

1_discretization

March 13, 2023

1 Discretization

The aim of the notebook is to evaluate the performance of CategoricalHMM on discretized data

```
[1]: import numpy as np
from hmmlearn import hmm
import matplotlib.pyplot as plt
import seaborn as sns
import pandas as pd
import urllib
import itertools
from scipy.stats import multivariate_normal
from source.model.discretized_HMM import DiscreteHMM, DISCRETIZATION_TECHNIQUES
```

```
[2]: import ssm
from ssm.util import find_permutation
from ssm.plots import gradient_cmap, white_to_color_cmap

import matplotlib.pyplot as plt
%matplotlib inline

import seaborn as sns
sns.set_style("white")
# sns.set_context("talk")

# https://xkcd.com/color/rgb/

with urllib.request.urlopen('https://xkcd.com/color/rgb.txt') as f:
    colors = f.readlines()
color_names = [str(c)[2:].split('\\t')[0] for c in colors[1:]]

colors = sns.xkcd_palette(color_names)
cmap = gradient_cmap(colors)
```

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[3]: colors
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```
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(0.49411764705882355, 0.11764705882352941, 0.611764705882353)]
```

[4]: # <https://hmmlearn.readthedocs.io/en/latest/tutorial.html>

```
np.random.seed(42)

true_model3 = hmm.GaussianHMM(n_components=3, covariance_type="full")
true_model3.startprob_ = np.array([0.6, 0.3, 0.1])
true_model3.transmat_ = np.array([[0.7, 0.2, 0.1],
                                 [0.3, 0.5, 0.2],
                                 [0.3, 0.3, 0.4]])

true_model3.means_ = np.array([[0.0, 0.0], [3.0, -3.0], [4.0, 3.0]])
true_model3.covars_ = np.array([[[[1, -.5], [-.5, 1.2]], [[.6, -.5], [-.5, 1.
    ↪2]], [[1.5, .5], [.5, 2.2]]]] * .8

true_model3.sample(10)
```

[4]: (array([[0.9694375 , -0.83691634],
 [-0.72845739, 1.72027549],
 [0.13888981, -0.73856628],
 [-1.19070703, -0.62107079],
 [-1.39713416, 0.05274311],
 [-0.09710759, -1.41211893],
 [0.77049997, -0.29208482],
 [0.45061783, -0.42945405],
 [0.93529815, -0.11410115],
 [1.41712714, 0.27877223]]),
 array([0, 0, 0, 0, 0, 0, 0, 0, 0, 0]))

[5]: true_model2 = hmm.GaussianHMM(n_components=2, covariance_type="full")
true_model2.startprob_ = true_model3.startprob_[:2] / true_model3.startprob_[:-
 ↪2].sum()
true_model2.transmat_ = true_model3.transmat_[:2, :2] / true_model3.transmat_[:-
 ↪2, :2].sum(axis=1).reshape(-1, 1)

true_model2.means_ = true_model3.means_[:2]
true_model2.covars_ = true_model3.covars_[:2]

true_model2.sample(10)

```

[5]: (array([[ 0.38101918, -0.47075945],
   [ 2.02364148, -2.5232011 ],
   [-1.36473817,  2.95830902],
   [-0.44026515,  0.72275594],
   [-0.11525016, -1.57529795],
   [ 0.89312562,  0.05620838],
   [ 0.88485255, -0.93366053],
   [ 3.63129627, -3.48988934],
   [ 0.05286998,  0.89759822],
   [ 0.43951158, -0.871224 ]]),
 array([0, 1, 0, 0, 0, 0, 0, 1, 0, 0]))

[6]: norm1 = multivariate_normal(true_model3.means_[0], true_model3.covars_[0])
norm2 = multivariate_normal(true_model3.means_[1], true_model3.covars_[1])
norm3 = multivariate_normal(true_model3.means_[2], true_model3.covars_[2])

[7]: sample_sizes = [20, 50, 100, 1000, 10000]
nodes_numbers = [3, 5, 7, 10, 15, 25, 50, 100]

[12]: def eval_model(n_components, number_of_nodes, discretizing, X, Z):
        if discretizing is None:
            myHMM = hmm.GaussianHMM(n_components, n_iter=150)
        else:
            myHMM = DiscreteHMM(discretizing, number_of_nodes, n_components,
        ↪n_iter=150)
            myHMM.fit(X)

        if discretizing is None:
            Z_hat = myHMM.predict(X)
            ll = myHMM.score(X)
        else:
            Xd = myHMM._discretize(X, False)
            Z_hat = myHMM.predict(Xd)
            ll = myHMM.score(Xd)

        if np.unique(Z_hat).shape[0] == n_components:
            Z_hat = find_permutation(Z, Z_hat)[Z_hat]
            acc = (Z_hat == Z).mean()
        else:
            print(f"Recognized {np.unique(Z_hat).shape[0]} instead of"
        ↪{n_components} states!")
            acc = -1
        return myHMM, ll, acc

def provide_grid(X):
    x1, y1 = X.min(axis=0) * 1.1

```

```

x2, y2 = X.max(axis=0) * 1.1

XX, YY = np.meshgrid(np.linspace(x1, x2, 100), np.linspace(y1, y2, 100))
data = np.column_stack((XX.ravel(), YY.ravel()))
lls = np.concatenate([norm1.pdf(data).reshape(-1, 1), norm2.pdf(data).
↪reshape(-1, 1), norm3.pdf(data).reshape(-1, 1)], axis=1)
return XX, YY, lls


def plot_true_HMM(X, Z, n_components, XX, YY, lls):
    plt.figure(figsize=(5, 5))
    for k in range(n_components):
        plt.plot(X[Z==k, 0], X[Z==k, 1], 'o', mfc=colors[k], mec='none', ms=4, u
↪alpha=.5)
        plt.contour(XX, YY, np.exp(lls[:,k]).reshape(XX.shape), u
↪cmap=white_to_color_cmap(colors[k]), levels=8)

    plt.plot(X[:,0], X[:,1], '-k', lw=1, alpha=.1)
    plt.xlabel("$x_1$")
    plt.ylabel("$x_2$")
    plt.title("Observation Distributions")
    plt.show()

    return XX, YY, lls


def compare_models(models, X, XX, YY, lls, target_ll):
    fig, axs = plt.subplots(len(nodes_numbers), len(DISCRETIZATION_TECHNIQUES), u
↪sharex=True, sharey=True, figsize=(10, 10))
    summary = []
    for i, j in itertools.product(range(len(DISCRETIZATION_TECHNIQUES) + 1), u
↪range(len(nodes_numbers))):
        discretizing, number_of_nodes = [*DISCRETIZATION_TECHNIQUES, None][i], u
↪nodes_numbers[j]
        if number_of_nodes > sample_size:
            continue
        summary.append({
            'discretizing': discretizing, 'number_of_nodes': number_of_nodes,
            'accuracy': models[discretizing, number_of_nodes]['acc'],
            'log-likelihood': models[discretizing, number_of_nodes]['ll'],
            'likelihood-ratio': np.exp(models[discretizing, u
↪number_of_nodes]['ll'] - target_ll)
        })

        if discretizing is not None:
            myHMM = models[discretizing, number_of_nodes]['model']
            Xd = myHMM._discretize(X, False)

```

