,t+1}$, independent of ϵ_t. The random variable $Q_{n,t+1}$ has a twice differentiable cumulative distribution function $F_{t+1}(\cdot)$ and a probability density function $f_{t+1}(\cdot)$ on support $[0, \infty)$ for all $n \leq N_r$.

At the beginning of period t+1, the available supply of each renewable firm is realized. Observing $Q_{n,t+1}$, each renewable firm n chooses its production quantity $q_{n,t+1}$ subject to $q_{n,t+1} \leq Q_{n,t+1}$. Because renewable firm n cannot produce more than $Q_{n,t+1}$ in period t+1, $Q_{n,t+1}$ can be seen as the period-(t+1) capacity constraint that is unobservable to firm n in period t. Simultaneously with all other firms, each inflexible firm k chooses its production quantity $q_{k,t+1} \leq K$ in period t+1. Based on the choice of production quantity in period t+1, a firm can incur two types of costs: undersupply or oversupply penalty, and the cost of production, all of which will be detailed below.

If a firm's production quantity in period t+1 is less than its cleared commitment in period t, the firm pays an *undersupply penalty rate* $\eta_{u,t+1}(p_t^*) = \beta_{u,t+1} p_t^* + b_{u,t+1}$ per its unit underproduction, where $b_{u,t+1} > 0$ and $\beta_{u,t+1} \in [0, 1]$ for all t. For example, if $q_{n,t+1} < S_{n,t}(p_t^*)$ for renewable firm n, then firm n must pay a total undersupply penalty of $\eta_{u,t+1}(p_t^*)(S_{n,t}(p_t^*) - q_{n,t+1})$ in period t+1. If a firm's production quantity in period t+1 is more than its cleared commitment in period t, the firm pays an *oversupply penalty rate* $\eta_{o,t+1}(p_t^*)$ per its unit overproduction. Here, $\eta_{o,t+1}(p) \doteq \beta_{o,t+1} p + b_{o,t+1}$ for $p \geq -b_{o,t+1}/\beta_{o,t+1}$, otherwise $\eta_{o,t+1}(p) = b_{t+1} > 0$ for $t \leq \tau$, where $b_{o,t+1} > 0$ and $\beta_{o,t+1} \in [0, 1]$.[5] In markets where penalty rates are linked to the realized price

[4] Here is a formal explanation of why p_t^* is the price that matches total day-ahead commitment and demand at minimum cost. Let $A_t(\cdot)$ be a period-t aggregate committed production schedule in equilibrium, and suppose that $A_t'(\cdot) > 0$. For any price $p < p_t^*$, $A_t(p) < D_t(p; \epsilon_t)$, which violates the ISO's requirement to match day-ahead demand and supply at all times. Thus, a price $p < p_t^*$ cannot be a market-clearing price. On the other hand, $A_t(p_t^*)p > A_t(p_t^*)p_t^*$ for $p > p_t^*$, implying that matching total day-ahead commitment and demand at price p is more costly than doing so at p_t^*. As a result, p_t^* is the unique price that achieves minimum cost in matching the day-ahead demand and total production commitment in period t.

[5] Note that $p < -b_{o,t+1}/\beta_{o,t+1}$ implies $\beta_{o,t+1}p + b_{o,t+1} < 0$. Thus, the condition $\eta_{o,t+1}(p) = b_{t+1} > 0$ for $p < -b_{o,t+1}/\beta_{o,t+1}$ ensures that firms are penalized rather than credited for their overproduction. The alternative case in which each firm receives a revenue based on its overproduction will be analyzed in Section EC.1.4 of the Electronic Companion.

in an electricity market that is related to the day-ahead market (e.g., as in MISO, PJM and ISO-NE (MISO 2016*b*, PJM 2015, UWIG 2011)), the parameters $\beta_{u,t+1}$ and $\beta_{o,t+1}$ represent the ultimate dependence of the undersupply and oversupply penalty rates in period t+1 on the day-ahead price p_t^*, respectively. The parameters $\beta_{u,t+1}$ and $\beta_{o,t+1}$ can also be viewed as a measure of dependence between the day-ahead price p_t^* and the realized price in the market based on which the undersupply and oversupply penalty rates are calculated, respectively.

The cost of production for each renewable firm is zero, and a renewable firm incurs no operating cost in reducing its output. For instance, wind generators can reduce their output by simply pitching the blades, which incurs no (or negligibly small) operating cost to wind generators (Wu and Kapuscinski 2013). In contrast, production is costly for inflexible firms. Each inflexible firm's production cost function in period t+1 is $C_{t+1}(\cdot)$ which is twice continuously differentiable, convex and strictly increasing, and it satisfies the following standard properties: $C_{t+1}(0) = 0$ and $C'_{t+1}(0) = 0$ for $t \leq \tau$.

In practice, an inflexible firm's average marginal cost of production is small relative to the average penalty rate (see, for instance, U.S. EIA (2013) and PJM (2015)) and inflexible firms are not generally exposed to supply uncertainty. Therefore, it is never profitable for an inflexible firm to produce less than its commitment to save from the production cost. Also, any positive oversupply penalty deters an inflexible firm from overproducing. As a result, each inflexible firm k's optimal production quantity is $q^*_{k,t+1} = \bar{S}_k$ for $t \leq \tau$, which is in line with the constant and continuous production schedules of inflexible firms observed in practice (U.S. EIA 2016). On the other hand, due to supply uncertainty, renewable firms might not be able to satisfy their production commitments, making their profits prone to the undersupply penalty rate. The objective of each type of firm will be formally defined in Section 2.1. The following table summarizes the explained sequence of events related to firms' production in period t+1.

Sequence of events related to the production in period t+1 (in the order of occurrence)

Day-ahead commitment and clearance in period t:

- **(1)** Each renewable firm n commits to a production schedule $S_{n,t}(\cdot)$ for $n = 1, \ldots, N_r$, and each inflexible firm k commits to a production quantity $\bar{S}_k$ for $k = N_r + 1, \ldots, N_r + N_i$.
- **(2)** Period-t day-ahead demand shock ϵ_t is realized.
- **(3)** Day-ahead market clearing price p_t^* is determined.

Production in period t+1 (based on the cleared day-ahead commitment in period t):

- **(4)** Available supply $Q_{n,t+1}$ is realized for $n = 1, \ldots, N_r$.
- **(5)** Each firm j chooses its production quantity $q_{j,t+1}$ for $j = 1, \ldots, N_r + N_i$.
- **(6)** Total penalty for each firm j is determined based on its cleared commitment in period t and $q_{j,t+1}$ for $j = 1, \ldots, N_r + N_i$.

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2.1. Firms' Objectives and the Equilibrium Definition

As explained above, an inflexible firm k's strategy is to commit to a production quantity $\bar{S}_k$ at $t=1$, and produce that quantity for all $t>1$; a renewable firm n's strategy is to choose (a) a committed production schedule $S_{n,t}(\cdot)$ in each day-ahead market t and (b) a production quantity $q_{n,t+1}$ for $t \le \tau$. Thus, in period 1, both renewable and inflexible firms simultaneously submit their day-ahead commitments; renewable firms simultaneously submit their production schedules $S_{n,t}(\cdot)$ in each day-ahead market $t \ge 2$ given period 1 commitments of inflexible firms. Definition 2 will formally define an equilibrium in this multi-period setting. To identify equilibrium strategies of firms, we shall analyze the problem backwards and start with the production decision of each renewable firm in period $t+1$ for any given $S_{n,t}(\cdot)$.

In period t+1, after observing $Q_{n,t+1}$, each renewable firm n chooses its production quantity $q_{n,t+1}$ to minimize the total realized net penalty associated with its cleared commitment $S_{n,t}(p_t^*)$. Hence, renewable firm n's optimal production quantity in period $t+1$ is

$$q^*_{n,t+1} \doteq \arg\min_{0\le q_{n,t+1}\le Q_{n,t+1}} \left[\eta_{u,t+1}(p_t^*)\left(S_{n,t}(p_t^*)-q_{n,t+1}\right)^+ + \eta_{o,t+1}(p_t^*)\left(q_{n,t+1}-S_{n,t}(p_t^*)\right)^+\right]. \tag{2}$$

In period t, given the optimal production quantity $q^*_{n,t+1}$ and any period-t commitment profiles $S_{-n,t} \doteq (S_{1,t},\dots,S_{n-1,t},S_{n+1,t},\dots,S_{N_r,t})$ and $\bar{\mathbb{S}} \doteq (\bar{S}_{N_r+1},\dots,\bar{S}_{N_r+N_i})$ of other firms, each renewable firm n chooses its production schedule $S_{n,t}(\cdot)$ that achieves its commitment-related ex-post maximum expected profit with respect to period-t day-ahead demand.[6] By doing so, given any $S_{-n,t}$ and $\bar{\mathbb{S}}$, firm n ex-ante achieves the commitment-related maximum expected profit it would have achieved if the random shock ϵ_t was known to the firm n before its period-t commitment. In period t, given any $S_{-n,t}$ and $\bar{\mathbb{S}}$, renewable firm n achieves its commitment-related ex-post maximum expected profit with respect to period-t day-ahead demand by choosing $S_{n,t}(\cdot)$ that satisfies the following:

$$\Pi_n(p_t^*(S_{n,t},S_{-n,t},\bar{\mathbb{S}},\epsilon_t);\epsilon_t,S_{-n,t},\bar{\mathbb{S}}) \ge \Pi_n(p_t^*(S,S_{-n,t},\bar{\mathbb{S}},\epsilon_t);\epsilon_t,S_{-n,t},\bar{\mathbb{S}}), \quad \text{for } S:\mathbb{R}\to\mathbb{R}_+ \tag{3}$$

and any realization of random shock ϵ_t. Here, $\Pi_n(p_t^*(S_{n,t},S_{-n,t},\bar{\mathbb{S}},\epsilon_t);\epsilon_t,S_{-n,t},\bar{\mathbb{S}})$ is renewable firm n's commitment-related expected profit in period t at the market clearing price $p_t^*(S_{n,t},S_{-n,t},\bar{\mathbb{S}},\epsilon_t)$, when firm n commits to $S_{n,t}$ and other firms commit to $S_{-n,t}$ and $\bar{\mathbb{S}}$ in period t, and the period-t day-ahead shock is ϵ_t. For example, given any $S_{-n,t}$ and $\bar{\mathbb{S}}$, renewable firm n's commitment-related period-t expected profit at price p and random shock ϵ_t is $\Pi_n(p;\epsilon_t,S_{-n,t},\bar{\mathbb{S}})$, which is equal to the revenue from period-t day-ahead commitment minus total expected penalty associated with that commitment. Specifically,

$$\Pi_n(p;\epsilon_t,S_{-n,t},\bar{\mathbb{S}}) = \mathbb{E}_{Q_{n,t+1}}\left[p\mathcal{R}_{n,t}(p;\epsilon_t) - \eta_{u,t+1}(p)\left(\mathcal{R}_{n,t}(p;\epsilon_t)-q^*_{n,t+1}\right)^+ - \eta_{o,t+1}(p)\left(q^*_{n,t+1}-\mathcal{R}_{n,t}(p;\epsilon_t)\right)^+\right], \tag{4}$$

where $q^*_{n,t+1}$ is a function of $Q_{n,t+1}$ by (2), and $\mathcal{R}_{n,t}(\cdot;\epsilon_t)$ is the firm n's period-t residual demand curve at random shock ϵ_t:

$$\mathcal{R}_{n,t}(p;\epsilon_t) \doteq D_t(p,\epsilon_t) - \sum_{j\ne n}^{N_r} S_{j,t}(p) - \sum_{k=N_r+1}^{N_r+N_i} \bar{S}_k. \tag{5}$$

[6] The term "commitment-related" expected profit in period t refers to the expected profit earned as a result of the commitment in period-t day-ahead market; the realized net penalty in period t should not be included in that profit because said penalty is a result of the actual production in period t and the cleared commitment in period $t-1$, which are unrelated to the period-t commitment decision.

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Renewable firms' production schedules that satisfy (3) constitute an ex-post period-t equilibrium with respect to day-ahead demand given any $\bar{\mathbb{S}}$. The formal definition of such an equilibrium is as follows.

DEFINITION 1. **(Period-t Supply Function Equilibrium)** For any given commitment profile of inflexible firms $\bar{\mathbb{S}}$, renewable firms' commitment profile $\mathbb{S}_t \doteq (S_{1,t}, \ldots, S_{N_r,t})$ is called "*period-t supply function equilibrium*" or in short "*period-t equilibrium*" if $S_{n,t}(\cdot)$ satisfies (3) for $n = 1, \ldots, N_r$.

It is perhaps worth noting that the production schedule $S_{n,t}(\cdot)$ that satisfies (3) also achieves the commitment-related *ex-ante* maximum expected profit $\mathbb{E}_{\epsilon_t}[\Pi_n(p_t^*(\cdot, S_{-n,t}, \bar{\mathbb{S}}, \epsilon_t); \epsilon_t, S_{-n,t}, \bar{\mathbb{S}})]$ given any $\bar{\mathbb{S}}$ and $S_{-n,t}$ because (3) holds for every realization of ϵ_t. Thus, given any $\bar{\mathbb{S}}$ and $S_{-n,t}$, $S_{n,t}(\cdot)$ that satisfies (3) achieves not only the ex-post maximum expected profit but also the ex-ante maximum expected profit with respect to period-t day-ahead demand.

In period 1, each inflexible firm k's best response is to choose its production commitment $\bar{S}_k \in [0, K]$ to maximize its total expected profit for any given commitment profiles $\bar{S}_{-k} \doteq (\bar{S}_{N_r+1}, \ldots, \bar{S}_{k-1}, \bar{S}_{k+1}, \ldots, \bar{S}_{N_r+N_i})$ and $(\mathbb{S}_1, \ldots, \mathbb{S}_\tau)$ of other firms:

$$\max_{\bar{S}_k \in [0,K]} \Pi_k(\bar{S}_k; \bar{S}_{-k}, \mathbb{S}_1, \ldots \mathbb{S}_\tau) \doteq \mathbb{E}_{\epsilon_1, \ldots, \epsilon_\tau}\left[\sum_{t=1}^{\tau} p_t^*(\bar{S}_k; \bar{S}_{-k}, \mathbb{S}_t, \epsilon_t)\bar{S}_k - C_{t+1}(\bar{S}_k)\right]. \tag{6}$$

Inflexible firm k's total expected profit is equal to the total revenue from day-ahead commitments minus total production cost. Note that firm k's production cost in period t+1 is $C_{t+1}(\bar{S}_k)$ because as explained earlier, $q_{k,t+1}^* = \bar{S}_k$. We focus on a parameter region in which the unconstrained optimizer of (6) is a real number, that is, for $t \le \tau$,

$$\partial\Big(C_{t+1}(x)(1 - \Phi_t(\tilde{\epsilon}_t(N_i x, 0)))\Big)/\partial x \ge a_t > 0, \quad x \ge \tilde{x}_t, \tag{7}$$

for some constants a_t and $\tilde{x}_t$, and (6) has a unique stationary point. The condition (7) means that the rate of increase in the production cost function $C_{t+1}(\cdot)$ ultimately dominates the rate of decrease in $(1 - \Phi_t(\tilde{\epsilon}_t(N_i x, 0)))$, which is a tail probability of the day-ahead demand shock in period t.

In this multi-period setting, we aim to characterize an *equilibrium*, which is formally defined below. To state Definition 2, let $\vec{q} \doteq (q_{j,t+1};\ t \le \tau,\ j \le N_r + N_i)$, which is the collection of production quantities of all firms in all periods.

DEFINITION 2. **(Equilibrium in the Multi-period Setting)** A strategy profile $(\mathbb{S}_1, \ldots \mathbb{S}_\tau, \bar{\mathbb{S}}, \vec{q})$ is an *equilibrium* in the explained multi-period setting if

$$\Pi_k(\bar{S}_k; \bar{S}_{-k}, \mathbb{S}_1, \ldots, \mathbb{S}_\tau) \ge \Pi_k(s; \bar{S}_{-k}, \mathbb{S}_1, \ldots, \mathbb{S}_\tau), \ \ s \in \mathbb{R}_+ \text{ and } k = N_r + 1, \ldots, N_r + N_i, \tag{8}$$

$$\Pi_n(p_t^*(S_{n,t}, S_{-n,t}, \bar{\mathbb{S}}, \epsilon_t); \epsilon_t, S_{-n,t}, \bar{\mathbb{S}}) \ge \Pi_n(p_t^*(S, S_{-n,t}, \bar{\mathbb{S}}, \epsilon_t); \epsilon_t, S_{-n,t}, \bar{\mathbb{S}}),\ S: \mathbb{R} \to \mathbb{R}_+,\ \epsilon_t,\ n \le N_r, \tag{9}$$

and $\vec{q} = (q_{j,t+1}^*;\ t \le \tau,\ j \le N_r + N_i)$ under the commitment profile $(\mathbb{S}_1, \ldots \mathbb{S}_\tau, \bar{\mathbb{S}})$.
Note that (9) is due to (3).

2.2. A performance metric for an ISO

An important performance metric for an ISO is the supply security for cleared day-ahead commitments. "Period-t reliability" is a measure of supply security for period-t day-ahead commitments, and it is defined as

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the probability of the aggregate cleared commitment in period t to be smaller than total production quantity of all firms in period t+1:

DEFINITION 3. (**Period-t Reliability**) Under an equilibrium strategy profile $(\mathbb{S}_1, \ldots \mathbb{S}_\tau, \bar{\mathbb{S}}, \vec{q})$, the *period-$t$ reliability* is defined as

$$\rho_t(\epsilon_t) \doteq \mathbb{P}_{Q_{1,t+1},\ldots,Q_{N_r,t+1}}\left(\sum_{n=1}^{N_r} S_{n,t}(p_t^*) + \sum_{k=N_r+1}^{N_r+N_i} \bar{S}_k \leq \sum_{j=1}^{N_r+N_i} q_{j,t+1}^* \Big| \epsilon_t\right), \quad t \leq \tau. \tag{10}$$

By (10), supply uncertainty of renewable firms is one of the major factors that put reliability at risk.[7]

In our analysis, we study an equilibrium in which same-type firms consider same commitment strategies:

$$\bar{S} \doteq \bar{S}_{N_r+1} = \bar{S}_{N_r+2} = \ldots = \bar{S}_{N_r+N_i} \text{ and} \tag{11}$$

$$S_t(p) \doteq S_{1,t}(p) = \ldots = S_{N_r,t}(p) \quad \text{for } t \leq \tau,\ p \in \mathbb{R}. \tag{12}$$

For ease of exposition, hereafter, we consider a linear demand function $D_t(p, \epsilon_t) = v_t - \alpha_t p + \epsilon_t$ where $v_t > 0$ and $\alpha_t > 0$ are constants. All problem parameters introduced in this section are common knowledge to all firms.

3. Analysis

We begin our analysis by characterizing firms' equilibrium strategies. To do so, we solve backwards. First, Lemma 1 identifies the optimal production quantity of renewable firm n in period t+1, given other firms' committed production schedules in a period-t day-ahead market. The proof of all formal results in our paper can be found in Appendices A through P of the Electronic Companion. Recall the residual demand notation $\mathcal{R}_{n,t}(p;\epsilon_t)$ in (5).

LEMMA 1. *Consider any renewable firm n. Given that committed production schedules of other firms are $\bar{\mathbb{S}}$ and $S_{-n,t}$ in a period-t day-ahead market, the renewable firm n's optimal production quantity in period $t+1$ is*

$$q_{n,t+1}^*(p, \epsilon_t, Q_{n,t+1}) = \min\{Q_{n,t+1}, \mathcal{R}_{n,t}(p;\epsilon_t)\} \tag{13}$$

for any realization of the renewable firm n's available supply $Q_{n,t+1}$ and period-t market clearing price $p > 0$. Furthermore, given ϵ_t, $p > 0$, $S_{-n,t}$ and $\bar{\mathbb{S}}$, firm n's commitment-related expected profit (4) in period t is equivalent to

$$\Pi_n(p;\epsilon_t, S_{-n,t}, \bar{\mathbb{S}}) = \mathbb{E}_{Q_{n,t+1}}\left[p\mathcal{R}_{n,t}(p;\epsilon_t) - \eta_{u,t+1}(p)\left(\mathcal{R}_{n,t}(p;\epsilon_t) - Q_{n,t+1}\right)^+\right]. \tag{14}$$

In the period-t day-ahead market, a renewable firm's residual demand at the market clearing price is equal to the firm's cleared production commitment by (1). Based on this, (13) implies that renewable firm n optimally produces at most its period-t cleared commitment in period t+1. The reason is that each firm must pay a penalty if the firm's production in period t+1 exceeds its period-t cleared commitment. This positive overproduction penalty deters a renewable firm from producing more than its cleared commitment. The dependence

[7] Note that (10) is equivalent to $\mathbb{P}_{Q_{1,t+1},\ldots,Q_{N_r,t+1}}\left(\sum_{n=1}^{N_r} S_{n,t}(p_t^*) \leq \sum_{n=1}^{N_r} q_{n,t+1}^* \big| \epsilon_t\right)$ because as explained before, $q_{k,t+1}^* = \bar{S}_k$ for $k = N_r + 1, \ldots N_r + N_i$ and $t \geq 1$. Regardless, period-t reliability is dependent on period-t commitments of both inflexible and renewable firms because p_t^* is a function of $\bar{S}_k$ and $S_{n,t}$ for $n = 1, \ldots, N_r$ and $k = N_r + 1, \ldots, N_r + N_i$ by (1).

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of the optimal production quantity (13) on the cleared commitment underscores the importance of day-ahead commitment strategies for renewable firms.

Using Lemma 1, we now present an intuitive approach to derive a period-t equilibrium defined in Definition 1. (The formal statement and analysis of a period-t equilibrium are provided in Proposition 1 and Appendix B of the Electronic Companion, respectively.) Recall from (4) that $\Pi_n(\cdot;\epsilon_t, S_{-n,t}, \bar{\mathbb{S}})$ represents renewable firm n's (commitment-related) expected profit in period t, given ϵ_t and other firms' commitments $S_{-n,t}$ and $\bar{\mathbb{S}}$. To maximize (4) for any ϵ_t given $S_{-n,t}$ and $\bar{\mathbb{S}}$, firm n must commit to a production schedule inducing a market clearing price that maximizes $\Pi_n(\cdot;\epsilon_t, S_{-n,t}, \bar{\mathbb{S}})$ for each ϵ_t. With such a commitment, firm n achieves the maximum expected profit it would achieve in period t if it observed the random shock ϵ_t before its commitment in period t. Note that a market clearing price $p < 0$ is never optimal for firm n as $\Pi_n(p;\epsilon_t, S_{-n,t}, \bar{\mathbb{S}}) < 0$ for $p < 0$. By Lemma 1, $\Pi_n(p;\epsilon_t, S_{-n,t}, \bar{\mathbb{S}})$ is equivalent to (14) for $p > 0$. Then, for any given ϵ_t and other firms' commitments $S_{-n,t}$ and $\bar{\mathbb{S}}$, the price p that maximizes (14) satisfies the following first order condition:

$$\mathcal{R}_{n,t}(p;\epsilon_t) + \mathcal{R}'_{n,t}(p;\epsilon_t)\Big[p - \eta_{u,t+1}(p)F_{t+1}(R_{n,t}(p;\epsilon_t))\Big] - \beta_{u,t+1}\int_0^{\mathcal{R}_{n,t}(p;\epsilon_t)} (\mathcal{R}_{n,t}(p;\epsilon_t) - x)dF_{t+1}(x) = 0. \tag{15}$$

Recall the notation $\tilde{\epsilon}_t(\cdot,\cdot)$ from Section 2, and denote by $A_t(p)$ the aggregate production commitment by all firms at p in period t. Then, from (1), (5) and (15), we have

$$S_{n,t}(p) + \left(\partial D_t\left(p, \tilde{\epsilon}_t\left(A_t(p), p\right)\right)/\partial p - \sum_{j\neq n}^{N_r} S'_{j,t}(p)\right)[p - \eta_{u,t+1}(p)F_{t+1}(S_{n,t}(p))]$$
$$- \beta_{u,t+1}\int_0^{S_{n,t}(p)} (S_{n,t}(p) - x)dF_{t+1}(x) = 0, \tag{16}$$

which characterizes the best-response committed production schedule of firm n in period t. Observe from (12) and the fact that $\partial D_t\left(p, \tilde{\epsilon}_t\left(A_t(p), p\right)\right)/\partial p = -\alpha_t$, (16) reduces to the following ordinary differential equation:

$$S'_t(p) = \frac{1}{N_r - 1}\left[\frac{S_t(p) - \beta_{u,t+1}\int_0^{S_t(p)}(S_t(p) - x)dF_{t+1}(x)}{p - (\beta_{u,t+1}p + b_{u,t+1})F_{t+1}(S_t(p))} - \alpha_t\right]. \tag{17}$$

The following proposition establishes the existence of a period-t equilibrium and formally characterizes renewable firms' committed production schedules in such an equilibrium.

PROPOSITION 1. (***Existence and Characterization of a Period-t Equilibrium***)
(i) *For any given commitment profile of inflexible firms $\bar{\mathbb{S}}$ and $t = 1,\ldots,\tau$, a function $S_t(\cdot)$ is each renewable firm's committed production schedule in a period-t supply function equilibrium that satisfies* (12) *if and only if $S_t(\cdot)$ satisfies the ordinary differential equation* (17) *subject to a monotonicity constraint and an initial condition, respectively:*

$$0 < S'_t(p) < \infty,\;\; p \geq p_{\ell,t}, \quad \textit{and} \quad S_t(p_{\ell,t}) = 0, \quad \textit{where} \quad p_{\ell,t} \doteq 0. \tag{18}$$

(ii) *For $t = 1,\ldots,\tau$, there exists a function $S_t(\cdot)$ that satisfies* (17) *subject to* (18). *Therefore, there exists a period-t supply function equilibrium that satisfies* (12) *for any given $\bar{\mathbb{S}}$.*

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Let us develop an understanding about period-t equilibrium conditions (17) and (18). Suppose that p is the period-t market clearing price. Then, the term $p-(\beta_{u,t+1}p+b_{u,t+1})F_{t+1}(S_t(p))$ in (17), which is equal to $p-\eta_{u,t+1}(p)F_{t+1}(S_t(p))$, can be interpreted as each renewable firm's expected marginal profit for an additional unit of commitment in period t. This is because if a renewable firm's period-t commitment increases by one unit, the firm gains p and incurs an expected cost of commitment $\eta_{u,t+1}(p)F_{t+1}(S_t(p))$. The reason for this cost is as follows. There are two possible cases about a renewable firm's available supply in period t+1: It is either (a) smaller than the firm's period-t cleared commitment $S_t(p)$ or (b) strictly larger than that. Case (a) occurs with probability $F_{t+1}(S_t(p))$, which equals the underproduction probability in period t+1. Given that the firm underproduces in period t+1, an additional unit of commitment at price p costs $\eta_{u,t+1}(p)$ to the firm. On the other hand, case (b) occurs with probability $(1-F_{t+1}(S_t(p)))$. In this case, the firm produces its cleared production commitment by Lemma 1, and thus the cost of an additional unit of commitment is equal to the firm's marginal cost of production, which is zero. Taking the expectation of the cost of an additional unit of commitment with respect to the random available supply in period t+1, we obtain expected marginal cost $\eta_{u,t+1}(p)F_{t+1}(S_t(p))$ in period t.

The term $S_t(p)-\beta_{u,t+1}\int_0^{S_t(p)}(S_t(p)-x)dF_{t+1}(x)$ in (17) can be interpreted as each renewable firm's expected marginal profit as a result of a unit increase in price. This is because for a given period-t commitment, if price p increases by one unit, a renewable firm's revenue increases by $S_t(p)$ whereas its expected undersupply penalty increases by $\beta_{u,t+1}\int_0^{S_t(p)}(S_t(p)-x)dF_{t+1}(x)$.

By (17), a smaller α_t implies a larger rate of increase in the period-t commitment with respect to price for any fixed pair of price and commitment quantity. The reason is that a smaller α_t implies a larger demand at any price $p>0$, motivating each renewable firm to sell more. The monotonicity constraint in (18) suggests that each renewable firm commits to a larger quantity at a higher price for any t. The conditions in (18) also show that each renewable firm commits to a positive production quantity only for $p>0$. Note that (17) and (18) are robust to changes in the day-ahead market size v_t. Specifically, $S_t(\cdot)$ is a period-t supply function equilibrium regardless of the value of v_t, or the day-ahead market size net from inflexible firms' production. This is because, by committing to $S_t(\cdot)$, each renewable firm achieves the maximum expected profit that can be achieved for any given potential day-ahead market clearing price. Since there is a one-to-one mapping between the realized day-ahead market clearing price and v_t in equilibrium, by committing to $S_t(\cdot)$, in fact, each renewable firm achieves the maximum expected profit for any v_t given any commitment profile of other firms.

Using Proposition 1, Proposition 2 explicitly identifies equilibrium strategies of all firms in all periods, including each inflexible firm's equilibrium production commitment at $t=1$. To state Proposition 2, we shall introduce the following notation:

$$\underline{z}_t(\bar{s}) \doteq N_i\bar{s}-v_t. \tag{19}$$

For $\bar{s}\le K$, the term $\underline{z}_t(\bar{s})$ represents period-t day-ahead demand shock that results in $p_t^*=0$ when each inflexible firm's production commitment is $\bar{s}$.

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PROPOSITION 2. *(i) There exists an equilibrium $(\mathbb{S}_1, \ldots \mathbb{S}_\tau, \bar{\mathbb{S}}, \vec{q})$ that satisfies* (11) *and* (12). *In such an equilibrium, firms' strategies are as follows.*

(ii) For $t = 1, \ldots, \tau$, each renewable firm n commits to a production schedule $S_t(\cdot)$ that satisfies (17) *and* (18) *in period t, and firm n produces $q^*_{n,t+1}(p^*_t, S_t, Q_{n,t+1}) = \min\{Q_{n,t+1}, S_t(p^*_t)\}$ in period $t+1$.*

(iii) At $t = 1$, each inflexible firm commits to a production quantity $\bar{S} = \min\{\bar{s}^*, K\}$ such that $\bar{s}^*$ satisfies

$$\Psi(\bar{s}) \doteq \sum_{t=1}^{\tau} \left[\int_{\underline{z}_t(\bar{s})}^{\infty} \left(p^*_t(\bar{s}; z) - \frac{\bar{s}}{\alpha_t + N_r S'_t(p^*_t(\bar{s}; z))} - C'_{t+1}(\bar{s}) \right) d\Phi_t(z) + C_{t+1}(\bar{s}) \phi_t(\underline{z}_t(\bar{s})) \right] = 0, \tag{20}$$

where $S_t(\cdot)$ is as identified in part (ii), and $p^*_t(\bar{s}; z)$ is the price that satisfies $v_t - \alpha_t p^*_t(\bar{s}; z) + z = N_r S_t(p^*_t(\bar{s}; z)) + N_i \bar{s}$.

PROPOSITION 3. *There exists a unique equilibrium $(\mathbb{S}_1, \ldots \mathbb{S}_\tau, \bar{\mathbb{S}}, \vec{q})$ that satisfies* (11) *and* (12).

By (20), an inflexible firm's committed production quantity $\bar{S}$ in equilibrium is dependent on renewable firms' committed production schedules in all periods. The reason is that, by (1), renewable firms' committed production schedules impact the market clearing price in each period, thereby affecting an inflexible firm's total expected profit (6).

Proposition 2 verifies that the interpretations related to $S_t(\cdot)$ explained below Proposition 1 are in fact valid in an equilibrium where inflexible firms also strategically commit to their production quantities. Hereafter, the term "*committed production schedule*" in period t is used to refer to the function $S_t(\cdot)$ that satisfies the equilibrium conditions (17) and (18).

Because renewable firms do not overproduce in the equilibrium by Proposition 2-(ii), the period-t committed production schedule is not dependent on the oversupply penalty parameters $\beta_{o,t+1}$ and $b_{o,t+1}$ for any t. On the other hand, the undersupply penalty rate significantly influences renewable firms' equilibrium commitments by (17) because supply uncertainty brings forth the possibility of underproduction for a renewable firm under the equilibrium production strategy identified in Proposition 2-(ii).

The penalty rules for renewable energy producers have been subject to various changes in the U.S. electricity markets (UWIG 2009, 2011). Yet, the implications of penalties for renewable firms' day-ahead committed production schedules and the reliability have not been investigated. Below, Propositions 4 through 7 in Section 3.1 will show that seemingly intuitive undersupply penalty rules can have unintended consequences.

3.1. Impact of the Undersupply Penalty Rate

In our setting, a higher $\beta_{u,t+1}$ implies a strictly higher undersupply penalty rate $\eta_{u,t+1}(p) = \beta_{u,t+1}p + b_{u,t+1}$ for $p > 0$. Therefore, one might expect that increasing $\beta_{u,t+1}$ or imposing a positive market-based penalty rate $\beta_{u,t+1}p$ in addition to the fixed rate $b_{u,t+1}$ would motivate renewable firms to lower their production commitments in period t to avoid a high underproduction penalty. Contrary to this intuition, Proposition 4 shows that if there is an increase in $\beta_{u,t+1}$ or if the market-based undersupply penalty rate $\beta_{u,t+1}p$ is imposed in addition to the fixed rate $b_{u,t+1}$, each renewable firm can commit to a strictly larger production quantity at *all* prices in equilibrium.

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PROPOSITION 4. (***Uniform Inflation of Committed Production Schedules***)
Consider the equilibrium characterized in Proposition 2 and any $t = 1, \ldots, \tau$.
(i) Each renewable firm's period-t committed production schedule is uniformly and strictly larger with a higher $\beta_{u,t+1}$, i.e., $\partial S_t(p; \beta_{u,t+1})/\partial\beta_{u,t+1} > 0$ for $p > 0$, if and only if the number of renewable firms N_r and the undersupply penalty parameter $b_{u,t+1}$ satisfy

$$N_r \leq \Gamma_{t+1} \quad \text{and} \quad b_{u,t+1} < \Delta_{t+1}, \tag{21}$$

where Γ_{t+1} and Δ_{t+1} are positive constants.
(ii) If (21), imposing a market-based penalty rate $\beta_{u,t+1}p > 0$ in addition to the fixed rate $b_{u,t+1}$ for underproduction results in a uniformly and strictly larger committed production schedule $S_t(\cdot)$ for each renewable firm in period t.

REMARK 1. This proposition and subsequent propositions in this section do not require the uniqueness of the equilibrium as results hold for any equilibrium that satisfies (11) and (12). Thus, to facilitate the extensions of Propositions 4 through 7 to other settings, proofs of Propositions 4 through 7 (in Appendices E through H of the Electronic Companion) are presented for any equilibrium that satisfies (11) and (12).

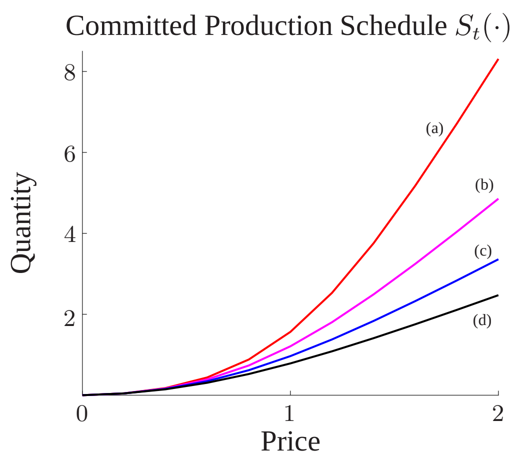

Figure 1 Period-t committed production schedule $S_t(\cdot)$ of each renewable firm in equilibrium. Here, the parameter $\beta_{u,t+1}$ for (a), (b), (c) and (d) are 0.9, 0.8, 0.7 and 0.6, respectively. The rest of the parameter set is as follows: there are two renewable firms, $\alpha_t = 0.001$, $b_{u,t+1} = 0.01$ and $F_{t+1}(\cdot)$ is a Lomax distribution with scale parameter 1 and shape parameter 2.

Let us explain the intuition behind the conditions in (21). If there is an increase in $\beta_{u,t+1}$, a renewable firm can mitigate the increase in the undersupply penalty rate $\eta_{u,t+1}(\cdot)$ by uniformly inflating its committed production schedule in period t, and thereby reducing the day-ahead market clearing price p_t^* for any given commitment profile of inflexible firms. However, a renewable firm benefits from such a commitment inflation in equilibrium if and only if (a) the market clearing price is highly sensitive to a change in the committed production schedule so that a slight increase in the committed production schedule can achieve a significant reduction in the realized undersupply penalty rate with the new $\beta_{u,t+1}$, and (b) the expected cost of commitment is low. Condition (b) is equivalent to having a sufficiently small $b_{u,t+1}$. Condition (a) is equivalent to having a

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small number of renewable firms in the market because if the market is extremely competitive (i.e., $N_r \to \infty$), a renewable firm has very limited or negligibly small market power to influence the market clearing price with its commitment.