```

        for k in range(2):
            axs[j, i].contour(XX, YY, np.exp(lls[:,k]).reshape(XX.shape),  

        ↪cmap=white_to_color_cmap(colors[k]), levels=5, alpha=.4)
            for k in range(number_of_nodes):
                axs[j, i].plot(X[Xd[:, 0] == k, 0], X[Xd[:, 0] == k, 1], 'o',  

        ↪mec='none', ms=2, alpha=.5, mfc=colors[k % len(colors)])
            for k in range(number_of_nodes):
                axs[j, i].plot(myHMM.nodes[0, k], myHMM.nodes[1, k], '*',  

        ↪mec='none', ms=4, color='black', mfc=colors[k % len(colors)])

    for ax, col in zip(axs[0], DISCRETIZATION_TECHNIQUES):
        ax.set_title(col)

    for ax, row in zip(axs[:,0], nodes_numbers):
        ax.set_ylabel(row, rotation=90, size='large')

fig.tight_layout()
plt.show()
summary = pd.DataFrame(summary).sort_values('number_of_nodes')
display(summary)
print(summary.to_latex())
sns.lineplot(data=summary, x='number_of_nodes', y='accuracy',  

        ↪hue='discretizing')
plt.title("Accuracy")
plt.show()
sns.lineplot(data=summary, x='number_of_nodes', y='log-likelihood',  

        ↪hue='discretizing')
plt.title("Log-likelihood")
plt.show()

def evaluation(sample_size, n_components):
    true_model = true_model2 if n_components == 2 else true_model3
    X, Z = true_model.sample(sample_size)
    ll_target = true_model.score(X)

    XX, YY, lls = provide_grid(X)
    models = dict()
    for discretizing, number_of_nodes in itertools.  

        ↪product([*DISCRETIZATION_TECHNIQUES, None], nodes_numbers):
        if number_of_nodes > sample_size:
            continue
        myHMM, ll, acc = eval_model(n_components, number_of_nodes,  

        ↪discretizing, X, Z)
        models[discretizing, number_of_nodes] = {'ll': ll, 'acc': acc, 'model':  

        ↪myHMM}

```

```
XX, YY, lls = provide_grid(X)
plot_true_HMM(X, Z, n_components, XX, YY, lls)
compare_models(models, X, XX, YY, lls, ll_target)
```

```
[13]: for sample_size in sample_sizes:
    print(f"sample_size = {sample_size}")
    evaluation(sample_size, 2)
    evaluation(sample_size, 3)
```

Fitting a model with 21 free scalar parameters with only 20 data points will result in a degenerate solution.

Fitting a model with 31 free scalar parameters with only 20 data points will result in a degenerate solution.

Fitting a model with 21 free scalar parameters with only 20 data points will result in a degenerate solution.

Fitting a model with 29 free scalar parameters with only 20 data points will result in a degenerate solution.

Fitting a model with 21 free scalar parameters with only 20 data points will result in a degenerate solution.

Fitting a model with 29 free scalar parameters with only 20 data points will result in a degenerate solution.

sample_size = 20

Fitting a model with 21 free scalar parameters with only 20 data points will result in a degenerate solution.

Fitting a model with 29 free scalar parameters with only 20 data points will result in a degenerate solution.

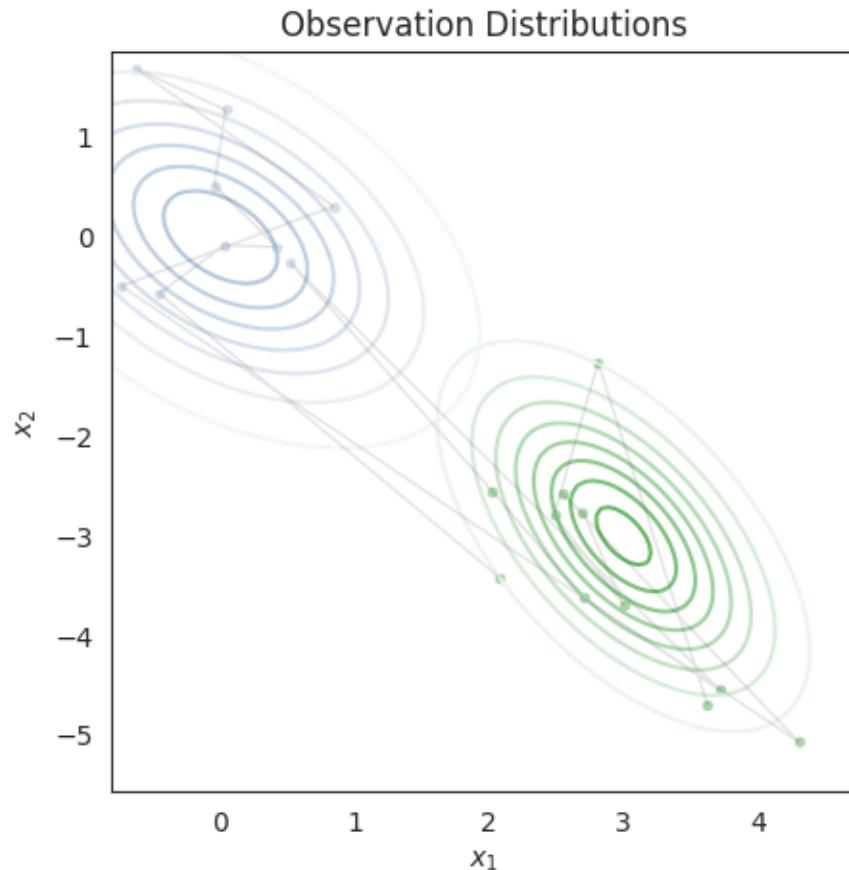
```
/home/kabalce/.local/lib/python3.10/site-
packages/sklearn/cluster/_kmeans.py:870: FutureWarning: The default value of
`n_init` will change from 10 to 'auto' in 1.4. Set the value of `n_init`
explicitly to suppress the warning
    warnings.warn(
```

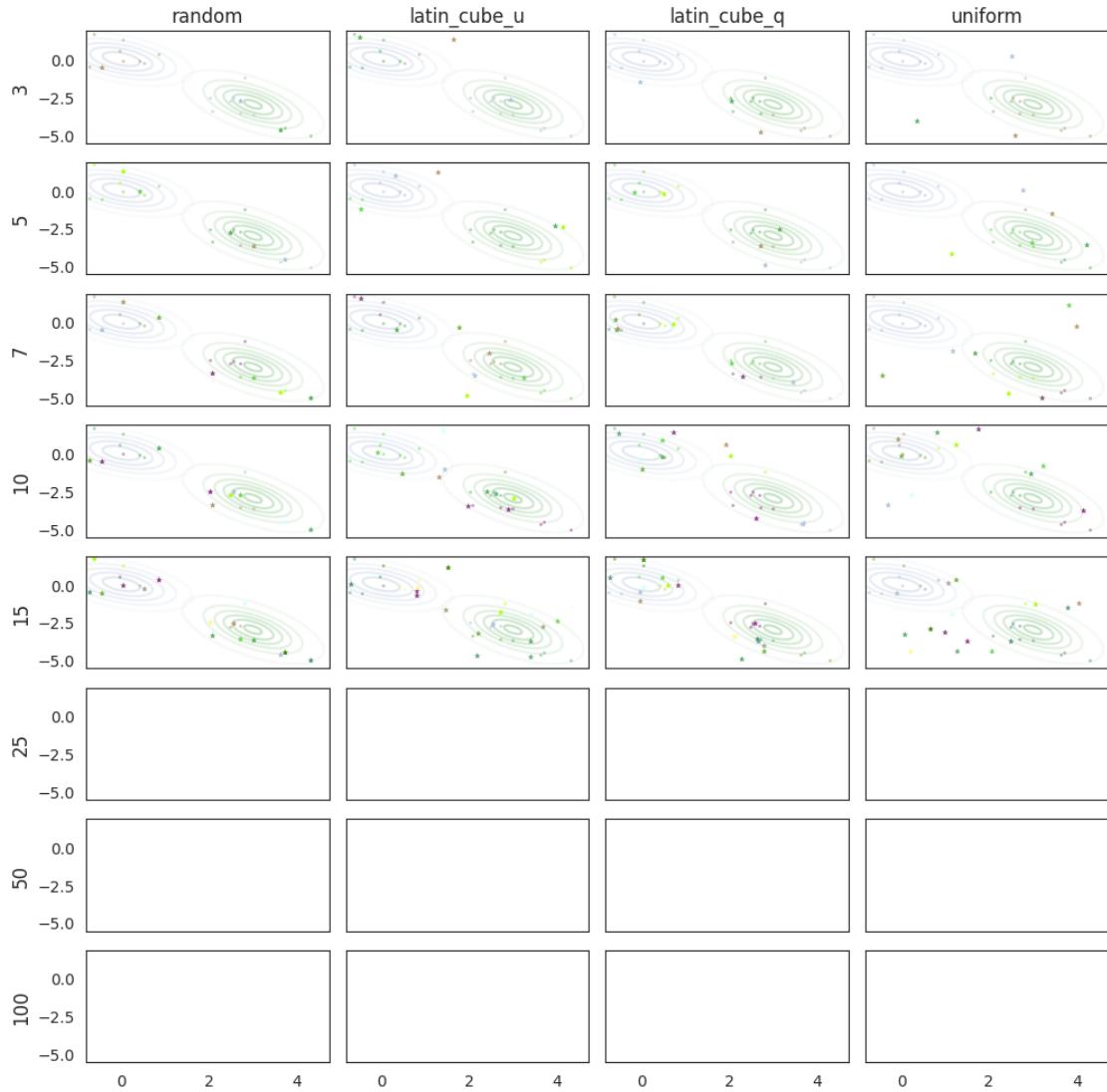
```
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```
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```

```
/home/kabalce/.local/lib/python3.10/site-
packages/sklearn/cluster/_kmeans.py:870: FutureWarning: The default value of
`n_init` will change from 10 to 'auto' in 1.4. Set the value of `n_init`
explicitly to suppress the warning
```

```
warnings.warn(  
/home/kabalce/.local/lib/python3.10/site-  
packages/sklearn/cluster/_kmeans.py:870: FutureWarning: The default value of  
'n_init' will change from 10 to 'auto' in 1.4. Set the value of 'n_init'  
explicitly to suppress the warning  
warnings.warn(
```





	discretizing	number_of_nodes	accuracy	log-likelihood	likelihood-ratio
0	random	3	0.60	-17.799542	8.083310e+16
20	None	3	1.00	-54.077037	1.420605e+01
5	latin_cube_u	3	0.90	-12.494001	1.628382e+19
15	uniform	3	0.90	-13.933288	3.860842e+18
10	latin_cube_q	3	0.65	-18.523026	3.920883e+16
1	random	5	0.90	-24.248245	1.279242e+14
21	None	5	1.00	-54.077037	1.420605e+01
6	latin_cube_u	5	0.65	-23.060436	4.195769e+14
16	uniform	5	0.55	-20.324024	6.474710e+15
11	latin_cube_q	5	0.60	-28.824406	1.316896e+12
22	None	7	1.00	-54.077037	1.420605e+01
17	uniform	7	0.90	-18.697363	3.293595e+16
12	latin_cube_q	7	0.60	-33.673223	1.032136e+10

```

7   latin_cube_u          7   0.50    -28.269755  2.293154e+12
2     random              7   0.90    -29.793548  4.996413e+11
13  latin_cube_q         10  0.90    -29.360866  7.701404e+11
8   latin_cube_u          10  0.55    -25.515272  3.603215e+13
18  uniform               10  0.90    -24.316958  1.194294e+14
3   random               10  0.90    -36.147299  8.694795e+08
23  None                  10  1.00    -54.077037  1.420605e+01
14  latin_cube_q         15  0.65    -39.227169  3.996584e+07
9   latin_cube_u          15  0.90    -31.000382  1.494642e+11
19  uniform               15  0.90    -23.921797  1.773076e+14
4   random               15  0.90    -45.082951  1.144340e+05
24  None                  15  1.00    -54.077037  1.420605e+01

/tmp/ipykernel_54422/2508230348.py:83: FutureWarning: In future versions
`DataFrame.to_latex` is expected to utilise the base implementation of
`Styler.to_latex` for formatting and rendering. The arguments signature may
therefore change. It is recommended instead to use `DataFrame.style.to_latex`
which also contains additional functionality.

print(summary.to_latex())

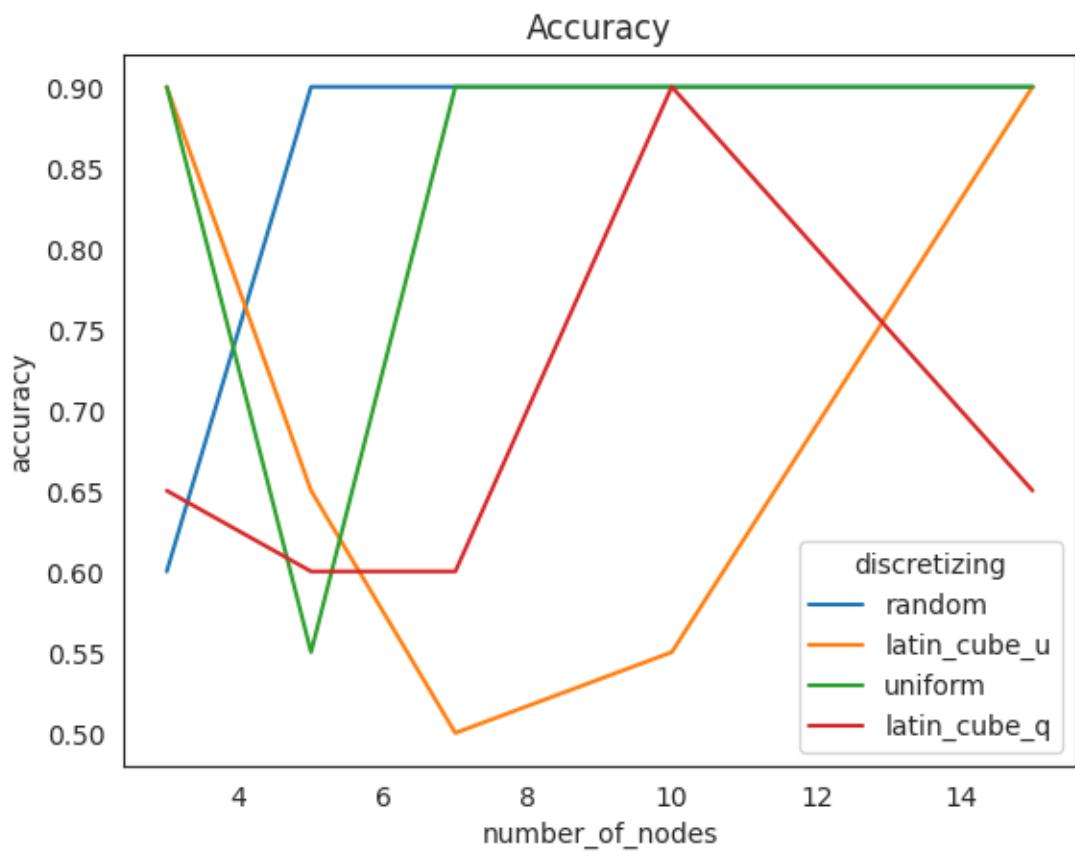
\begin{tabular}{llrrrr}
\toprule
{} & discretizing & number\_of\_nodes & accuracy & log-likelihood &
likelihood-ratio \\
\midrule
0 & random & 3 & 0.60 & -17.799542 &
8.083310e+16 \\
20 & None & 3 & 1.00 & -54.077037 &
1.420605e+01 \\
5 & latin\_cube\_u & 3 & 0.90 & -12.494001 &
1.628382e+19 \\
15 & uniform & 3 & 0.90 & -13.933288 &
3.860842e+18 \\
10 & latin\_cube\_q & 3 & 0.65 & -18.523026 &
3.920883e+16 \\
1 & random & 5 & 0.90 & -24.248245 &
1.279242e+14 \\
21 & None & 5 & 1.00 & -54.077037 &
1.420605e+01 \\
6 & latin\_cube\_u & 5 & 0.65 & -23.060436 &
4.195769e+14 \\
16 & uniform & 5 & 0.55 & -20.324024 &
6.474710e+15 \\
11 & latin\_cube\_q & 5 & 0.60 & -28.824406 &
1.316896e+12 \\
22 & None & 7 & 1.00 & -54.077037 &
1.420605e+01 \\
17 & uniform & 7 & 0.90 & -18.697363 &
3.293595e+16 \\

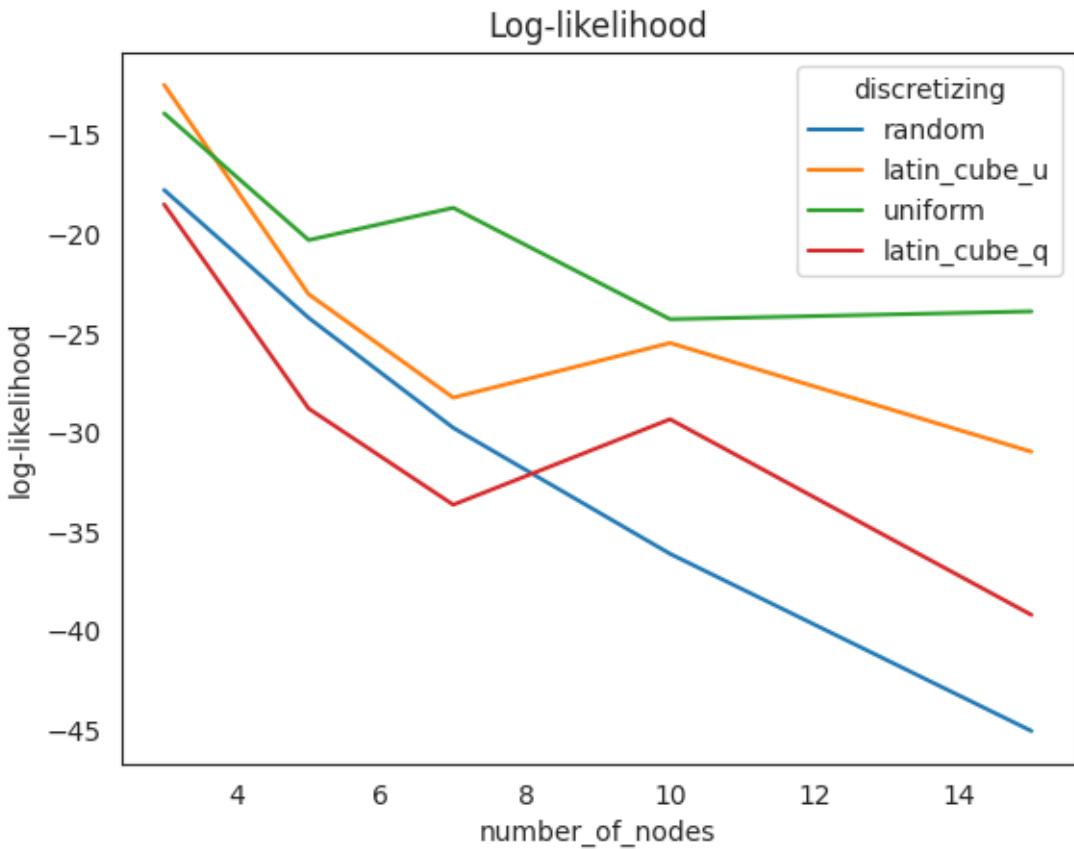
```

```

12 & latin\_cube\_q & 7 & 0.60 & -33.673223 &
1.032136e+10 \\
7 & latin\_cube\_u & 7 & 0.50 & -28.269755 &
2.293154e+12 \\
2 & random & 7 & 0.90 & -29.793548 &
4.996413e+11 \\
13 & latin\_cube\_q & 10 & 0.90 & -29.360866 &
7.701404e+11 \\
8 & latin\_cube\_u & 10 & 0.55 & -25.515272 &
3.603215e+13 \\
18 & uniform & 10 & 0.90 & -24.316958 &
1.194294e+14 \\
3 & random & 10 & 0.90 & -36.147299 &
8.694795e+08 \\
23 & None & 10 & 1.00 & -54.077037 &
1.420605e+01 \\
14 & latin\_cube\_q & 15 & 0.65 & -39.227169 &
3.996584e+07 \\
9 & latin\_cube\_u & 15 & 0.90 & -31.000382 &
1.494642e+11 \\
19 & uniform & 15 & 0.90 & -23.921797 &
1.773076e+14 \\
4 & random & 15 & 0.90 & -45.082951 &
1.144340e+05 \\
24 & None & 15 & 1.00 & -54.077037 &
1.420605e+01 \\
\bottomrule
\end{tabular}