Figure 1 pictures a numerical example where in equilibrium, a uniformly larger $\eta_{u,t+1}(\cdot)$ (due to a higher $\beta_{u,t+1}$) results in a uniformly and strictly larger committed production schedule for each renewable firm in period t, as suggested by Proposition 4. For any given $\bar{\mathbb{S}}$, although such commitment inflations result in a smaller market clearing price, the realized undersupply penalty rate $\eta_{u,t+1}(p_t^*)$ with the new $\beta_{u,t+1}$ can be strictly larger than the one with the original $\beta_{u,t+1}$. This is because, in certain cases, the resulting decrease in the market clearing price might not be large enough to suppress the increase in $\beta_{u,t+1}$. In fact, Proposition EC.9 (in Appendix P) shows that, if (21), for any given commitment profile of inflexible firms, an increase in $\beta_{u,t+1}$ results in a *strictly larger* realized undersupply penalty rate $\eta_{u,t+1}(p_t^*)$ in equilibrium when $\beta_{u,t+1}$ is small and ϵ_t is moderate.

We now study the implications of $\beta_{u,t+1}$ on period-t reliability (10) in equilibrium, which is an important performance metric for an ISO (MISO 2014*a*). The rationale behind the existence of an undersupply penalty is to improve reliability by motivating firms to commit to production quantities they can deliver in the following day. The following two propositions show that an undersupply penalty rate can defeat this purpose.

PROPOSITION 5. (***Period-t Reliability Degradation with a Larger*** $\eta_{u,t+1}(\cdot)$)
Consider any $t = 1, \dots, \tau$ and $\bar{\mathbb{S}}$. (i) An increase in $\beta_{u,t+1}$ results in strictly lower period-t reliability (10) for every realization of ϵ_t (i.e., with probability 1) in equilibrium if and only if (21). (ii) If (21), imposing a positive market-based penalty rate $\beta_{u,t+1}p$ in addition to the fixed rate $b_{u,t+1}$ per unit of underproduction results in strictly lower period-t reliability (10) for every realization of ϵ_t (i.e., with probability 1) in equilibrium.

Proposition 6 proves a stronger result on reliability with some additional conditions.

PROPOSITION 6. (***Reliability Degradation in All Periods with a Larger*** $\eta_{u,t+1}(\cdot)$)
Suppose that (21) holds for some $t = 1, \dots, \tau$. (i) In equilibrium, an increase in $\beta_{u,t+1}$ results in lower reliability in all *periods for every realization of ϵ_j $j = 1, \dots, \tau$ (i.e., with probability 1), if either (1) $K < K_L$ where $K_L > 0$ is a constant, or (2) the following set of conditions hold for a particular $\gamma > 0$ and some positive constants $\tilde{s}$ and K_H such that $\tilde{s} \le K$ and $K_L < K_H$:*

$$\mathbb{E}_{\epsilon_t}\left[\sqrt{\frac{\partial S_t(p_t^*(\tilde{s}))}{\partial \beta_{u,t+1}}}\;\middle|\; \epsilon_t \ge \underline{z}_t(\tilde{s})\right] > \gamma, \;\; K > K_H \quad \text{and} \quad S_t''(\cdot) > -\alpha_t^2/(N_r K). \tag{22}$$

(ii) In equilibrium, if either the conditions in (22) hold for $\beta_{u,t+1} \in [0,1)$ or $K < K_L$, imposing a positive market-based penalty rate $\beta_{u,t+1}p$ (in addition to the fixed penalty rate $b_{u,t+1}$) per unit of underproduction results in lower reliability (10) in all *periods with probability 1.*

Let us explain the conditions in (22). Proposition 4 proved that, if (21), each renewable firm uniformly inflates its committed production schedule in period t as a response to an increase in $\beta_{u,t+1}$. Based on this,

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the first condition in (22) introduces a measure for the magnitude of such inflations, and represents a scenario in which the aforementioned inflation measure is sufficiently large. The second condition in (22) implies that each inflexible firm's capacity is non-binding in equilibrium for all $\beta_{u,t+1} \in [0, 1)$ so that inflexible firms' commitments are sensitive to changes in $\beta_{u,t+1}$. The third condition in (22) ensures that, when $\beta_{u,t+1}$ increases, the period-t market clearing price does not become too insensitive to the production commitment of inflexible firms.[8] If the magnitude of renewable commitment inflation is large (which corresponds to the first condition in (22)), and if the market clearing price is sufficiently sensitive to the inflexible firms' production commitment (which is guaranteed by the third condition in (22)), each inflexible firm commits strictly less to increase the equilibrium market clearing price for any ϵ_t. Note from (1) that the reduction in inflexible firms' commitments results in strictly larger cleared commitment for each renewable firm in every period with probability 1, leading to strictly lower reliability in *all* periods with probability 1.

It could also be valuable to understand the impact of the undersupply penalty rate on the expected mismatch between the aggregate day-ahead commitment and the total production quantity in the next period. Because $q^*_{k,t+1} = \bar{S}$ for $t \leq \tau$, and in equilibrium, renewable firms do not overproduce by Proposition 2, the expected mismatch in period t (after the clearance of period-t day-ahead market) is $\mathbb{E}_{Q_{1,t+1}\cdots Q_{n,t+1}}\left[\sum_{n=1}^{N_r}(S_t(p^*_t) - Q_{n,t+1})^+\right]$. Then, by Proposition 4, it is trivial to show that Propositions 5 and 6 hold as stated when the term "lower (period-t) reliability" is replaced with "larger expected mismatch (in period t)."

Propositions 4 through 6 hold when the competition among renewable firms is not very intense. In a more general setting, Proposition 7 below shows that each renewable firm can inflate its equilibrium period-t commitment due to an increase in the undersupply penalty rate $\eta_{u,t+1}(\cdot)$ even if there is intense competition among renewable firms. However, such an inflation occurs only in a certain price range in equilibrium.

PROPOSITION 7. (***Partial Commitment Inflation with a Larger*** $\eta_{u,t+1}(\cdot)$) *For any $t = 1, \ldots, \tau$, consider the following two alternative schemes for the undersupply penalty rate in period $t+1$: $\eta_1(p) = \beta_1 p + b_1$ and $\eta_2(p) = \beta_2 p + b_2$ such that $\beta_1 > \beta_2 \geq 0$ and $0 < b_1 < b_2$. Then, in equilibrium, there exists a price interval in which each renewable firm commits to a strictly larger production quantity in period t under the penalty scheme with a larger $\eta_{u,t+1}(\cdot)$.*

COROLLARY 1. *Under the setting stated in Proposition 7, there exists a nonempty period-t day-ahead demand shock interval in which the scheme with a larger $\eta_{u,t+1}(\cdot)$ results in a strictly lower period-t reliability (10) for any given $\bar{\mathbb{S}}$ in equilibrium.*

[8] To see this, suppose $\beta_{u,t+1}$ is increased. For a given $\bar{\mathbb{S}}$, this would reduce period-t market clearing price due to the renewable firms' commitment inflation (explained in Proposition 4) when (21) holds. Under the third condition in (22), the inverse of $S_t(p)$ does not become very flat as p decreases, or equivalently the inflexible firms can still influence the period-t market clearing price via their production commitments. (If the inverse of $S_t(p)$ became very flat for smaller p, then period-t market clearing price would have been virtually unresponsive to the inflexible firms' production commitments.)

The proof of Proposition 7 shows that the scenario in which $S_t(p;\eta_1) > S_t(p;\eta_2)$ despite $\eta_1(p) > \eta_2(p)$ can occur when the price is not too small. The numerical example in Section 6 demonstrates that when parameters are set to realistic values, such type of a commitment inflation can be observed for a wide range of prices (i.e., for $p > \$4.7$ per MWh). From a reliability standpoint, this means that even when there is intense competition among renewable firms, period-t reliability can decrease for a very wide range of day-ahead demand shocks due to an increase in the undersupply penalty rate. The numerical example (in Section 6) verifies this, and shows that in fact, the aforementioned type of commitment inflations can strictly decrease the expected reliability in the associated day-ahead market.

3.2. Key Insights Related to Propositions 4 through 7

Propositions 4 through 7 offer valuable insights for settings in which firms choose their production schedules in the day-ahead market and pay their undersupply penalty based on the market clearing price in the day-ahead market or a related electricity market such as a balancing market. Examples of such settings include regional markets operated by MISO, PJM and ISO-NE (MISO 2016*b*, PJM 2015, UWIG 2011). As explained in Section 2, in said settings, the day-ahead price influences the penalty rate (directly or indirectly), and the coefficient $\beta_{u,t+1}$ represents the ultimate dependence of the undersupply penalty rate on the day-ahead market clearing price. The magnitude of $\beta_{u,t+1}$ can be viewed as a measure of dependence between the day-ahead price and the realized price in the market based on which the undersupply penalty is calculated.[9] In light of this, a higher $\beta_{u,t+1}$ corresponds to a higher dependence between these two market prices.

In this context, Propositions 4 and 7 have important implications for firms. If the undersupply penalty rate is linked to the realized price of another electricity market whose outcomes are strongly dependent on the outcomes of the day-ahead market, firms can manipulate the undersupply penalty rate by changing their production schedules in the day-ahead market. As a result, with a higher dependence, despite the undersupply penalty rate is higher at all $p > 0$, firms can have the financial incentive to uniformly or partially inflate their committed production schedules, thereby mitigating the increase in the undersupply penalty rate, as shown in Propositions 4 and 7.

Propositions 5 and 6 have key implications for reliability. Linking the undersupply penalty rate to the realized price of a market that is more closely related to the day-ahead electricity market can strictly decrease reliability (10) for any realization of the day-ahead demand shock. It is straightforward to show that the magnitude of such type of reliability degradation in period t increases with the realization of random shock ϵ_t. Thus, for an independent system operator, one way to mitigate said reliability degradation is to implement more accurate forecasting techniques to reduce ϵ_t. Another way to mitigate this type of reliability degradation is to link the undersupply penalty rate to a market that is less related to the day-ahead electricity market. In fact,

[9] The positive $\beta_{u,t+1}$ suggests a positive dependence. The intuition behind the positive dependence can be explained as follows. Holding all other problem parameters the same, if the random shock ϵ increases (respectively, decreases), a higher (respectively, lower) day-ahead market clearing price is realized. This implies a larger (respectively, smaller) total production commitment in equilibrium by the monotonicity constraint in (18). As a result, the total available supply net from the production commitment is smaller (respectively, larger), implying a higher (respectively, a lower) net demand in the rest of the system and hence a higher (respectively, a lower) market-clearing price in the closely related electricity market.

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Proposition 6 suggests that for markets in which the aforementioned type of reliability degradation is possible, letting $\beta_{u,t+1} = 0$ maximizes the reliability (10) for any realization of the day-ahead demand shock in *all* periods. Thus, for such type of markets, using a fixed undersupply penalty rate $b_{u,t+1}$ instead of a market-based undersupply rate achieves the maximum reliability in all periods with probability 1.

Proposition EC.10 (in Appendix P) analyzes the impact of $b_{u,t+1}$ on renewable firms' equilibrium commitments and the discussion following it explains the reliability implications. In contrast to Proposition 4, Proposition EC.10 shows that if there is an increase in $b_{u,t+1}$, each renewable firm's equilibrium production commitment in period t decreases in a non-empty price interval. Thus, in equilibrium, renewable firms do not uniformly inflate their committed production schedules as a response to an increase in $b_{u,t+1}$. This suggests that a larger undersupply penalty rate $\eta_{u,t+1}(\cdot)$ can provide financial incentives to renewable firms for uniformly inflating their equilibrium production commitments only when the undersupply penalty rate is market-based (as in MISO, PJM and ISO-NE (MISO 2016*b*, PJM 2015, UWIG 2011)); a fixed undersupply penalty rate $b_{u,t+1}$ does not offer such incentives.

4. Firms with both Renewable and Nonrenewable Technologies

Section 3 analyzed a day-ahead market with N_r renewable and N_i inflexible firms. To understand the implications of firms with multiple diversified technologies, this section considers a setting where there are N firms, each of which owns one renewable generator and one inflexible generator. All other modeling elements are the same as in Section 2, with the exception that the period-t market clearing equation (1) must be modified as $D_t(p_t^*, \epsilon_t) = \sum_{j=1}^{N} S_{j,t}(p_t^*) + \sum_{j=1}^{N} \bar{S}_j$.

In this setting, firm j's optimal inflexible energy production is $\bar{S}_j$. Because firm j's (production-related) total realized net penalty in period t+1 is (2), the optimal renewable energy production of firm j is as identified in Lemma 1. (Proof of Lemma 1 remains the same.) Therefore, firm j's period-t expected profit from renewable energy commitment is (4), implying that firm j's committed renewable energy production schedule $S_{j,t}(\cdot)$ is as identified in Proposition 1 in a period-t supply function equilibrium that satisfies (12). (The proof of Proposition 1 remains the same.) Firm j's inflexible energy commitment at $t = 1$ impacts the market-clearing price in every day-ahead market; hence, it affects firm j's expected profit from both renewable and inflexible energy in every period. Thus, at $t = 1$, each firm j chooses an inflexible energy commitment $\bar{S}_j \in [0, K]$ to maximize its total expected profit from its inflexible and renewable energy commitments, considering any given commitment profile of other firms:

$$\mathbb{E}_{\vec{\epsilon}}\left[\sum_{t=1}^{\tau} p_t^* \bar{S}_j - C_{t+1}(\bar{S}_j)\right] + \mathbb{E}_{\vec{\epsilon}}\left[\sum_{t=1}^{\tau} p_t^* S_{j,t}(p_t^*) - \eta_{u,t+1}(p_t^*)\mathbb{E}_{Q_{j,t+1}}\left[\left(S_{j,t}(p_t^*) - Q_{j,t+1}\right)^+\right]\right]. \tag{23}$$

Here, $\vec{\epsilon} \doteq (\epsilon_1, \epsilon_2, \ldots, \epsilon_\tau)$ is a vector of day-ahead random shocks, and the period-t market clearing price p_t^* is a function of ϵ_t, $\bar{S}_j$ and other firms' period-t commitment profiles $\bar{S}_{-j}$ and $\mathbb{S}_t$.

Proposition 8 below identifies firms' equilibrium strategies. To state Proposition 8, we shall define $\delta_1(p) \doteq \beta_{u,t+1}\int_0^{S_t(p)}(S_t(p) - x)dF_{t+1}(x)$ and $\delta_2(p) \doteq p - \eta_{u,t+1}(p)F_{t+1}(S_t(p))$ where $S_t(\cdot)$ is a function that satisfies (17) and (18).

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PROPOSITION 8. (i) *There exists an equilibrium* $(\mathbb{S}_1, \ldots \mathbb{S}_\tau, \bar{\mathbb{S}}, \vec{q})$ *that satisfies* (11) *and* (12). *In such an equilibrium, firms' strategies are as follows.*
(ii) *For* $t = 1, \ldots, \tau$, *each firm* j *commits to a renewable production schedule* $S_t(\cdot)$ *that satisfies* (17) *and* (18) *in the period-*t *day-ahead market, and firm* j*'s equilibrium renewable energy production in period* $t+1$ *is* $\min\{S_t(p_t^*), Q_{j,t+1}\}$.
(iii) *At* $t = 1$, *each firm's inflexible energy commitment in equilibrium is* $\bar{S} = \min\{K, \bar{s}^*\}$ *such that* $\bar{s}^*$ *is the solution of*

$$\Psi_M(\bar{s}) \doteq \sum_{t=1}^{\tau} \int_{\underline{z}_t(\bar{s})}^{\infty} -\left(\frac{S_t(p_t^*(\bar{s};z)) - \delta_1(p_t^*(\bar{s};z)) + S_t'(p_t^*(\bar{s};z))\delta_2(p_t^*(\bar{s};z))}{\alpha_t + NS_t'(p_t^*(\bar{s};z))} \right) d\Phi_t(z)$$
$$+ \sum_{t=1}^{\tau} \int_{\underline{z}_t(\bar{s})}^{\infty} \left(p_t^*(\bar{s};z) - \frac{\bar{s}}{\alpha_t + NS_t'(p_t^*(\bar{s};z))} - C_{t+1}'(\bar{s}) \right) d\Phi_t(z) + C_{t+1}(\bar{s})\phi_t(\underline{z}_t(\bar{s})) = 0, \qquad (24)$$

where $\underline{z}_t(\bar{s})$ *is as defined in* (19), $S_t(\cdot)$ *is as in Proposition 8-*(ii) *and* $p_t^*(\bar{s};z)$ *satisfies* $v_t - \alpha_t p_t^*(\bar{s};z) + z = N(\bar{s} + S_t(p_t^*(\bar{s};z)))$.

To compare the equilibrium characterized in Proposition 8 with the one identified in Proposition 2, we let $N_r = N_i = N$. Proposition 8 shows that, compared to Proposition 2, each firm's inflexible energy commitment is the only element that differs in equilibrium. Regarding other results in Section 3, the statements and the proofs of Propositions 3, 4, 5 and 7 remain unchanged in this setting. Proposition 6 holds with a larger lower bound on $S_t''(\cdot)$ in (22). As a result, the main results and insights in Section 3 hold in this setting.

Finally, the following proposition studies the implications of these multi-technology firms for inflexible energy commitments and reliability in equilibrium.

PROPOSITION 9. (i) *A multi-technology firm's equilibrium inflexible energy commitment* $($*identified in Proposition 8-*$(iii))$ *is smaller than an inflexible-only firm's equilibrium commitment* $($*identified in Proposition 2-*$(iii))$.
(ii) *For any* $t = 1, \ldots, \tau$, *a day-ahead electricity market with multi-technology firms results in lower period-*t *reliability* (10) *for any realization of period-*t *day-ahead demand shock* ϵ_t $($*i.e., with probability 1*$)$ *in equilibrium, compared to the day-ahead electricity market explained in Section 2.*

If firms have multiple diversified technologies, each restricts its inflexible energy commitment to increase the market clearing price in all periods with probability 1, thereby gaining a larger expected profit in equilibrium. As a result, when firms have multiple diversified technologies, a larger renewable energy commitment is cleared to satisfy a particular day-ahead demand in each period. This immediately implies lower equilibrium reliability in all periods with probability 1.

5. Subsidy

In the U.S., various renewable firms receive subsidies for their production. For example, U.S. wind energy producers receive a "production tax credit" for their production (DSIRE 2015). Motivated by this fact, this section extends our base model (in Section 2) to consider production-based subsidies for renewable firms.

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For any t, each renewable firm n receives a total subsidy of $T_t(q_{n,t})$ by producing $q_{n,t}$ in period t. Here, $T_t(\cdot)$ is a twice continuously differentiable and concave function that satisfies $T_t(0)=0$ and $T_t'(\cdot)>0$. All other modeling elements are the same as the ones explained in Section 2, except the following: In this setting, the optimal production quantity of renewable firm n in period t+1 is the one that minimizes the firm's realized net penalty in that period:

$$q^*_{n,t+1} \doteq \operatorname*{arg\,min}_{0\le q_{n,t+1}\le Q_{n,t+1}} \eta_{u,t+1}(p_t^*)\left(S_{n,t}(p_t^*)-q_{n,t+1}\right)^+ + \eta_{o,t+1}(p_t^*)\left(q_{n,t+1}-S_{n,t}(p_t^*)\right)^+ - T_{t+1}(q_{n,t+1}). \tag{25}$$

Note that $q^*_{n,t+1}$ is a function of $Q_{n,t+1}$, and unlike the setting in Section 2, by (25), overproduction is a potentially viable option for renewable firms due to subsidies. Based on (25), in period t, renewable firm n chooses a production schedule $S_{n,t}(\cdot)$ to maximize its commitment-related expected profit (26) for any random shock ϵ_t, given any commitment profiles $S_{-n,t}$ and $\bar{\mathbb{S}}$ of other firms:

$$\mathbb{E}_{Q_{n,t+1}}\left[p\mathcal{R}_{n,t}(p;\epsilon_t)+T_{t+1}(q^*_{n,t+1})-\eta_{u,t+1}(p)\left(\mathcal{R}_{n,t}(p;\epsilon_t)-q^*_{n,t+1}\right)^+-\eta_{o,t+1}(p)\left(q^*_{n,t+1}-\mathcal{R}_{n,t}(p;\epsilon_t)\right)^+\right]. \tag{26}$$

Accounting for these differences, our aim is to characterize firms' day-ahead commitments and production strategies in an equilibrium that satisfies (11) and (12). The analysis in this section requires considering two possible cases about the initial subsidy rate $T_t'(0)$ for each t: $b_{o,t}<T_t'(0)$ and $b_{o,t}\ge T_t'(0)$. To state our results for $b_{o,t}<T_t'(0)$, we define the critical supply level $\hat{s}_t$ as the solution of

$$T_t'(\hat{s}_t)=b_{o,t}. \tag{27}$$

For $b_{o,t}<T_t'(0)$, the attention is restricted to $T_t(\cdot)$ that ensures the uniqueness of $\hat{s}_t$. Proposition 10 below establishes the existence of a solution function $S_t(\cdot)$ to a particular set of conditions. Later, Proposition 11 will verify that $S_t(\cdot)$ characterizes each renewable firm's committed production schedule in equilibrium.

PROPOSITION 10. *Consider any $t=1,\dots,\tau$. (i) If $b_{o,t+1}<T'_{t+1}(0)$, there exists a function $S_t(\cdot)$ that satisfies the following two-piece ordinary differential equation*

$$S_t'(p)=\begin{cases}\left[\dfrac{S_t(p)-\beta_{u,t+1}\int_0^{S_t(p)}(S_t(p)-x)dF_{t+1}(x)}{p-\eta_{u,t+1}(p)F_{t+1}(S_t(p))+b_{o,t+1}\left(1-F_{t+1}(S_t(p))\right)}-\alpha_t\right]\dfrac{1}{(N_r-1)} & \text{if } S_t(p)<\hat{s}_{t+1}, \quad (28a)\\[2ex] \left[\dfrac{S_t(p)-\beta_{u,t+1}\int_0^{S_t(p)}(S_t(p)-x)dF_{t+1}(x)}{p-\eta_{u,t+1}(p)F_{t+1}(S_t(p))+T'_{t+1}(S_t(p))(1-F_{t+1}(S_t(p)))}-\alpha_t\right]\dfrac{1}{(N_r-1)} & \text{if } S_t(p)\ge\hat{s}_{t+1}, \quad (28b)\end{cases}$$

subject to a monotonicity constraint and an initial condition, respectively:

$$0<S_t'(p)<\infty \text{ for } p\in(p_{\ell,t},\infty) \quad \text{and} \quad S_t(p_{\ell,t})=0, \tag{29}$$

where $p_{\ell,t}\doteq -b_{o,t+1}$.
(ii) There exists a function $S_t(\cdot)$ that satisfies $(28b)$ subject to the following monotonicity constraint and initial condition, respectively: $0<S_t'(p)<\infty$ for $p>p_{\ell,t}$ and $S_t(p_{\ell,t})=0$ where $p_{\ell,t}\doteq -T'_{t+1}(0)$.

Recall from Proposition 2 that, in our base model, each renewable firm's committed production schedule in a period is characterized by the solution of a *single* ODE subject to certain conditions. Proposition 11 and the

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discussion at the end of Appendix K (in the Electronic Companion) establish a new structure: In stark contrast to Proposition 2, Proposition 11 proves that if renewable firms receive subsidies, the characterization of a renewable firm's committed production schedule in a period can require *more than one* ODE in equilibrium. For ease of exposition, we focus on a fixed oversupply penalty rate in Proposition 11, and explain at the end of Appendix K that the multiple-piece structure of an equilibrium committed production schedule is preserved with a market-based oversupply penalty rate.

PROPOSITION 11. (i) *There exists an equilibrium* $(\mathbb{S}_1, \ldots \mathbb{S}_\tau, \bar{\mathbb{S}}, \vec{q})$ *that satisfies* (11) *and* (12). *In such an equilibrium, firms' strategies are as follows.*
(ii) *Consider any* $t = 1, \ldots, \tau$. *If* $b_{o,t+1} < T'_{t+1}(0)$, *then each renewable firm commits to a production schedule* $S_t(\cdot)$ *that satisfies* $(28a)$ *through* (29) *in period-*t *day-ahead market, and the equilibrium production quantity of renewable firm* n *in period* $t+1$ *is*

$$q^*_{n,t+1} = \begin{cases} Q_{n,t+1} & \text{if } Q_{n,t+1} \le S_t(p^*_t), & (30a) \\ S_t(p^*_t) & \text{if } Q_{n,t+1} > S_t(p^*_t) \quad \& \quad b_{o,t+1} > T'_{t+1}(S_t(p^*_t)), & (30b) \\ \hat{s}_{t+1} & \text{if } Q_{n,t+1} > S_t(p^*_t) \quad \& \quad T'_{t+1}(Q_{n,t+1}) < b_{o,t+1} \le T'_{t+1}(S_t(p^*_t)), & (30c) \\ Q_{n,t+1} & \text{if } Q_{n,t+1} > S_t(p^*_t) \quad \& \quad b_{o,t+1} \le T'_{t+1}(Q_{n,t+1}). & (30d) \end{cases}$$

(iii) *Consider any* $t = 1, \ldots, \tau$. *If* $b_{o,t+1} \ge T'_{t+1}(0)$, *then each renewable firm commits to a production schedule* $S_t(\cdot)$ *that satisfies said conditions in Proposition 10-*(ii), *and the equilibrium production quantity of renewable firm* n *in period* $t+1$ *is* $q^*_{n,t+1} = \min\{Q_{n,t+1}, S_t(p^*_t)\}$.
(iv) *At* $t = 1$, *each inflexible firm commits to a production quantity* $\bar{S} = \min\{\bar{s}^*, K\}$ *such that* $\bar{s}^*$ *satisfies*

$$0 = \sum_{t=1}^{\tau} \left[\int_{\underline{z}_t(\bar{s})}^{\infty} \left(p^*_t(\bar{s}; z) - \frac{\bar{s}}{\alpha_t + N_r S'_t(p^*_t(\bar{s}; z)) \mathbb{I}\{z \ge \bar{z}_t(\bar{s})\}} - C'_{t+1}(\bar{s}) \right) d\Phi_t(z) \right] + [C_{t+1}(\bar{s}) - p_\ell \bar{s}]\, \phi_t(\underline{z}_t(\bar{s})), \tag{31}$$

where $\underline{z}_t(\bar{s}) \doteq N_i \bar{s} + \alpha_t p_\ell - v_t$, $\bar{z}_t(\bar{s}) \doteq N_i \bar{s} + \alpha_t p_{\ell,t} - v_t$, $p^*_t(\bar{s}; z)$ *is the solution of* $v_t - \alpha_t p^*_t(\bar{s}; z) + z = N_r S_t(p^*_t(\bar{s}; z)) + N_i \bar{s}$, *and* $S_t(\cdot)$ *is as identified in part* (ii) *if* $b_{o,t+1} < T'_{t+1}(0)$; *otherwise,* $S_t(\cdot)$ *is as in part* (iii).

REMARK 2. Proposition 11 demonstrates that with subsidies, each renewable firm commits to a positive production quantity over a range of negative prices.[10]

By (27), if the initial subsidy rate is large in period t+1 (i.e., $T'_{t+1}(0) > b_{o,t+1}$), the quantity space is divided into two intervals - the *subsidy-prevalent interval* $[0, \hat{s}_{t+1})$ and the *penalty-prevalent interval* $[\hat{s}_{t+1}, \infty)$ - as the subsidy rate is larger than the oversupply penalty rate if and only if the quantity is in $[0, \hat{s}_{t+1})$. Proposition 11-(ii) shows that $S_t(\cdot)$ satisfies a different ODE in each of these intervals. The difference between (28a) and (28b)

[10] A negative day-ahead market clearing price means that each firm with a positive production commitment at that price pays for its commitment. Negative electricity prices are observed in practice, especially when the market demand is low and there are production subsidies (Malik and Weber 2016, U.S. EIA 2012). U.S. EIA (2012) explains that subsidizing renewable energy is a reason for negative prices observed in practice. With production subsidies, renewable firms can profitably produce even at negative prices due to said additional revenue stream. As a result, they can profitably offer positive production quantities at negative prices in day-ahead markets, leading to a negative market clearing price when the day-ahead demand is low (that is, when either v_t or ϵ_t is low). This is consistent with the finding in Proposition 11.

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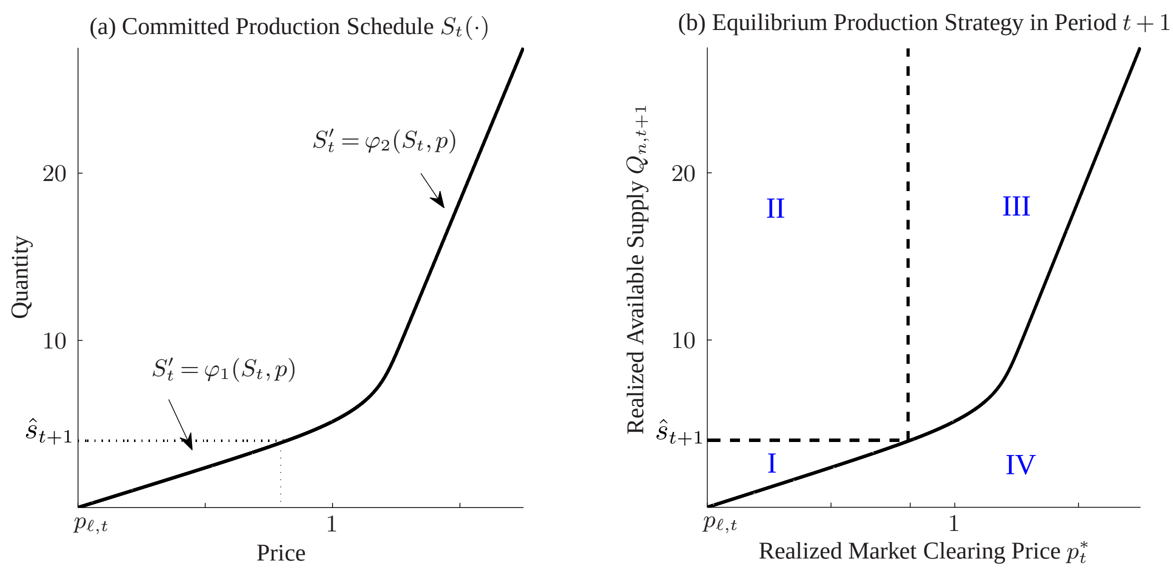

Figure 2 Panel (a) pictures $S_t(\cdot)$ characterized in Proposition 11-(ii), where $\varphi_1(\cdot,\cdot)$ and $\varphi_2(\cdot,\cdot)$ represent the mappings in (28a) and (28b), respectively, and panel (b) displays four regions identified by (30a) through (30d) with the following data: $N_r = 2$, $\alpha_t = 0.001$, $b_{u,t+1} = b_{o,t+1} = 1$, $T_{t+1}(\cdot) = 4\sqrt{\cdot}$ and $F_{t+1}(\cdot)$ is a normal distribution with mean 6 and standard deviation 1.5 truncated at 0. By Proposition 11-(ii), in equilibrium, renewable firm n produces $Q_{n,t+1}$ in region I, $\hat{s}_{t+1}$ in region II, $S_t(p_t^*)$ in region III and $Q_{n,t+1}$ in region IV.

is that the parameter $b_{o,t+1}$ in (28a) is replaced with $T'_{t+1}(S_t(p))$ in (28b). This is because when $Q_{n,t+1} \geq S_t(p)$, depending on which interval the firm's cleared commitment is in, renewable firm n gains a different benefit on top of price p per an additional unit of period-t commitment. To see this, suppose that $S_t(p) \in [0, \hat{s}_{t+1})$. In this subsidy-prevalent interval, a renewable firm optimally overproduces to benefit from the large subsidy rate. Thus, by committing to an additional unit in period t, the firm decreases its overproduction by one unit in period t+1, thereby saving the associated oversupply penalty rate $b_{o,t+1}$. On the other hand, if the firm's commitment $S_t(p)$ is in the penalty-prevalent interval $[\hat{s}_{t+1}, \infty)$, the firm's equilibrium production quantity is $S_t(p)$ in period t+1. As a result, by increasing $S_t(p)$ by one unit, the firm also increases its production in period t+1 by one unit and gains an additional benefit of $T'_{t+1}(S_t(p))$.

Proposition 11 explicitly characterizes each renewable firm's equilibrium production strategy. Accordingly, in equilibrium, a renewable firm's production quantity in period t+1 is highly dependent on the firm's committed production schedule and the day-ahead market clearing price in period t, as well as the realized available supply of the firm in period t+1. If the initial subsidy rate is sufficiently small (i.e., $T'_{t+1}(0) \leq b_{o,t+1}$), the oversupply penalty rate always dominates the subsidy rate in period t+1. This deters a renewable firm from overproduction in period t+1, as shown in Proposition 11-(iii).

If the initial subsidy rate is not too small, overproduction becomes a viable option for a renewable firm. In this case, Proposition 11-(ii) shows that the space for the day-ahead market clearing price and the available supply is divided into four regions and in each of these regions, a particular production strategy is implemented in equilibrium. Figure 2-(b) pictures these four regions (i.e., regions I through IV) for a numerical example.

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(Regions I, II, III and IV are characterized by conditions in (30d), (30c), (30b) and (30a), respectively.) Note that overproduction is a feasible strategy in regions I through III where the firm's available supply is larger than its commitment. In these regions, the firm faces a tradeoff between paying an oversupply penalty versus receiving a subsidy in evaluating its overproduction option. In region I, the firm optimally produces all of its available supply because the firm's available supply is in the subsidy-prevalent interval $[0, \hat{s}_{t+1})$ and underproduction is never optimal due to the undersupply penalty. In contrast, in regions II and III, the firm's available supply is in the (oversupply) penalty-prevalent interval $[\hat{s}_{t+1}, \infty)$. In region II, the firm optimally produces $\hat{s}_{t+1}$ because it is suboptimal to underproduce and $\hat{s}_{t+1}$ is the largest quantity at which the subsidy rate (weakly) dominates the oversupply penalty rate. In region III, the firm optimally produces its day-ahead commitment because underproduction is never optimal and the oversupply penalty rate dominates the subsidy rate at any production quantity in that region. In region IV, the firm's available supply is smaller than the firm's commitment. Thus, to minimize the undersupply penalty, the firm optimally produces the maximum feasible quantity, which is its available supply.

Regarding other results in Section 3, proofs of Propositions 4 through 6 extend to this setting in a straightforward fashion, meaning that our insights in Section 3 hold with subsidies. Specifically, the uniform inflation of $S_t(\cdot)$ with a larger $\eta_{u,t+1}(\cdot)$ (in Proposition 4), the period-t reliability degradation with probability 1 due to a larger $\eta_{u,t+1}(\cdot)$ (in Proposition 5) occur if (21) and the fixed oversupply penalty rate is not too large. Furthermore, under aforementioned conditions, Proposition 6 holds as stated.

The discussion at the end of Appendix K (of the Electronic Companion) explains that with a market-based oversupply penalty rate, the aforementioned insights related to the commitment and production strategies of a renewable firm remain the same, except with a market-based oversupply penalty rate, the critical supply $\hat{s}_{t+1}$ is a decreasing function of price rather than a constant. Therefore, in this case, the lower boundary of region II is a decreasing curve rather than a flat line.

In this section, we provided an analysis of general concave subsidy functions. It is perhaps worth noting that the analysis of a linear subsidy is a special case of said analysis.

6. Numerical Example

This section presents a numerical example based on MISO's day-ahead electricity market. The purpose of the numerical example is to demonstrate the following two insights: (i) even a slight change in the undersupply penalty rate can have a large impact on firms' equilibrium commitments (and hence reliability), and (ii) unexpected consequences of a larger undersupply penalty rate for firms' commitments and reliability can be observed at plausible prices when the parameters are set to realistic values. Below, we first explain how these realistic values are obtained for parameters of interest. Then, using these parameters, we solve for the equilibrium characterized in Proposition 2 under two alternative penalty schemes.

In MISO's day-ahead electricity market, a day is divided into 24 time blocks: 12:00am - 1:00am, 1:00am - 2:00am,. . . , 11:00pm - 12:00am. For any given time block, a firm can commit to a production schedule in the

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day-ahead market to deliver electricity in the given time block of the following day. The day-ahead market is cleared for each time block, resulting in 24 pairs of MISO system day-ahead electricity prices and total cleared production commitments.

In MISO system, there are $N_i = 45$ major inflexible firms (U.S. EIA 2014*b*). Using the U.S. fuel price data set (U.S. EIA 2014*a*) and the U.S. power plant operations database (U.S. EIA 2014*b*), we calculate monthly total cost and monthly total electricity generation of each inflexible firm. Assuming that the variable cost of production for each inflexible firm in month $m = 1, \ldots, 12$ is $c_m q^2$ at output q, where c_m is the variable cost coefficient in month m, we estimate c_m by running an ordinary least squares regression between the monthly total fuel cost and the monthly total electricity generation data for these 45 major inflexible firms.