```





Fitting a model with 26 free scalar parameters with only 20 data points will result in a degenerate solution.

Fitting a model with 35 free scalar parameters with only 20 data points will result in a degenerate solution.

Fitting a model with 50 free scalar parameters with only 20 data points will result in a degenerate solution.

Fitting a model with 26 free scalar parameters with only 20 data points will result in a degenerate solution.

Fitting a model with 35 free scalar parameters with only 20 data points will result in a degenerate solution.

Fitting a model with 50 free scalar parameters with only 20 data points will result in a degenerate solution.

Fitting a model with 23 free scalar parameters with only 20 data points will result in a degenerate solution.

Fitting a model with 35 free scalar parameters with only 20 data points will result in a degenerate solution.

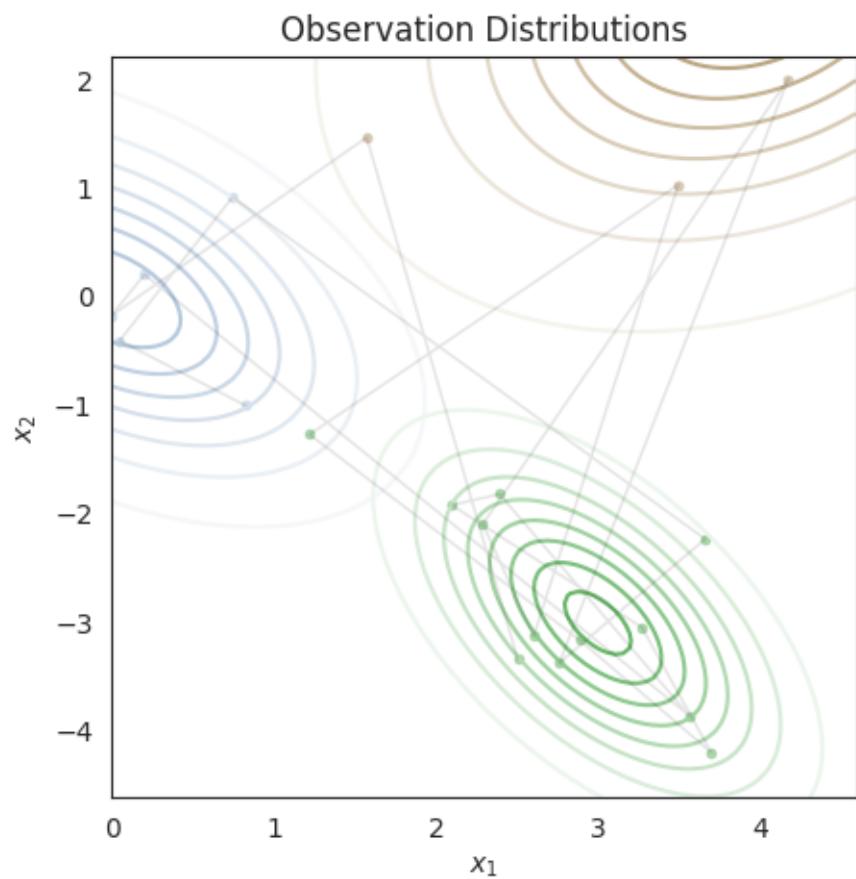
Fitting a model with 50 free scalar parameters with only 20 data points will result in a degenerate solution.

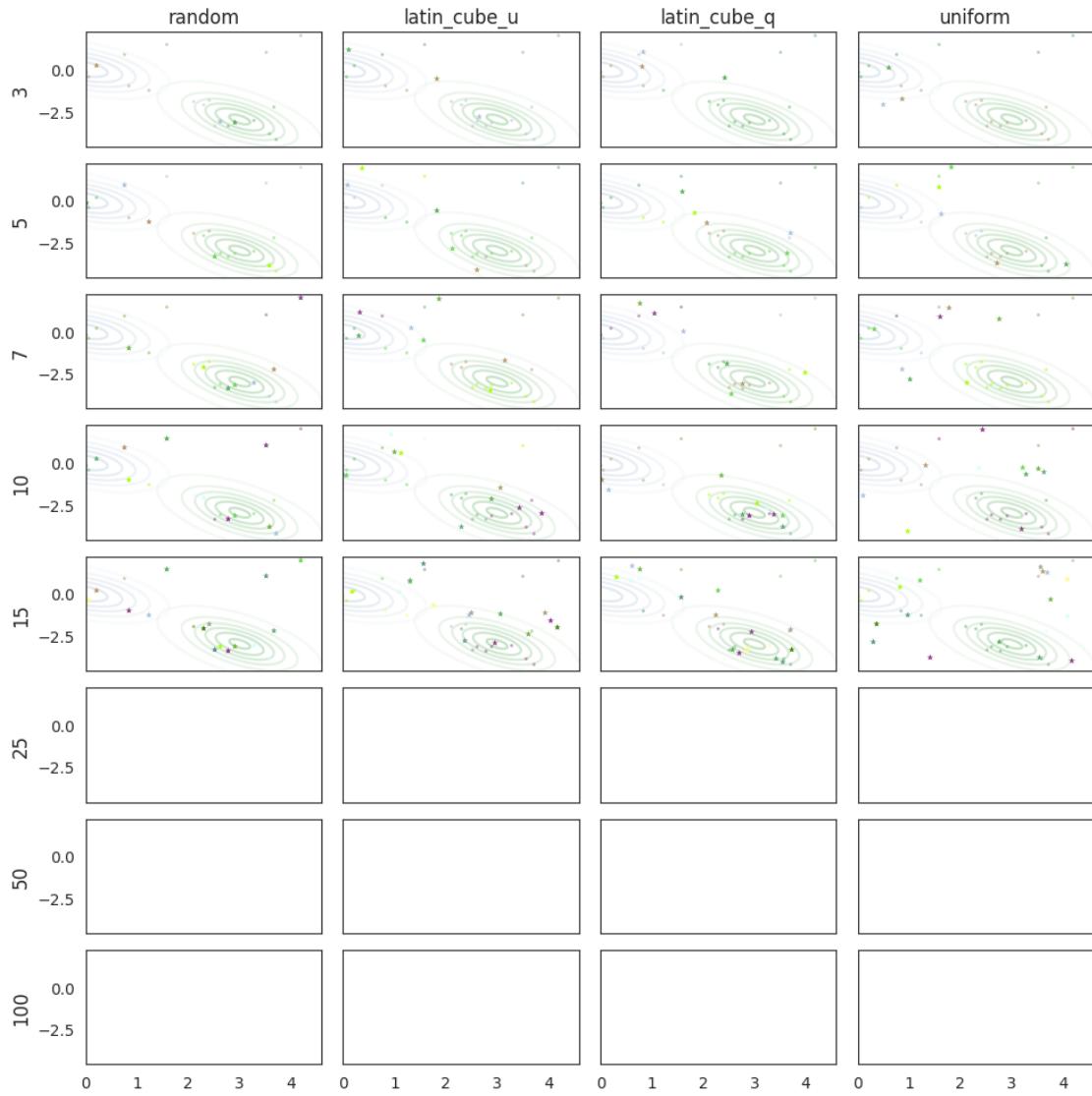
Fitting a model with 26 free scalar parameters with only 20 data points will result in a degenerate solution.

```
Fitting a model with 35 free scalar parameters with only 20 data points will
result in a degenerate solution.
Fitting a model with 50 free scalar parameters with only 20 data points will
result in a degenerate solution.

Recognized 1 instead of 3 states!

/home/kabalce/.local/lib/python3.10/site-
packages/sklearn/cluster/_kmeans.py:870: FutureWarning: The default value of
`n_init` will change from 10 to 'auto' in 1.4. Set the value of `n_init`
explicitly to suppress the warning
    warnings.warn(
/home/kabalce/.local/lib/python3.10/site-
packages/sklearn/cluster/_kmeans.py:870: FutureWarning: The default value of
`n_init` will change from 10 to 'auto' in 1.4. Set the value of `n_init`
explicitly to suppress the warning
    warnings.warn(
/home/kabalce/.local/lib/python3.10/site-
packages/sklearn/cluster/_kmeans.py:870: FutureWarning: The default value of
`n_init` will change from 10 to 'auto' in 1.4. Set the value of `n_init`
explicitly to suppress the warning
    warnings.warn(
/home/kabalce/.local/lib/python3.10/site-
packages/sklearn/cluster/_kmeans.py:870: FutureWarning: The default value of
`n_init` will change from 10 to 'auto' in 1.4. Set the value of `n_init`
explicitly to suppress the warning
    warnings.warn(
/home/kabalce/.local/lib/python3.10/site-
packages/sklearn/cluster/_kmeans.py:870: FutureWarning: The default value of
`n_init` will change from 10 to 'auto' in 1.4. Set the value of `n_init`
explicitly to suppress the warning
    warnings.warn(
```