MISO's rule on "dispatchable intermittent resources (DIR)," which requires all major wind producers to register as a dispatchable intermittent resource and actively participate in electricity markets, was not in full effect at the beginning of 2013 (MISO 2014*a*). Thus, to estimate the price sensitivity parameter α for each time block in every month of 2013, we assume that the day-ahead electricity prices in 2013 are driven by N_i inflexible firms (each of which commit to a particular production quantity that remains the same for a certain time) and $N_c = 25$ other conventional firms that can commit to a different production schedule in every day-ahead electricity market (U.S. EIA 2014*b*).(It is perhaps worth noting that in 2013, the generation by the latter type of firms was only around 10% of the generation by inflexible firms in MISO system (MISO 2015*a*).) Firms' equilibrium commitment strategies in this market are characterized by (EC.113) and (EC.114) in the Electronic Companion. Based on these, we find the estimate $\widehat{\alpha}_{m,\ell}$ of the true sensitivity parameter $\alpha_{m,\ell}$ for each month $m = 1, \ldots, 12$ and time block $\ell = 1, \ldots, 24$ by using a method that minimizes residual sum of squares for every time block of each month. Details of this method can be found in Appendix O of the Electronic Companion. For instance, as a result of the aforementioned procedure, we have $\widehat{\alpha}_{5,10} = 0.0043$ for the 9:00am - 10:00am time block in May.

The cumulative probability distribution $F_{m,\ell}(\cdot)$ of a wind producer for each time block $\ell = 1, 2, \ldots, 24$ in month $m = 1, 2, \ldots, 12$ is identified as follows. MISO publishes the hourly wind generation data on a daily basis (MISO 2015*c*). The wind generation is usually smaller than the wind power potential due to curtailment. The MISO data do not include the curtailment percentages. Hence, we estimate the monthly curtailment percentages in 2013 using the curtailment data in (Ruud 2013) and (Wiser and Bolinger 2014). Using the estimated curtailment percentages, actual hourly wind generation data throughout 2013 (MISO 2015*c*) and the number of wind power producers in MISO system (U.S. EIA 2014*b*), we fit a Lomax distribution to estimate the wind power potential for each time block in each month, and estimate the scale and shape parameters via maximum likelihood estimation.

The penalty rules and rates vary from market to market. Using the explained parameter estimates for May, we now consider the market explained in Section 2 with $N_r = 25$ renewable firms and $N_i = 45$ inflexible firms, and solve for the equilibrium identified in Proposition 2 under the following two alternative rules for the

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undersupply penalty rate. Under the first rule (i.e., rule A), the undersupply penalty rate at day-ahead price p is $\eta_A(p) = \$(p+0.047)$/MWh, which is the day-ahead price p plus the average fixed penalty rate in May (MISO 2016*a*). With the rule A, a firm receives the day-ahead payment only based on its actual delivery. Under the second rule (i.e., rule B), the undersupply penalty rate is equal to the real-time price, which is estimated as $\eta_B(p) = \$(0.962p+0.225)$/MWh for the time block 9:00am - 10:00am in May. The average MISO day-ahead market clearing price in 2013 was \$31.94/MWh (MISO 2014*c*). Thus, in equilibrium characterization, we restrict attention to a plausible range of day-ahead prices where p is in between \$0/MWh and \$150/MWh under each of the undersupply penalty rules A and B. (An example of a renewable firm's equilibrium committed production schedule under penalty rule B is included at the end of Appendix O in the Electronic Companion.)

Note that penalty coefficients are very close to each other under rules A and B in the time block 9:00am - 10:00am of May. Specifically, the fixed penalty terms under two rules differ only by \$0.18 per MWh (which is around 0.5% of the average market clearing price in 2013), and the change in the weight assigned to day-ahead price is less than 4%. Such close penalty coefficients also imply very close penalty rates. For instance, at the 2013 average market clearing price $p = \$31.94$/MWh, $\eta_A(31.94) - \eta_B(31.94) =$ \$1.04/MWh, which is just 3% of $\eta_A(31.94) =$ \$31.99/MWh. Despite such small differences, renewable firms' equilibrium committed production schedules are considerably different under the rules A and B. Starting from very small prices (i.e., $p \geq \$4$ per MWh), each renewable firm's equilibrium commitment under the rule A is more than 15% larger, relative to the one under the rule B.

In fact, average penalty coefficients in May also do not differ too much with respect to the penalty rule. However, because renewable firms' commitments are very sensitive to the penalty rule, inflexible firms' equilibrium commitments are also (indirectly) very sensitive to the penalty rule. Assuming τ is one month, an inflexible firm's equilibrium production commitment is 900 MW under rule A whereas the corresponding figure under rule B is 920 MW, which is around a 22% increase from the former commitment. The increase in the electricity production by all inflexible firms due to switching from rule A to B is sufficient to meet more than 991,000 residential customers' 2014 electricity demand in Michigan (U.S. EIA 2015). This and the large difference in renewable firms' equilibrium commitments under rules A and B demonstrate that the penalty rule can have a drastic effect on equilibrium production commitments in day-ahead markets.

The unintended consequences of a larger undersupply penalty scheme for reliability and renewable firms' commitments are prevalent in the time block 9:00am - 10:00am of May. Note that $\eta_A(p) > \eta_B(p)$ for $p > \$4.7$ per MWh. However, based on our observation above, the equilibrium commitment of each renewable firm under η_A is strictly larger than the one under η_B for $p > \$4.7$ per MWh in said time block. These imply that for a wide range of prices, the production commitment of each renewable firm is strictly larger under the rule that results in a higher undersupply penalty rate, as suggested by Proposition 7. Furthermore, despite that $\eta_A(\cdot)$ is strictly larger than $\eta_B(\cdot)$ for $p > \$4.7$ per MWh, penalty rule A results in strictly lower equilibrium reliability than penalty rule B for any nonnegative random shock ϵ in said time block. (Such random shocks imply an

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equilibrium market clearing price larger than \$4.7 per MWh under both penalty rules in the time block 9:00am - 10:00am of May.) Because rule A results in strictly lower reliability in such a wide range of ϵ, it also results in strictly lower expected reliability than rule B in said time block.

7. Further Extensions

In the electronic companion, Sections EC.1.1 through EC.1.4 analyze various variants of the model in Section 2, and show that the main results and insights developed in Section 3 are robust.

Recall from Section 2 that renewable firm n's available supply $Q_{n,t+1}$ in period t+1 can be seen as a period-(t+1) capacity constraint that is unobservable to firm n in period t. The formulation in Section 2 allows for a cumulative probability distribution $F_{t+1}(\cdot)$ such that $Q_{n,t+1}$ is smaller than a finite number with almost probability 1. Section EC.1.1 analyzes a setting where the available supply $Q_{n,t+1}$ is smaller than a finite constant with probability 1. Proposition EC.1 in Section EC.1.1 characterizes the equilibrium in this setting, and the last paragraph of Section EC.1.1 explains that the main results and insights in Section 3 extend to this setting.

Section EC.1.2 studies a model where the penalty rates in period t are also affected by the realization of $\sum_{n=1}^{N_r} Q_{n,t}$ for all t. Proposition EC.2 in Section EC.1.2 identifies the equilibrium in this extended setting. The last paragraph of Section EC.1.2 explains that the main results and insights in Section 3 hold in this setting, as well.

Section EC.1.3 explains that all results and proofs hold as stated if some of the renewable firms sell their supply through long term contracts. Finally, Section EC.1.4 allows renewable firms to receive revenue for their overproduction, and establishes that the main results and insights in Section 3 hold for a more general set of conditions in this extension (see Propositions EC.3 through EC.5 for formal statements in Section EC.1.4).

8. Discussions and Future Research

In this paper, we consider a day-ahead market that consists of both conventional firms such as coal-based or nuclear power plants, which are inflexible in their production commitments, and renewable firms, which can change their commitments in every day-ahead market. One of our main insights is that increasing and imposing a market-based undersupply penalty rate, which is common in many electricity markets in the U.S., can result in commitment inflation by renewable firms and expected reliability degradation in markets. If the relative manipulation power of renewable firms is large (i.e., when the competition among renewable firms is not intense), this effect is observed severely: As a response to imposing or increasing a market-based undersupply penalty rate, each renewable firm *uniformly* inflates its production schedule, and that can result in strictly lower reliability in all periods with probability 1. Even if there is intense competition among renewable firms, Proposition 7 and our numerical example based on the MISO's day-ahead market show that a larger market-based penalty rate can lead to partially inflated commitments by renewable firms, and strictly *smaller* expected reliability in the associated day-ahead market. In light of these findings, we conjecture that if other types

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of firms (such as flexible conventional firms) are also considered in the market, under similar conditions, expected reliability will still decrease due to an increase in the current market-based penalties, and intuitively, the resulting expected reliability degradation due to imposing or increasing the market-based undersupply penalty rate would be smaller because of the intensified competition in the market.

Our findings on reliability have implications for black-out probability. The black-out probability in period $t+1$, which is defined as the probability that the total actual supply fails to meet the actual electricity demand in period $t+1$, is a decreasing function of the period-t reliability (10) for any given state of the system in period $t+1$. Thus, all our results related to period-t reliability can also be written in terms of the period-$(t+1)$ black-out probability using the observation that if one increases, the other decreases.

Recall that, in our formulation, the available supply of each renewable firm has the same distribution function; that property is necessary for the analytical tractability as the explicit analysis of an asymmetric SFE with general supply functions is well known to be intractable (see Section 1.2 for a discussion). However, our extensive numerical study suggests that the unexpected consequences of the undersupply penalty rate (in Propositions 4, 5 and 6) extend even if renewable firms are heterogenous with respect to the distribution functions of their available supplies.

This paper analyzes the current practice in electricity markets. Our finding that a market-based undersupply penalty rate can defeat its purpose by reducing the reliability leads to an interesting market design question for future research: In the long-run, what is the penalty design that eliminates the aforementioned commitment inflations? Or, what is the penalty rule that maximizes the reliability? Proposition 6 suggests that for markets in which severe reliability degradation (as in Proposition 6) is possible, imposing only the fixed part of the undersupply penalty rate results in strictly higher reliability in all periods with probability 1. Then, how does an increase in the fixed penalty rate affect the reliability and the renewable firms' commitments? Proposition EC.10 shows that, perhaps interestingly, the aforementioned uniform inflation behavior never occurs with an increase in the fixed undersupply penalty rate. Thus, one way to eliminate the *uniform* commitment inflation by renewable firms is to use a fixed undersupply penalty rate, which does not depend on the market price. Furthermore, the vast majority of our numerical examples show that imposing or increasing the fixed undersupply penalty rate results in *strictly lower* commitment by renewable firms for any potential day-ahead market clearing price, and larger reliability with probability 1. Thus, we conjecture that imposing or increasing a fixed undersupply penalty rate is more effective in improving the reliability than a market-based penalty rate.

Another interesting future research direction could be to identify the penalty rule that maximizes social welfare. The social welfare formulation requires considering payoffs of all related parties, including all market participants, the ISO, and end-consumers. ISOs are revenue-neutral non-profit organizations (PJM 2017, FERC 2015, MISO 2014*b*). This means that if an ISO collects any surplus or revenue from the market (e.g., in the form of a penalty or a fraction of price), it has to distribute that surplus back to the relevant parties in the market (Sioshansi 2008). Thus, it is sufficient to consider payoffs of market participants and end-consumers

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in the social welfare formulation. Because the revenue of a generating firm is the cost of a buying firm, the social welfare can be simplified to the consumer surplus from electricity consumption minus the sum of the total monetary value of damage due to reliability issues and the production cost of electricity. We conjecture that if the monetary value of damage due to reliability issues is sufficiently large, the reliability should have a dominating effect on social welfare. Thus, the penalty rule that maximizes the reliability would also be the one that maximizes social welfare. If that monetary value is relatively small, the penalty rule that maximizes consumer surplus minus total cost of production would be the one that maximizes social welfare. If it is the latter, then a market-based penalty rate could be the one that maximizes social welfare, as the uniform or partial commitment inflation by renewable firms may help reduce the reliance on expensive conventional resources with a positive probability.

Acknowledgments

The authors thank Professor Serguei Netessine, anonymous Associate Editor and three referees for their valuable feedback. The authors acknowledge the support of UNC Kenan-Flagler Business School and the University of Chicago Booth School of Business in this research.

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Electronic Companion for "Strategic Commitment to a Production Schedule with Uncertain Supply and Demand: Renewable Energy in Day-Ahead Electricity Markets"

This supplementary material consists of two main parts. Section EC.1 includes further extensions mentioned in Section 7. Section EC.2 includes the proofs of all propositions and supplementary results stated in Sections 3 through 5 and Sections EC.1.1 through EC.1.4. Section EC.2 also provides more details about Section 6.

EC.1. Further Extensions and Discussions

EC.1.1. Constrained Capacity for Renewable Firms

As explained earlier, the available supply $Q_{n,t}$ can be seen as the realized capacity of renewable firm n in period t because firm n cannot produce more than $Q_{n,t}$ in period t. Recall that $Q_{n,t}$ has a cumulative distribution function $F_t(\cdot)$ and density function $f_t(\cdot)$ for $n \leq N_r$. Note that Section 2 allowed $f_t(\cdot)$ to be arbitrarily small for large realizations of $Q_{\cdot,t}$, implying that probability of available supply to be larger than a finite number could be arbitrarily close to zero. This section studies a variant of the model in Section 2 to consider a constrained support for $F_t(\cdot)$ for all t. Specifically, suppose that $F_t : [0, \xi_t] \to [0, 1]$ for all t where ξ_t is a positive constant. This means that each renewable firm's realized capacity in period t is smaller than ξ_t. Other than this element, all modelling elements are the same as the ones explained in Section 2. This suggests that every renewable firm is allowed to choose any twice differentiable function as its committed production schedule in each day-ahead market.

PROPOSITION EC.1. (i) *There exists an equilibrium* $(\mathbb{S}_1, \ldots \mathbb{S}_\tau, \bar{\mathbb{S}}, \vec{q})$ *that satisfies* (11) *and* (12). *In such an equilibrium, firms' strategies are as follows.*
(ii) *For* $t = 1, \ldots, \tau$, *in period-*t *day-ahead market, there exists a constant* $\bar{p}_t > 0$ *such that each renewable firm commits to a production schedule* $S_t(\cdot)$ *that satisfies*

$$S_t'(p) = \begin{cases} \dfrac{1}{N_r - 1}\left[\dfrac{S_t(p) - \beta_{u,t+1}\int_0^{S_t(p)}(S_t(p) - x)dF_{t+1}(x)}{p - (b_{u,t+1} + \beta_{u,t+1}p)F_{t+1}(S_t(p))} - \alpha_t\right] & \text{if } p < \bar{p}_t, \quad \text{(EC.1a)} \\[2ex] \dfrac{1}{N_r - 1}\left[\dfrac{S_t(p) - \beta_{u,t+1}\int_0^{\xi_{t+1}}(S_t(p) - x)dF_{t+1}(x)}{(1 - \beta_{u,t+1})p - b_{u,t+1}} - \alpha_t\right] & \text{if } p \geq \bar{p}_t, \quad \text{(EC.1b)} \end{cases}$$

subject to $S_t'(p) \in (0, \infty)$ *and* $S_t(0) = 0$. *The equilibrium production quantity of renewable firm* n *in period t+1 is* $q^*_{n,t+1} = \min\{S_t(p), Q_{n,t+1}\}$.
(iii) *At* $t = 1$, *each inflexible firm commits to* $\min\{K, \bar{s}^*\}$ *where* $\bar{s}^*$ *satisfies* (20) *and* $S_t(\cdot)$ *is as identified in Proposition EC.1-*(ii).

Proposition EC.1 shows that each renewable firm n considers ξ_{t+1} in choosing its period-t committed production schedule. The equilibrium structure identified in Proposition EC.1 is similar to the one in Proposition 11. Specifically, each renewable firm's period-t committed production schedule $S_t(\cdot)$ is characterized by two

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ordinary differential equations. To form a period-t equilibrium, the solutions of these two differential equations must be smoothly pasted at $p=\bar{p}_t$, that is, both the values and the first derivative of these solutions must match at $p=\bar{p}_t$.

The main results in Section 3 extend to this setting: Propositions 4 through 6 hold as stated, with the exception that $S_t(\cdot)$ in Propositions 4 through 6 must be replaced with $S_t(\cdot)$ characterized in Proposition EC.1-(ii). The proof of Proposition 4 in this setting is included at the end of Appendix L. The proofs of Propositions 5 and 6 remain the same, except $S_t(\cdot)$ in those proofs must refer to $S_t(\cdot)$ identified in Proposition EC.1-(ii).

EC.1.2. $\sum_{n=1}^{N_r} Q_{n,\cdot}$-Sensitive Penalty Rates

This section extends the original model in Section 2 to consider penalty rates that are sensitive to the realized total available supply of renewable firms. This extension is relevant to settings where the undersupply and oversupply penalty rates in period $t+1$ are linked to the realized price in another electricity market which is related to the period-t day-ahead market and affected by renewable firms' total available supply in period t+1. Formally, suppose that the undersupply and oversupply penalty rates in period t are as follows, respectively:

$$\tilde{\eta}_{u,t}(p)=\beta_{u,t}p+b_{u,t}-\gamma_{u,t}\sum_{n=1}^{N_r}Q_{n,t} \quad \text{and} \quad \tilde{\eta}_{o,t}(p)=\beta_{o,t}p+b_{o,t}-\gamma_{o,t}\sum_{n=1}^{N_r}Q_{n,t}, \tag{EC.2}$$

where $\gamma_{u,t}>0$ and $\gamma_{o,t}$ are non-zero constants. Based on (EC.2), if the realization of $\sum_{n=1}^{N_r} Q_{n,\cdot}$ is low in a period, each firm must pay a larger undersupply penalty rate in that period. To avoid any inconsistent equilibrium in which penalty rates are negative for some $p\geq 0$, we focus on a setting in which the support of $F_t(\cdot)$ is $[0,\xi_t]$ for all t where $\xi_t<\min\{b_{u,t}/\gamma_{u,t}, b_{o,t}/|\gamma_{o,t}|\}/N_r$. For tractability, we assume that the available supply of each renewable firm is independent from others' available supplies.

PROPOSITION EC.2. (i) *There exists an equilibrium* $(\mathbb{S}_1,\ldots\mathbb{S}_\tau,\bar{\mathbb{S}},\vec{q})$ *that satisfies* (11) *and* (12). *In such an equilibrium, firms' strategies are as follows.*

(ii) *For* $t=1,\ldots,\tau$, *in period-t day-ahead market, there exists a constant* $\bar{p}_t>0$ *such that each renewable firm commits to a production schedule* $S_t(\cdot)$ *that satisfies*

$$S_t'(p)=\begin{cases}\dfrac{1}{N_r-1}\left[\dfrac{S_t(p)-\beta_{u,t+1}\int_0^{S_t(p)}(S_t(p)-x)dF_{t+1}(x)}{p-(b_{u,t+1}+\beta_{u,t+1}p)F_{t+1}(S_t(p))+\chi(S_t,\mu_{t+1})\gamma_{u,t+1}}-\alpha_t\right] & \text{if } p<\bar{p}_t,\\ \dfrac{1}{N_r-1}\left[\dfrac{S_t(p)-\beta_{u,t+1}\int_0^{\xi_{t+1}}(S_t(p)-x)dF_{t+1}(x)}{p-(b_{u,t+1}+\beta_{u,t+1}p)+\gamma_{u,t+1}N_r\mu_{t+1}}-\alpha_t\right] & \text{if } p\geq\bar{p}_t,\end{cases} \tag{EC.3}$$

subject to $S_t'(p)\in(0,\infty)$ *and* $S_t(0)=0$ *where* $\chi(S_t,\mu_{t+1})\doteq\int_0^{S_t(p)}(x+(N_r-1)\mu_{t+1})dF_{t+1}(x)$. *The equilibrium production quantity of renewable firm* n *in period* t+1 *is* $q^*_{n,t+1}=\min\{S_t(p),Q_{n,t+1}\}$.

(iii) *At* $t=1$, *each inflexible firm commits to a production quantity* $\bar{S}=\min\{K,\bar{s}^*\}$ *such that* $\bar{s}^*$ *satisfies* (20) *and* $S_t(\cdot)$ *is as identified in Proposition EC.2-*(ii).

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In this setting, the main results and insights in Section 3 hold. Specifically, Propositions 4 through 6 hold as stated. The proof of Proposition 4 when the penalty rates are as defined in (EC.2) can be found in Appendix M. The proofs of Propositions 5 and 6 remain unchanged, except $S_t(\cdot)$ in those proofs must refer to $S_t(\cdot)$ identified in Proposition EC.2-(ii).

EC.1.3. The Existence of Power Purchase Agreements

In the U.S., power purchase agreement (PPA) is an important tool in renewable energy investment. Under a PPA, a buyer (such as a corporation or a utility) signs up a long term contract with a renewable power producer, and the buyer owns the entire available supply of the renewable power producer for the contract duration (Windustry 2007). Based on this, consider a variant of our base model where in addition to $N_r + N_i$ firms, there are $N_p \geq 1$ renewable power producers that sell their available supply through PPAs and buyers use this procured energy to satisfy their electricity demands. Then, given any set of PPAs with $j = 1, \ldots, N_p$ renewable firms, all propositions in Section 3 hold as stated. The proofs of these results remain unchanged with the exception that period-t day-ahead demand defined in Section 2 must be replaced with the period-t day-ahead demand net of the total expected available supply of N_p renewable firms in period t+1. Specifically, v_t must be replaced with $\tilde{v}_t \doteq v_t - \sum_{j=1}^{N_p} \mathbb{E}[\tilde{Q}_{j,t+1}]$, where $\tilde{Q}_{j,t+1}$ is the available supply of a renewable firm j that sells under a PPA in period t+1. This is because at the time of day-ahead market clearing in period t, it is known that $\sum_{j=1}^{N_p} \mathbb{E}[\tilde{Q}_{j,t+1}]$ of the demand is satisfied through PPAs in period t+1. Thus, there is no need to schedule supply commitment for that amount in the period-t day-ahead market.

EC.1.4. Credit-dominated Market

Section 3 analyzed a day-ahead market where renewable firms pay an oversupply penalty for their overproduction. This type of market will be called a *penalty-dominated market* (regarding overproduction). This section studies a day-ahead market where renewable firms gain revenue for their excess production and the revenue rate per unit overproduction is larger than the oversupply penalty rate, implying a positive net revenue per unit overproduction. This net revenue per unit overproduction is called the *credit rate* and this type of market is referred to as the *credit-dominated market*. A renewable firm might receive such additional revenues for example if it sells its excess production to a buyer at a price linked to the day-ahead price or at the realized price in another electricity market that is related to the day-ahead market. We formalize a credit-dominated market as follows.

If renewable firm n's cleared commitment in period t is strictly smaller than its production quantity in period t+1 (i.e., $S_{n,t}(p_t^*) < q_{n,t+1}$), the firm receives a credit rate $c_{o,t+1}(p_t^*)$ per unit overproduction in period t+1, where $c_{o,t+1}(p) \doteq \beta_{c,t+1} p + b_{c,t+1}$, $b_{c,t+1} \geq 0$ and $\beta_{c,t+1} \in (0,1)$. The constants $\beta_{c,t+1}$ and $b_{c,t+1}$ also satisfy $b_{c,t+1} \leq b_{u,t+1}$ and $\beta_{c,t+1} \leq \beta_{u,t+1}$, implying that the credit rate cannot exceed the undersupply penalty rate at any price. For settings in which the credit rate is linked to the realized price in another electricity market that is related to the day-ahead market (such as a balancing market), $\beta_{c,t+1}$ can be interpreted as a measure of

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dependence between the period-t day-ahead market clearing price and the price based on which the credit rate is calculated in period t+1.

After observing its available supply $Q_{n,t+1}$, each renewable firm n chooses a production quantity $q_{n,t+1}$ in period t+1 to minimize its total realized net penalty in that period:

$$\min_{0\le q_{n,t+1}\le Q_{n,t+1}} \eta_{u,t+1}(p_t^*)\left(S_{n,t}(p_t^*)-q_{n,t+1}\right)^+ - c_{o,t+1}(p_t^*)\left(q_{n,t+1}-S_{n,t}(p_t^*)\right)^+ . \tag{EC.4}$$

Based on (EC.4), in this setting, (4) must be replaced with the following expression:

$$\mathbb{E}_{Q_{n,t+1}}\left[p\mathcal{R}_{n,t}(p;\epsilon_t)-\eta_{u,t+1}(p)\left(\mathcal{R}_{n,t}(p;\epsilon_t)-q^*_{n,t+1}\right)^+ + c_{o,t+1}(p)\left(q^*_{n,t+1}-\mathcal{R}_{n,t}(p;\epsilon_t)\right)^+\right], \tag{EC.5}$$

where $q^*_{n,t+1}$ is a function of $Q_{n,t+1}$ by (EC.4). Other than (EC.4) and (EC.5), all modeling elements are the same as in Section 2. (In this section, inflexible firms do not receive revenue for their overproduction, which is consistent with the practice (MISO 2016*b*). In fact, if inflexible firms are allowed to receive credit for their overproduction, an equilibrium in which $\bar{S}_k = 0$ and $q^*_{k,t+1} = K$ for all k and t emerges, which is unrealistic (MISO 2015*b*). To restrict attention to a realistic equilibrium, for inflexible firms, we consider the same setting as in Section 2.) In this setting, the following proposition establishes the existence of an equilibrium and characterizes firms' commitment and production strategies in that equilibrium.

PROPOSITION EC.3. (i) *There exists an equilibrium that satisfies* (11) *and* (12). *In such an equilibrium, firms' strategies are as follows.*
(ii) *For* $t=1,\ldots,\tau$, *in a period-*t *day-ahead market, each renewable firm* n *commits to a production schedule* $S_t(\cdot)$ *that satisfies the following ordinary differential equation*

$$S_t'(p)=\left[\frac{S_t(p)-\beta_{u,t+1}\int_0^{S_t(p)}(S_t(p)-x)dF_{t+1}(x)+\beta_{c,t+1}\int_{S_t(p)}^{\infty}(x-S_t(p))dF_{t+1}(x)}{p-\eta_{u,t+1}(p)F_{t+1}(S_t(p))-c_{o,t+1}(p)(1-F_{t+1}(S_t(p)))}-\alpha_t\right]\lambda_r, \tag{EC.6}$$

subject to a monotonicity constraint and an initial condition, respectively,

$$0<S_t'(p)<\infty,\ \ p\in(p_{\ell,t},\infty), \quad \textit{and} \quad S_t(p_{\ell,t})=0, \tag{EC.7}$$

where $p_{\ell,t}\in\Big(b_{c,t+1}/(1-\beta_{c,t+1}),(\alpha_t b_{c,t+1}+\beta_{c,t+1}\mu_{t+1})/((1-\beta_{c,t+1})\alpha_t)\Big)$ *and* $\lambda_r \doteq 1/(N_r-1)$. *Furthermore, renewable firm* n*'s equilibrium production quantity in period* t+1 *is* $q^*_{n,t+1}=Q_{n,t+1}$.
(iii) *At* $t=1$, *each inflexible firm commits to a production quantity* $\bar{S}=\min\{\bar{s}^*,K\}$ *such that* $\bar{s}^*$ *satisfies* (20) *where* $S_t(\cdot)$ *is as identified in Proposition EC.3-*(ii).

By Proposition EC.3-(ii), each renewable firm optimally produces all its available supply because of the positive credit rate per unit overproduction. Moreover, the minimum price $p_{\ell,t}$ at which each renewable firm commits to a positive quantity is positive. The reason is as follows. At zero price, zero commitment is optimal for a renewable firm because it does not receive any revenue for its commitment but receives credit rate for each unit of overproduction. Consider any positive price that is arbitrarily close to zero. By continuity of a committed

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production schedule, at such a small price, the firm's commitment (if any) must be sufficiently small and hence the probability of overproduction must be very close to 1. Therefore, by increasing the period-t commitment by one unit, the firm decreases its overproduction by about one unit, thereby forfeiting approximately the credit rate in period t+1. Because the credit rate dominates the marginal benefit of unit commitment at very small prices, the firm optimally commits to zero quantity at such small prices in period t, overproducing all its available supply in period t+1.

Proposition EC.8 (at the end of Appendix N in the Electronic Companion) compares the equilibrium commitments and reliability in credit- versus penalty-dominated markets. Specifically, it shows that compared to a penalty-dominated market, a credit-dominated market results in less commitment by renewable firms for prices not extremely large. Thus, for any given commitment profile of inflexible firms, a credit-dominated market results in higher period-t reliability than a penalty-dominated market when the period-t day-ahead random shock ϵ_t is not extremely large. (The proof of Proposition EC.8 can be found in Appendix N.)

Regarding our original reliability results in Section 3, Proposition 7 holds as stated in a credit-dominated market. (The proof of this statement can be found in Appendix N.) Proposition EC.4 below extends Propositions 4 through 6 in a credit-dominated market by considering any two period-t equilibria with different $\beta_{u,t+1}$ and a particular $p_{\ell,t}$. Proposition EC.4 establishes that Propositions 4 through 6 hold in a more general setting when renewable firms receive a credit rate for overproduction.

PROPOSITION EC.4. *Consider any $t = 1, \ldots, \tau$. Then, increasing $\beta_{u,t+1}$ or imposing a market-based undersupply penalty rate (in addition to the fixed rate $b_{u,t+1}$) results in (i) a uniformly and strictly larger period-t committed production schedule for each renewable firm in any equilibrium (i.e., $\partial S_t(p;\beta_{u,t+1})/\partial\beta_{u,t+1} > 0$ for $p > 0$ and $\beta_{u,t+1} \in [0,1)$), and (ii) strictly lower period-t reliability with probability 1 in any equilibrium with any given $\bar{\mathbb{S}}$. Furthermore, (iii) increasing $\beta_{u,t+1}$ results in lower reliability in* all *periods with probability 1 if either $K < K_{L,c}$ or the following set of conditions hold for a particular $\gamma_c > 0$: $K > K_{H,c}$, $\mathbb{E}_{\epsilon_t}\left[\sqrt{\partial S_t(p_t^*(\tilde{s}_c))/\partial\beta_{u,t+1}}|\epsilon_t \geq \underline{z}_t(\tilde{s}_c)\right] > \gamma_c$ and $S_t''(\cdot) > -\alpha_t^2/(N_r K)$, where constants $\tilde{s}_c$, $K_{L,c}$ and $K_{H,c}$ satisfy $\tilde{s}_c \leq K$ and $0 < K_{L,c} < K_{H,c}$.*

The conditions in Propositions EC.4 are different than the ones in Propositions 4 through 6 because the (relative) manipulative power of a renewable firm on the undersupply penalty rate is different in credit- and penalty-dominated markets. Intuitively, in a credit-dominated market, the oversupply credit encourages renewable firms to commit to smaller quantities compared to a penalty-dominated market. Hence, due to smaller aggregate commitment quantities, a renewable firm's (relative) manipulative power is larger in a credit-dominated market. Proposition EC.4 formalizes that even with a large number of firms or a high constant penalty rate, a single renewable firm can still influence the market outcomes through its supply inflation decisions in a credit-dominated market due to the aforementioned larger manipulative power. On the other hand, in a penalty-dominated market, a renewable firm has such manipulative power on the undersupply penalty rate only when the number of competitors and the constant undersupply penalty rate are not extremely large.

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Proposition EC.4 analyzes the implications of a higher $\beta_{u,t+1}$ *in isolation*. Proposition EC.5 and the discussion following it provide a robustness analysis by studying the absolute values of commitment inflation and reliability degradation with a higher $\beta_{u,t+1}$ when $\beta_{c,t+1}$ is dependent on $\beta_{u,t+1}$. This analysis is relevant to settings where the market-based credit rate and the market-based undersupply penalty rate are the same or they are based on the realized prices of two related electricity markets. In the former setting, $\beta_{c,t+1} = \beta_{u,t+1}$. Therefore, $\partial\beta_{c,t+1}/\partial\beta_{u,t+1} = 1$, which corresponds to the maximum dependence between the market-based credit rate and the market-based undersupply penalty rate. On the other hand, if $\partial\beta_{c,t+1}/\partial\beta_{u,t+1} = 0$, $\beta_{c,t+1}$ is independent of $\beta_{u,t+1}$, and this corresponds to the minimum dependence between said two rates. Based on these examples, we consider a general dependence by letting $\beta_{c,t+1} = g_{t+1}(\beta_{u,t+1})$ where g_{t+1} is a differentiable function. Here, $g'_{t+1}(\beta_{u,t+1})$ can be interpreted as a measure of dependence between the market-based credit rate and the market-based undersupply penalty rate.

PROPOSITION EC.5. *For any realization of period-t day-ahead demand shock ϵ_t and in any equilibrium, the magnitude of period-t commitment inflation by each renewable firm $($due to a higher $\beta_{u,t+1})$ strictly increases with $g'_{t+1}(\beta_{u,t+1})$.*

An immediate corollary of Proposition EC.5 is the following: For any given commitment profile of inflexible firms $\bar{\mathbb{S}}$, the magnitude of period-t reliability degradation (due to a higher $\beta_{u,t+1}$) in equilibrium strictly increases with $g'_{t+1}(\beta_{u,t+1})$ with probability 1 (that is, for every realization of period-t day-ahead demand shock ϵ_t). This and Proposition EC.4-(iii) immediately imply that the aforementioned relation between period-t reliability degradation and $g'_{t+1}(\beta_{u,t+1})$ holds for any equilibrium (as in Definition 2) when the capacity of each inflexible firm is not too large. Thus, for these cases, if the market-based credit and undersupply penalty rates are more dependent, the magnitude of period-t reliability degradation (due to a higher $\beta_{u,t+1}$) is larger with probability 1. As a result, when said rates are the same, the period-t reliability degradation due to a higher $\beta_{u,t+1}$ is the maximum. (Those rates can be the same for instance if they are linked to the realized price in the same electricity market.) Such intensified reliability degradation can be mitigated by linking the credit and undersupply penalty rates to the realized prices of two less related electricity markets, thereby reducing the dependence between the market-based credit and undersupply penalty rates. In contrast to a credit-dominated market, in a penalty-dominated market, the aforementioned type of dependence between the market-based oversupply and undersupply penalty rates does not affect the magnitude of commitment inflation or reliability degradation. This is because, by Proposition 2, renewable firms that operate in a penalty-dominated market optimally choose not to overproduce. Hence, the magnitude of the oversupply penalty rate does not influence committed production schedules of renewable firms.

EC.2. Proof of All Results in Sections 3, 4, 5 and EC.1

This section includes the proofs of all formal results included in the main body of the paper and in Section EC.1 of the Electronic Companion.

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Appendix A: Proof of Lemma 1

Recall that renewable firm n chooses its period-(t+1) production quantity after ϵ_t, p and $Q_{n,t+1}$ are realized. Observe from (1) and (5) that for any given ϵ_t, $\mathcal{R}_{n,t}(p;\epsilon_t)$ at the market-clearing price p is equal to the firm n's cleared commitment in period-t day-ahead market. This and (2) imply that $q^*_{n,t+1}(p,\epsilon_t,Q_{n,t+1})$ is equal to

$$\arg\min_{0\le q_{n,t+1}\le Q_{n,t+1}} \left\{\eta_{u,t+1}(p)(\mathcal{R}_{n,t+1}(p;\epsilon_t)-q_{n,t+1})^+ + \eta_{o,t+1}(p)(q_{n,t+1}-\mathcal{R}_{n,t+1}(p;\epsilon_t))^+\right\}.$$

Then, from simple algebra, (13) follows. By replacing $q^*_{n,t+1}(p,\epsilon_t,Q_{n,t+1})$ in (4) with $\min\{Q_{n,t+1},\mathcal{R}_{n,t}(p;\epsilon_t)\}$ in (13), we have (14). $\square$

Appendix B: Proof of Proposition 1

We first show the following existence result. Then, we will prove parts (i) and (ii) of Proposition 1.

LEMMA EC.1. *For any $t=1,\ldots,\tau$, there exists a function $S_t(\cdot)$ that satisfies* (17) *subject to* (18).

Proof of Lemma EC.1: Take any $t=1,\ldots,\tau$. Let $p_{\ell,t}\doteq 0$. Define the mapping $d:[p_{\ell,t},\infty)\times\mathbb{R}_+/\{(p_{\ell,t},0)\}\to\mathbb{R}$ as

$$d(p,s)\doteq\left(\left(s-\beta_{u,t+1}\int_0^s(s-x)f_{t+1}(x)dx\right)/\left(p-\eta_{u,t+1}(p)F_{t+1}(s)\right)-\alpha_t\right)/(N_r-1). \tag{EC.8}$$

We prove our claim in three steps:

Step 1: Take any point $(\tilde{p},\tilde{s})\in[p_{\ell,t},\infty)\times\mathbb{R}_+/\{(p_{\ell,t},0)\}$. We will prove that there exists a unique trajectory $S_t(\cdot)$ that satisfies $S_t(\tilde{p})=\tilde{s}$ and (17) for $p\in[\tilde{p}-\delta,\tilde{p}+\delta]$ where $\delta>0$ is a constant. By simple algebra, we have

$$\frac{\partial d(p,s)}{\partial s}=\frac{(1-\beta_{u,t+1}F_{t+1}(s))(p-\eta_{u,t+1}(p)F_{t+1}(s))+\left(s-\beta_{u,t+1}\int_0^s(s-x)f_{t+1}(x)dx\right)\eta_{u,t+1}(p)f_{t+1}(s)}{(N_r-1)B^2(p,s)}, \tag{EC.9}$$

where $B(p,s)\doteq p-\eta_{u,t+1}(p)F_{t+1}(s)$. Suppose that $\tilde{p}-\eta_{u,t+1}(\tilde{p})F_{t+1}(\tilde{s})\neq 0$. Then, observe from (EC.9) that d is Lipschitz continuous in s on $[\tilde{s}-\hat{\delta},\tilde{s}+\hat{\delta}]$ for some constant $\hat{\delta}>0$. This and the elementary ODE theory imply that there exists a unique trajectory $S_t(\cdot)$ that satisfies $S_t(\tilde{p})=\tilde{s}$ and (17) for $p\in[\tilde{p}-\delta,\tilde{p}+\delta]$ where $\delta>0$ is a constant. Suppose now that $\tilde{p}-\eta_{u,t+1}(\tilde{p})F_{t+1}(\tilde{s})=0$. From (17), we have $p'(s)=z(p,s)\doteq 1/d(p,s)$, for $s>0$. Similarly, z is Lipschitz continuous on $[\tilde{p}-\delta,\tilde{p}+\delta]$ for some constant $\delta>0$. Therefore, there exists a unique function $p(\cdot)$ that satisfies $p(\tilde{s})=\tilde{p}$ and $p'(s)=z(p,s)$ for $s\in[\tilde{s}-\tilde{\delta},\tilde{s}+\tilde{\delta}]$ where $\tilde{\delta}>0$ is a constant.