	discretizing	number_of_nodes	accuracy	log-likelihood	likelihood-ratio
0	random	3	-1.00	-21.318041	1.413020e+18
20	None	3	0.30	-58.613771	8.970534e+01
5	latin_cube_u	3	0.55	-16.712338	1.413773e+20
15	uniform	3	0.50	-12.445624	1.007836e+22
10	latin_cube_q	3	0.85	-6.298051	4.712454e+24
1	random	5	0.55	-22.396156	4.807605e+17
21	None	5	0.70	-58.864933	6.978147e+01
6	latin_cube_u	5	0.75	-24.255382	7.489917e+16
16	uniform	5	0.15	-24.523332	5.729384e+16
11	latin_cube_q	5	0.65	-23.555936	1.507450e+17
22	None	7	0.70	-52.751678	3.152776e+04
17	uniform	7	0.70	-20.171292	4.448103e+18

```

12 latin_cube_q      7   0.75    -30.174809  2.012346e+14
7  latin_cube_u     7   0.65    -26.922399  5.202433e+15
2   random          7   0.15    -29.975228  2.456856e+14
13 latin_cube_q     10  0.55    -28.866958  7.442098e+14
8  latin_cube_u     10  0.35    -31.852868  3.757778e+13
18  uniform         10  0.65    -27.349530  3.393952e+15
3   random          10  0.25    -32.023251  3.169089e+13
23  None            10  0.60    -57.916892  1.800816e+02
14 latin_cube_q     15  0.50    -33.959858  4.569586e+12
9  latin_cube_u     15  0.60    -30.245811  1.874421e+14
19  uniform         15  0.40    -29.731891  3.133715e+14
4   random          15  0.30    -39.310372  2.168594e+10
24  None            15  0.90    -53.068534  2.296593e+04

/tmp/ipykernel_54422/2508230348.py:83: FutureWarning: In future versions
`DataFrame.to_latex` is expected to utilise the base implementation of
`Styler.to_latex` for formatting and rendering. The arguments signature may
therefore change. It is recommended instead to use `DataFrame.style.to_latex`
which also contains additional functionality.

print(summary.to_latex())