We now introduce some notation and preliminary results that will be used in the remainder of the proof. For each $\hat{k}\ge 0$, define function $g_{\hat{k}}(\cdot)$ as

$$g_{\hat{k}}(s)\doteq\left[\left(s-\beta_{u,t+1}\int_0^s(s-x)f_{t+1}(x)dx\right)/((N_r-1)\hat{k}+\alpha_t)+b_{u,t+1}F_{t+1}(s)\right]/(1-\beta_{u,t+1}F_{t+1}(s)), \tag{EC.10}$$

for $s\in\mathbb{R}_+$. Two important functions we use below are

$$g_0(s)=\left[\left(s-\beta_{u,t+1}\int_0^s(s-x)f_{t+1}(x)dx\right)/\alpha_t+b_{u,t+1}F_{t+1}(s)\right]/(1-\beta_{u,t+1}F_{t+1}(s)),\quad s\in\mathbb{R}_+, \tag{EC.11}$$

$$g_\infty(s)\doteq\lim_{\hat{k}\to\infty}g_{\hat{k}}(s)=b_{u,t+1}F_{t+1}(s)/(1-\beta_{u,t+1}F_{t+1}(s)),\quad s\in\mathbb{R}_+. \tag{EC.12}$$

Define the set of points that lie in between the curves $g_0(\cdot)$ and $g_\infty(\cdot)$ as

$$\mathcal{D}\doteq\{(p,s):p\in(g_\infty(s),g_0(s)),s>0\}. \tag{EC.13}$$

Observe from (17) and (EC.8) that for each $(p,s)\in[p_{\ell,t},\infty)\times\mathbb{R}_+/\{(p_{\ell,t},0)\}$, the unique trajectory $S_{t,(p,s)}(\cdot)$ that passes through (p,s) (the existence and the uniqueness of which are proved above) satisfies

$$d(p,S_{t,(p,s)}(p))=S'_{t,(p,s)}(p). \tag{EC.14}$$

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Step 2: We now proceed to show that if $(\tilde{p},\tilde{s})\in\mathcal{D}$, then the unique trajectory $S_t(\cdot)$ that passes through $(\tilde{p},\tilde{s})$ (the existence and the uniqueness of which are proved in Step 1) also passes through $(p_{\ell,t},0)$ and has the following property: $0<S_t'(p)<\infty$ for $p\in(p_{\ell,t},\tilde{p}]$. To show this claim, take any $(\tilde{p},\tilde{s})\in\mathcal{D}$. For any point (p,s) that lies above g_0 or below g_∞ (i.e., $p\in(-\infty,g_\infty(s))\cup(g_0(s),\infty)$), $d(p,s)<0$, which implies that

$$S'_{t,(p,s)}(p)=d(p,S_{t,(p,s)}(p))<0, \tag{EC.15}$$

for the unique trajectory $S_{t,(p,s)}(\cdot)$ that passes through (p,s). From (EC.11) and (EC.12), it follows that $g_0(\cdot)$ and $g_\infty(\cdot)$ are strictly increasing in s. This implies that the unique trajectory $S_t(\cdot)$ passing through $(\tilde{p},\tilde{s})$ cannot intersect with neither g_0 nor g_∞ for $p\le\tilde{p}$ because if $S_t(\cdot)$ crosses either of these two functions, then $S_t(\cdot)$ either remains below g_∞ or above g_0 due to (EC.15). This and the fact that $\lim_{s\to0}g_0(s)=\lim_{s\to0}g_\infty(s)=p_{\ell,t}$ imply that $(p,S_t(p))\in\mathcal{D}$ for $p\in(p_{\ell,t},\tilde{p}]$ and the unique trajectory $S_t(\cdot)$ needs to pass through $(p_{\ell,t},0)$. (Because there exists a common local Lipschitz constant for any set of points in $\mathcal{D}$ whose price components form a finite compact price interval, the trajectory on $(p_{\ell,t},\tilde{p}]$ can be uniquely constructed in finitely many steps by using unique trajectories on price intervals of length 2δ from $\tilde{p}$ to $p_{\ell,t}$.) Observe from (EC.8) and (EC.10) that

$$d(g_{\hat{k}}(s),s)=\hat{k}\quad\text{for } s>0 \text{ and } \hat{k}\ge0. \tag{EC.16}$$

Because $g_{\hat{k}}(s)$ is strictly decreasing and continuous in $\hat{k}$ for each $s>0$, for every $(p,s)\in\mathcal{D}$, there exists a constant $\hat{k}\in(0,\infty)$ such that $p=g_{\hat{k}}(s)$. This and (EC.16) imply that $d(p,s)=d(g_{\hat{k}}(s),s)=\hat{k}\in(0,\infty)$ for $(p,s)\in\mathcal{D}$. Then, because the unique trajectory $S_t(\cdot)$ lies in between g_0 and g_∞ for $p\in(p_{\ell,t},\tilde{p}]$, it follows from (EC.14) that $d(p,S_t(p))=S_t'(p)\in(0,\infty)$ for $p\in(p_{\ell,t},\tilde{p}]$.

Step 3: It remains to prove that there exists a solution $S_t(\cdot)$ that satisfies the following three conditions: (i) the ODE in (17) for $p\in(p_{\ell,t},\infty)$, (ii) $S_t'(p)\in(0,\infty)$ for $p\in(p_{\ell,t},\infty)$, and (iii) $S_t(p_{\ell,t})=0$. Take any $\widehat{s}>0$. Consider the price interval $\mathcal{P}\doteq(g_\infty(\widehat{s}),g_0(\widehat{s}))$. We claim and show below that there does not exist a maximum price $\overline{p}$ in $\mathcal{P}$ such that the unique trajectory passing through $(\overline{p},\widehat{s})$ crosses g_∞ at some point. Similarly, there does not exist a minimum price $\underline{p}$ in $\mathcal{P}$ such that the unique trajectory passing through $(\underline{p},\widehat{s})$ crosses g_0 at some point. Therefore, the set of prices in $\mathcal{P}$ resulting in a unique trajectory that crosses g_0 is an open set in $\mathcal{P}$, and is disjoint from the set of prices in $\mathcal{P}$ resulting in a unique trajectory that crosses g_∞, which is also an open set in $\mathcal{P}$. Denoting the former set by G_0 and the latter set by G_∞, an element of G_0 is strictly higher than any element of G_∞. As a result, for each $\widehat{s}>0$, there exists a closed set $\mathcal{C}_{\widehat{s}}\subset\mathcal{P}$, which could have a single element, such that the unique trajectory $S_t(\cdot)$ passing through $(\widetilde{p},\widehat{s})$ does not ever cross either g_0 or g_∞ for $\widetilde{p}\in\mathcal{C}_{\widehat{s}}$. Because the set $\mathcal{C}_{\widehat{s}}$ is non-empty for any $\widehat{s}>0$, there exists a function $S_t(\cdot)$ that satisfies (17) for $p\in(p_{\ell,t},\infty)$. Because $S_t(\cdot)$ lies between g_0 and g_∞, $S_t'(p)\in(0,\infty)$ for $p\in(p_{\ell,t},\infty)$ and $S_t(p_{\ell,t})=0$. It only remains to prove our claim about the non-existence of a maximum price $\overline{p}$. Suppose for a contradiction that there exists a maximum price $\overline{p}\in\mathcal{P}$ such that the unique trajectory $S_{t,(\overline{p},\widehat{s})}(\cdot)$ passing through $(\overline{p},\widehat{s})$ crosses g_∞ at some point (p_1,s_1). Take a point (p_2,s_2) on g_∞ such that $s_2>s_1$ (which implies that $p_2>p_1$). We know from Step 1 and similar arguments in Step 2 that there exists a unique trajectory $S_{t,(p_2,s_2)}(\cdot)$ that passes through (p_2,s_2), and $(p,S_{t,(p_2,s_2)}(p))\in\mathcal{D}$ for $p\in(p_{\ell,t},p_2)$. This and $s_2>s_1$ imply $\overline{\overline{p}}>\overline{p}$ where $\overline{\overline{p}}\doteq\{p\in\mathcal{P}:S_{t,(p_2,s_2)}(p)=\widehat{s}\}$, which contradicts with the fact that $\overline{p}$ is the maximum price. $\square$

Proof of Proposition 1-(i): To prove part (i), we state and prove Lemmas EC.2 and EC.3. Lemma EC.2 identifies the best-response committed production schedule of a renewable firm n in period t, given all other firms' committed production schedules in that period. Lemma EC.2 will be used to prove Lemma EC.3, which identifies necessary and sufficient conditions for a period-t supply function equilibrium that satisfies (12).

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LEMMA EC.2. *Suppose that the commitment profile of inflexible firms is $\bar{\mathbb{S}}$. Given that renewable firm j's period-t production schedule $S_{j,t}(\cdot)$ satisfies conditions in (17) and (18) for $j \neq n$, the renewable firm n achieves its optimal commitment-related ex-post expected profit (with respect to period-t day-ahead demand) in period t by committing to $S_{n,t}(\cdot)$ that satisfies the following conditions:*

$$S_{n,t}(p) - \beta_{u,t+1}\left[\int_0^{S_{n,t}(p)}(S_{n,t}(p)-x)dF_{t+1}(x)\right] - \left(\alpha_t + \sum_{j\neq n}^{N_r} S'_{j,t}(p)\right)\left[p - \eta_{u,t+1}(p)F_{t+1}(S_{n,t}(p))\right] = 0, \quad \text{(EC.17)}$$

$S'_{n,t}(p) \in (0,\infty)$ for $p \in (p_{\ell,t},\infty)$, and $S_{n,t}(p_{\ell,t}) = 0$ where $p_{\ell,t} \doteq 0$.

Proof of Lemma EC.2: Recall (5) and $\Pi_n(p;\epsilon_t, S_{-n,t},\bar{\mathbb{S}})$ in (14). Take any realization ϵ of the period-t day-ahead demand shock ϵ_t. (Observe that such ϵ satisfies (16) for all $j \neq n$ by the assumption in the lemma). By taking the derivative of $\Pi_n(p;\epsilon, S_{-n,t},\bar{\mathbb{S}})$ with respect to p, we obtain the first order condition that is necessary to identify the price that maximizes $\Pi_n(p;\epsilon, S_{-n,t},\bar{\mathbb{S}})$:

$$\begin{aligned}\partial\Pi_n(p;\epsilon,S_{-n,t},\bar{\mathbb{S}})/\partial p = \mathcal{R}_{n,t}(p;\epsilon) - \beta_{u,t+1}\int_0^{\mathcal{R}_{n,t}(p;\epsilon)}(\mathcal{R}_{n,t}(p;\epsilon)-x)dF_{t+1}(x)\\ - \left(\alpha_t + \sum_{j\neq n}^{N_r} S'_{j,t}(p)\right)\left[p - \eta_{u,t+1}(p)F_{t+1}(\mathcal{R}_{n,t}(p;\epsilon))\right] = 0. \quad \text{(EC.18)}\end{aligned}$$

We will confirm the sufficiency of the first order condition by showing that $\partial^2\Pi_n(p;\epsilon,S_{-n,t},\bar{\mathbb{S}})/\partial p^2 < 0$. To show this, we first need to find $S''_{j,t}(\cdot)$ for $j\neq n$. To express $S''_{j,t}(\cdot)$ and $\partial^2\Pi_n(p;\epsilon,S_{-n,t},\bar{\mathbb{S}})/\partial p^2$ in a compact form, we define functions $\tilde{\gamma}(\cdot,\cdot)$, $A_j(\cdot)$, and $B_j(\cdot)$ for $p \geq p_{\ell,t}$ as $\tilde{\gamma}(y,p) \doteq -\eta_{u,t+1}(p)f_{t+1}(y(p))$ for $y:[p_{\ell,t},\infty)\to\mathbb{R}$, $A_j(p) \doteq S_{j,t}(p) - \beta_{u,t+1}\int_0^{S_{j,t}(p)}(S_{j,t}(p)-x)dF_{t+1}(x)$ and $B_j(p) \doteq p - \eta_{u,t+1}(p)F_{t+1}(S_{j,t}(p))$. For brevity, we drop arguments of functions in calculating $S''_{j,t}(\cdot)$. Using the fact that $S_{j,t}(\cdot)$ satisfies (17) for $j\neq n$, which is assumed in the statement of lemma, it follows from simple algebra and the observation that $S'_{j,t} = (A_j/B_j - \alpha_t)/(N_r-1)$ that

$$\begin{aligned}(N_r-1)S''_{j,t} &= (1-\eta'_{u,t+1}F_{t+1})\left[\frac{S'_{j,t}}{B_j} - \frac{A_j}{B_j^2}\right] + \frac{\eta_{u,t+1}f_{t+1}S'_{j,t}A_j}{B_j^2}\\ &= \frac{(1-\eta'_{u,t+1}F_{t+1})}{B_j}\left[-\frac{N_r-2}{N_r-1}\frac{A_j}{B_j} - \frac{\alpha_t}{N_r-1}\right] + \frac{\eta_{u,t+1}f_{t+1}S'_{j,t}A_j}{B_j^2}\\ &= -(1-\eta'_{u,t+1}F_{t+1})(N_r-2)\frac{S'_{j,t}}{B_j} - (1-\eta'_{u,t+1}F_{t+1})\frac{\alpha_t}{B_j} + \frac{\eta_{u,t+1}f_{t+1}S'_{j,t}A_j}{B_j^2}. \quad \text{(EC.19)}\end{aligned}$$

Then, taking the derivative of (EC.18) with respect to p, using (EC.19) and (17) for $j\neq n$, and observing that $\mathcal{R}'_{n,t}(p;\epsilon) = -\alpha_t - \sum_{j\neq n}^{N_r} S'_{j,t}(p)$ and $\mathcal{R}''_{n,t}(p;\epsilon) = -\sum_{j\neq n}^{N_r} S''_{j,t}(p)$ we calculate $\partial^2\Pi_n(p;\epsilon,S_{-n,t},\bar{\mathbb{S}})/\partial p^2$ as

$$-\left[1-\beta_{u,t+1}F_{t+1}(\mathcal{R}_{n,t}(p;\epsilon))\right]\left(\alpha_t + \sum_{j\neq n}^{N_r}\frac{N_r S'_{j,t}(p)}{N_r-1}\right) + \tilde{\gamma}(\mathcal{R}_{n,t},p)\left(\alpha_t + \sum_{j\neq n}^{N_r} S'_{j,t}(p)\right)^2 + \sum_{j\neq n}^{N_r}\frac{\tilde{\gamma}(S_{j,t},p)S'_{j,t}(p)A_j(p)}{(N_r-1)B_j(p)}, \quad \text{(EC.20)}$$

where $\mathcal{R}_{n,t}(p;\epsilon) = v_t - \alpha_t p + \epsilon - \sum_{j\neq n}^{N_r} S_{j,t}(p)$. Recall that $S_{j,t}(\cdot)$ satisfies conditions in (17) and (18) for $j\neq n$. Therefore, $S'_{j,t}(\cdot) > 0$ for $j \neq n$. This, and the facts that $\beta_{u,t+1} \leq 1$ and $F_{t+1}(\cdot) \leq 1$ imply that the first term in (EC.20) is strictly negative. Also, observe that $\tilde{\gamma}(y,p) < 0$ for $p \geq p_{\ell,t}$ and any function $y:[p_{\ell,t},\infty)\to\mathbb{R}$. As a result, the second term in (EC.20) is also strictly negative for $p \geq p_{\ell,t}$. Note that $A_j(p) = S_{j,t}(p)(1-\beta_{u,t+1}F_{t+1}(S_{j,t}(p))) + \beta_{u,t+1}\int_0^{S_{j,t}(p)} x dF_{t+1}(x) > 0$ for $p > p_{\ell,t}$, and $B_j(p) > 0$ for $p > p_{\ell,t}$. This and the fact that $\tilde{\gamma}(S_{j,t},p) < 0$ for $p \geq p_{\ell,t}$ complete our argument that $\partial^2\Pi_n(p;\epsilon,S_{-n,t},\bar{\mathbb{S}})/\partial p^2 < 0$ for $p \geq p_{\ell,t}$. A price $p < p_{\ell,t}$ is never optimal since (EC.18) is positive for $p < p_{\ell,t}$. As a result, for each ϵ, there exists a unique price $\widehat{p}(\epsilon)$ that maximizes $\Pi_n(p;\epsilon, S_{-n,t}, \bar{\mathbb{S}})$, and that price is the one that satisfies the first order condition in (EC.18). Comparing (EC.17) and (EC.18), we have $S_{n,t}(\widehat{p}(\epsilon)) = \mathcal{R}_{n,t}(\widehat{p}(\epsilon);\epsilon)$ for any ϵ, which means that by committing to the production schedule $S_{n,t}(\cdot)$ that satisfies (EC.17) and $S'_{n,t}(p) \in (p_{\ell,t}, \infty)$, the renewable firm n guarantees that for each ϵ the residual demand curve crosses $S_{n,t}(\cdot)$ only at the unique price $\widehat{p}(\epsilon)$. By that way, firm n achieves its optimal commitment-related ex-post expected profit with respect to period-t day-ahead demand (that is, the maximum $\Pi_n(\cdot;\epsilon, S_{-n,t}, \bar{\mathbb{S}})$) for each ϵ given $S_{-n,t}$ and $\bar{\mathbb{S}}$). From this, our claim follows. □

LEMMA EC.3. *For any given $\bar{\mathbb{S}}$, a function $S_t(\cdot)$ is each renewable firm's committed production schedule in a period-t supply function equilibrium that satisfies* (12) *if and only if $S_t(\cdot)$ satisfies conditions in* (17) *and* (18).

Proof of Lemma EC.3: We will first show that if a function $S_t(\cdot)$ satisfies (17) and (18), then it is a period-t supply function equilibrium that satisfies (12). Lemma EC.2 identifies the best response committed production schedule of a renewable firm n given that inflexible firms' commitment profile is $\bar{\mathbb{S}}$ and the period-t committed production schedule $S_{j,t}(\cdot)$ of any other renewable firm j satisfies (17) and (18). If renewable firms consider identical commitment strategies, $S_t = S_{1,t} = S_{2,t} = \ldots = S_{N_r,t}$ by (12). Applying this in Lemma EC.2, the differential equation in (EC.17) reduces to (17); $S'_{n,t}(\cdot) \in (p_{\ell,t}, \infty)$ and $S_{n,t}(p_{\ell,t}) = 0$ reduce to $S'_t(\cdot) \in (p_{\ell,t}, \infty)$ and $S_t(p_{\ell,t}) = 0$, respectively. Thus, by Lemma EC.2, such an $S_t(\cdot)$ traces through points that maximize each renewable firm's optimal (commitment-related) ex-post expected profit (with respect to period-t day-ahead demand) for any ϵ_t, given $\bar{\mathbb{S}}$ and any other renewable firm's commitment $S_t(\cdot)$. As a result, given $\bar{\mathbb{S}}$, $S_t(\cdot)$ is a period-t supply function equilibrium that satisfies (12) by Definition 1. With this, we prove the sufficiency of (17) and (18) for a period-t supply function equilibrium that satisfies (12). The necessity of the stated conditions follows from arguments that use first order necessary conditions of the optimality. □

Lemma EC.3 completes our argument for the proof of part (i). □

Proof of Proposition 1-(ii): The first sentence of the claim is proved in Lemma EC.1 at the beginning of Appendix B. The second sentence of the claim follows from Lemma EC.1 and Proposition 1-(i). □

Appendix C: Proof of Proposition 2

Proof of Proposition 2-(i): Consider a strategy profile $(\mathbb{S}_1, \ldots, \mathbb{S}_\tau, \bar{\mathbb{S}}, \vec{q})$ that consists of following components: $S_t(\cdot)$ that satisfies (17) and (18) for $t \le \tau$, $\bar{S} = \min\{\bar{s}^*, K\}$ where $\bar{s}^*$ is a solution of (20), and the production profile $\vec{q}$ such that $q^*_{n,t+1} = \min\{S_t(p^*_t), Q_{n,t+1}\}$ for $n \le N_r$ and $q^*_{k,t+1} = \bar{S}$ for $k = N_r+1, \ldots, N_r+N_i$ and $t \le \tau$. In this proof, we will show that this strategy profile exists and is an equilibrium (as defined in Definition 2) that satisfies (11) and (12).

Proposition 1 explicitly characterizes and establishes the existence of a period-t supply function equilibrium for any given commitment profile $\bar{\mathbb{S}}$ of inflexible firms. Observe from (17) and (18) that in a period-t supply function equilibrium, renewable firms' committed production schedules are not affected by $\bar{\mathbb{S}}$. Thus, any period-t supply function equilibrium (along with its resulting production strategy identified in Lemma 1) already satisfies (9). As a result, to characterize and show the existence of an equilibrium that satisfies (11) and (12), it is sufficient to characterize and show the existence of $\bar{\mathbb{S}}$ that satisfies (8) subject to (11) and (12). Based on this, take any $\{S_t(\cdot); t \le \tau\}$ such that $S_t(\cdot)$ satisfies (17) and (18) for $t \le \tau$. Then, given all other firms' committed production schedules $(\bar{S}_{-k}, S_1, \ldots, S_\tau)$, inflexible firm k's expected profit in (6) is equivalent to the following if $\bar{S}_k = \bar{s}$:

$$\Pi_k(\bar{s}; \bar{S}_{-k}, S_1, \ldots, S_\tau) = \sum_{t=1}^{\tau} \int_{\tilde{z}_t}^{\infty} \left(p^*_t(\bar{s}; \bar{S}_{-k}, S_t, z)\bar{s} - C_{t+1}(\bar{s})\right) d\Phi_t(z). \tag{EC.21}$$

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Here, z is the generic notation for the realization of ϵ_t, $\tilde{z}_t \doteq \bar{s} + \sum_{j\in\{N_r+1,\ldots,N_r+N_i\}\setminus\{k\}} \bar{S}_j - v_t$ is the ϵ_t that induces the market clearing price to be the minimum possible price $p_\ell = 0$ (based on renewable firms' commitments in Proposition 1). By (1), $p_t^*(\bar{s};\bar{S}_{-k},S_t,z)$, that is, the period-$t$ market clearing price at day-ahead demand shock z, satisfies

$$v_t - \alpha_t p_t^*(\bar{s};\bar{S}_{-k},S_t,z) + z - \bar{s} - \sum_{\substack{j=N_r+1\\ j\neq k}}^{N_r+N_i} \bar{S}_j - N_r S_t(p_t^*(\bar{s};\bar{S}_{-k},S_t,z)) = 0. \tag{EC.22}$$

Applying the implicit function theorem on (EC.22), we have

$$\partial p_t^*(\bar{s};\bar{S}_{-k},S_t,z)/\partial\bar{s} = -1/\left(\alpha_t + N_r S_t'(p_t^*(\bar{s};\bar{S}_{-k},S_t,z))\right).$$

Using this, the definition of $\tilde{z}_t$ and the fact that $p_t^*(\bar{s};\bar{S}_{-k},S_t,\tilde{z}_t) = 0$, it follows by an application of the Leibniz rule that $\partial\Pi_k(\bar{s};\bar{S}_{-k},S_1,\ldots,S_\tau)/\partial\bar{s}$ is equal to

$$\sum_{t=1}^{\tau}\left\{\int_{\tilde{z}_t}^{\infty}\left(p_t^*(\bar{s};\bar{S}_{-k},S_t,z) - \frac{\bar{s}}{\alpha_t + N_r S_t'(p_t^*(\bar{s};\bar{S}_{-k},S_t,z))} - C_{t+1}'(\bar{s})\right)d\Phi_t(z) + C_{t+1}(\bar{s})\phi_t(\tilde{z}_t)\right\}. \tag{EC.23}$$

Applying (11) to (EC.23), it follows that (20) is the first order condition for the unconstrained optimizer of (EC.21) where $p_t^*(\bar{s};z)$ is the price that satisfies $v_t - \alpha_t p_t^*(\bar{s};z) + z = N_r S_t(p_t^*(\bar{s};z)) + N_i\bar{s}$. Recalling the notation $\Psi(\bar{s})$ in (20), we now show that there exists an $\bar{s}^*$ that satisfies (20), that is, $\Psi(\bar{s}^*) = 0$. Observe from (20) that

$$\lim_{\bar{s}\to 0}\Psi(\bar{s}) = \sum_{t=1}^{\tau}\int_{-v_t}^{\infty} p_t^*(\bar{s};z)d\Phi_t(z) > 0, \tag{EC.24}$$

which suggests that $\bar{s}^* > 0$. We claim and show below that $\limsup_{\bar{s}\to\infty}\Psi(\bar{s}) < 0$. Then, it follows from (EC.24) that $\bar{s}^* \in (0,\infty)$ and $\bar{s}^*$ satisfies the first order condition $\Psi(\bar{s}^*) = 0$. As a result, $\bar{s}^*$ is the same as the unique stationary point of (EC.21) subject to (11), and hence it is the global unconstrained maximizer of (EC.21) subject to (11). With the addition of capacity constraint $\bar{S}_k \le K$, it follows from elementary optimization arguments that the global maximizer for (EC.21) subject to (11) exists and equals $\bar{S}_k^* = \min\{\bar{s}^*, K\}$. This completes the proof for the existence of the strategy profile mentioned at the beginning of proof, and establishes that the aforementioned strategy profile is an equilibrium.

We now proceed to show our claim that $\limsup_{\bar{s}\to\infty}\Psi(\bar{s}) < 0$. Recall the notation $\underline{z}_t(\bar{s})$ in (19). For $\bar{s} > 0$, we have

$$\begin{aligned}
\Psi(\bar{s}) &= \sum_{t=1}^{\tau}\left\{\int_{\underline{z}_t(\bar{s})}^{\infty}\left(p_t^*(\bar{s};z) - \frac{\bar{s}}{\alpha_t + N_r S_t'(p_t^*(\bar{s};z))}\right)d\Phi_t(z) - C_{t+1}'(\bar{s})\left(1 - \Phi(\underline{z}_t(\bar{s}))\right) + C_{t+1}(\bar{s})\phi_t(\underline{z}_t(\bar{s}))\right\}\\
&\overset{(a)}{<} \sum_{t=1}^{\tau}\left\{\int_{\underline{z}_t(\bar{s})}^{\infty} p_t^*(\bar{s};z)d\Phi_t(z) - C_{t+1}'(\bar{s})\left(1 - \Phi(\underline{z}_t(\bar{s}))\right) + N_i C_{t+1}(\bar{s})\phi_t(\underline{z}_t(\bar{s}))\right\}\\
&\overset{(b)}{=} \sum_{t=1}^{\tau}\left\{\int_{\underline{z}_t(\bar{s})}^{\infty} p_t^*(\bar{s};z)d\Phi_t(z) - \frac{\partial\left(C_{t+1}(\bar{s})(1 - \Phi(N_i\bar{s} - v_t))\right)}{\partial\bar{s}}\right\}.
\end{aligned}$$

Here, the inequality (a) is because for $\bar{s} > 0$

$$\int_{\underline{z}_t(\bar{s})}^{\infty} -\bar{s}/(\alpha_t + N_r S_t'(p_t^*(\bar{s};z))) < 0, \tag{EC.25}$$

and $C_{t+1}(\bar{s})\phi_t(\underline{z}_t(\bar{s})) > 0$. Because $\limsup_{\bar{s}\to\infty}\underline{z}_t(\bar{s}) = \infty$ and $\limsup_{\bar{s}\to\infty} p_t^*(\bar{s};z) \le 0$, we have $\limsup_{\bar{s}\to\infty} p_t^*(\bar{s};z)d\Phi_t(z) \le 0$. Combining this, (EC.25) and (7), by equation (b), we have $\limsup_{\bar{s}\to\infty}\Psi(\bar{s}) < 0$. □

Proofs of Propositions 2-(ii) and 2-(iii): Proposition 2-(i) characterizes an equilibrium and establishes the existence of it. Parts (ii) and (iii) immediately follow from the first paragraph in the proof of Proposition 2-(i). □

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Appendix D: Proof of Proposition 3

To show this claim, it is sufficient to prove the uniqueness of a period-t supply function equilibrium for any $t = 1,\dots,\tau$ and $\bar{\mathbb{S}}$, as the uniqueness of $\bar{S}$ identified in Proposition 2-(iii) with any sequence of supply function equilibria $\{S_t(\cdot); t \le \tau\}$ follows from the arguments in Proposition 2-(i).

Take any $t \le \tau$. We first show the uniqueness of a period-t supply function equilibrium for $N_r > 2$. To do so, we will now solve the ODE (17) for certain values of p. Define $\tilde{p} \doteq p^{-1}$ and $\tilde{S}_t \doteq S_t^{-1}$. Using these definitions and by elementary analysis, we have the following when $\tilde{p} \to 0$ and $\tilde{S}_t \to 0$:

$$d\tilde{S}_t/d\tilde{p} = \left(d\tilde{S}_t/dS_t\right) S_t'(p)\,(dp/d\tilde{p}) \overset{(\star)}{=} \left[\tilde{S}_t^{-1}/\tilde{p}^{-1} - \alpha_t\right] \tilde{S}_t^2/(\tilde{p}^2(N_r-1)). \tag{EC.26}$$

The equation ($\star$) in (EC.26) is because $S_t'(p)$ in (17) reduces to $\left[\tilde{S}_t^{-1}/\tilde{p}^{-1} - \alpha_t\right]/(N_r-1)$ when $\tilde{p} \to 0$ and $\tilde{S}_t \to 0$. The reason is that $\lim_{\tilde{S}_t \to 0} F_{t+1}(1/\tilde{S}_t) = 1$, $b_{u,t+1}$ is negligible compared to $\tilde{p}^{-1}$ when $\tilde{p} \to 0$, and as $\tilde{S}_t \to 0$, $\int_0^{1/\tilde{S}_t} x dF_{t+1}(x)$ converges to a finite constant μ_{t+1}, which is negligible compared to $1/\tilde{S}_t$ when $\tilde{S}_t \to 0$. Let $\tilde{S}_t = g(\tilde{p})\tilde{p}$. Using this relation in (EC.26) and by rearranging terms, we conclude that the solution of (EC.26) must satisfy

$$d\tilde{p}/\tilde{p} = -dg/((1-1/(N_r-1))g + \alpha_t g^2/(N_r-1)). \tag{EC.27}$$

Observe that this is a separable ODE. Then, by standard ODE theory arguments, it follows that

$$g(\tilde{p}) = (1-(N_r-1)^{-1})/\left(\hat{c}^{-1}\tilde{p}^{\left(1-(N_r-1)^{-1}\right)} - \alpha_t(N_r-1)^{-1}\right), \tag{EC.28}$$

where $\hat{c}$ is a constant of integration. Considering the expansion of $\tilde{S}_t$ around $\tilde{p} = 0$, $g(\tilde{p})$ can be interpreted as the slope of $\tilde{S}_t$ when $\tilde{p} \to 0$. For the inverse of $\tilde{S}_t$ to form a period-t supply function equilibrium, the aforementioned slope (EC.28) must be nonnegative by the monotonicity constraint in (18). If it is negative, there exists a strictly positive $\tilde{p}$ such that $\tilde{S}_t'(\tilde{p}) < 0$, and this implies by the definitions of $\tilde{p}$ and $\tilde{S}_t$ that there exists a price $p > 0$ such that $S_t'(p) < 0$, which contradicts the equilibrium condition in (18). We claim and show below that there exists a unique $\hat{c}$ that results in a nonnegative slope (EC.28) when $\tilde{p} \to 0$. From this, the uniqueness of a period-t supply function equilibrium immediately follows for any given $\bar{\mathbb{S}}$. We now show our claim about the uniqueness of such $\hat{c}$. Suppose $\hat{c} \neq 0$. Then, the slope (EC.28) is $g(0) = -(1-(N_r-1)^{-1})(N_r-1)/\alpha_t < 0$ when $\tilde{p} \to 0$. Observe that $g(0)$ is non-negative only for $\hat{c} = 0$.

We now show the uniqueness of a period-t supply function equilibrium for $N_r = 2$. By similar arguments explained above, we have $d\tilde{S}_t/d\tilde{p} = \left(\tilde{S}_t^2/\tilde{p}^2\right)\left(\tilde{p}/\tilde{S}_t - \alpha_t\right)$. Substituting $\tilde{S}_t = u(\tilde{p})\tilde{p}$ in the aforementioned ODE where $u(\cdot)$ is a function, we conclude that the solution must satisfy $d\tilde{p}/\tilde{p} = -du/(\alpha_t u^2)$. This is a separable ODE, and its solution is $\tilde{p} = e^{1/(\alpha_t u)}\tilde{c}$ where $\tilde{c}$ is a constant, which is strictly monotone in the constant of integration. According to this solution, if $\tilde{c} \neq 0$, then $\tilde{p} \to 0$ implies $1/(\alpha_t u) \to -\infty$. Based on this, because $\alpha_t > 0$, the slope u is strictly negative when $\tilde{p} \to 0$. Hence, a solution with $\tilde{c} \neq 0$ cannot be a period-t supply function equilibrium as it violates (18). From this, it follows that the solution with $\tilde{c} = 0$ is the unique period-t supply function equilibrium when $N_r = 2$. $\square$

Appendix E: Proof of Proposition 4

Proposition 4-(ii) immediately follows from part (i) and the facts that Γ_{t+1} and Δ_{t+1} do not depend on $\beta_{u,t+1}$. Therefore, in this section, we prove Proposition 4-(i). Fix any $t = 1,\dots,\tau$. We first state and prove Lemmas EC.4 and EC.5. The statements of these lemmas will be used in the subsequent parts of the proof.

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LEMMA EC.4. *A function $S_t(\cdot)$ that satisfies the conditions in (17) and (18) also satisfies*

$$S_t'(0) = \frac{1}{2}\left(\frac{(N_r-2)-\alpha_t\psi(0)}{(N_r-1)\psi(0)} + \sqrt{\frac{((N_r-2)-\alpha_t\psi(0))^2}{((N_r-1)\psi(0))^2} + \frac{4\alpha_t}{(N_r-1)\psi(0)}}\right), \tag{EC.29}$$

where the mapping $\psi(\cdot)$ is defined as $\psi(p) \doteq \eta_{u,t+1}(p) f_{t+1}(S_t(p))$ for $p \ge p_{\ell,t} \doteq 0$, and hence $\psi(0) = b_{u,t+1} f_{t+1}(0) > 0$.

Proof of Lemma EC.4: Consider $S_t(\cdot)$ that satisfies the conditions in (17) and (18). We find $S_t'(0)$ by letting $p \searrow 0$ on both sides of (17). Observe that as $p \searrow 0$, both numerator and denominator of the fraction in brackets on the right hand side of (17) go to zero. Applying L'Hôpital's rule in (17), we have

$$S_t'(0) = \lim_{p\searrow 0} \frac{1}{N_r-1}\left[\frac{(1-\beta_{u,t+1}F_{t+1}(S_t(p)))S_t'(p)}{1-\beta_{u,t+1}F_{t+1}(S_t(p)) - S_t'(p)(\beta_{u,t+1}p + b_{u,t+1})f_{t+1}(S_t(p))} - \alpha_t\right]. \tag{EC.30}$$

Because $S_t(0) = 0$ by (18), and from structural properties of functions $\eta_{u,t+1}$, f_{t+1} and F_{t+1}, it follows that

$$S_t'(0) = \left[S_t'(0)/(1 - S_t'(0)\psi(0)) - \alpha_t\right]/(N_r - 1). \tag{EC.31}$$

Solving for $S_t'(0)$ in (EC.31), we have the following two solutions:

$$S_t'(0) = \frac{1}{2}\left(\frac{(N_r-2)-\alpha_t\psi(0)}{(N_r-1)\psi(0)} \mp \sqrt{\frac{((N_r-2)-\alpha_t\psi(0))^2}{((N_r-1)\psi(0))^2} + \frac{4\alpha_t}{(N_r-1)\psi(0)}}\right). \tag{EC.32}$$

The relevant solution is the positive one since a negative $S_t'(0)$ contradicts the monotonicity constraint in (18). As a result, $S_t'(0)$ is as in (EC.29). $\square$

In the remainder of the proof, for clarity, we include $\beta_{u,t+1}$ as an argument of $S_t(\cdot)$. In particular, let $S_t(\cdot;\beta_{u,t+1})$ be any function that satisfies conditions in (17) and (18) for a given $\beta_{u,t+1}$.

LEMMA EC.5. *$S_t''(0;\beta_{u,t+1})$ strictly increases with $\beta_{u,t+1}$ if and only if $N_r \le \Gamma_{t+1}$ and $b_{u,t+1} < \Delta_{t+1}$ for some positive constants Γ_{t+1} and Δ_{t+1}.*

Proof of Lemma EC.5: Recall from (17) that

$$S_t'(p;\beta_{u,t+1}) = \frac{1}{N_r-1}\left[\frac{S_t(p;\beta_{u,t+1})(1-\beta_{u,t+1}F_{t+1}(S_t(p;\beta_{u,t+1}))) + \beta_{u,t+1}\int_0^{S_t(p;\beta_{u,t+1})} x f_{t+1}(x)dx}{p(1-\beta_{u,t+1}F_{t+1}(S_t(p;\beta_{u,t+1}))) - b_{u,t+1}F_{t+1}(S_t(p;\beta_{u,t+1}))} - \alpha_t\right]. \tag{EC.33}$$

If we take the derivative of both sides in (EC.33) with respect to p and let p go to zero, the numerator and the denominator of the resulting fraction on the right hand side goes to zero. Then, applying the L'Hôpital rule, we have

$$S_t''(0;\beta_{u,t+1}) = \lim_{p\searrow 0} \frac{S_t''(p;\beta_{u,t+1})\delta(p) + S_t'(p;\beta_{u,t+1})\delta'(p) - \theta'(p)\left(S_t'(p;\beta_{u,t+1})(N_r-1)+\alpha_t\right)}{2(N_r-1)\theta(p)}, \tag{EC.34}$$

where $\theta(p) \doteq 1 - \beta_{u,t+1}F_{t+1}(S_t(p;\beta_{u,t+1})) - f_{t+1}(S_t(p;\beta_{u,t+1}))\eta_{u,t+1}(p)S_t'(p;\beta_{u,t+1})$ and $\delta(p) \doteq 1 - \beta_{u,t+1}F_{t+1}(S_t(p;\beta_{u,t+1}))$ for $p \ge 0$. From elementary algebra, it follows that

$$\lim_{p\searrow 0}\theta'(p) = -2\beta_{u,t+1}f_{t+1}(0)S_t'(0;\beta_{u,t+1}) - b_{u,t+1}\left[f_{t+1}'(0)\left(S_t'(0;\beta_{u,t+1})\right)^2 + f_{t+1}(0)S_t''(0;\beta_{u,t+1})\right], \tag{EC.35}$$

$$\lim_{p\searrow 0}\delta'(p) = -\beta_{u,t+1}f_{t+1}(0)S_t'(0;\beta_{u,t+1}), \quad \lim_{p\searrow 0}\theta(p) = 1 - f_{t+1}(0)b_{u,t+1}S_t'(0;\beta_{u,t+1}), \quad \lim_{p\searrow 0}\delta(p) = 1. \tag{EC.36}$$

Substituting (EC.35) and (EC.36) in (EC.34), and solving for $S_t''(0;\beta_{u,t+1})$, we find that

$$S_t''(0;\beta_{u,t+1}) = \frac{((2N_r-3)S_t'(0;\beta_{u,t+1}) + 2\alpha_t)f_{t+1}(0)S_t'(0;\beta_{u,t+1})}{2N_r - 3 - f_{t+1}(0)\left(3S_t'(0;\beta_{u,t+1})(N_r-1)+\alpha_t\right)b_{u,t+1}}\beta_{u,t+1} + b_{u,t+1}f_{t+1}'(0)(S_t'(0;\beta_{u,t+1}))^2\Xi_t, \tag{EC.37}$$

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where $\Xi_t \doteq \left(S_t'(0;\beta_{u,t+1})(N_r-1)+\alpha_t\right) / \left(2N_r-3-f_{t+1}(0)\left(3S_t'(0;\beta_{u,t+1})(N_r-1)+\alpha_t\right)b_{u,t+1}\right)$. By (EC.29) in Lemma EC.4, $S_t'(0;\beta_{u,t+1})$ does not change with $\beta_{u,t+1}$. Therefore, $S_t''(0;\beta_{u,t+1})$ strictly increases in $\beta_{u,t+1}$ if and only if the coefficient of $\beta_{u,t+1}$ in (EC.37) - which is the first fraction on the right hand side of (EC.37) - is positive. Observe that the numerator of that coefficient, which is $((2N_r-3)S_t'(0;\beta_{u,t+1})+2\alpha_t)f_{t+1}(0)S_t'(0;\beta_{u,t+1}))$, is already positive as $S_t'(0;\beta_{u,t+1})>0$. Also, substituting (EC.29) in (EC.37), it follows from elementary analysis that there exists a constant $\Gamma_{t+1}>0$ such that the denominator of the coefficient of $\beta_{u,t+1}$ in (EC.37) is positive if and only if $N_r \le \Gamma_{t+1}$ and $b_{u,t+1} < \Delta_{t+1} \doteq \left(-N_r+\sqrt{N_r^2+0.5}\right)/(\alpha_t f_{t+1}(0))$. From this, the statement in Lemma EC.5 follows. Note that because the aforementioned denominator is not dependent on $\beta_{u,t+1}$, the constants Γ_{t+1} and Δ_{t+1} are not dependent on $\beta_{u,t+1}$. □

Take any two undersupply penalty coefficients $\beta_{u,t+1}=\beta_1,\beta_2$ such that $0\le\beta_2<\beta_1\le 1$. Because $S_t(0;\beta_{u,t+1})=0$ for $\beta_{u,t+1}\in[0,1]$ by (18) and $S_t'(0;\beta_{u,t+1})$ is the same for $\beta_{u,t+1}\in[0,1]$ by (EC.29), it follows from expanding $S_t(h;\beta_1)$ and $S_t(h;\beta_2)$ around 0 and using Lemma EC.5 that $S_t(h;\beta_1)>S_t(h;\beta_2)$ for an arbitrarily small constant $h>0$ if and only if the stated conditions in Lemma EC.5 holds. We claim and show below that if $S_t(\tilde p;\beta_1)>S_t(\tilde p;\beta_2)$ for $\tilde p>0$, then $S_t(p;\beta_1)>S_t(p;\beta_2)$ for all $p\ge\tilde p$. Using this claim, and recalling the fact that $S_t(h;\beta_1)>S_t(h;\beta_2)$ for an arbitrarily small constant $h>0$ if and only if the stated conditions in Lemma EC.5 hold, we complete our argument for the proof of Proposition 4-(i).

It only remains to prove our claim that if $S_t(\tilde p;\beta_1)>S_t(\tilde p;\beta_2)$ for $\tilde p>0$, then $S_t(p;\beta_1)>S_t(p;\beta_2)$ for all $p\ge\tilde p$. Define the mapping $d(p,s,\beta)$ as $d(p,s,\beta)\doteq\frac{1}{N_r-1}\left[\frac{s(1-\beta F_{t+1}(s))+\beta\int_0^s xf_{t+1}(x)dx}{(1-\beta F_{t+1}(s))p-b_{u,t+1}F_{t+1}(s)}-\alpha_t\right]$ for $(p,s)\in\mathbb{R}_+\times\mathbb{R}_+/\{(p_{\ell,t},0)\}$ and $\beta\in[0,1]$. Comparing (17) and the mapping d, we have

$$d(p,S_t(p;\beta_{u,t+1}),\beta_{u,t+1})=S_t'(p;\beta_{u,t+1}),\quad p>0,\ \beta_{u,t+1}\in[0,1]. \tag{EC.38}$$

Then, it follows from elementary algebra that

$$\frac{\partial d(p,s,\beta_{u,t+1})}{\partial\beta_{u,t+1}}=\frac{p\int_0^s xf_{t+1}(x)dx+b_{u,t+1}F_{t+1}(s)\left(sF_{t+1}(s)-\int_0^s xf_{t+1}(x)dx\right)}{(N_r-1)\left(p(1-\beta_{u,t+1}F_{t+1}(s))-b_{u,t+1}F_{t+1}(s)\right)^2}>0, \tag{EC.39}$$

$$\frac{\partial d(p,s,\beta_{u,t+1})}{\partial s}=\frac{\kappa_{t+1}(s)\left(p\kappa_{t+1}(s)-b_{u,t+1}F_{t+1}(s)\right)+\eta_{u,t+1}(p)f_{t+1}(s)\left(s\kappa_{t+1}(s)+\beta_{u,t+1}\int_0^s xf_{t+1}(x)dx\right)}{(N_r-1)\left(p\kappa_{t+1}(s)-b_{u,t+1}F_{t+1}(s)\right)^2}, \tag{EC.40}$$

where $\kappa_{t+1}(s)\doteq(1-\beta_{u,t+1}F_{t+1}(s))$. Observe that (EC.40) is positive for all (p,s) such that $p\kappa_{t+1}(s)-b_{u,t+1}F_{t+1}(s)>0$. Because $p\kappa_{t+1}(S_t(p;\beta_{u,t+1}))-b_{u,t+1}F_{t+1}(S_t(p;\beta_{u,t+1}))>0$ for $p>0$ and $\beta_{u,t+1}\in[0,1]$, it follows that $\partial d(p,S_t(p;\beta_{u,t+1}),\beta_{u,t+1})/\partial s>0$. This, (EC.38) and (EC.39) imply that if $S_t(p;\beta_1)>S_t(p;\beta_2)$, then $S_t'(p;\beta_1)>S_t'(p;\beta_2)$. From this, our claim immediately follows. □

Appendix F: Proof of Proposition 5

We first provide the proof of part (i). Fix a period-t day-ahead demand shock ϵ_t and take any given $\bar{\mathbb{S}}$. Then, comparing Propositions 1 and 2-(ii), the period-t reliability (10) in equilibrium is the same as the period-t reliability under a period-t supply function equilibrium for any realization of ϵ_t. Thus, we focus on the analysis of a period-t supply function equilibrium for the reliability. Define $\mathbb{Q}_{t+1}\doteq(Q_{1,t+1},\dots,Q_{n,t+1})$. Given $\bar{\mathbb{S}}$, because $q^*_{k,t+1}=\bar S_k$ for $k=N_r+1,\dots,N_r+N_i$ (as explained in Section 2), in any period-t supply function equilibrium, we have

$$\rho_t(\epsilon_t)=\mathbb{P}_{\mathbb{Q}_{t+1}}\left(N_rS_t(p_t^*)+\sum_{k=N_r+1}^{N_r+N_i}\bar S_k\le\sum_{j=1}^{N_r+N_i}q^*_{j,t+1}\Big|\epsilon_t\right)=\mathbb{P}_{\mathbb{Q}_{t+1}}\left(N_rS_t(p_t^*)\le\sum_{n=1}^{N_r}q^*_{n,t+1}\Big|\epsilon_t\right)$$

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$$\stackrel{(a)}{=} \mathbb{P}_{\mathbb{Q}_{t+1}}\left(N_r S_t(p_t^*) \le \sum_{n=1}^{N_r} \min\{S_t(p_t^*), Q_{n,t+1}\}\Big|\epsilon_t\right)$$
$$= \mathbb{P}_{\mathbb{Q}_{t+1}}\left(0 \le \sum_{n=1}^{N_r} \min\{0, Q_{n,t+1} - S_t(p_t^*)\}\Big|\epsilon_t\right). \quad \text{(EC.41)}$$

The equation (a) holds because $q_{n,t+1}^* = \min\{S_t(p_t^*), Q_{n,t+1}\}$ for $n = 1, \ldots, N_r$ by Proposition 2, and the equation (EC.41) follows from reorganizing terms inside the large parentheses of (a). By elementary probability arguments, we have

$$\partial\mathbb{P}_{\mathbb{Q}_{t+1}}\left(0 \le \sum_{n=1}^{N_r} \min\{0, Q_{n,t+1} - x\}\Big|\epsilon_t\right)/\partial x < 0. \quad \text{(EC.42)}$$

Because $S_t'(\cdot) > 0$ by (18), it follows from (1) and Proposition 4 that for any $\beta_{u,t+1}$, an increase in $\beta_{u,t+1}$ results in a strictly larger $S_t(p_t^*)$ for every realization of ϵ_t if and only if (21). Using this and (EC.42), we conclude that an increase in $\beta_{u,t+1}$ results in a strictly lower period-t reliability (EC.41) if and only if (21), which completes the proof of part (i). The proof of part (ii) immediately follows from part (i) because part (i) holds for all $\beta_{u,t+1} \in [0,1]$. $\square$

Appendix G: Proof of Proposition 6

The proof of part (ii) immediately follows from part (i). Therefore, in this section, we provide the proof of part (i). Fix any vector of day-ahead demand shocks $\vec{\epsilon} \doteq (\epsilon_1, \epsilon_2, \ldots, \epsilon_\tau)$. Suppose that (21) holds for some $t \le \tau$.

We first introduce some notation and preliminary analysis that will be used in the remainder of the proof. Let $\bar{s}^*(\beta_{u,t+1})$ be the solution of (20). Observe from the proof of Proposition 2-(i) that $\bar{s}^*(\beta_{u,t+1})$ is each inflexible firm's unconstrained committed production quantity in equilibrium that satisfies (11) and (12). Define

$$K_L \doteq \min\{\bar{s}^*(\beta_{u,t+1}) : \beta_{u,t+1} \in [0,1]\} \quad \text{and} \quad K_H \doteq \max\{\bar{s}^*(\beta_{u,t+1}) : \beta_{u,t+1} \in [0,1]\}. \quad \text{(EC.43)}$$

Also, define the period-j residual demand curve for renewable firms in equilibrium as

$$\overline{D}_j(p, \epsilon_j) \doteq v_t - \alpha_j p + \epsilon_j - N_i \bar{S}, \ j = 1, \ldots, \tau. \quad \text{(EC.44)}$$

Then, by (1), in equilibrium, the period-j day-ahead market clearing price satisfies

$$\overline{D}_j(p, \epsilon_j) = N_r S_j(p), \ j = 1, \ldots, \tau. \quad \text{(EC.45)}$$

We now prove conditions (1) and (2) stated in part (i). **Condition (1):** Suppose that $K < K_L$. Then, by (EC.43), $K < \bar{s}^*(\beta_{u,t+1})$ for $\beta_{u,t+1} \in [0,1]$. This implies by Proposition 2-(iii) that in any equilibrium with any $\beta_{u,t+1} \in [0,1]$, an inflexible firm's production commitment at $t = 1$ is $\bar{S} = \min\{K, \bar{s}^*(\beta_{u,t+1})\} = K$. Thus, in each period j, $\overline{D}_j(p, \epsilon_j) = v_j - \alpha_j p + \epsilon_j - N_i K$, which does not change with $\beta_{u,t+1}$. This, (21) and Proposition 4 immediately imply the claim for period-t reliability in equilibrium. For other periods $j \ne t$, period-j reliability in equilibrium is not affected by changes in $\beta_{u,t+1}$. This is because if there is an increase in $\beta_{u,t+1}$, both $S_j(\cdot)$ and $\overline{D}_j(p, \epsilon_j)$ remain unchanged for $j \ne t$. As a result, cleared aggregate commitment of renewable firms in period j, which is $N_r S_j(p_j^*)$ remains the same, and hence the period-j reliability in equilibrium does not change by (EC.41). **Condition (2):** Suppose now that $K > K_H$, which implies by (EC.43) that $K > \bar{s}^*(\beta_{u,t+1})$ for $\beta_{u,t+1} \in [0,1]$. Then, by Proposition 2, in any equilibrium with any $\beta_{u,t+1} \in [0,1]$, an inflexible firm's production commitment at $t = 1$ is

$$\bar{S} = \bar{s}^*(\beta_{u,t+1}). \quad \text{(EC.46)}$$

We now state a result that will be used later in the proof. The proof of the following lemma is relegated to the end of this section.

LEMMA EC.6. *If conditions in* (22) *hold,* $\partial\bar{s}^*(\beta_{u,t+1})/\partial\beta_{u,t+1} < 0$ *for* $\beta_{u,t+1} \in [0,1]$.

Suppose that there is an increase in $\beta_{u,t+1}$, as stated in the proposition and the conditions in (22) hold. Let us first analyze the impact of that increase for period-t reliability in equilibrium. By Lemma EC.6, (EC.44) and (EC.46), such an increase results in a uniformly larger $\overline{D}_j(p,\epsilon_j)$ for $j = 1,\ldots,\tau$. This, the inflated committed production schedule $S_t(\cdot)$ with the larger $\beta_{u,t+1}$ by Proposition 4 and (EC.45) imply that each renewable firm's period-t cleared commitment $S_t(p_t^*)$ strictly increases with $\beta_{u,t+1}$. Then, by (EC.41) and (EC.42), equilibrium period-t reliability strictly decreases with $\beta_{u,t+1}$. Let us now analyze the impact of a higher $\beta_{u,t+1}$ for equilibrium period-j reliability $j \neq t$. A uniformly larger $\overline{D}_j(p,\epsilon_j)$ (due to a higher $\beta_{u,t+1}$) results in a strictly larger cleared production commitment $S_j(p_j^*)$ for each renewable firm as renewable firms' aggregate committed production schedule $N_r S_j(\cdot)$ in each period $j \neq t$ remains the same regardless of any change in $\beta_{u,t+1}$. Strictly larger $S_j(p_j^*)$, (EC.41) and (EC.42) imply that the equilibrium period-j reliability strictly decreases with $\beta_{u,t+1}$ for $j \neq t$.

Proof of Lemma EC.6: Recall the notation $\Psi(\cdot)$ in (20). Then, by definition of $\bar{s}^*(\beta_{u,t+1})$, $\Psi(\bar{s}^*(\beta_{u,t+1})) = 0$ for $\beta_{u,t+1} \in [0,1]$. Applying the implicit function theorem on $\Psi(\bar{s}^*(\beta_{u,t+1})) = 0$, we have

$$\partial\bar{s}^*(\beta_{u,t+1})/\partial\beta_{u,t+1} = -\frac{\partial\Psi(\bar{s})/\partial\beta_{u,t+1}}{\partial\Psi(\bar{s})/\partial\bar{s}}\Big|_{\bar{s}=\bar{s}^*(\beta_{u,t+1})}. \tag{EC.47}$$

We know from second order necessary conditions that $\partial\Psi(\bar{s})/\partial\bar{s}|_{\bar{s}=\bar{s}^*(\beta_{u,t+1})} < 0$. Then, by (EC.47), it follows that $\partial\bar{s}^*(\beta_{u,t+1})/\partial\beta_{u,t+1} < 0$ if and only if

$$\partial\Psi(\bar{s})/\partial\beta_{u,t+1}|_{\bar{s}=\bar{s}^*(\beta_{u,t+1})} < 0. \tag{EC.48}$$

Suppose that (22) holds. Based on this, we now show that (EC.48) holds and hence $\partial\bar{s}^*(\beta_{u,t+1})/\partial\beta_{u,t+1} < 0$. To show (EC.48), we first derive an expression for $\partial\Psi(\bar{s})/\partial\beta_{u,t+1}$. Because $K > K_H$ by (22), $\bar{s}^*(\beta_{u,t+1}) < K$ for all $\beta_{u,t+1} \in [0,1]$ by (EC.43). Therefore, it is sufficient to focus on $\bar{s} \in [0,K]$ as a feasible set. By elementary analysis,

$$\frac{\partial\Psi(\bar{s})}{\partial\beta_{u,t+1}} = \int_{\underline{z}_t(\bar{s})}^{\infty}\left(\frac{\partial p_t^*}{\partial\beta_{u,t+1}} + \frac{\bar{s}\,N_r}{(\alpha_t+N_rS_t'(p_t^*))^2}\left[\frac{\partial S_t'(p)}{\partial\beta_{u,t+1}}\Big|_{p=p_t^*} + S_t''(p_t^*)\frac{\partial p_t^*}{\partial\beta_{u,t+1}}\right]\right)d\Phi_t(z) \tag{EC.49}$$

$$= \int_{\underline{z}_t(\bar{s})}^{\infty}\left\{-\underbrace{\left(\frac{\partial S_t(p)}{\partial\beta_{u,t+1}}\Big|_{p=p_t^*}\right)}_{\doteq X(\bar{s})}\underbrace{\left[\frac{N_r}{(\alpha_t+N_rS_t'(p_t^*))}\left(1+\frac{\bar{s}N_r}{(\alpha_t+N_rS_t'(p_t^*))^2}S_t''(p_t^*)\right)\right]}_{\doteq Y(\bar{s})}\right\}d\Phi_t(z) \tag{EC.50}$$

$$+ \int_{\underline{z}_t(\bar{s})}^{\infty}\underbrace{\left(\frac{\partial S_t'(p)}{\partial\beta_{u,t+1}}\Big|_{p=p_t^*}\right)\frac{\bar{s}N_r}{(\alpha_t+N_rS_t'(p_t^*))^2}}_{\doteq\theta(\bar{s})}d\Phi_t(z). \tag{EC.51}$$

Here, p_t^* is a function of $\bar{s}$ and period-t day-ahead demand shock realization z; (EC.50) is due to the fact that $\partial p_t^*(\epsilon_t)/\partial\beta_{u,t+1} = -\big(\partial S_t(p_t^*(\epsilon_t))/\partial\beta_{u,t+1}\big)N_r/\Big(\alpha_t+N_rS_t'(p_t^*(\epsilon_t))\Big)$. By (EC.50) and (EC.51), (EC.48) is equivalent to

$$\mathbb{E}_{\epsilon_t}\Big[\theta(\bar{s})\big|\epsilon_t \geq \underline{z}_t(\bar{s})\Big] < \mathbb{E}_{\epsilon_t}\Big[X(\bar{s})Y(\bar{s})\big|\epsilon_t \geq \underline{z}_t(\bar{s})\Big]. \tag{EC.52}$$

We now show that if (22), then we have (EC.52). We claim and show at the end of this proof that if the third condition in (22) holds, $Y(\bar{s}) > 0$ for $\epsilon_t \geq \underline{z}_t(\bar{s})$ and $\bar{s} \in [0,K]$, and $X(\bar{s}) > 0$ and $\theta(\bar{s}) \geq 0$ for $\epsilon_t \geq \underline{z}_t(\bar{s})$ and $\bar{s} \in [0,K]$. Define

$$\tilde{s} \doteq \min_{\bar{s}\in[0,K]}\left\{\left(\mathbb{E}_{\epsilon_t}\Big[\sqrt{X(\bar{s})}\big|\epsilon_t \geq \underline{z}_t(\bar{s})\Big]\right)^2 - \mathbb{E}_{\epsilon_t}\Big[\theta(\bar{s})\big|\epsilon_t \geq \underline{z}_t(\bar{s})\Big]\mathbb{E}_{\epsilon_t}\Big[Y(\bar{s})^{-1}\big|\epsilon_t \geq \underline{z}_t(\bar{s})\Big]\right\}, \tag{EC.53}$$

$$\gamma \doteq \sqrt{\mathbb{E}_{\epsilon_t}\Big[\theta(\tilde{s})\big|\epsilon_t \geq \underline{z}_t(\tilde{s})\Big]\mathbb{E}_{\epsilon_t}\Big[Y(\tilde{s})^{-1}\big|\epsilon_t \geq \underline{z}_t(\tilde{s})\Big]}. \tag{EC.54}$$

Then, the first condition in (22) implies that

$$\left(\mathbb{E}_{\epsilon_t}\left[\sqrt{X(\bar{s})}|\epsilon_t \geq \underline{z}_t(\bar{s})\right]\right)^2 - \mathbb{E}_{\epsilon_t}\left[\theta(\bar{s})\big|\epsilon_t \geq \underline{z}_t(\bar{s})\right]\mathbb{E}_{\epsilon_t}\left[Y(\bar{s})^{-1}\big|\epsilon_t \geq \underline{z}_t(\bar{s})\right] > 0,\ \bar{s}\in[0,K],$$

which is equivalent to

$$\mathbb{E}_{\epsilon_t}\left[\theta(\bar{s})\big|\epsilon_t \geq \underline{z}_t(\bar{s})\right] < \frac{\left(\mathbb{E}_{\epsilon_t}\left[\sqrt{X(\bar{s})}\big|\epsilon_t \geq \underline{z}_t(\bar{s})\right]\right)^2}{\mathbb{E}_{\epsilon_t}\left[Y(\bar{s})^{-1}\big|\epsilon_t \geq \underline{z}_t(\bar{s})\right]},\ \ \bar{s}\in[0,K]. \tag{EC.55}$$

Because $X(\bar{s})>0$ and $Y(\bar{s})>0$ for $\bar{s}\in[0,K]$ and $\epsilon_t \geq \underline{z}_t(\bar{s})$, we have the following from the Cauchy-Schwarz inequality for conditional expectations:

$$\left(\mathbb{E}_{\epsilon_t}\left[\sqrt{X(\bar{s})}|\epsilon_t \geq \underline{z}_t(\bar{s})\right]\right)^2 \leq \mathbb{E}_{\epsilon_t}\left[X(\bar{s})Y(\bar{s})|\epsilon_t \geq \underline{z}_t(\bar{s})\right]\mathbb{E}_{\epsilon_t}\left[Y(\bar{s})^{-1}|\epsilon_t \geq \underline{z}_t(\bar{s})\right],\ \bar{s}\in[0,K], \tag{EC.56}$$

which is equivalent to

$$\frac{\left(\mathbb{E}_{\epsilon_t}\left[\sqrt{X(\bar{s})}\big|\epsilon_t \geq \underline{z}_t(\bar{s})\right]\right)^2}{\mathbb{E}_{\epsilon_t}\left[Y(\bar{s})^{-1}\big|\epsilon_t \geq \underline{z}_t(\bar{s})\right]} \leq \mathbb{E}_{\epsilon_t}\left[X(\bar{s})Y(\bar{s})\big|\epsilon_t \geq \underline{z}_t(\bar{s})\right],\ \bar{s}\in[0,K]. \tag{EC.57}$$

This and (EC.55) imply (EC.52), which concludes our argument for $\partial\Psi(\bar{s})/\partial\beta_{u,t+1}|_{\bar{s}=\bar{s}^*(\beta_{u,t+1})}<0$ under (22).

It remains to prove our claim that $Y(\bar{s})>0$ for $\epsilon_t \geq \underline{z}_t(\bar{s})$ and $\bar{s}\in[0,K]$ if the third condition in (22) holds, and $X(\bar{s})>0$ and $\theta(\bar{s})\geq 0$ for $\epsilon_t \geq \underline{z}_t(\bar{s})$ and $\bar{s}\in[0,K]$. Note that $\theta(\bar{s})\geq 0$ for $\epsilon_t \geq \underline{z}_t(\bar{s})$ and $\bar{s}\in[0,K]$ since, by (17), $\frac{\partial S_t'(p)}{\partial\beta_{u,t+1}}$ at $p=p_t^*$ is equal to

$$\frac{b_{u,t+1}F_{t+1}(S_t(p_t^*))\int_0^{S_t(p_t^*)}(S_t(p_t^*)-x)dF_{t+1}(x)+p_t^*\left(S_t(p_t^*)F_{t+1}(S_t(p_t^*))-\int_0^{S_t(p_t^*)}(S_t(p_t^*)-x)dF_{t+1}(x)\right)}{(N_r-1)(p_t^*-\eta_{u,t+1}(p_t^*)F_{t+1}(S_t(p_t^*)))^2}, \tag{EC.58}$$

which is positive. Observe from Proposition 4 that $X(\bar{s})>0$ for $\epsilon_t \geq \underline{z}_t(\bar{s})$ and $\bar{s}\in[0,K]$. Finally, $Y(\bar{s})>0$ for $\epsilon_t \geq \underline{z}_t(\bar{s})$ and $\bar{s}\in[0,K]$ if and only if

$$S_t''(p_t^*) > -\frac{(\alpha_t+N_rS_t'(p_t^*))^2}{\bar{s}N_r},\ \epsilon_t \geq \underline{z}_t(\bar{s}) \text{ and } \bar{s}\in[0,K]. \tag{EC.59}$$

Because $\bar{s}\leq K$ and $S_t'(\cdot)>0$,

$$-\frac{\alpha_t^2}{KN_r} > -\frac{(\alpha_t+N_rS_t'(p_t^*))^2}{\bar{s}N_r}. \tag{EC.60}$$

This and the third condition in (22) imply (EC.59). Thus, under (22), $Y(\bar{s})>0$ for $\epsilon_t \geq \underline{z}_t(\bar{s})$ and $\bar{s}\in[0,K]$. □

Appendix H: Proof of Proposition 7

Denote by $S_t(\cdot;\eta_1)$ and $S_t(\cdot;\eta_2)$ any two equilibrium committed production schedules of renewable firm n with under-supply penalty rates $\eta_1(\cdot)$ and $\eta_2(\cdot)$, respectively. Define $\widehat{p}\doteq\frac{b_2-b_1}{\beta_1-\beta_2}$, which is the unique price that satisfies $\eta_1(\widehat{p})=\eta_2(\widehat{p})$. Because $b_1<b_2$, it follows from Lemma EC.4 that $S_t'(0;\eta_1)>S_t'(0;\eta_2)$. This and $S_t(0;\eta_1)=S_t(0;\eta_2)=0$ imply that either of the following two scenarios holds: either $S_t(p;\eta_1)>S_t(p;\eta_2)$ for all $p>0$ or $S_t(\cdot;\eta_1)$ and $S_t(\cdot;\eta_2)$ intersect at least at one point. Suppose that the former scenario holds. Then, for $p>\widehat{p}$, $S_t(p;\eta_1)>S_t(p;\eta_2)$ and $\eta_1(p)>\eta_2(p)$ because $\eta_1(p)>\eta_2(p)$ if and only if $p>\widehat{p}$. Suppose now that the latter scenario holds. Denote by $\underline{p}$, the minimum price at which $S_t(\cdot;\eta_1)$ and $S_t(\cdot;\eta_2)$ intersect. From the definition of $\underline{p}$, we have $S_t(p;\eta_1)>S_t(p;\eta_2)$ for $p\in(0,\underline{p})$ and $S_t(p;\eta_1)<S_t(p;\eta_2)$ for $p\in(\underline{p},\overline{p})$ where $\overline{p}$ is either the second minimum price at which $S_t(\cdot;\eta_1)$ and $S_t(\cdot;\eta_2)$ intersect or $\overline{p}$ is $+\infty$. If $\underline{p}>\widehat{p}$, then $S_t(p;\eta_1)>S_t(p;\eta_2)$ and $\eta_1(p)>\eta_2(p)$ for $p\in(\widehat{p},\underline{p})$. If $\underline{p}<\widehat{p}$, then $S_t(p;\eta_2)>S_t(p;\eta_1)$ and $\eta_2(p)>\eta_1(p)$ for $p\in(\underline{p},\min\{\overline{p},\widehat{p}\})$. Here, the interval $(\underline{p},\min\{\overline{p},\widehat{p}\})$ is non-empty by the definition of $\overline{p}$. The parameter set in which $\underline{p}=\widehat{p}$ is a set of Lebesgue measure zero, which completes the proof of Proposition 7. □

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Appendix I: Proofs of Results in Section 4

Proof of Proposition 8: ***Part*** (i)***:*** Because Proposition 1 holds as stated in this setting, we know that a period-t supply function equilibrium that satisfies (12) exists for any committed inflexible energy profile $\bar{\mathbb{S}}$, and each firm's renewable production schedule is not dependent on $\bar{\mathbb{S}}$ in any equilibrium. Based on this and (23), each firm j chooses its inflexible energy commitment at $t = 1$ as follows.

$$\max_{0 \le \bar{S}_j \le K} \quad \Bigg\{ \sum_{t=1}^{\tau} \int_{\underline{\tilde{z}}_t}^{\infty} \Big[(S_t(p_t^*(\bar{\mathbb{S}}; z)) + \bar{S}_j) p_t^*(\bar{\mathbb{S}}; z) - C_{t+1}(\bar{S}_j) \Big] d\Phi_t(z) \\ - \sum_{t=1}^{\tau} \int_{\underline{\tilde{z}}_t}^{\infty} \Big[\eta_{u,t+1}(p_t^*(\bar{\mathbb{S}}; z)) \int_0^{S_t(p_t^*(\bar{\mathbb{S}};z))} \big(S_t(p_t^*(\bar{\mathbb{S}}; z)) - x \big)\, dF_{t+1}(x) \Big] d\Phi_t(z) \Bigg\}, \tag{EC.61}$$

where $\underline{\tilde{z}}_t \doteq \sum_{n=1}^N \bar{S}_n - v_t$ and $p_t^*(\bar{\mathbb{S}}; z)$ satisfies $v_t - \alpha_t p_t^*(\bar{\mathbb{S}}; z) + z = \sum_{n=1}^N \bar{S}_n + N S_t(p)$. We claim and show below that if the feasible set for $\bar{S}_j$ was $\mathbb{R}_+$, an equilibrium (as in Definition 2) subject to (11) and (12) would exist. Then, with the capacity constraint ($\bar{S}_j \le K$), the minimum of K and a firm's inflexible energy commitment in the aforementioned unconstrained equilibrium is the firm's inflexible energy commitment $\bar{S}$ in the actual equilibrium. This immediately implies part (i).

We now show our above claim that if the feasible set for $\bar{S}_j$ is $\mathbb{R}_+$, an unconstrained equilibrium subject to (11) and (12) exists. To do so, by Proposition 1, it is sufficient to show that given all other firms' inflexible energy commitments, firm j's unconstrained inflexible energy commitment at $t = 1$ that maximizes (EC.61) is finite when (11) and (12) hold. In the remainder of the proof, $\bar{s}^*$ will represent the aforementioned unconstrained inflexible commitment. Applying the implicit function theorem on $v_t - \alpha_t p_t^*(\bar{\mathbb{S}}; z) + z = \sum_{n=1}^N \bar{S}_n + N S_t(p_t^*(\bar{\mathbb{S}}; z))$, we have $\partial p_t^*(\bar{\mathbb{S}}; z)/\partial \bar{S}_j = -1/(\alpha_t + N S_t'(p))$. Then, taking the derivative of (EC.61) with respect to $\bar{S}_j$, using $\partial p_t^*(\bar{\mathbb{S}}; z)/\partial \bar{S}_j = -1/(\alpha_t + N S_t'(p))$, and applying the condition $\bar{S}_1 = \ldots = \bar{S}_N = \bar{s}$ in (11), we get $\Psi_M(\bar{s})$ in (24). By (17) and the fact that $S_t'(\cdot) > 0$, the term in the first line of $\Psi_M(\cdot)$ is negative. From this and the definition of $\Psi(\bar{s})$ in Proposition 2, it follows that $\Psi_M(\bar{s}) < \Psi(\bar{s})$ for all $\bar{s}$. This and the fact that $\lim_{\bar{s} \to \infty} \Psi(\bar{s}) < 0$ imply that

$$\lim_{\bar{s} \to \infty} \Psi_M(\bar{s}) < 0. \tag{EC.62}$$

As a result, each firm's inflexible energy commitment $\bar{s}^*$ exists and it is finite. ***Part*** (ii)***:*** Part (ii) immediately follows from part (i), Proposition 1 and Lemma 1. ***Part*** (iii)***:*** Recall the notation $\bar{s}^*$ in part (i). Because (23) subject to (11) and (12) has a unique stationary point by our formulation in Section 4, by (EC.62), $\bar{s}^*$ is the unique stationary point of (EC.61) subject to (11) and hence it is the unique global maximizer of (EC.61) subject to (11). Therefore, $\Psi_M(\bar{s}^*) = 0$. By the proof of part (i), we already know that $\bar{S} = \min\{K, \bar{s}^*\}$, which immediately implies part (iii). □

Proof of Proposition 9: We first prove part (i). In any equilibrium, because $S_t'(p) > 0$ for $p \ge 0$ by (18), it follows from (17) that $\delta_2(p) > 0$. This with $S_t'(\cdot) > 0$ imply that the first line of $\Psi_M(\bar{s})$ in (24) is strictly negative. From Proposition 8-(iii), we already know that $\bar{S} = \min\{K, \bar{s}^*\}$ where $\bar{s}^*$ satisfies $\Psi_M(\bar{s}^*) = 0$. Comparing $\Psi(\cdot)$ with $\Psi_M(\cdot)$, we conclude that $\Psi_M(\cdot) < \Psi(\cdot)$. From this, Proposition 2-(iii) and Proposition 8-(iii), part (i) follows. We now prove part (ii). Part (i) and (1) imply that $p_t^*(\epsilon_t)$ is larger in a day-ahead electricity market with multi-technology firms for any t and ϵ_t. Because $S_t(\cdot)$ remains the same in both type of day-ahead markets, a larger $p_t^*(\epsilon_t)$ immediately implies a larger cleared renewable energy commitment $S_t(p_t^*(\epsilon_t))$ for each firm in period-t day-ahead market. Then, by similar arguments in the proof of Proposition 5, the equilibrium reliability claim in part (ii) follows. □

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Appendix J: Proof of Proposition 10

In this section, we prove Proposition 10-(i); the proof of Proposition 10-(ii) is a special case of the proof presented below, and thus it is omitted. Suppose that $b_{o,t+1} < T'_{t+1}(0)$. Then, from (27) we have

$$\hat{s}_{t+1} \doteq (T'_{t+1})^{-1}(b_{o,t+1}). \tag{EC.63}$$

Recall that the structural properties of $T_{t+1}(\cdot)$ ensure the uniqueness of $\hat{s}_{t+1}$. Let $p_{\ell,t} \doteq -b_{o,t+1}$ and $\lambda_r \doteq 1/(N_r - 1)$. Define the mapping $d : [p_{\ell,t}, \infty) \times \mathbb{R}_+ / \{(p_{\ell,t}, 0)\} \to \mathbb{R}$ as

$$d(p,s) \doteq \begin{cases} \left[\dfrac{s - \beta_{u,t+1}\int_0^s (s-x)dF_{t+1}(x)}{p - \eta_{u,t+1}(p)F_{t+1}(s) + b_{o,t+1}(1 - F_{t+1}(s))} - \alpha_t\right]\lambda_r & \text{if } s < \hat{s}_{t+1} \quad \text{(EC.64a)} \\ \left[\dfrac{s - \beta_{u,t+1}\int_0^s (s-x)dF_{t+1}(x)}{p - \eta_{u,t+1}(p)F_{t+1}(s) + T'_{t+1}(s)\,(1 - F_{t+1}(s))} - \alpha_t\right]\lambda_r & \text{if } s \geq \hat{s}_{t+1}. \quad \text{(EC.64b)} \end{cases}$$

Define the complementary cumulative distribution $\overline{F}_{t+1}(\cdot)$ as

$$\overline{F}_{t+1}(x) \doteq 1 - F_{t+1}(x), \quad x \geq 0. \tag{EC.65}$$

Proposition 10-(i) follows from the same arguments explained in the proof of Lemma EC.1 with the following exceptions in the Step 1 of that proof:

Step 1: From the same arguments explained in Step 1 of the proof of Lemma EC.1, it follows that for any point $(\tilde{p}, \tilde{s}) \in [p_{\ell,t}, \infty) \times \mathbb{R}_+ / \{(p_{\ell,t}, 0)\}$, there exists a unique trajectory $\underline{S}_t(\cdot)$ that satisfies $\underline{S}_t(\tilde{p}) = \tilde{s}$ and the ODE in (28a) for $p \in [\tilde{p} - \delta, \tilde{p} + \delta]$. Similarly, for any point $(\tilde{p}, \tilde{s}) \in [-T'_{t+1}(0), \infty) \times \mathbb{R}_+ / \{(-T'_{t+1}(0), 0)\} \to \mathbb{R}$, there exists a unique trajectory $\overline{S}_t(\cdot)$ that satisfies both $\overline{S}_t(\tilde{p}) = \tilde{s}$ and the ODE in (28b) for $p \in [\tilde{p} - \delta, \tilde{p} + \delta]$.