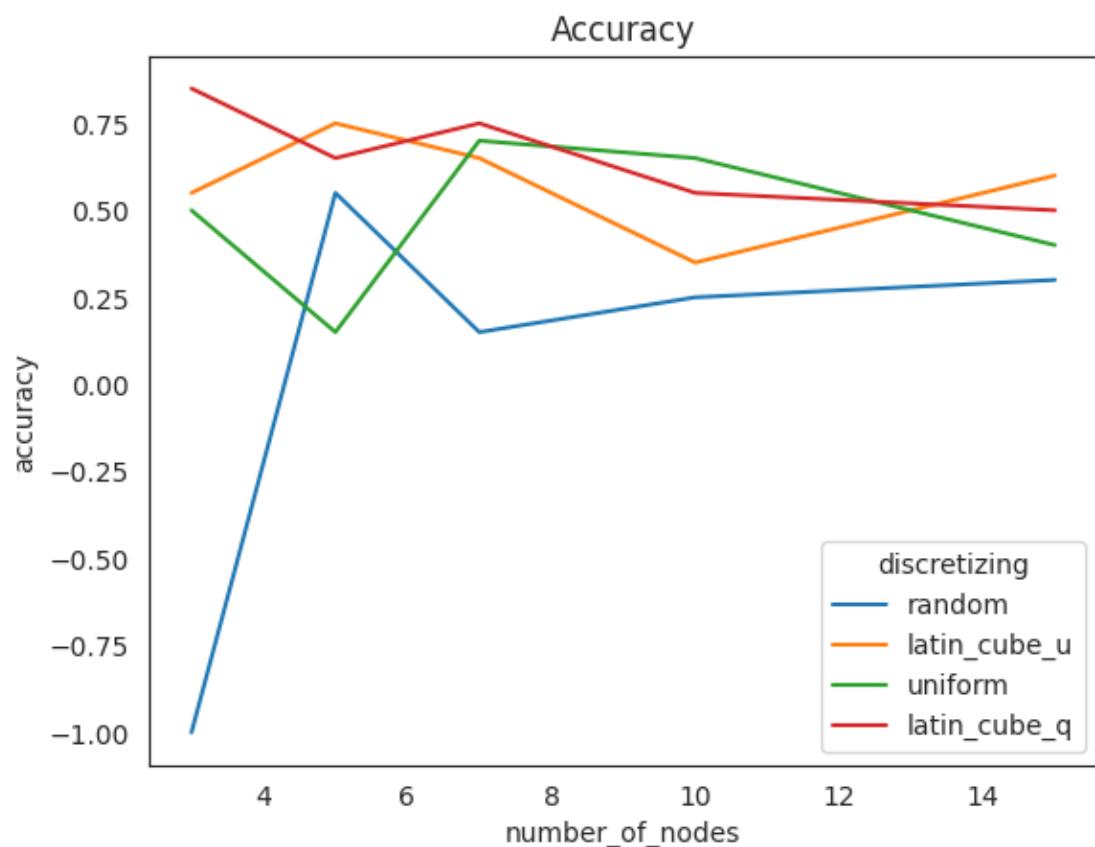
\begin{tabular}{llrrrr}
\toprule
{} & discretizing & number\_of\_nodes & accuracy & log-likelihood &
likelihood-ratio \\
\midrule
0 & random & 3 & -1.00 & -21.318041 &
1.413020e+18 \\
20 & None & 3 & 0.30 & -58.613771 &
8.970534e+01 \\
5 & latin\_cube\_u & 3 & 0.55 & -16.712338 &
1.413773e+20 \\
15 & uniform & 3 & 0.50 & -12.445624 &
1.007836e+22 \\
10 & latin\_cube\_q & 3 & 0.85 & -6.298051 &
4.712454e+24 \\
1 & random & 5 & 0.55 & -22.396156 &
4.807605e+17 \\
21 & None & 5 & 0.70 & -58.864933 &
6.978147e+01 \\
6 & latin\_cube\_u & 5 & 0.75 & -24.255382 &
7.489917e+16 \\
16 & uniform & 5 & 0.15 & -24.523332 &
5.729384e+16 \\
11 & latin\_cube\_q & 5 & 0.65 & -23.555936 &
1.507450e+17 \\
22 & None & 7 & 0.70 & -52.751678 &
3.152776e+04 \\
17 & uniform & 7 & 0.70 & -20.171292 &

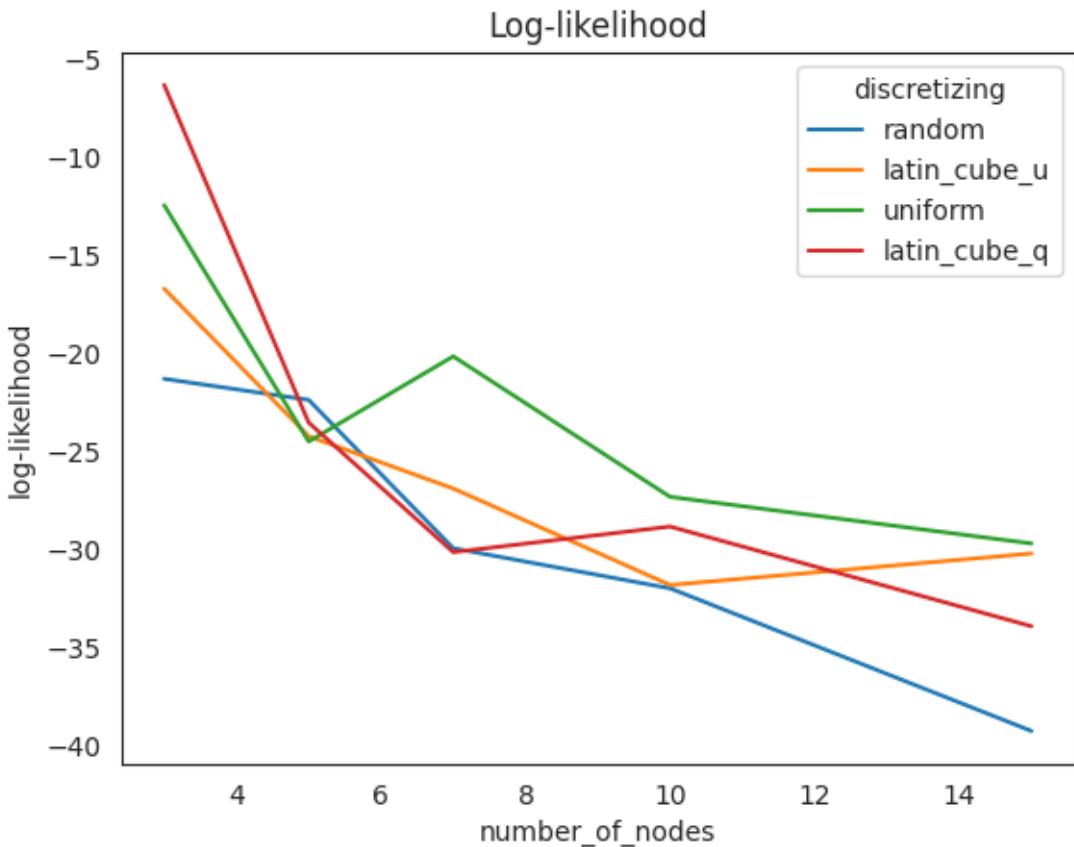
```

```

4.448103e+18 \\
12 & latin\_cube\_q & 7 & 0.75 & -30.174809 &
2.012346e+14 \\
7 & latin\_cube\_u & 7 & 0.65 & -26.922399 &
5.202433e+15 \\
2 & random & 7 & 0.15 & -29.975228 &
2.456856e+14 \\
13 & latin\_cube\_q & 10 & 0.55 & -28.866958 &
7.442098e+14 \\
8 & latin\_cube\_u & 10 & 0.35 & -31.852868 &
3.757778e+13 \\
18 & uniform & 10 & 0.65 & -27.349530 &
3.393952e+15 \\
3 & random & 10 & 0.25 & -32.023251 &
3.169089e+13 \\
23 & None & 10 & 0.60 & -57.916892 &
1.800816e+02 \\
14 & latin\_cube\_q & 15 & 0.50 & -33.959858 &
4.569586e+12 \\
9 & latin\_cube\_u & 15 & 0.60 & -30.245811 &
1.874421e+14 \\
19 & uniform & 15 & 0.40 & -29.731891 &
3.133715e+14 \\
4 & random & 15 & 0.30 & -39.310372 &
2.168594e+10 \\
24 & None & 15 & 0.90 & -53.068534 &
2.296593e+04 \\
\bottomrule
\end{tabular}

```





Fitting a model with 51 free scalar parameters with only 50 data points will result in a degenerate solution.

Fitting a model with 101 free scalar parameters with only 50 data points will result in a degenerate solution.

Fitting a model with 97 free scalar parameters with only 50 data points will result in a degenerate solution.

```
sample_size = 50
```

Fitting a model with 51 free scalar parameters with only 50 data points will result in a degenerate solution.

Fitting a model with 101 free scalar parameters with only 50 data points will result in a degenerate solution.

Fitting a model with 99 free scalar parameters with only 50 data points will result in a degenerate solution.

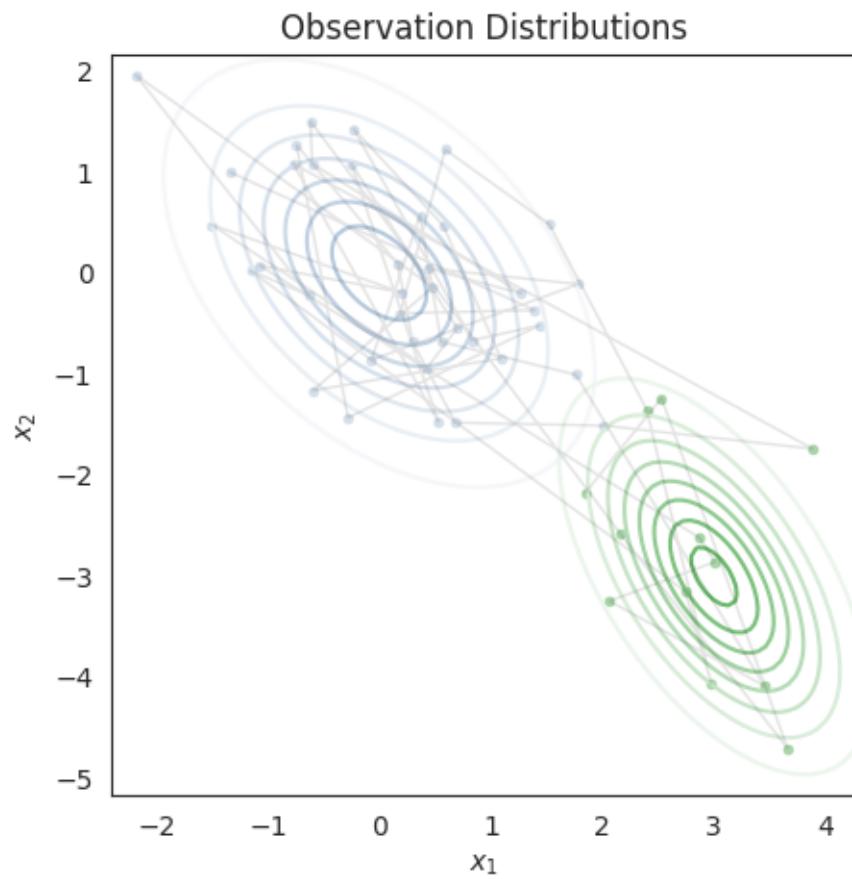
```
/home/kabalce/.local/lib/python3.10/site-
packages/sklearn/cluster/_kmeans.py:870: FutureWarning: The default value of
`n_init` will change from 10 to 'auto' in 1.4. Set the value of `n_init`
explicitly to suppress the warning
```