Let $\vartheta_{t+1}(s) \doteq 1 - \beta_{u,t+1}F_{t+1}(s)$. Based on this, for each $\hat{k} \geq 0$, define the function $g_{\hat{k}}(\cdot)$ as

$$g_{\hat{k}}(s) \doteq \begin{cases} \left[\frac{\left(s - \beta_{u,t+1}\int_0^s (s-x)dF_{t+1}(x)\right)}{((N_r - 1)\hat{k} + \alpha_t)} + b_{u,t+1}F_{t+1}(s) - b_{o,t+1}\overline{F}_{t+1}(s)\right]/\vartheta_{t+1}(s) & \text{if } s < \hat{s}_{t+1} \\ \left[\frac{\left(s - \beta_{u,t+1}\int_0^s (s-x)dF_{t+1}(x)\right)}{((N_r - 1)\hat{k} + \alpha_t)} + b_{u,t+1}F_{t+1}(s) - T'_{t+1}(s)\overline{F}_{t+1}(s)\right]/\vartheta_{t+1}(s) & \text{if } s \geq \hat{s}_{t+1}, \end{cases} \tag{EC.66}$$

which implies that

$$g_0(s) = \begin{cases} \left[\left(s - \beta_{u,t+1}\int_0^s (s-x)dF_{t+1}(x)\right)/\alpha_t + b_{u,t+1}F_{t+1}(s) - b_{o,t+1}\,\overline{F}_{t+1}(s)\right]/\vartheta_{t+1}(s) & \text{if } s < \hat{s}_{t+1} \\ \left[\left(s - \beta_{u,t+1}\int_0^s (s-x)dF_{t+1}(x)\right)/\alpha_t + b_{u,t+1}F_{t+1}(s) - T'_{t+1}(s)\overline{F}_{t+1}(s)\right]/\vartheta_{t+1}(s) & \text{if } s \geq \hat{s}_{t+1}, \end{cases} \tag{EC.67}$$

$$g_\infty(s) \doteq \lim_{\hat{k}\to\infty} g_{\hat{k}}(s) = \begin{cases} (b_{u,t+1}F_{t+1}(s) - b_{o,t+1}\,\overline{F}_{t+1}(s))/\vartheta_{t+1}(s) & \text{if } s < \hat{s}_{t+1} \\ (b_{u,t+1}F_{t+1}(s) - T'_{t+1}(s)\overline{F}_{t+1}(s))/\vartheta_{t+1}(s) & \text{if } s \geq \hat{s}_{t+1}. \end{cases} \tag{EC.68}$$

Observe from (EC.63), (EC.67) and (EC.68) that $g_0(\cdot)$ and $g_\infty(\cdot)$ are continuous and increasing functions. From the arguments above, we know that for each $(p,s) \in [p_{\ell,t}, \infty) \times [0, \hat{s}_{t+1}) / \{(p_{\ell,t}, 0)\}$, there exists a unique trajectory $S_{t,(p,s)}(\cdot)$ that passes through (p,s) and satisfies (28a). Notice that this trajectory also satisfies

$$d(p, S_{t,(p,s)}(p)) = S'_{t,(p,s)}(p), \quad (p,s) \in [p_{\ell,t}, \infty) \times [0, \hat{s}_{t+1}) / \{(p_{\ell,t}, 0)\}. \tag{EC.69}$$

Similarly, for each $(p,s) \in [p_{\ell,t}, \infty) \times [\hat{s}_{t+1}, \infty)$, the unique trajectory $S_{t,(p,s)}(\cdot)$ that passes through (p,s) and satisfies (28b) - the existence and the uniqueness of which is proved above - also satisfies (EC.69). With these observations, the proof of Proposition 10-(i) can be completed by applying the same arguments explained in Steps 2 and 3 of the proof of Lemma EC.1. $\square$

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Appendix K: Proof of Proposition 11

The general outline of the proof: In this section, we state and prove Proposition EC.6 that identifies a period-t supply function equilibrium that satisfies (12) and the resulting production profile of renewable firms for a given $\bar{\mathbb{S}}$. Proposition EC.6 also establishes the existence of the aforementioned period-t supply function equilibrium. Using period-t supply function equilibrium for $t = 1, \ldots, \tau$ and (6), it is straightforward to verify that inflexible firms' equilibrium commitment would be as identified in Proposition 11-(iv). The existence of equilibrium commitment of inflexible firms subject to (11) immediately follows from similar arguments explained in the proof of Proposition 2-(iii). Combining this and the existence of a period-t supply function equilibrium for $t = 1, \ldots, \tau$ establishes Proposition 11-(i). Because the proof of Proposition EC.6 explicitly identify the equilibrium strategies of each renewable firm at every t, Proposition 11-(ii) and (iii) immediately follow. Based on the above explanation, the proof of Proposition 11-(iv) is straightforward and hence omitted.

PROPOSITION EC.6. *Consider any* $t = 1, \ldots, \tau$. (i) *Suppose that* $b_{o,t+1} < T'_{t+1}(0)$. *Then, for any given* $\bar{\mathbb{S}}$, *a function* $S_t(\cdot)$ *is each renewable firm's committed production schedule in a period-t supply function equilibrium that satisfies* (12) *if and only if* $S_t(\cdot)$ *satisfies* $(28a)$ *through* (29). *For any given* $\bar{\mathbb{S}}$, *there exists a period-t supply function equilibrium that satisfies* (12).
(ii) *Suppose that* $b_{o,t+1} \geq T'_{t+1}(0)$. *Then, for any given* $\bar{\mathbb{S}}$, *a function* $S_t(\cdot)$ *is each firm's committed production schedule in a period-t supply function equilibrium that satisfies* (12) *if and only if* $S_t(\cdot)$ *satisfies* $(28b)$ *subject to* $0 < S'_t(p) < \infty$ *for* $p > p_{\ell,t}$ *and* $S_t(p_{\ell,t}) = 0$ *where* $p_{\ell,t} \doteq -T'_{t+1}(0)$. *For any given* $\bar{\mathbb{S}}$, *there exists a period-t supply equilibrium that satisfies* (12).

Proof of Proposition EC.6: The outline of this proof is as follows. The proof of Proposition EC.6-(ii) is a special case of Proposition EC.6-(i), and hence omitted. To prove Proposition EC.6-(i), we state and prove Lemmas EC.7 through EC.9 and Proposition EC.7. Lemma EC.9 shows the second sentence of Proposition 11-(i). The last sentence of Proposition 11-(i) follows from Lemmas EC.7 through EC.9 and Proposition 10.

Recall that each renewable firm decides its period-(t+1) production quantity after the period-t day-ahead market is cleared and its available supply in period t+1 is realized. Therefore, to characterize a renewable firm's equilibrium production schedule in period t, we first solve for its optimal production quantity in period t+1.

LEMMA EC.7. *Consider renewable firm n and suppose that other firms' period-t commitments are* $S_{-n,t}$ *and* $\bar{\mathbb{S}}$. *Then, for any* ϵ_t, *available supply* $Q_{n,t+1}$ *and the period-t day-ahead market clearing price p such that* $\eta_{u,t+1}(p) > 0$, *renewable firm n's optimal production quantity in period t+1 is*

$$q^*_{n,t+1}(p, \epsilon_t, Q_{n,t+1}) = \begin{cases} Q_{n,t+1} & \text{if } Q_{n,t+1} \leq \mathcal{R}_{n,t}(p;\epsilon_t), \\ \mathcal{R}_{n,t}(p;\epsilon_t) & \text{if } Q_{n,t+1} > \mathcal{R}_{n,t}(p;\epsilon_t) \quad \& \quad b_{o,t+1} > T'_{t+1}\left(\mathcal{R}_{n,t}(p;\epsilon_t)\right), \\ \hat{s}_{t+1} & \text{if } Q_{n,t+1} > \mathcal{R}_{n,t}(p;\epsilon_t) \quad \& \quad T'_{t+1}(Q_{n,t+1}) < b_{o,t+1} \leq T'_{t+1}\left(\mathcal{R}_{n,t}(p;\epsilon_t)\right), \\ Q_{n,t+1} & \text{if } Q_{n,t+1} > \mathcal{R}_{n,t}(p;\epsilon_t) \quad \& \quad b_{o,t+1} \leq T'_{t+1}(Q_{n,t+1}), \end{cases} \tag{EC.70}$$

where $\hat{s}_{t+1}$ *is a critical supply level that satisfies* $T'_{t+1}(\hat{s}_{t+1}) = b_{o,t+1}$.

Proof of Lemma EC.7: Recall (25), and define the function $\Gamma(\cdot)$ such that the optimal production quantity in period t+1 is $\arg\max_{0 \leq q_{n,t+1} \leq Q_{n,t+1}} \Gamma(q_{n,t+1})$. Observe from (1) and (5) that $\mathcal{R}_{n,t}(p^*_t;\epsilon) = S_{n,t}(p^*_t)$ for each

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period-t day-ahead shock ϵ. We prove our claim under the following two cases: Case 1: Suppose that $Q_{n,t+1} \leq \mathcal{R}_{n,t}(p;\epsilon)$. This and the fact that $q_{n,t+1} \leq Q_{n,t+1}$ imply that $q_{n,t+1} \leq \mathcal{R}_{n,t}(p;\epsilon)$. Therefore, $\Gamma(q_{n,t+1}) = T_{t+1}(q_{n,t+1}) - \eta_{u,t+1}(p)(\mathcal{R}_{n,t}(p;\epsilon) - q_{n,t+1})$. Because $T_{t+1}(\cdot)$ is increasing and $\eta_{u,t+1}(p) > 0$, $\Gamma'(x) = T'_{t+1}(x) + \eta_{u,t+1}(p) > 0$ for $x > 0$. Hence, $q^*_{n,t+1}(p,\epsilon,Q_{n,t+1}) = Q_{n,t+1}$. Case 2: Suppose that $Q_{n,t+1} > \mathcal{R}_{n,t}(p;\epsilon)$. Then, because $T_{t+1}(\cdot)$ is increasing, $\Gamma(\mathcal{R}_{n,t}(p;\epsilon)) > \Gamma(q_{n,t+1})$ for $q_{n,t+1} < \mathcal{R}_{n,t}(p;\epsilon)$. As a result, it is strictly suboptimal to produce $q_{n,t+1} < \mathcal{R}_{n,t}(p;\epsilon)$. Therefore, $q^*_{n,t+1}(p,\epsilon,Q_{n,t+1}) \in [\mathcal{R}_{n,t}(p;\epsilon), Q_{n,t+1}]$. Note that $\Gamma(q_{n,t+1}) = T_{t+1}(q_{n,t+1}) - b_{o,t+1}(q_{n,t+1} - \mathcal{R}_{n,t}(p;\epsilon))$ for $q_{n,t+1} \in [\mathcal{R}_{n,t}(p;\epsilon), Q_{n,t+1}]$. This and the concavity of $\Gamma(\cdot)$ in $[\mathcal{R}_{n,t}(p;\epsilon), Q_{n,t+1}]$ imply that $q^*_{n,t+1}(p,\epsilon,Q_{n,t+1}) = \mathcal{R}_{n,t}(p;\epsilon)$ if $\Gamma'(\mathcal{R}_{n,t}(p;\epsilon)) = T'_{t+1}(\mathcal{R}_{n,t}(p;\epsilon)) - b_{o,t+1} < 0$, $q^*_{n,t+1}(p,\epsilon,Q_{n,t+1}) = Q_{n,t+1}$ if $T'_{t+1}(Q_{n,t+1}) - b_{o,t+1} \geq 0$, and $q^*_{n,t+1}(p,\epsilon,Q_{n,t+1}) = \hat{s}_{t+1}$ where $T'_{t+1}(\hat{s}_{t+1}) = b_{o,t+1}$ if $\Gamma'(\mathcal{R}_{n,t}(p;\epsilon)) \geq 0$ and $\Gamma'(Q_{n,t+1}) < 0$. By rearranging terms, we have the last three lines of (EC.70). $\square$

It follows from (26) and (EC.70) that given other firm's period-t commitments $S_{-n,t}$ and $\bar{\mathbb{S}}$, renewable firm n's commitment-related period-t expected profit at price p and random shock ϵ is $\Pi_n(p;\epsilon,S_{-n,t},\bar{\mathbb{S}})$, which is equivalent to

$$\begin{aligned}
&\mathbb{E}_{Q_{n,t+1}}\Big(\underbrace{\mathbb{I}(\mathcal{A})\big\{p\mathcal{R}_{n,t}(p;\epsilon) + T_{t+1}(Q_{n,t+1}) - \eta_{u,t+1}(p)(\mathcal{R}_{n,t}(p;\epsilon) - Q_{n,t+1})\big\}}_{\text{supply}_{t+1} = \text{production}_{t+1} \leq \text{commitment}_t} + \underbrace{\mathbb{I}(\mathcal{B}_1)\big\{p\mathcal{R}_{n,t}(p;\epsilon) + T_{t+1}(\mathcal{R}_{n,t}(p;\epsilon))\big\}}_{\text{supply}_{t+1} > \text{production}_{t+1} = \text{commitment}_t}\Big) \\
&+ \mathbb{E}_{Q_{n,t+1}}\Big(\underbrace{\mathbb{I}(\mathcal{B}_2)\big\{p\mathcal{R}_{n,t}(p;\epsilon) + T_{t+1}(\hat{s}_{t+1}) - b_{o,t+1}(\hat{s}_{t+1} - \mathcal{R}_{n,t}(p;\epsilon))\big\}}_{\text{supply}_{t+1} > \text{production}_{t+1} > \text{commitment}_t}\Big) \\
&+ \mathbb{E}_{Q_{n,t+1}}\Big(\underbrace{\mathbb{I}(\mathcal{B}_3)\big\{p\mathcal{R}_{n,t}(p;\epsilon) + T_{t+1}(Q_{n,t+1}) - b_{o,t+1}(Q_{n,t+1} - \mathcal{R}_{n,t}(p;\epsilon))\big\}}_{\text{supply}_{t+1} = \text{production}_{t+1} > \text{commitment}_t}\Big) \qquad \text{(EC.71)}
\end{aligned}$$

where

$$\begin{aligned}
\mathcal{A} &\doteq \Big\{Q_{n,t+1} \leq \mathcal{R}_{n,t}(p;\epsilon)\Big\},\ \mathcal{B}_1 \doteq \Big\{Q_{n,t+1} > \mathcal{R}_{n,t}(p;\epsilon)\Big\} \bigcap \Big\{b_{o,t+1} > T'_{t+1}\big(\mathcal{R}_{n,t}(p;\epsilon)\big)\Big\}, \\
\mathcal{B}_2 &\doteq \Big\{Q_{n,t+1} > \mathcal{R}_{n,t}(p;\epsilon)\Big\} \bigcap \Big\{T'_{t+1}(Q_{n,t+1}) < b_{o,t+1} \leq T'_{t+1}\big(\mathcal{R}_{n,t}(p,\epsilon)\big)\Big\}, \\
\mathcal{B}_3 &\doteq \Big\{Q_{n,t+1} > \mathcal{R}_{n,t}(p;\epsilon)\Big\} \bigcap \Big\{b_{o,t+1} \leq T'_{t+1}(Q_{n,t+1})\Big\}.
\end{aligned}$$

PROPOSITION EC.7. *Suppose that $S_{j,t}(\cdot)$ satisfies $(28a)$ through (29) for $j \leq N_r$ and $j \neq n$. Then, in period t, renewable firm n maximizes $\Pi_n(\cdot;\epsilon_t,S_{-n,t},\bar{\mathbb{S}})$ for all ϵ_t if the firm n commits to the production schedule $S_{n,t}(\cdot)$ that satisfies*

$$\begin{aligned}
&S_{n,t}(p) - \beta_{u,t+1}\int_0^{S_{n,t}(p)}(S_{n,t}(p)-x)dF_{t+1}(x) \\
&-\left(\alpha_t + \sum_{j\neq n}^{N_r} S'_{j,t}(p)\right)\left[p - \eta_{u,t+1}(p)F_{t+1}(S_{n,t}(p)) + \min\left\{T'_{t+1}(S_{n,t}(p)), b_{o,t+1}\right\}\overline{F}_{t+1}(S_{n,t}(p))\right] = 0, \qquad \text{(EC.72)} \\
&S'_{n,t}(p) \in (0,\infty) \text{ for } p \in (-b_{o,t+1},\infty), \text{ and } S_{n,t}(-b_{o,t+1}) = 0.
\end{aligned}$$

Proof of Proposition EC.7: Recall (EC.63) and (EC.65). Take any $\epsilon_t = \epsilon$ and production schedule profile $S_{-n,t}$ of other renewable firms such that $S_{j,t}(\cdot)$ satisfies conditions in $(28a)$ through (29) for $j \leq N_r$ and $j \neq n$. Define functions $\pi_1(\cdot)$ and $\pi_2(\cdot)$ as

$$\begin{aligned}
\pi_1(p) \doteq & p\mathcal{R}_{n,t}(p;\epsilon) + \int_0^{\mathcal{R}_{n,t}(p;\epsilon)} T_{t+1}(x)dF_{t+1}(x) - \eta_{u,t+1}(p)\int_0^{\mathcal{R}_{n,t}(p;\epsilon)}(\mathcal{R}_{n,t}(p;\epsilon)-x)dF_{t+1}(x) \\
&+ \int_{\mathcal{R}_{n,t}(p;\epsilon)}^{\infty} T_{t+1}(\mathcal{R}_{n,t}(p;\epsilon))dF_{t+1}(x), \qquad \text{(EC.73)} \\
\pi_2(p) \doteq & p\mathcal{R}_{n,t}(p;\epsilon) + \int_0^{\mathcal{R}_{n,t}(p;\epsilon)} T_{t+1}(x)dF_{t+1}(x) - \eta_{u,t+1}(p)\int_0^{\mathcal{R}_{n,t}(p;\epsilon)}(\mathcal{R}_{n,t}(p;\epsilon)-x)dF_{t+1}(x) \\
&+ \int_{\mathcal{R}_{n,t}(p;\epsilon)}^{\hat{s}_{t+1}} \left(T_{t+1}(x) - b_{o,t+1}(x-\mathcal{R}_{n,t}(p;\epsilon))\right)dF_{t+1}(x) + \int_{\hat{s}_{t+1}}^{\infty}\left(T_{t+1}(\hat{s}_{t+1}) - b_{o,t+1}(\hat{s}_{t+1}-\mathcal{R}_{n,t}(p;\epsilon))\right)dF_{t+1}(x). \\
&\qquad \text{(EC.74)}
\end{aligned}$$

Define the critical price $\hat{p}$ as the unique price that satisfies $\mathcal{R}_{n,t}(\hat{p};\epsilon) = \hat{s}_{t+1}$. Because $\mathcal{R}'_{n,t}(\cdot;\epsilon) < 0$, it follows from (EC.71) and the definition of $\hat{p}$ that the renewable firm n's (commitment-related) expected period-t profit at price p is

$$\Pi_n(p;\epsilon, S_{-n,t}, \bar{\mathbb{S}}) = \begin{cases} \pi_1(p) & \text{if} \quad p < \hat{p}, \\ \pi_2(p) & \text{if} \quad p \geq \hat{p}. \end{cases} \tag{EC.75}$$

We now state and prove the following lemma that shows the concavity of $\pi_1(\cdot)$ and $\pi_2(\cdot)$.

LEMMA EC.8. *For any ϵ and $S_{-n,t}$ such that $S_{j,t}(\cdot)$ satisfies conditions in (28a), (28b) and (29) for $j \leq N_r$ and $j \neq n$, $\pi''_1(\cdot) < 0$ and $\pi''_2(\cdot) < 0$. Therefore, there exist unique prices p_1^* and p_2^* that maximize $\pi_1(\cdot)$ and $\pi_2(\cdot)$, respectively.*

Proof of Lemma EC.8: Recall (EC.73) and (EC.74). By elementary analysis,

$$\begin{aligned} \pi'_1(p) = &\mathcal{R}_{n,t}(p;\epsilon) - \beta_{u,t+1}\int_0^{\mathcal{R}_{n,t}(p;\epsilon)} (\mathcal{R}_{n,t}(p;\epsilon) - x) dF_{t+1}(x) \\ &+ \mathcal{R}'_{n,t}(p;\epsilon)\left[p - \eta_{u,t+1}(p)F_{t+1}(\mathcal{R}_{n,t}(p;\epsilon)) + T'_{t+1}(\mathcal{R}_{n,t}(p;\epsilon))\overline{F}_{t+1}(\mathcal{R}_{n,t}(p;\epsilon))\right], \qquad \text{(EC.76)} \\ \pi'_2(p) = &\mathcal{R}_{n,t}(p;\epsilon) - \beta_{u,t+1}\int_0^{\mathcal{R}_{n,t}(p;\epsilon)} (\mathcal{R}_{n,t}(p;\epsilon) - x) dF_{t+1}(x) \\ &+ \mathcal{R}'_{n,t}(p;\epsilon)\left[p - \eta_{u,t+1}(p)F_{t+1}(\mathcal{R}_{n,t}(p;\epsilon)) + b_{o,t+1}\,\overline{F}_{t+1}(\mathcal{R}_{n,t}(p;\epsilon))\right], \qquad \text{(EC.77)} \end{aligned}$$

which imply that

$$\begin{aligned} \pi''_1(p) = &2\mathcal{R}'_{n,t}(p;\epsilon)(1 - \beta_{u,t+1}F_{t+1}(\mathcal{R}_{n,t}(p;\epsilon))) + \left(\mathcal{R}'_{n,t}(p;\epsilon)\right)^2 T''_{t+1}(\mathcal{R}_{n,t}(p;\epsilon))\overline{F}_{t+1}(\mathcal{R}_{n,t}(p;\epsilon)) \\ &+ \left(\mathcal{R}'_{n,t}(p;\epsilon)\right)^2\left[-\eta_{u,t+1}(p)f_{t+1}(\mathcal{R}_{n,t}(p;\epsilon)) - T'_{t+1}(\mathcal{R}_{n,t}(p;\epsilon))f_{t+1}(\mathcal{R}_{n,t}(p;\epsilon))\right] \\ &+ \mathcal{R}''_{n,t}(p;\epsilon)[p - \eta_{u,t+1}(p)F_{t+1}(\mathcal{R}_{n,t}(p;\epsilon)) + T'_{t+1}(\mathcal{R}_{n,t}(p;\epsilon))\overline{F}_{t+1}(\mathcal{R}_{n,t}(p;\epsilon))], \qquad \text{(EC.78)} \\ \pi''_2(p) = &2\mathcal{R}'_{n,t}(p;\epsilon)(1 - \beta_{u,t+1}F_{t+1}(\mathcal{R}_{n,t}(p;\epsilon))) + \left(\mathcal{R}'_{n,t}(p;\epsilon)\right)^2[-\eta_{u,t+1}(p)f_{t+1}(\mathcal{R}_{n,t}(p;\epsilon)) - b_{o,t+1}f_{t+1}(\mathcal{R}_{n,t}(p;\epsilon))] \\ &+ \mathcal{R}''_{n,t}(p;\epsilon)[p - \eta_{u,t+1}(p)F_{t+1}(\mathcal{R}_{n,t}(p;\epsilon)) + b_{o,t+1}\overline{F}_{t+1}(\mathcal{R}_{n,t}(p;\epsilon))]. \end{aligned}$$

We now prove that $\pi''_1(\cdot) < 0$ by using (EC.78). For that, we first derive $S''_{j,t}(\cdot)$ for $j \neq n$ since $\mathcal{R}''_{n,t}(p;\epsilon) = -\sum_{j\neq n}^{N_r} S''_{j,t}(p)$. For brevity, we drop the arguments of functions in this step. Because $S_{j,t}(\cdot)$ satisfies (28a) and (28b) for $j \neq n$, it follows from elementary analysis that

$$(N_r - 1)S''_{j,t}(p) = \begin{cases} -\frac{(1-\beta_{u,t+1}F_{t+1})}{\overline{B}_j}[(N_r-2)S'_{j,t} + \alpha_t] + \frac{(\eta_{u,t+1}(p)+b_{o,t+1})f_{t+1}A_jS'_{j,t}}{\overline{B}_j^2} & \text{if} \quad p < \hat{p} \\ -\frac{(1-\beta_{u,t+1}F_{t+1})}{\underline{B}_j}[(N_r-2)S'_{j,t} + \alpha_t] + \frac{(\eta_{u,t+1}f_{t+1}+T'_{t+1}f_{t+1}-T''_{t+1}\overline{F}_{t+1})A_jS'_{j,t}}{\underline{B}_j^2} & \text{if} \quad p \geq \hat{p}, \end{cases} \tag{EC.79}$$

where functions $\underline{B}_j(\cdot)$, $\overline{B}_j(\cdot)$, $A_j(\cdot)$ are defined as $\underline{B}_j(p) \doteq p - \eta_{u,t+1}(p)F_{t+1}(S_{j,t}(p)) + T'_{t+1}(S_{j,t}(p))\overline{F}_{t+1}(S_{j,t}(p))$, $\overline{B}_j(p) \doteq p - \eta_{u,t+1}(p)F_{t+1}(S_{j,t}(p)) + b_{o,t+1}\overline{F}_{t+1}(S_{j,t}(p))$, $A_j(p) \doteq S_{j,t}(p) - \beta_{u,t+1}\int_0^{S_{j,t}(p)}(S_{j,t}(p) - x)f_{t+1}(x)dx$.

We now show that $\pi''_1(\cdot) < 0$ by analyzing the sign of $\pi''_1(\cdot)$ for $p < \hat{p}$ and $p \geq \hat{p}$ separately. Let us first focus on prices $p \geq \hat{p}$. Define the function $\gamma(\cdot,\cdot)$ as $\gamma(y,p) \doteq -\eta_{u,t+1}(p)f_{t+1}(y(p)) - T'_{t+1}(y(p))f_{t+1}(y(p)) + T''_{t+1}(y(p))\overline{F}_{t+1}(y(p))$, for $y : \mathbb{R} \to \mathbb{R}$. Because $S_{j,t}(\cdot)$ satisfies (28a), (28b) and (EC.79) for $j \neq n$, $\mathcal{R}'_{n,t}(p;\epsilon) = -\alpha_t - \sum_{j\neq n}^{N_r} S'_{j,t}(p)$ and $\mathcal{R}''_{n,t}(p;\epsilon) = -\sum_{j\neq n}^{N_r} S''_{j,t}(p)$, it follows that for $p \geq \hat{p}$,

$$\begin{aligned} \pi''_1(p) = &-\left(\alpha_t + \frac{N_r}{N_r - 1}\sum_{j\neq n}^{N_r} S'_{j,t}(p)\right)(1 - \beta_{u,t+1}F_{t+1}(\mathcal{R}_{n,t})) \\ &+ \left(\alpha_t + \sum_{j\neq n}^{N_r} S_{j,t}(p)\right)^2 \gamma(\mathcal{R}_{n,t}, p) + \sum_{j\neq n}^{N_r} \frac{\gamma(S_{j,t}, p)A_j(p)S'_{j,t}(p)}{(N_r - 1)\underline{B}_j(p)} \overset{(\star)}{<} 0. \end{aligned} \tag{EC.80}$$

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The inequality ($\star$) is because $\gamma(\mathcal{R}_{n,t},p)<0$ for $p\geq -T'_{t+1}(0)$, $S'_{j,t}(\cdot)>0$ by the monotonicity condition in (29) and $\underline{B}_j(p)>0$ for $p>-T'_{t+1}(0)$ (otherwise $S_{j,t}$ would violate the monotonicity condition in (29)).

Let us now focus attention to prices $p<\hat{p}$. Because $b_{o,t+1}>T'_{t+1}(\mathcal{R}_{n,t}(p;\epsilon))$ for $p<\hat{p}$, it follows from (EC.78) and (EC.79) that

$$\begin{aligned}\pi_1''(p)=&(-2+k(p))(1-\beta_{u,t+1}F_{t+1}(S_{j,t}(p)))\alpha_t+\left(-2+k(p)\frac{N_r-2}{N_r-1}\right)(1-\beta_{u,t+1}F_{t+1}(\mathcal{R}_{n,t}))\sum_{j\neq n}^{N_r}S'_{j,t}\\&+\left(\alpha+\sum_{j\neq n}^{N_r}S_{j,t}\right)^2\gamma(\mathcal{R}_{n,t},p)+k(p)\sum_{j\neq n}^{N_r}\frac{-(\eta_{u,t+1}(p)+b_{o,t+1})A_j(p)f_{t+1}(S_{j,t})S'_{j,t}(p)}{(N_r-1)\overline{B}_j(p)}<0,\end{aligned}\tag{EC.81}$$

for $p<\hat{p}$ where $k(p)\in(0,1)$. Combining (EC.80) and (EC.81) and from continuity of π_1', we conclude that $\pi_1''(\cdot)<0$. Similar logic can be used to prove that $\pi_2''(p)<0$ for all p. Therefore, both π_1 and π_2 are strictly concave and have unique maximizers p_1^* and p_2^*, respectively. □

Observe from (EC.63), (EC.76), (EC.77) and the definition of $\hat{p}$ that

$$\pi_1'(p)>\pi_2'(p) \text{ for } p<\hat{p}, \text{ and } \pi_1'(p)<\pi_2'(p) \text{ for } p>\hat{p}\tag{EC.82}$$

$$\pi_1(\hat{p})=\pi_2(\hat{p}) \quad \text{and} \quad \pi_1'(\hat{p})=\pi_2'(\hat{p}).\tag{EC.83}$$

These and strict concavity of $\pi_1(\cdot)$ and $\pi_2(\cdot)$ imply that there exists a unique price $p^*(\epsilon)$ that maximizes $\Pi_n(p;\epsilon,S_{-n,t},\bar{\mathbb{S}})$.

We now proceed to show that the unique price $p^*(\epsilon)$ that maximizes $\Pi_n(\cdot;\epsilon,S_{-n,t},\bar{\mathbb{S}})$ is the solution of (EC.72). From (EC.75) and (EC.82), the optimal price $p^*(\epsilon)$ should satisfy

$$0=\max\{\pi_1'(p),\pi_2'(p)\}.\tag{EC.84}$$

By (EC.76) and (EC.77) and using the fact that $\mathcal{R}'_{n,t}(p;\epsilon)<0$, (EC.84) is equivalent to

$$\begin{aligned}0=&\mathcal{R}_{n,t}(p;\epsilon)-\beta_{u,t+1}\int_0^{\mathcal{R}_{n,t}(p;\epsilon)}(\mathcal{R}_{n,t}(p;\epsilon)-x)dF_{t+1}(x)\\&+\mathcal{R}'_{n,t}(p;\epsilon)[p-\eta_{u,t+1}(p)F_{t+1}(\mathcal{R}_{n,t}(p;\epsilon))+\min\{b_{o,t+1},T'_{t+1}(\mathcal{R}_{n,t}(p;\epsilon)\}\overline{F}_{t+1}(\mathcal{R}_{n,t}(p;\epsilon))].\end{aligned}\tag{EC.85}$$

By market clearing condition (1) and the definition of $\mathcal{R}_{n,t}(\cdot;\cdot)$ in (5), $S_{n,t}(p)=\mathcal{R}_{n,t}(p;\epsilon)$ for all p. Using this and (EC.85), we conclude that $S_{n,t}$ should satisfy (EC.72). It is never profitable for a renewable firm to produce at price $p\leq p_{\ell,t}$ and $S'_{n,t}\in(0,\infty)$ guarantees that the residual demand crosses the renewable firm's supply curve at unique price. From these, our claim follows. □

LEMMA EC.9. *For any $\bar{\mathbb{S}}$, a function $S_t(\cdot)$ is each renewable firm's committed production schedule in a period-t supply function equilibrium that satisfies* (12) *if and only if $S_t(\cdot)$ satisfies conditions* (28a) *through* (29).

Proof of Lemma EC.9: This lemma can be proved by using the same arguments in Lemma EC.3 except that the words "Lemma EC.2" in the proof of Lemma EC.3 should be replaced with "Proposition EC.7". □

With these results, we conclude the proof of Proposition 11. □

Equilibrium with a market-based oversupply penalty: Define $A(p)\doteq S_t(p)-\beta_{u,t+1}\int_0^{S_t(p)}(S_t(p)-x)dF_{t+1}(x)-\beta_{o,t+1}\int_{S_t(p)}^{\tilde{s}(p)}(x-S_t(p))dF_{t+1}(x)$, and let $\tilde{s}(p)$ be the solution of $T'_{t+1}(\tilde{s}(p))=\eta_{o,t+1}(p)$. Assume that $\lim_{p\to p_{\ell,t}}\tilde{s}(p)=\infty$ and $\lim_{x\to 0}x\overline{F}_{t+1}(x)=0$. Then, using the similar arguments explained above, one can show that each renewable firm's equilibrium committed production schedule $S_t(\cdot)$ must satisfy

$$S_t'(p)=\frac{1}{N_r-1}\left[\frac{A(p)+(1-\eta_{o,t+1}(p)/T'_{t+1}(\tilde{s}(p))-(\tilde{s}(p)-S_t(p)))\beta_{o,t+1}\overline{F}_{t+1}(\tilde{s}(p))}{p-\eta_{u,t+1}(p)F_{t+1}(S_t(p)+\eta_{o,t+1}(p)(1-F_{t+1}(S_t(p)))}-\alpha_t\right]\tag{EC.86}$$

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if $\eta_{u,t+1}(p) < T'_{t+1}(S_t(p))$; $S_t(\cdot)$ satisfies

$$S'_t(p) = \frac{1}{N_r - 1}\left[\frac{S_t(p) - \beta_{u,t+1}\int_0^{S_t(p)}(S_t(p)-x)dF_{t+1}(x)}{p - \eta_{u,t+1}(p)F_{t+1}(S_t(p) + T'_{t+1}(S_t(p))(1 - F_{t+1}(S_t(p)))} - \alpha_t\right] \tag{EC.87}$$

if $\eta_{u,t+1}(p) \geq T'_{t+1}(S_t(p))$, subject to monotonicity and initial conditions, respectively:

$$S'_t(p) \in (0,\infty), \;\; p > p_{\ell,t}, \quad S_t(p_{\ell,t}) = 0, \tag{EC.88}$$

where $p_{\ell,t} \in (-(\beta_{o,t+1}\mu_{t+1} + \alpha_t b_{o,t+1})/(\alpha_t(1+\beta_{o,t+1})), -b_{o,t+1}/(1+\beta_{o,t+1}))$. Production commitment of each inflexible firm and production strategy of each renewable firm in equilibrium are both as identified in Proposition 11, with the exception that $\hat{s}_{t+1}$ should be replaced with $\tilde{s}(p)$, $b_{o,t+1}$ should be replaced with $\eta_{o,t+1}(p)$, and $S_t(\cdot)$ must refer to a function that satisfies (EC.86) through (EC.88).

Appendix L: Proofs of Results in Section EC.1.1

Proof of Proposition EC.1: The outline of the proof is as follows. We first introduce some analysis in three steps, namely Steps A through C. Then, using these steps, we show Proposition EC.1 (i) through (iii) at the end of the proof.

Step A:*Characterization of a Period-t Supply Function Equilibrium* Take any commitment profile $\bar{\mathbb{S}}$ for inflexible firms. This step shows that $S_t(\cdot)$ is each renewable firm's committed production schedule in a period-t supply equilibrium that satisfies (12) if and only if $S_t(\cdot)$ satisfies (EC.1a) and (EC.1b) subject to the monotonicity constraint and the initial condition stated in Proposition EC.1-(ii). (The existence of such a function $S_t(\cdot)$ will be proved in Step B.) To show this, we first claim (and prove below) the following statement: Given that $S_{j,t}(\cdot)$ satisfies (EC.1a), (EC.1b), the monotonicity constraint and the initial condition stated in part (ii) for $j \leq N_r$ and $j \neq n$, the function $S_{n,t}(\cdot)$ maximizes renewable firm n's commitment-related period-t expected profit $\Pi_n(\cdot;\epsilon_t, S_{-n,t}, \bar{\mathbb{S}})$ for any ϵ_t if it satisfies

$$S_{n,t}(p) - \beta_{u,t+1}\int_0^{S_{n,t}(p)}(S_{n,t}(p)-x)dF_{t+1}(x) - \left(\alpha_t + \sum_{j\neq n} S'_{j,t}(p)\right)[p - \eta_{u,t+1}(p)F_{t+1}(S_{n,t}(p))] = 0, \; p < \tilde{p}_t \tag{EC.89}$$

$$S_{n,t}(p) - \beta_{u,t+1}\int_0^{\xi_{t+1}}(S_{n,t}(p)-x)dF_{t+1}(x) - \left(\alpha_t + \sum_{j\neq n} S'_{j,t}(p)\right)[p - \eta_{u,t+1}(p)] = 0, \; p \geq \tilde{p}_t \tag{EC.90}$$

$$\text{subject to } S'_{n,t}(p) \in (0,\infty) \text{ for } p > 0, \text{ and } S_{n,t}(0) = 0, \tag{EC.91}$$

for some constant $\tilde{p}_t > 0$.

The conditions (EC.89), (EC.90) and (EC.91) identify the best response production schedule of renewable firm n in period t. Applying (12) on (EC.89), (EC.90) and (EC.91), it follows from similar arguments in the proof of Lemma EC.3 that $S_t(\cdot)$ is each renewable firm's committed production schedule in a period-t supply function equilibrium that satisfies (12) if and only if $S_t(\cdot)$ satisfies (EC.1a) and (EC.1b) subject to the monotonicity constraint and the initial condition stated in Proposition EC.1-(ii).

We now show our claim about $S_{n,t}(\cdot)$ that maximizes $\Pi_n(\cdot;\epsilon_t, S_{-n,t}, \bar{\mathbb{S}})$ for every ϵ_t. Note that Lemma 1 holds as stated in this setting because of the positive oversupply penalty rate. Observe from (4) that it is never profitable for renewable firm n to commit to a positive quantity at $p \leq 0$. As a result, by Lemma 1, given all other firms' commitment profiles, renewable firm n's (commitment-related) expected profit $\Pi_n(\cdot;\epsilon_t, S_{-n,t}, \bar{\mathbb{S}})$ in period t is

$$\begin{cases} \pi_1(p) \doteq p\mathcal{R}_{n,t}(p;\epsilon_t) - \eta_{u,t+1}(p)\int_0^{\mathcal{R}_{n,t}(p;\epsilon_t)}(\mathcal{R}_{n,t}(p;\epsilon_t) - x)dF_{t+1}(x) & \text{if } \mathcal{R}_{n,t}(p;\epsilon_t) < \xi_{t+1}, \\ \pi_2(p) \doteq p\mathcal{R}_{n,t}(p;\epsilon_t) - \eta_{u,t+1}(p)\int_0^{\xi_{t+1}}(\mathcal{R}_{n,t}(p;\epsilon_t) - x)dF_{t+1}(x) & \text{if } \mathcal{R}_{n,t}(p;\epsilon_t) \geq \xi_{t+1}. \end{cases} \tag{EC.92}$$

Based on this, we have

$$\pi_1'(p) = \mathcal{R}_{n,t}(p;\epsilon_t) - \beta_{u,t+1}\int_0^{\mathcal{R}_{n,t}(p;\epsilon_t)}(\mathcal{R}_{n,t}(p;\epsilon_t)-x)dF_{t+1}(x) + \mathcal{R}_{n,t}'(p;\epsilon_t)(p-\eta_{u,t+1}(p)F_{t+1}(\mathcal{R}_{n,t}(p;\epsilon_t)), \tag{EC.93}$$

$$\pi_2'(p) = \mathcal{R}_{n,t}(p;\epsilon_t) - \beta_{u,t+1}\int_0^{\xi_{t+1}}(\mathcal{R}_{n,t}(p;\epsilon_t)-x)dF_{t+1}(x) + \mathcal{R}_{n,t}'(p;\epsilon_t)(p-\eta_{u,t+1}(p)). \tag{EC.94}$$

Using (EC.93), (EC.94) and by elementary analysis, we have the following counterpart of Proposition EC.7: For any period-t day-ahead shock ϵ_t and any $S_{-n,t}(\cdot)$ such that $S_{j,t}(\cdot)$ satisfies (EC.1a) and (EC.1b) subject to the monotonicity constraint and the initial condition stated in part (ii) for $j \le N_r$ and $j \ne n$, $\pi_1(\cdot)$ and $\pi_2(\cdot)$ are strictly concave. Therefore, there exists a unique price p_m that maximizes $\pi_m(\cdot)$ for $m = 1,2$. (The proof of this claim is very similar to the proof of Proposition EC.7, and hence omitted.) The price $p_t^*(\epsilon_t)$ that maximizes $\Pi_n(p;\epsilon_t,S_{-n,t},\bar{\mathbb{S}})$ can either result in $\mathcal{R}_{n,t}(p_t^*(\epsilon_t)) < \xi_{t+1}$ or $\mathcal{R}_{n,t}(p_t^*(\epsilon_t)) \ge \xi_{t+1}$. If it is the former case, from structural properties of $\pi_1(\cdot)$ and $\pi_2(\cdot)$, $p_t^*(\epsilon_t)$ satisfies $\pi_1'(p) = 0$; if it is the latter one, $p_t^*(\epsilon_t)$ satisfies $\pi_2'(p) = 0$. Defining $\tilde{p}_t$ as the unique price that satisfies $\mathcal{R}_{n,t}(\tilde{p}_t) = \xi_{t+1}$, the aforementioned conditions ($\pi_1'(p) = 0$ and $\pi_2'(p) = 0$) reduces to (EC.89) and (EC.90) because $S_{n,t}(p) = \mathcal{R}_{n,t}(p;\epsilon_t)$ for any ϵ_t. Based on (EC.93) and (EC.94), it is never profitable for renewable firm n to commit to a positive quantity at $p<0$ because both (EC.93) and (EC.94) are positive for $p \ge 0$; $S_{n,t}' \in (0,\infty)$ ensures that the residual demand crosses the firm n's supply curve at a unique price. This completes our arguments to prove (EC.89), (EC.90) and (EC.91).

Step B: *Existence of a function $S_t(\cdot)$* In this step, we show the existence of a solution $S_t(\cdot)$ to (EC.1a) and (EC.1b) subject to the monotonicity and the initial condition stated in Proposition EC.1-(ii). The proof of this claim is similar to the proof of Lemma EC.1, with the following exceptions. Define the mapping d as

$$\begin{aligned} d(s,p) \doteq & \left(\left(s - \beta_{u,t+1}\int_0^s (s-x)dF_{t+1}(x)\right) / (p-\eta_{u,t+1}(p)F_{t+1}(s)) - \alpha_t\right)\mathbb{I}(s<\xi_{t+1})/(N_r-1) \\ & + \left(\left(s - \beta_{u,t+1}\int_0^{\xi_{t+1}} (s-x)dF_{t+1}(x)\right) / (p-\eta_{u,t+1}(p)) - \alpha_t\right)\mathbb{I}(s\ge\xi_{t+1})/(N_r-1), \end{aligned} \tag{EC.95}$$

where $\mathbb{I}$ is an indicator function. Step 1 in the proof of Lemma EC.1 should be modified as follows. We already know from the proof of Lemma EC.1 that for any $(\tilde{p},\tilde{s}) \in [p_{\ell,t},0]\times\mathbb{R}_+ \setminus \{(0,0)\}$ there exists a unique trajectory $\underline{S}_{t,(\tilde{p},\tilde{s})}(\cdot)$ that satisfies (EC.1a) for $p\in[\tilde{p}-\delta,\tilde{p}+\delta]$ for some $\delta>0$. In addition, from similar arguments in the proof of Lemma EC.1, it follows that for any $(\tilde{p},\tilde{s}) \in \mathbb{R}_+\times\mathbb{R}_+ \setminus \{(b_{u,t+1}/(1-\beta_{u,t+1}),0)\}$, there exists a unique trajectory $\hat{S}_{t,(\tilde{p},\tilde{s})}(\cdot)$ that satisfies (EC.1b) for $p\in[\tilde{p}-\hat{\delta},\tilde{p}+\hat{\delta}]$ for some $\hat{\delta}>0$. For each $\hat{k}\ge 0$, define the function $g_{\hat{k}}(\cdot)$ as $g_{\hat{k}}(s) \doteq \left[\left(s-\beta_{u,t+1}\int_0^s(s-x)f_{t+1}(x)dx\right)/((N_r-1)\hat{k}+\alpha_t)+b_{u,t+1}F_{t+1}(s)\right]\mathbb{I}(s<\xi_{t+1})/(1-\beta_{u,t+1}F_{t+1}(s)) + \left[\left(s-\beta_{u,t+1}\int_0^{\xi_{t+1}}(s-x)f_{t+1}(x)dx\right)/((N_r-1)\hat{k}+\alpha_t)+b_{u,t+1}\right]\mathbb{I}(s\ge\xi_{t+1})/(1-\beta_{u,t+1})$, which implies that

$$\begin{aligned} g_0(s) \doteq & \left[\left(s-\beta_{u,t+1}\int_0^s(s-x)f_{t+1}(x)dx\right)/\alpha_t + b_{u,t+1}F_{t+1}(s)\right]\mathbb{I}(s<\xi_{t+1})/(1-\beta_{u,t+1}F_{t+1}(s)) \\ & + \left[\left(s-\beta_{u,t+1}\int_0^{\xi_{t+1}}(s-x)f_{t+1}(x)dx\right)/\alpha_t + b_{u,t+1}\right]\mathbb{I}(s\ge\xi_{t+1})/(1-\beta_{u,t+1}), \end{aligned} \tag{EC.96}$$

$$g_\infty(s) \doteq b_{u,t+1}F_{t+1}(s)\mathbb{I}(s<\xi_{t+1})/(1-\beta_{u,t+1}F_{t+1}(s)) + b_{u,t+1}\mathbb{I}(s\ge\xi_{t+1})/(1-\beta_{u,t+1}). \tag{EC.97}$$

Notice from (EC.96) and (EC.97) that $g_0(\cdot)$ and $g_\infty(\cdot)$ are continuous and increasing. We already know from the arguments above that there exists a unique trajectory $S_{t,(p,s)}(\cdot)$ that passes through (p,s) and satisfies (EC.1a) for any $(p,s)\in[0,\infty)\times[0,\xi_{t+1})\setminus\{(0,0)\}$. Such a trajectory also satisfies $d(S_{t,(p,s)}(p),p) = S_{t,(p,s)}'(p)$ for $(p,s)\in[0,\infty)\times[0,\xi_{t+1})\setminus$

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$\{(0,0)\}$. Similarly, there exists a unique trajectory $S_{t,(p,s)}(\cdot)$ that passes through (p,s) and satisfies (EC.1b) for $(p,s) \in [0,\infty) \times [\xi_{t+1},\infty) \setminus \{(b_{u,t+1}/(1-\beta_{u,t+1}),0)\}$, and $S_{t,(p,s)}$ satisfies $d(S_{t,(p,s)}(p),p) = S'_{t,(p,s)}(p)$ for such pairs of (p,s). Using these facts, and observing that $d(s, g_{\hat{k}}(s)) = \hat{k} > 0$ for $(p,s) \in \{(p,s) : g_\infty(s) < p < g_0(s)\}$, the existence proof follows by applying the same arguments in Steps 2 and 3 of the proof of the Lemma EC.1.

Step C: *Existence and Characterization of Inflexible Firms' Equilibrium Commitments* For any $\bar{\mathbb{S}}$, Step A characterizes a period-t supply function equilibrium that satisfies (12). Step B establishes the existence of the aforementioned period-t supply function equilibrium. Then, using said period-t supply function equilibrium and (6), by elementary optimization arguments, the equilibrium commitment of each inflexible firm exists subject to (11) and is characterized as in part (iii). The proof is identical to the proof of Proposition 2-(iii).

Based on these steps, proofs of parts (i) through (iii) are as follows. **Proof of part (i):** Step A characterizes a period-t supply function equilibrium that satisfies (12) for any $\bar{\mathbb{S}}$ and Step B establishes the existence of it. Based on these, Step C establishes the existence of an equilibrium $(\mathbb{S}_1, \ldots \mathbb{S}_\tau, \bar{\mathbb{S}}, \vec{q})$ that satisfies (11) and (12). **Proof of part (ii):** Each renewable firm's period-t committed production schedule in equilibrium is identified in Step A. Because Lemma 1 holds as stated in this setting, renewable firm n's optimal production quantity in period t+1 is $\min\{S_t(\cdot), Q_{n,t+1}\}$. **Proof of part (iii):** It immediately follows from Step C. □

Proof of Proposition 4 with Constrained Available Supply: By Propositions 2-(ii) and EC.1-(ii), for $p < \bar{p}_t$, each renewable firm n's period-t committed production schedule $S_t(p)$ is the same with and without a constraint on the support of $F_{t+1}(\cdot)$. Therefore, for $p < \bar{p}_t$, the proofs of the statements in Proposition 4 with constrained available supply is identical to the proof of Proposition EC.1-(ii). To show the claim for $p \geq \bar{p}_t$, as in the last step of the proof of Proposition 4, it is sufficient to verify that $\partial d(s,p)/\partial s > 0$ and $\partial d(s,p)/\partial \beta_{u,t+1} > 0$ for $p \geq \bar{p}_t$ in any equilibrium. Only the second line of the mapping d (EC.95) is relevant for $p \geq \bar{p}_t$. By elementary analysis, we have

$$\partial d(s,p)/\partial \beta_{u,t+1} = \frac{\left(b_{u,t+1}\int_0^{\xi_{t+1}}(s-x)dF_{t+1}(x) + p\left(s - \int_0^{\xi_{t+1}}(s-x)dF_{t+1}(x)\right)\right)}{(N_r-1)(p-\eta_{u,t+1}(p))^2},$$
$$\partial d(s,p)/\partial s = (1-\beta_{u,t+1})/\left((N_r-1)(p-\eta_{u,t+1}(p))\right).$$

Because $p - \eta_{u,t+1}(p) > 0$ by the monotonicity condition in Proposition EC.1-(ii) and $S_t(p) - \int_0^{\xi_{t+1}}(S_t(p) - x)dF_{t+1}(x) > 0$ in any equilibrium, it follows that $\partial d(S_t(p),p)/\partial s > 0$ and $\partial d(S_t(p),p)/\partial \beta_{u,t+1} > 0$ for $p \geq \bar{p}_t$. This implies that Proposition 4 holds as stated when $F_{t+1}(\cdot)$ is defined on the support $[0,\xi_{t+1}]$ for all t. □

Appendix M: Proofs of Results in Section EC.1.2

Proof of Proposition EC.2: The claim can be proved by using the same arguments as in the proof of Proposition EC.1. Below we will highlight the main differences from the proof of Proposition EC.1.

To characterize a period-t supply function equilibrium, Step A in the proof of Proposition EC.1 should be modified as follows. Lemma 1 holds in this setting, except that $\eta_{u,t+1}(\cdot)$ must be replaced with $\tilde{\eta}_{u,t+1}(\cdot)$ in (14). Then, using the modified (14) and (EC.2), π_1 and π_2 in (EC.92) must be redefined as follows: $\pi_1(p) \doteq p\mathcal{R}_{n,t}(p;\epsilon) - \eta_{u,t+1}(p)\int_0^{\mathcal{R}_{n,t}(p;\epsilon)}(\mathcal{R}_{n,t}(p;\epsilon) - x)dF_{t+1}(x) + \gamma_{u,t+1}\int_0^{\mathcal{R}_{n,t}(p;\epsilon)}(x + (N_r-1)\mu_{t+1})(\mathcal{R}_{n,t}(p;\epsilon) - x)dF_{t+1}(x)$ for $\mathcal{R}_{n,t}(p;\epsilon) < \xi_{t+1}$, and $\pi_2(p) \doteq p\mathcal{R}_{n,t}(p;\epsilon) - \eta_{u,t+1}(p)\int_0^{\xi_{t+1}}(\mathcal{R}_{n,t}(p;\epsilon) - x)dF_{t+1}(x) + \gamma_{u,t+1}\int_0^{\xi_{t+1}}(x + (N_r - 1)\mu_{t+1})(\mathcal{R}_{n,t}(p;\epsilon) - x)dF_{t+1}(x)$ for $\mathcal{R}_{n,t}(p;\epsilon) \geq \xi_{t+1}$. Using these and applying the same ideas as in the Step A of the proof of Proposition EC.1, it follows that a period-t supply function equilibrium that satisfies (12) is characterized by

(EC.3) subject to the monotonicity constraint and the initial condition in Proposition EC.2-(ii) where $\bar{p}_t$ is the unique price that satisfies $\pi_1(\bar{p}_t) = \pi_2(\bar{p}_t)$ when (1) and (12) are applied to π_1 and π_2.

We now explain how Step B in the proof of Proposition EC.1 must be modified. The definition of the mapping d in (EC.95) must be modified as

$$d(s,p) \doteq \left[\frac{s - \beta_{u,t+1}\int_0^s (s-x)dF_{t+1}(x)}{p - (b_{u,t+1}+\beta_{u,t+1}p)F_{t+1}(s) + \gamma_{u,t+1}\int_0^s (x+(N_r-1)\mu_{t+1})dF_{t+1}(x)} - \alpha_t\right]\frac{\mathbb{I}(s<\xi_{t+1})}{(N_r-1)} \tag{EC.98}$$

$$+\left[\frac{s - \beta_{u,t+1}\int_0^{\xi_{t+1}} (s-x)dF_{t+1}(x)}{p - (b_{u,t+1}+\beta_{u,t+1}p) + \gamma_{u,t+1}N_r\mu_{t+1}} - \alpha_t\right]\frac{\mathbb{I}(s\geq\xi_{t+1})}{(N_r-1)}, \tag{EC.99}$$

where $\mathbb{I}$ is an indicator function. The definition of the function $g_{\hat{k}}(s)$ must be changed to the following: $g_{\hat{k}}(s) \doteq \left[\left(s - \beta_{u,t+1}\int_0^s (s-x)dF_{t+1}(x)\right)/\theta_1(\hat{k}) + b_{u,t+1}F_{t+1}(s) - \gamma_{u,t+1}\int_0^s (x+(N_r-1)\mu_{t+1})dF_{t+1}(x)\right]\mathbb{I}(s < \xi_{t+1})/\theta_2(s) + \left[\left(s - \beta_{u,t+1}\int_0^{\xi_{t+1}} (s-x)dF_{t+1}(x)\right)/\theta_1(\hat{k}) + b_{u,t+1} - \gamma_{u,t+1}N_r\mu_{t+1}\right]\mathbb{I}(s \geq \xi_{t+1})/\theta_2(\xi_{t+1})$ where $\theta_1(\hat{k}) \doteq \left((N_r-1)\hat{k} + \alpha_t\right)$ and $\theta_2(s) \doteq (1-\beta_{u,t+1}F_{t+1}(s))$. Using these and applying the same arguments as in the Step B of the proof of Proposition EC.2, the existence of a period-t supply function equilibrium characterized in Step A follows.

Step C in the proof of Proposition EC.1 remains unchanged in this setting. This concludes our arguments for the proof of Proposition EC.2. □

Proof of Proposition 4 with (EC.2): Letting $p \to 0$ on both sides of (EC.3) and applying L'Hôpital rule, we conclude that $S_t'(0)$ is as in (EC.29) where $\psi(0) \doteq (b_{u,t+1} - \gamma_{u,t+1}(N_r-1)\mu_{t+1})f_{t+1}(0) > 0$. As a result, $S_t'(0)$ does not change with $\beta_{u,t+1}$. Similar to Lemma EC.5, $\partial S_t''(0)/\partial\beta_{u,t+1} > 0$ if and only if $N_r \leq \tilde{\Gamma}_{t+1}$ and $\beta_{u,t+1} < \tilde{\Delta}_{t+1}$ for some positive constants $\tilde{\Gamma}_{t+1}$ and $\tilde{\Delta}_{t+1}$. (The proof of this statement is the same as the proof of Lemma EC.5 in Appendix E, with the exception that all $b_{u,t+1}$ terms in the proof of Lemma EC.5 should be replaced with $(b_{u,t+1} - \gamma_{u,t+1}(N_r-1)\mu_{t+1})$.) This implies that at an arbitrarily small price $h > 0$, a renewable firm's production commitment is strictly larger with a higher $\beta_{u,t+1}$ in any equilibrium. Based on this, as explained in the last paragraph of the proof of Proposition 4, it is sufficient to prove that $\partial d(s,p)/\partial s > 0$ and $\partial d(s,p)/\partial\beta_{u,t+1} > 0$ for $p \geq 0$ in any equilibrium. We claim and show below that $\partial d(s,p)/\partial s|_{s=S_t(p)} > 0$ and $\partial d(p,s)/\partial\beta_{u,t+1} > 0$ for all $p \geq 0$. From this, it follows that the statements in Proposition 4 hold when the penalty rates in period t are as in (EC.2).

It only remains to show our above claim that $\partial d(s,p)/\partial s > 0$ and $\partial d(s,p)/\partial\beta_{u,t+1} > 0$ for $p \geq 0$ in any equilibrium. Recall (EC.98) and (EC.99). By the definition of $\bar{p}_t$ and (EC.3), (EC.98) is relevant for $p < \bar{p}_t$ whereas (EC.99) is relevant for $p \geq \bar{p}_t$ in the analysis of $d(s,p)$. Recall also the notation $\eta_{u,t+1}(p) = b_{u,t+1} + \beta_{u,t+1}p$. Then, from elementary analysis, it follows that the signs of $\partial d(s,p)/\partial s|_{s=S_t(p)}$ for $p \in [0,\bar{p}_t)$, $\partial d(p,s)/\partial\beta_{u,t+1}$ for $p \in [0,\bar{p}_t)$, $\partial d(s,p)/\partial s|_{s=S_t(p)}$ for $p \geq \bar{p}_t$ and $\partial d(p,s)/\partial\beta_{u,t+1}$ for $p \geq \bar{p}_t$ are the same as the signs of $A_1(p)$ through $A_4(p)$, respectively, where

$$\begin{aligned}A_1(p) \doteq{}& (1-\beta_{u,t+1}F_{t+1}(S_t(p)))\left(p - \eta_{u,t+1}(p)F_{t+1}(S_t(p)) + \gamma_{u,t+1}\int_0^{S_t(p)}(x+(N_r-1)\mu_{t+1})dF_{t+1}(x)\right)\\ &+\left[\eta_{u,t+1}(p) - \gamma_{u,t+1}(S_t(p)+(N_r-1)\mu_{t+1})\right]f_{t+1}(S_t(p))\left(S_t(p) - \beta_{u,t+1}\int_0^{S_t(p)}(S_t(p)-x)dF_{t+1}(x)\right),\end{aligned} \tag{EC.100}$$

$$\begin{aligned}A_2(p) \doteq{}& pS_t(p)F_{t+1}(S_t(p)) - p\int_0^{S_t(p)}(S_t(p)-x)dF_{t+1}(x)\\ &+\left(\int_0^{S_t(p)}(S_t(p)-x)dF_{t+1}(x)\right)\left[b_{u,t+1}F_{t+1}(S_t(p)) - \gamma_{u,t+1}\int_0^{S_t(p)}(x+(N_r-1)\mu_{t+1})dF_{t+1}(x)\right],\end{aligned} \tag{EC.101}$$

$$A_3(p) \doteq (1-\beta_{u,t+1})\left(p-\eta_{u,t+1}(p)+\gamma_{u,t+1}N_r\mu_{t+1}\right), \tag{EC.102}$$

$$A_4(p) \doteq pS_t(p) - p\int_0^{\xi_{t+1}}(S_t(p)-x)dF_{t+1}(x) + [b_{u,t+1}-\gamma_{u,t+1}N_r\mu_{t+1}]\left(\int_0^{\xi_{t+1}}(S_t(p)-x)dF_{t+1}(x)\right). \tag{EC.103}$$

We now show two inequalities (i.e., (EC.104) and (EC.105)) that will be used to show the signs of A_1 through A_4. Notice that the term $p-\eta_{u,t+1}(p)F_{t+1}(S_t(p))+\gamma_{u,t+1}\int_0^{S_t(p)}(x+(N_r-1)\mu_{t+1})dF_{t+1}(x)$ appears on the right hand side of (EC.3). Because $S_t'(p)>0$ by Proposition EC.2-(ii), it follows from (EC.3) that

$$p-\eta_{u,t+1}(p)F_{t+1}(S_t(p))+\gamma_{u,t+1}\int_0^{S_t(p)}(x+(N_r-1)\mu_{t+1})dF_{t+1}(x)>0, \quad p\in[0,\bar{p}_t), \tag{EC.104}$$

which is our first inequality. Furthermore, because $\mu_{t+1}<\xi_{t+1}$ for all t by definition of ξ_{t+1}, our assumption $\xi_{t+1}<b_{u,t+1}/(N_r\gamma_{u,t+1})$ implies that

$$b_{u,t+1}-\gamma_{u,t+1}(\xi_{t+1}+(N_r-1)\mu_{t+1})>0, \tag{EC.105}$$

which is our second inequality. Based on (EC.104) and (EC.105), let us first prove that $A_1(p)>0$ for $p\in[0,\bar{p}_t)$. Because $S_t(p)<\xi_{t+1}$ for $p\in[0,\bar{p}_t)$, we have the following by (EC.105):

$$\eta_{u,t+1}(p)-\gamma_{u,t+1}(S_t(p)+(N_r-1)\mu_{t+1}) \geq b_{u,t+1}-\gamma_{u,t+1}(S_t(p)+(N_r-1)\mu_{t+1}) \tag{EC.106}$$

$$> b_{u,t+1}-\gamma_{u,t+1}(\xi_{t+1}+(N_r-1)\mu_{t+1})>0. \tag{EC.107}$$

Then, by (EC.104) and (EC.107), we have $A_1(p)>0$ for $p\in[0,\bar{p}_t)$ by (EC.100). By (EC.105), the definitions of A_2 and A_4 and elementary algebra, we have $A_2(p)>0$ for $p\in[0,\bar{p}_t)$ and $A_4(p)>0$ for $p\geq\bar{p}_t$. Finally, $A_3(p)>0$ for $p\geq\bar{p}_t$ by (EC.3) and the fact that $S_t'(p)>0$. □

Appendix N: Proofs of Results in Section EC.1.4

We first state and prove Lemma EC.10, which will be used in the remainder of this section.

LEMMA EC.10. *There exists a function $S_t(\cdot)$ that satisfies the ordinary differential equation $(EC.6)$ subject to the conditions in $(EC.7)$, where $p_{\ell,t}\in(b_{c,t+1}/(1-\beta_{c,t+1}),(\alpha_t b_{c,t+1}+\beta_{c,t+1}\mu_{t+1})/(\alpha_t(1-\beta_{c,t+1})))$ and $\lambda_r\doteq 1/(N_r-1)$.*

Proof of Lemma EC.10: The proof of this claim follows from similar arguments explained in the proof of Lemma EC.1 with the following main differences: The mapping d defined in (EC.8) should be replaced with $\frac{1}{N_r-1}\left[\frac{s-\beta_{u,t+1}\int_0^s(s-x)f_{t+1}(x)dx+\beta_{c,t+1}\int_s^\infty(x-s)f_{t+1}(x)dx}{p-\eta_{u,t+1}(p)F_{t+1}(s)-c_{o,t+1}(p)(1-F_{t+1}(s))}-\alpha_t\right]$. The definitions of functions $g_{\hat{k}}$, g_0 and g_∞ should be replaced with the following expressions:

$$g_{\hat{k}}(s)\doteq\frac{[b_{u,t+1}F_{t+1}(s)+b_{c,t+1}(1-F_{t+1}(s))]}{(1-\beta_{u,t+1}F_{t+1}(s)-\beta_{c,t+1}(1-F_{t+1}(s)))}+\frac{s-\beta_{u,t+1}\int_0^s(s-x)f_{t+1}(x)dx+\beta_{c,t+1}\int_s^\infty(x-s)f_{t+1}(x)dx}{(1-\beta_{u,t+1}F_{t+1}(s)-\beta_{c,t+1}(1-F_{t+1}(s)))(\alpha_t+(N_r-1)\hat{k})},$$

$$g_0(s)=\frac{[b_{u,t+1}F_{t+1}(s)+b_{c,t+1}(1-F_{t+1}(s))]}{(1-\beta_{u,t+1}F_{t+1}(s)-\beta_{c,t+1}(1-F_{t+1}(s)))}+\frac{s-\beta_{u,t+1}\int_0^s(s-x)f_{t+1}(x)dx+\beta_{c,t+1}\int_s^\infty(x-s)f_{t+1}(x)dx}{\alpha_t(1-\beta_{u,t+1}F_{t+1}(s)-\beta_{c,t+1}(1-F_{t+1}(s)))},$$

$$g_\infty(s)\doteq\lim_{\hat{k}\to\infty}g_{\hat{k}}(s)=\frac{[b_{u,t+1}F_{t+1}(s)+b_{c,t+1}(1-F_{t+1}(s))]}{(1-\beta_{u,t+1}F_{t+1}(s)-\beta_{c,t+1}(1-F_{t+1}(s)))},$$

for $s\in\mathbb{R}_+$. And finally, one should note that $\lim_{s\to0}g_0=(\alpha_t b_{c,t+1}+\beta_{c,t+1}\mu_{t+1})/(\alpha_t(1-\beta_{c,t+1}))$ and $\lim_{s\to0}g_\infty=b_{c,t+1}/(1-\beta_{c,t+1})$, and hence $p_{\ell,t}\in(b_{c,t+1}/(1-\beta_{c,t+1}),(\alpha_t b_{c,t+1}+\beta_{c,t+1}\mu_{t+1})/(\alpha_t(1-\beta_{c,t+1})))$. □

Proof of Proposition EC.3: The outline of our proof is as follows. Below, we will show that for any given $\bar{\mathbb{S}}$, a function $S_t(\cdot)$ is each renewable firm's committed production schedule in a period-t supply function equilibrium if and only if it satisfies (EC.6) and (EC.7). This and Lemma EC.10 imply that there exists a period-t supply function equilibrium for any given $\bar{\mathbb{S}}$. Because (EC.6) and (EC.7) establish that each renewable firm's committed production schedule in a period is not dependent on $\bar{\mathbb{S}}$, using the identified period-t equilibrium and (6), it follows from the same arguments as in the proof of Proposition 2-(iii) that each inflexible firm's equilibrium commitment exists and is characterized by the conditions stated in Proposition 2-(iii), except $S_t(\cdot)$ is as identified in Lemma EC.10. With these, parts (i) through (iii) follow.

We now show that for any $\bar{\mathbb{S}}$, a function $S_t(\cdot)$ is each renewable firm's committed production schedule in a period-t supply function equilibrium if and only if it satisfies (EC.6) and (EC.7). This claim is proved by applying similar arguments explained in the proof of Proposition 1 with the following differences. First, throughout the proof of Proposition 1, (17) and (18) should be replaced with $(EC.6)$ and $(EC.7)$, respectively. In the statement of Lemma EC.2, (EC.17) should be replaced with $S_{n,t}(p) - \beta_{u,t+1}\int_0^{S_{n,t}(p)}(S_{n,t}(p)-x)dF_{t+1}(x) + \beta_{c,t+1}\int_{S_{n,t}(p)}^{\infty}(x-S_{n,t}(p))dF_{t+1}(x) - \left(\alpha_t + \sum_{j\neq n}^{N_r} S'_{j,t}(p)\right)[p-\eta_{u,t+1}(p)F_{t+1}(S_{n,t}(p)) - c_{o,t+1}(p)(1-F_{t+1}(S_{n,t}(p)))] = 0$, and the definition of $p_{\ell,t}$ should be updated to $p_{\ell,t} \in (b_{c,t+1}/(1-\beta_{c,t+1}), (\alpha_t b_{c,t+1} + \beta_{c,t+1}\mu_{t+1})/((1-\beta_{c,t+1})\alpha_t))$.

In the proof of Lemma EC.2, the following modifications are required: (4) must be replaced with (EC.5) for $\Pi_n(p;\epsilon_t, S_{-n,t}, \bar{\mathbb{S}})$. From similar arguments in the proof of Lemma 1, it follows that the optimal production quantity is $q^*_{n,t+1} = Q_{n,t+1}$ and hence $\Pi_n(p;\epsilon_t, S_{-n,t}, \bar{\mathbb{S}})$ is equivalent to $\mathcal{R}_{n,t}(p;\epsilon_t)p - \eta_{u,t+1}(p)\int_0^{\mathcal{R}_{n,t}(p;\epsilon_t)}(\mathcal{R}_{n,t}(p;\epsilon_t) - x)dF_{t+1}(x) + c_{o,t+1}(p)\int_{\mathcal{R}_{n,t}(p;\epsilon_t)}^{\infty}(x-\mathcal{R}_{n,t}(p;\epsilon_t))dF_{t+1}(x)$. As a result, the first order condition (EC.18) must be replaced with $\mathcal{R}_{n,t}(p;\epsilon) - \beta_{u,t+1}\int_0^{\mathcal{R}_{n,t}(p;\epsilon)}(\mathcal{R}_{n,t}(p;\epsilon) - x)dF_{t+1}(x) + \beta_{c,t+1}\int_{\mathcal{R}_{n,t}(p;\epsilon)}^{\infty}(x-\mathcal{R}_{n,t}(p;\epsilon))dF_{t+1}(x) - \mathcal{R}'_{n,t}(p;\epsilon)[p-\eta_{u,t+1}(p)F_{t+1}(\mathcal{R}_{n,t}(p;\epsilon)) - c_{o,t+1}(p)(1-F_{t+1}(\mathcal{R}_{n,t}(p;\epsilon)))] = 0$. The functions $\tilde{\gamma}$, A_j and B_j should be redefined as $\tilde{\gamma}(y,p) \doteq (c_{o,t+1}(p) - \eta_{u,t+1}(p))f_{t+1}(y(p))$ for $y \in [p_{\ell,t},\infty)$, $A_j(p) \doteq S_{j,t}(p) - \beta_{u,t+1}\int_0^{S_{j,t}(p)}(S_{j,t}(p) - x)dF_{t+1}(x) + \beta_{c,t+1}\int_{S_{j,t}(p)}^{\infty}(x - S_{j,t}(p))dF_{t+1}(x)$ and $B_j(p) \doteq p - \eta_{u,t+1}(p)F_{t+1}(S_{j,t}(p)) - c_{o,t+1}(p)(1 - F_{t+1}(S_{j,t}(p)))$. Then, by elementary analysis, (EC.19) must be replaced with $-(1-\beta_{u,t+1}F_{t+1} - \beta_{c,t+1}(1-F_{t+1}))[(N_r-2)S'_{j,t} + \alpha_t]/B_j + \frac{(\eta_{u,t+1}-c_{o,t+1})f_{t+1}S'_{j,t}A_j}{B_j^2}$, which implies that (EC.20) must be replaced with $-(1-\beta_{u,t+1}F_{t+1} - \beta_{c,t+1}(1-F_{t+1}))\left(\alpha_t + N_r\sum_{j\neq n}^{N_r}S'_{j,t}/(N_r-1)\right) + \tilde{\gamma}(\mathcal{R}_{n,t},p)(\alpha_t + \sum_{j\neq n}^{N_r}S'_{j,t}(p))^2 + \sum_{j\neq n}^{N_r}\frac{\tilde{\gamma}(S_{j,t},p)S'_{j,t}(p)A_j(p)}{(N_r-1)B_j(p)}$. All other arguments in the proof of Lemma EC.2 remain the same. The claim and the proof of Lemma EC.3 remain unchanged. $\square$

Proof of Proposition EC.4: Below, we only prove part (i). Parts (ii) and (iii) can be shown by using part (i) and the same arguments as in the proofs of Propositions 5 and 6, respectively. To show part (i), we will compare a renewable firm's period-t committed production schedules in any two equilibria such that the initial points of these committed production schedules are equal to a particular $p_{\ell,t}$, but one of them is the solution of (EC.6) and (EC.7) with a higher $\beta_{u,t+1}$.

The proof consists of two steps. In Step 1, we analyze how $S'_t(p_{\ell,t})$ and $S''_t(p_{\ell,t})$ change with $\beta_{u,t+1}$ to understand the local behaviour of the committed production schedules around $p_{\ell,t}$. In Step 2, we extend our analysis to the entire domain. Step 1: Because $S_t(p_{\ell,t}) = 0$ by (EC.7), it follows from (EC.6) that $S'_t(p_{\ell,t}) = (\beta_{c,t+1}\mu_{t+1}/(p_{\ell,t} - c_{o,t+1}(p_{\ell,t})) - \alpha_t)/(N_r-1)$. ($S'_t(p_{\ell,t})$ is positive as $p_{\ell,t} \in (b_{c,t+1}/(1-\beta_{c,t+1}), (\alpha_t b_{c,t+1} + \beta_{c,t+1}\mu_{t+1})/(\alpha_t(1-\beta_{c,t+1})))$.) Thus, $S'_t(p_{\ell,t})$ does not change with $\beta_{u,t+1}$. We now proceed to analyze $S''_t(p_{\ell,t})$. Recall from (EC.65) that $\overline{F}_{t+1}(\cdot)$ represents the complementary cumulative distribution of $Q_{j,t+1}$ for $j = 1,\ldots,N_r$. Then, taking the derivative of both sides in (EC.6), we have

$$S''_t(p) = \frac{-(1-\beta_{u,t+1}F_{t+1}(S_t(p)) - \beta_{c,t+1}\overline{F}_{t+1}(S_t(p)))((N_r-2)S'_t(p)+\alpha_t) + \theta_1(p)}{(N_r-1)(p-\eta_{u,t+1}(p)F_{t+1}(S_t(p)) - c_{o,t+1}(p)\overline{F}_{t+1}(S_t(p)))},$$

where $\theta_1(p) \doteq (\eta_{u,t+1}(p) - c_{o,t+1}(p)) f_{t+1}(S_t(p)) S_t'(p)((N_r-1)S_t'(p)+\alpha_t)$. Letting $p \to p_{\ell,t}$ on both sides of the above equation, it follows that

$$\begin{aligned} S_t''(p_{\ell,t}) = & \frac{p_{\ell,t} f_{t+1}(0) S_t'(p_{\ell,t})((N_r-1)S_t'(p_{\ell,t})+\alpha_t)}{(N_r-1)(p_{\ell,t}-c_{o,t+1}(p_{\ell,t}))}\beta_{u,t+1} \\ & - \frac{\theta_2(p_{\ell,t}) + (c_{o,t+1}(p_{\ell,t}) - b_{u,t+1}) f_{t+1}(0) S_t'(p_{\ell,t})((N_r-1)S_t'(p_{\ell,t})+\alpha_t)}{(N_r-1)(p_{\ell,t}-c_{o,t+1}(p_{\ell,t}))}, \end{aligned} \tag{EC.108}$$

where $\theta_2(p_{\ell,t}) \doteq (1-\beta_{c,t+1})((N_r-2)S_t'(p_{\ell,t})+\alpha_t)$. The equation (EC.108) implies that $S_t''(p_{\ell,t})$ always increases with $\beta_{u,t+1}$ as the coefficient of $\beta_{u,t+1}$ (which is the first fraction on the right hand side of (EC.108)) is positive. Step 2: By expanding $S_t(p_{\ell,t}+h)$ around $p_{\ell,t}$, it follows that a higher $\beta_{u,t+1}$ results in a strictly larger $S_t(p_{\ell,t}+h)$ for an arbitrarily small $h>0$ because by Step 1, $S_t'(p_{\ell,t})$ does not change with $\beta_{u,t+1}$ and $S_t''(p_{\ell,t})$ always increases with $\beta_{u,t+1}$. Similar to the proof of Proposition 4, we claim and show below that if $S_t(\tilde{p})$ is strictly larger with a higher $\beta_{u,t+1}$ at a price $\tilde{p} > p_{\ell,t}$, then $S_t(p)$ is strictly larger with a higher $\beta_{u,t+1}$ for all $p \ge \tilde{p}$. Using this, and recalling the fact that $S_t(p_{\ell,t}+h)$ is strictly larger with a higher $\beta_{u,t+1}$ for an arbitrarily small constant $h>0$, we complete the proof of Proposition EC.4.

We now proceed to prove our aforementioned claim. Define the mapping $\lambda(p,s,\beta_{u,t+1},\beta_{c,t+1})$ as $\lambda(p,s,\beta_{u,t+1},\beta_{c,t+1}) \doteq \frac{1}{N_r-1}\left[\frac{s-\beta_{u,t+1}\int_0^s(s-x)dF_{t+1}(x)+\beta_{c,t+1}\int_s^\infty(x-s)dF_{t+1}(x)}{p-\eta_{u,t+1}(p)F_{t+1}(s)-c_{o,t+1}(p)\overline{F}_{t+1}(s)} - \alpha_t\right]$ for $(p,s) \in [p_{\ell,t},\infty) \times \mathbb{R}_+ \setminus \{(p_{\ell,t},0)\}$. Comparing (EC.6) and the mapping λ, we have

$$\lambda(p, S_t(p), \beta_{u,t+1}, \beta_{c,t+1}) = S_t'(p), \quad p > p_{\ell,t}. \tag{EC.109}$$

Define $I_1 \doteq \int_0^s(s-x)dF_{t+1}(x)$, $I_2 \doteq \int_s^\infty(x-s)dF_{t+1}(x)$, $A(s) \doteq s - \beta_{u,t+1}\int_0^s(s-x)dF_{t+1}(x) + \beta_{c,t+1}\int_s^\infty(x-s)dF_{t+1}(x)$ and $B(s) \doteq p - \eta_{u,t+1}(p)F_{t+1}(s) - c_{o,t+1}(p)\overline{F}_{t+1}(s)$. Then, by elementary analysis, $\frac{\partial\lambda(p,s,\beta_{u,t+1},\beta_{c,t+1})}{\partial\beta_{u,t+1}}$ and $\frac{\partial\lambda(p,s,\beta_{u,t+1},\beta_{c,t+1})}{\partial s}$ are equivalent to the following expressions, respectively:

$$\frac{p(sF_{t+1}(s) - I_1 + \beta_{c,t+1}F_{t+1}(s)I_2) + (b_{u,t+1}F_{t+1}(s) + c_{o,t+1}(p)\overline{F}_{t+1}(s))I_1}{(N_r-1)B^2(s)} > 0, \ s > 0, \tag{EC.110}$$

$$\left[(1-\beta_{u,t+1}F_{t+1}(s) - \beta_{c,t+1}\overline{F}_{t+1}(s))B(s) + f_{t+1}(s)(\eta_{u,t+1}(p) - c_{o,t+1}(p))A(s)\right]/(N_r-1)B^2(s). \tag{EC.111}$$

Observe that (EC.111) is positive for all (p,s) such that $B(s)>0$, since $\eta_{u,t+1}(p) > c_{o,t+1}(p)$ for all p. Because $B(S_t(p)) > 0$ for $p > p_{\ell,t}$ in any equilibrium, $\partial\lambda(p,S_t(p),\beta_{u,t+1},\beta_{c,t+1})/\partial s > 0$. From this, (EC.109) and (EC.110), it follows that if $S_t(p)$ is strictly larger with a higher $\beta_{u,t+1}$, then $S_t'(p)$ is strictly larger with the higher $\beta_{u,t+1}$ at the same price p. This completes the proof for our claim. □

Proof of Proposition 7 for a Credit-dominated Market: Recall the notations $S_t(\cdot;\eta_1)$ and $S_t(\cdot;\eta_2)$ from the proof of Proposition 7. Let the initial points of $S_t(\cdot;\eta_1)$ and $S_t(\cdot;\eta_2)$ be some $p_{\ell,t}$. Because $S_t'(p_{\ell,t}) = (\beta_{c,t+1}\mu_{t+1}/(p_{\ell,t} - c_{o,t+1}(p_{\ell,t})) - \alpha_t)/(N_r-1)$, $S_t'(p_{\ell,t})$ is not affected by the change in the undersupply penalty rate. Rearranging (EC.108), observe that at a fixed $p_{\ell,t} > 0$ a larger $\eta_{u,t+1}(p_{\ell,t})$ implies a larger $S_t''(p_{\ell,t})$. Using these facts and recalling the notation $\widehat{p}$ from the proof of Proposition 7, we prove our claim under each of the following two cases for $p_{\ell,t}$. Case 1: Suppose that $p_{\ell,t} < \widehat{p}$. Then, either $S_t(\cdot;\eta_1)$ and $S_t(\cdot;\eta_2)$ do not intersect or $S_t(\cdot;\eta_1)$ and $S_t(\cdot;\eta_2)$ intersect at least at one point. If the former scenario holds, $\eta_1(p) < \eta_2(p)$ and $S_t(\cdot;\eta_1) < S_t(\cdot;\eta_2)$ for $p \in (p_{\ell,t},\widehat{p})$ because $S_t''(p_{\ell,t};\eta_1) < S_t''(p_{\ell,t};\eta_2)$ as $\eta_1(p_{\ell,t}) < \eta_2(p_{\ell,t})$. Suppose now the latter scenario holds. Denote by $\widetilde{p}$ the minimum price at which $S_t(\cdot;\eta_1)$ and $S_t(\cdot;\eta_2)$ intersect. Then, $\widetilde{p} > \widehat{p}$. (Suppose for a contradiction that $\widetilde{p} \le \widehat{p}$, which implies that $S_t'(\widetilde{p};\eta_1) > S_t'(\widetilde{p};\eta_2)$ since $S_t(\cdot;\eta_1) < S_t(\cdot;\eta_2)$ for $p \in (p_{\ell,t},\widetilde{p})$. However, by (EC.6), $\eta_1(\widetilde{p}) < \eta_2(\widetilde{p})$, $\beta_1 > \beta_2$ and $S_t(\widetilde{p};\eta_1) = S_t(\widetilde{p};\eta_2)$ imply that $S_t'(\widetilde{p};\eta_1) < S_t'(\widetilde{p};\eta_2)$, which is a contradiction.) Therefore, $\eta_1(p) < \eta_2(p)$ and $S_t(p;\eta_1) < S_t(p;\eta_2)$ for $p \in (p_{\ell,t},\widehat{p})$, and

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$\eta_1(p) > \eta_2(p)$ and $S_t(p;\eta_1) > S_t(p;\eta_2)$ for $p \in (\widetilde{p}, p^o)$ where $p^o \in \bar{\mathbb{R}}_+ \doteq \mathbb{R}_+ \cup \{\infty\}$ and $p^o > \widetilde{p}$. Case 2: Suppose now that $\widehat{p} < p_{\ell,t}$, which implies that $\eta_1(p_{\ell,t}) > \eta_2(p_{\ell,t})$. This inequality and the definition of $\widehat{p}$ imply that $\eta_1(p) > \eta_2(p)$ for all $p > p_{\ell,t}$. Combining this and the facts that $S_t''(p_{\ell,t};\eta_1) > S_t''(p_{\ell,t};\eta_2)$, $S_t'(p_{\ell,t};\eta_1) = S_t'(p_{\ell,t};\eta_2)$ and $S_t(p_{\ell,t};\eta_1) = S_t(p_{\ell,t};\eta_2) = 0$, we conclude that $\eta_1(p) > \eta_2(p)$ and $S_t(p;\eta_1) > S_t(p;\eta_2)$ for $p \in (p_{\ell,t}, p_o)$ where $p_o \in \bar{\mathbb{R}}_+$ and $p_o > p_{\ell,t}$. Because the parameter set in which $\widehat{p} = p_{\ell,t}$ has a Lebesgue measure zero, our claim follows from the analysis in Cases 1 and 2. $\square$

Proof of Proposition EC.5: Recall the mapping λ and $S_t'(p_{\ell,t})$ in the proof of Proposition EC.4. Recall also that $\beta_{c,t+1} = g_{t+1}(\beta_{u,t+1})$. By the definition of λ and elementary analysis, we have

$$\frac{d\lambda(p,s,\beta_{u,t+1},\beta_{c,t+1})}{d\beta_{u,t+1}} = \frac{\partial\lambda(p,s,\beta_{u,t+1},\beta_{c,t+1})}{\partial\beta_{u,t+1}} + \frac{\partial\lambda(p,s,\beta_{u,t+1},\beta_{c,t+1})}{\partial\beta_{c,t+1}} g_{t+1}'(\beta_{u,t+1}). \tag{EC.112}$$

From the proof of Proposition EC.4, we already know that $\frac{\partial\lambda(p,s,\beta_{u,t+1},\beta_{c,t+1})}{\partial\beta_{u,t+1}} > 0$. Because $\frac{\partial\lambda(p,S_t(p;\beta_{u,t+1}),\beta_{u,t+1},\beta_{c,t+1})}{\partial\beta_{c,t+1}} > 0$ in equilibrium, (EC.112) implies that $\frac{d\lambda(p,S_t(p;\beta_{u,t+1}),\beta_{u,t+1},\beta_{c,t+1})}{d\beta_{u,t+1}} > 0$ and $\frac{d\lambda(p,S_t(p;\beta_{u,t+1}),\beta_{u,t+1},\beta_{c,t+1})}{d\beta_{u,t+1}}$ strictly increases with $g_{t+1}'(\beta_{u,t+1})$. Combining this, (EC.109), $\partial S_t'(p_{\ell,t})/\partial\beta_{c,t+1} > 0$ and $\frac{\partial\lambda(p,S_t(p;\beta_{u,t+1}),\beta_{u,t+1},\beta_{c,t+1})}{\partial s} > 0$ by the proof of Proposition EC.4, we conclude that $dS_t(p;\beta_{u,t+1})/d\beta_{u,t+1}$ strictly increases with $g_{t+1}'(\beta_{u,t+1})$ for all $p > p_{\ell,t}$. $\square$

PROPOSITION EC.8. *Consider any $t = 1, \ldots, \tau$ and pair of equilibria in credit- and penalty-dominated markets, which are identified in Propositions 2 and EC.3. (i) Compared to a penalty-dominated market, a credit-dominated market results in a smaller period-t production commitment for each renewable firm if and only if $p \le \tilde{p}$, where $\tilde{p} \in \bar{\mathbb{R}}_+$. (ii) For any given commitment profile of inflexible firms, a credit-dominated market results in a larger period-t reliability than a penalty-dominated market if and only if the period-t day-ahead demand shock $\epsilon_t \le \tilde{\epsilon}$, where $\tilde{\epsilon} \in \bar{\mathbb{R}}$.*

Proof of Proposition EC.8: Let $S_{c,t}(\cdot)$ be a committed production schedule of a renewable firm in any equilibrium with a credit-dominated market, and denote by $S_t(\cdot)$, the equilibrium committed production schedule of a renewable firm in a penalty-dominated market. Recall that $S_{c,t}(\cdot)$ and $S_t(\cdot)$ satisfy the conditions in Proposition EC.3-(ii) and Proposition 2-(ii), respectively. We first prove part (i) by showing that $S_{c,t}(p) \le S_t(p)$ if and only if $p \le \tilde{p}$ where $\tilde{p} \in \bar{\mathbb{R}}$. For period t, let $p_{\ell,t}^c$ and $p_{\ell,t}$ be the minimum prices at which a renewable firm commits to a positive quantity in equilibrium with a credit-dominated market and a penalty-dominated market, respectively. We already know from Propositions EC.3-(ii) and 2-(ii) that $p_{\ell,t}^c > p_{\ell,t}$. This and the monotonicity of the committed production schedules imply that either $S_{c,t}(p) < S_t(p)$ for all $p > 0$ or there exists at least one positive price at which $S_{c,t}(\cdot)$ and $S_t(\cdot)$ intersect. We will show part (i) under each of these two scenarios. Note that the first scenario immediately implies part (i) as $\{\infty\} \subseteq \bar{\mathbb{R}}_+$. We now focus on the second scenario. Suppose for a contradiction that there exists at least two positive prices at which $S_{c,t}(\cdot)$ and $S_t(\cdot)$ intersect. Let the minimum and second minimum of these prices be p_c and $\bar{p}_c$, respectively. Comparing (17) and (EC.6) at $p \in \{p_c, \bar{p}_c\}$, we conclude that $S_{c,t}'(p) > S_t'(p)$ for $p \in \{p_c, \bar{p}_c\}$ because $S_{c,t}(p) = S_t(p)$ for $p \in \{p_c, \bar{p}_c\}$. The facts that $S_{c,t}'(p_c) > S_t'(p_c)$ and $S_{c,t}(p_c) = S_t(p_c)$ imply that $S_{c,t}(p) > S_t(p)$ for $p \in (p_c, \bar{p}_c)$. Then, for $S_t(\cdot)$ to catch back $S_{c,t}(\cdot)$ at $\bar{p}_c$, $S_{c,t}'(\bar{p}_c) \le S_t'(\bar{p}_c)$ must hold. But, this contradicts the fact that $S_{c,t}'(\bar{p}_c) > S_t'(\bar{p}_c)$. Therefore, in this scenario, there exists a unique price p_c at which $S_{c,t}(\cdot)$ and $S_t(\cdot)$ intersect. Because $p_{\ell,t}^c > p_{\ell,t}$, it follows from the monotonicity of the committed production schedules and the definition of p_c that $S_{c,t}(p) < S_t(p)$ if and only if $p \in (0, p_c)$. This completes our argument for part (i). Part (ii) is an immediate result of part (i), and can be shown using the definition of reliability (10) and elementary probability analysis. $\square$

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Appendix O: Further Details for Section 6

We first provide details about the day-ahead market considered to obtain the price sensitivity parameter α in Section 6. Please note that the market that will be explained below is only used to obtain the α parameter in Section 6. Then, we discuss firms' equilibrium strategies in this market. Later in this section, we will explain how we use this equilibrium to obtain the price sensitivity estimate $\widehat{\alpha}_{m,\ell}$ for each time block $\ell = 1, \ldots, 24$ in every month $m = 1, \ldots, 12$.

As explained in the fourth paragraph in Section 6, consider a day-ahead electricity market with N_i inflexible firms and N_c other conventional firms (such as oil-fueled power plants) each of which is capable of committing to a different production schedule in every day-ahead market. The latter type of firms are indexed by $z = N_i + 1, \ldots, N_i + N_c$. Period t corresponds to a day as in Section 2. Apart from the following two aspects, the set-up for this type of market is similar to the one explained in Section 2: (1) each period t consists of 24 time blocks and (2) renewable producers (if any) do not commit to a production schedule in this type of day-ahead market; rather, for every time block, their expected production in that time block of the following period is taken into account to determine the net day-ahead demand in the same time block of period t.

For a given month m, the day-ahead market operates as follows. At the beginning of each day-ahead market $t \leq \tau$, that is, before the uncertainty in the period-t day-ahead demand is resolved, each inflexible firm k commits to a production quantity, and each of the other conventional firms z commits to its twice continuously differentiable production schedule $\tilde{S}_{z,t,\ell} : \mathbb{R} \to \mathbb{R}_+$ for each of 24 time blocks without observing other firms' commitments in any time block of period-t day-ahead market. As in Section 2, if $t = 1$, the production commitment $\bar{S}_k$ is a decision variable for each inflexible firm k; for $t > 1$, inflexible firm k's day-ahead commitment remains the same and is equal to $\bar{S}_k$. After the realization of period-t day-ahead demand for each of 24 time blocks, the market clearing price is set for every 24 time block separately so that the aggregate period-t day-ahead commitment and period-t day-ahead demand net of the expected renewable production in period t+1 match for each time block. As explained in Section 1, day-ahead market clearance in a time block determines how much each day-ahead participant must produce in that time block of the following period.

In period t+1, the total production cost for each of the N_c firms is $L_{t+1}(q)$ at output $q \geq 0$, where $L_{t+1}(\cdot)$ is twice differentiable, convex and satisfies the following standard properties: $L_{t+1}(0) = 0$, $L'_{t+1}(0) = 0$. Note that $L_{\cdot}(\cdot)$ does not include an index for the time block. This is because in practice, the hourly variation in the production cost of inflexible firms and conventional firms is nonexistent or negligibly small. For any time block, if there is a mismatch between a firm's day-ahead commitment in period t and its actual production quantity in period t+1, the firm must pay an undersupply or oversupply penalty explained in Section 2. None of $(N_i + N_c)$ firms is exposed to supply uncertainty; each can produce the quantity of their choice. This and the fact that the marginal cost of production of each of these firms is low compared to the undersupply penalty rate imply that it is never profitable for any firm to underproduce to save from the production cost. Similarly, the overproduction penalty rate deters these $(N_i + N_c)$ firms from producing more than their commitments. As a result, each firm's optimal production quantity in a particular time block of period t+1 is equal to its day-ahead cleared production quantity in that time block of period t. Then, given day-ahead random shock $\epsilon_{t,\ell}$ and other firms' any commitment profiles $\bar{\mathbb{S}}$ and $\tilde{S}_{-z,t,\ell}$ in the time block ℓ of period t, each firm z's profit in said time block is equal to $p\mathcal{R}_{z,t,\ell}(p;\epsilon_{t,\ell}) - L_{t+1}(\mathcal{R}_{z,t,\ell}(p;\epsilon_{t,\ell}))$, where $\mathcal{R}_{z,t,\ell}(p;\epsilon_{t,\ell}) \doteq \tilde{v}_{t,\ell} - \alpha_{t,\ell}p + \epsilon_{t,\ell} - \sum_{k=1}^{N_i} \bar{S}_k - \sum_{j \neq z} \tilde{S}_{j,t,\ell}(p)$ is the firm z's residual demand curve in time block ℓ of period t and $\tilde{v}_{t,\ell}$ is equal to $v_{t,\ell}$ net of expected renewable production in time block ℓ of period t+1. The objective of each inflexible firm is similar to (6), with the exception that the following commitment profile should be considered: $\tilde{\mathbb{S}}_{t,\ell} \doteq (\tilde{S}_{N_i+1,t,\ell}, \ldots, \tilde{S}_{N_i+N_c,t,\ell})$.

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Using similar arguments as in the proof of Proposition 2, it is straightforward to verify that, in this setting, there exists a unique equilibrium that satisfies (11) and $\tilde{S}_{t,\ell} = \tilde{S}_{N_i+1,t,\ell} = \tilde{S}_{N_i+2,t,\ell} = \ldots = \tilde{S}_{N_i+N_c,t,\ell}$ for all ℓ. Firms' equilibrium commitment strategies are as follows. In period-t day-ahead market, in time block $\ell = 1,\ldots,24$, each of the N_c firms commits to a production schedule $\tilde{S}_{t,\ell}$ that satisfies the following properties:

$$\tilde{S}'_{t,\ell}(p) = \left(\tilde{S}_{t,\ell}(p)/(p - L'_{t+1}(\tilde{S}_{t,\ell}(p))) - \alpha_{t,\ell}\right)/(N_c - 1),\ \tilde{S}'_{t,\ell}(p) > 0 \text{ for } p \geq 0, \text{ and } \tilde{S}_{t,\ell}(0) = 0. \tag{EC.113}$$

At $t = 1$, each inflexible firm commits to a production quantity $\bar{S} = \min\{\bar{s}^*, K\}$ such that $\bar{s}^*$ satisfies

$$\sum_{t=1}^{\tau}\sum_{\ell=1}^{24}\left[\int_{\underline{z}_{t,\ell}(\bar{s})}^{\infty}\left(p^*_{t,\ell}(\bar{s};z) - \frac{\bar{s}}{\alpha_{t,\ell} + N_c\tilde{S}'_{t,\ell}(p^*_{t,\ell}(\bar{s};z))} - L'_{t+1}(\bar{s})\right)d\Phi_{t,\ell}(z) + L_{t+1}(\bar{s})\phi_{t,\ell}(\underline{z}_{t,\ell}(\bar{s}))\right] = 0, \tag{EC.114}$$

where $\underline{z}_{t,\ell}(\bar{s}) \doteq N_i\bar{s} - \tilde{v}_{t,\ell}$, $\tilde{S}_{t,\ell}(\cdot)$ is as in (EC.113), and $p^*_{t,\ell}(\bar{s};z)$ is the price that satisfies $\tilde{v}_{t,\ell} - \alpha_t p^*_{t,\ell}(\bar{s};z) + z = N_c\tilde{S}_{t,\ell}(p^*_{t,\ell}(\bar{s};z)) + N_i\bar{s}$.

Procedure for obtaining the estimate $\widehat{\alpha}_{m,\ell}$: Based on (EC.113), the committed production schedule of each of the N_c firms in month $m = 1,\ldots,12$ depends on its marginal cost of commitment $U'_m(\cdot)$, which is estimated by letting the variable cost of production be $U_m(q) = u_m q^2$ for all time blocks every day in month m and using the same procedure explained for inflexible firms. Using this, we estimate the price sensitivity parameter α for each time block $\ell = 1,\ldots,24$ in every month m of 2013 by employing the following method: In time block ℓ of month m, we have $\mathcal{N}(m)$ observations, where $\mathcal{N}(m)$ is the number of days in month m. For example, for $m = 5$ (May) and $\ell = 10$ (9:00am - 10:00am), we have 31 different observations because there are 31 days in May. The estimated cost curve of each of the N_c firms in month m is denoted by $\widehat{U}_m(\cdot)$. Let $\tilde{S}(\cdot;m,\ell)$ be the committed production schedule of each of N_c firms in equilibrium, and $\alpha_{m,\ell}$ be the true α value in time block ℓ of month m. Denote by $Y(m,\ell,d)$ and $p(m,\ell,d)$, each of N_c firms' observed cleared day-ahead production commitment and the observed day-ahead price in time block ℓ, day d, month m, respectively. Then, based on (EC.113), the estimate $\widehat{\alpha}_{m,\ell}$ of $\alpha_{m,\ell}$ for $m = 1,\ldots,12$ and $\ell = 1,\ldots,24$ is the one that minimizes residual sum of squares for every time block of each month:

$$\begin{aligned}
\widehat{\alpha}_{m,\ell} = \arg\min_{\alpha_{m,\ell}\geq 0} & \sum_{d\in\mathcal{N}(m)}\left(Y(m,\ell,d) - \tilde{S}(p(m,\ell,d);m,\ell)\right)^2 \\
\text{s.t.}\quad & \tilde{S}'(p;m,\ell) = \left(\tilde{S}(p;m,\ell)/(p - \widehat{U}'_m(\tilde{S}(p;m,\ell))) - \alpha_{m,\ell}\right)/(N_c - 1), \\
& \tilde{S}'(p;m,\ell) \in (0,\infty) \quad \text{for } p \geq 0, \text{ and } \tilde{S}(0;m,\ell) = 0.
\end{aligned}$$

Example equilibrium committed production schedule of a renewable firm under the penalty rule B: Consider the 9:00am - 10:00am time block of a particular day in May. For that time block, the calibrated committed production schedule of a renewable firm in equilibrium, which is denoted by $S(\cdot)$, is given by

$$S'(p) = \frac{1}{N_r - 1}\left[\frac{S(p) - 0.962\int_0^{S(p)}(S(p) - x)d\widehat{F}(x)}{p - (0.962p + 0.225)\widehat{F}(S(p))} - 0.0043\right] \tag{EC.115}$$

subject to the initial condition $S(0) = 0$ and the monotonicity constraint $S'(p) \in (0,\infty)$. Here, $N_r = 25$, 0.0043 corresponds to the estimated α parameter for the 9:00am - 10:00am time block in May, and $\widehat{F}(\cdot)$ is the Lomax distribution of the available supply for 9:00am - 10:00am time block in May with the estimated scale and shape parameter. Note that, day-ahead demand and available supply for a particular time block of a day are realized on distinct days and at distinct locations in a wide geographical region, implying the independence of $\widehat{F}(\cdot)$ from $\epsilon_{m,\ell}$ in the construction of (EC.115).

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Appendix P: The Statements and Proofs of Propositions EC.9 and EC.10

PROPOSITION EC.9. *Suppose that* (21) *holds for some* $t = 1, \ldots, \tau$. *Then, for any given* $\bar{\mathbb{S}}$, *the realized undersupply penalty rate in period t+1 increases with* $\beta_{u,t+1}$ *in equilibrium, that is,* $d\eta_{u,t+1}(p_t^*(\epsilon_t))/d\beta_{u,t+1} > 0$, *if* $\epsilon_t \in [\underline{\epsilon}, \bar{\epsilon}]$ *and* $\beta_{u,t+1} < \bar{\beta}$ *where* $\bar{\beta} \in (0,1]$, *and* $\bar{\epsilon}$ *and* $\underline{\epsilon}$ *are constants such that* $\underline{\epsilon} < \bar{\epsilon} < \infty$.

Proof of Proposition EC.9: Take any $S_t(\cdot)$ that satisfies (17) and (18), which is each renewable firm's committed production schedule in an equilibrium by Proposition 2-(ii). In this equilibrium, by an application of the implicit function theorem on (1), we have

$$\partial p_t^*(\epsilon_t)/\partial \beta_{u,t+1} = -\big(\partial S_t(p_t^*(\epsilon_t))/\partial \beta_{u,t+1}\big) N_r / \Big(\alpha_t + N_r S_t'(p_t^*(\epsilon_t))\Big). \tag{EC.116}$$

Recall that $\eta_{u,t+1}(p_t^*(\epsilon_t)) = \beta_{u,t+1} p_t^*(\epsilon_t) + b_{u,t+1}$. Then, from elementary analysis and (EC.116), we have

$$\begin{aligned} d\eta_{u,t+1}(p_t^*(\epsilon_t))/d\beta_{u,t+1} = H(p_t^*(\epsilon_t), \beta_{u,t+1}) &\doteq p_t^*(\epsilon_t) + (\partial p_t^*(\epsilon_t)/\partial \beta_{u,t+1})\beta_{u,t+1} \\ &= p_t^*(\epsilon_t) - \big(\partial S_t(p_t^*(\epsilon_t))/\partial \beta_{u,t+1}\big) N_r \beta_{u,t+1} / \left(\alpha_t + N_r S_t'(p_t^*(\epsilon_t))\right), \end{aligned} \tag{EC.117}$$

where $H(\cdot,\cdot): \mathbb{R}_+ \times [0,1] \to \mathbb{R}$ is a continuous mapping of $p_t^*(\epsilon_t)$ and $\beta_{u,t+1}$. Below we show that $H(p_t^*(\epsilon_t), \beta_{u,t+1}) > 0$ under the stated conditions in Proposition EC.9.

Define $\underline{\epsilon} \doteq \sum_{j=N_r+1}^{N_r+N_i} \bar{S}_j + \delta - v_t$ where $\delta > 0$ is arbitrarily small, and take any $\bar{\epsilon}$ such that $\underline{\epsilon} < \bar{\epsilon} < \infty$. Consider a period-$t$ day-ahead demand shock $\epsilon_t \in [\underline{\epsilon}, \bar{\epsilon}]$. Note that $\epsilon_t \geq \underline{\epsilon}$ implies $p_t^*(\epsilon_t) > 0$ for $\beta_{u,t+1} \in [0,1]$. Thus, $H(p_t^*(\epsilon_t), 0) > 0$ for $\epsilon_t \geq \underline{\epsilon}$ by (EC.117). Then, either of the following two scenarios is true by the continuity of $H(\cdot,\cdot)$: (i) $H(p_t^*(\epsilon_t), \beta_{u,t+1}) > 0$ for $\beta_{u,t+1} \in [0,1]$, or (ii) there exists a constant $\tilde{\beta}(\epsilon_t) \in (0,1)$ such that $H(p_t^*(\epsilon_t), \beta_{u,t+1}) > 0$ for $\beta_{u,t+1} < \tilde{\beta}(\epsilon_t)$. In either case, there exists a threshold $\tilde{\beta}(\epsilon_t) \in (0,1]$ such that

$$\beta_{u,t+1} < \tilde{\beta}(\epsilon_t) \;\Rightarrow\; H(p_t^*(\epsilon_t), \beta_{u,t+1}) > 0. \tag{EC.118}$$

Define $\bar{\beta} \doteq \inf\{\tilde{\beta}(\epsilon_t) : \epsilon_t \in [\underline{\epsilon}, \bar{\epsilon}]\}$. Since $[\underline{\epsilon}, \bar{\epsilon}]$ is a compact interval and $\tilde{\beta}(\epsilon_t) > 0$ for $\epsilon_t \in [\underline{\epsilon}, \bar{\epsilon}]$, it follows that $\bar{\beta} = \min\{\tilde{\beta}(\epsilon_t) : \epsilon_t \in [\underline{\epsilon}, \bar{\epsilon}]\} > 0$. Based on this and (EC.118), $\beta_{u,t+1} < \bar{\beta}$ implies $H(p_t^*(\epsilon_t), \beta_{u,t+1}) > 0$ for $\epsilon_t \in [\underline{\epsilon}, \bar{\epsilon}]$. □

PROPOSITION EC.10. *Let* $S_t(\cdot; y)$ *be each renewable firm's period-t committed production schedule in the equilibrium identified in Proposition 2-(ii) with* $b_{u,t+1} = y$. *Then, for any* $\underline{b}$ *and* $\bar{b}$ *such that* $\underline{b} < \bar{b}$, $S_t(p; \underline{b}) > S_t(p; \bar{b})$ *if and only if* $p < \bar{p}$ *for some* $\bar{p} \in \overline{\mathbb{R}}_+$.

This proposition shows that a higher $b_{u,t+1}$ results in a strictly smaller period-t committed production quantity for each renewable firm if and only if the price is smaller than some constant $\bar{p} \in \overline{\mathbb{R}}_+$. The rationale can be explained as follows. To mitigate the increase in $b_{u,t+1}$, the reduction in period-t market clearing price must be relatively large because of the weight $\beta_{u,t+1}$ for price in the undersupply penalty rate. Achieving such a large reduction in price is simply infeasible at low prices (i.e., when $p < \bar{p}$). Based on Proposition EC.10, it is straightforward to show that for any given commitment profile of inflexible firms, a higher $b_{u,t+1}$ results in a strictly higher period-t system reliability when the period-t day-ahead demand shock is not extremely large.

Proof of Proposition EC.10: Observe from (EC.29) that $S_t'(\cdot; b_{u,t+1})$ strictly decreases in $b_{u,t+1}$. This and the fact that $S_t(0; \underline{b}) = S_t(0; \bar{b}) = 0$ by (18) imply that $S_t(h; \underline{b}) > S_t(h; \bar{b})$ for $h > 0$ arbitrarily close to zero. Based on this, we prove Proposition EC.10 under each of the following two cases. Case 1: Suppose that there exists at least one positive price

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at which $S_t(\cdot;\underline{b})$ and $S_t(\cdot;\bar{b})$ intersect. We claim and show below that there exists a unique positive price $\bar{p}$ at which $S_t(\cdot;\underline{b})$ and $S_t(\cdot;\bar{b})$ intersect. Then, because $S_t(h;\underline{b}) > S_t(h;\bar{b})$, it follows from the continuity of $S_t(\cdot;\underline{b})$ and $S_t(\cdot;\bar{b})$ that $S_t(p;\underline{b}) > S_t(p;\bar{b})$ if and only if $p < \bar{p}$. Case 2: Suppose that there is no price at which $S_t(\cdot;\underline{b})$ and $S_t(\cdot;\bar{b})$ intersect. Then, the fact that $S_t(h;\underline{b}) > S_t(h;\bar{b})$ implies $S_t(p;\underline{b}) > S_t(p;\bar{b})$ for $p > 0$. From Cases 1 and 2, Proposition EC.10 immediately follows. It only remains to prove our claim in Case 1 about the uniqueness of the price at which $S_t(\cdot;\underline{b})$ and $S_t(\cdot;\bar{b})$ intersect. Suppose for a contradiction that there exist at least two positive prices at which $S_t(\cdot;\underline{b})$ and $S_t(\cdot;\bar{b})$ intersect. Denote by p_1 and p_2 the minimum and the second minimum of the aforementioned prices, respectively. Because $S_t(p;\underline{b}) = S_t(p;\bar{b})$ for $p \in \{p_1, p_2\}$, it follows from (17) that $S_t'(p;\underline{b}) < S_t'(p;\bar{b})$ for $p \in \{p_1, p_2\}$. Because $S_t'(p_1;\underline{b}) < S_t'(p_1;\bar{b})$ and $S_t(p_1;\underline{b}) = S_t(p_1;\bar{b})$, we have $S_t(p;\underline{b}) < S_t(p;\bar{b})$ for $p \in (p_1, p_2)$. However, this and the fact that $S_t'(p_2;\underline{b}) < S_t'(p_2;\bar{b})$ imply that $S_t(p_2;\underline{b}) < S_t(p_2;\bar{b})$, which is a contradiction. $\square$

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