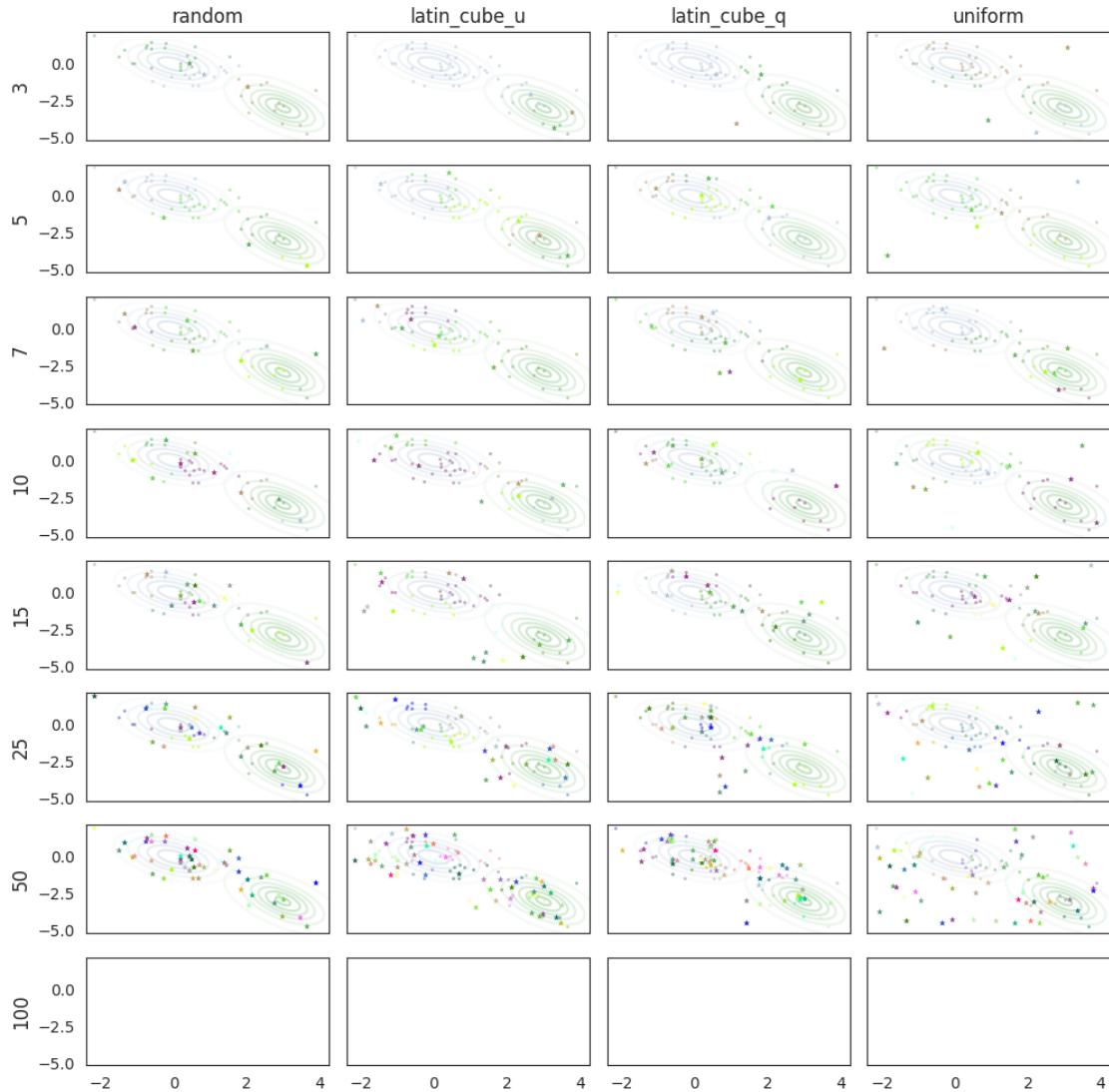
```
    warnings.warn(
```

```
/home/kabalce/.local/lib/python3.10/site-
```

```
packages/sklearn/cluster/_kmeans.py:870: FutureWarning: The default value of
`n_init` will change from 10 to 'auto' in 1.4. Set the value of `n_init`
explicitly to suppress the warning
    warnings.warn(
Recognized 1 instead of 2 states!

/home/kabalce/.local/lib/python3.10/site-
packages/sklearn/cluster/_kmeans.py:870: FutureWarning: The default value of
`n_init` will change from 10 to 'auto' in 1.4. Set the value of `n_init`
explicitly to suppress the warning
    warnings.warn(
/home/kabalce/.local/lib/python3.10/site-
packages/sklearn/cluster/_kmeans.py:870: FutureWarning: The default value of
`n_init` will change from 10 to 'auto' in 1.4. Set the value of `n_init`
explicitly to suppress the warning
    warnings.warn(
/home/kabalce/.local/lib/python3.10/site-
packages/sklearn/cluster/_kmeans.py:870: FutureWarning: The default value of
`n_init` will change from 10 to 'auto' in 1.4. Set the value of `n_init`
explicitly to suppress the warning
    warnings.warn(
/home/kabalce/.local/lib/python3.10/site-
packages/sklearn/cluster/_kmeans.py:870: FutureWarning: The default value of
`n_init` will change from 10 to 'auto' in 1.4. Set the value of `n_init`
explicitly to suppress the warning
    warnings.warn(
/home/kabalce/.local/lib/python3.10/site-
packages/sklearn/cluster/_kmeans.py:870: FutureWarning: The default value of
`n_init` will change from 10 to 'auto' in 1.4. Set the value of `n_init`
explicitly to suppress the warning
    warnings.warn(
/home/kabalce/.local/lib/python3.10/site-
packages/sklearn/cluster/_kmeans.py:870: FutureWarning: The default value of
`n_init` will change from 10 to 'auto' in 1.4. Set the value of `n_init`
explicitly to suppress the warning
    warnings.warn(
```





	discretizing	number_of_nodes	accuracy	log-likelihood	likelihood-ratio
0	random	3	0.96	-43.065071	2.863734e+45
14	latin_cube_q	3	0.94	-38.422619	2.972515e+47
21	uniform	3	0.62	-43.734166	1.466723e+45
7	latin_cube_u	3	0.78	-16.165127	1.378573e+57
28	None	3	0.98	-148.209396	6.213444e-01
22	uniform	5	-1.00	-47.376320	3.842209e+43
15	latin_cube_q	5	0.92	-64.804632	1.036481e+36
8	latin_cube_u	5	0.98	-69.495502	9.513541e+33
1	random	5	0.50	-67.705601	5.697529e+34
29	None	5	0.98	-148.209380	6.213542e-01
30	None	7	0.98	-148.209383	6.213524e-01
9	latin_cube_u	7	0.50	-75.845488	1.661798e+31
23	uniform	7	0.58	-71.202538	1.725783e+33

2	random	7	0.66	-88.517243	5.215608e+25
16	latin_cube_q	7	0.84	-69.994853	5.774000e+33
24	uniform	10	0.98	-82.761885	1.647500e+28
31	None	10	0.98	-148.209411	6.213350e-01
17	latin_cube_q	10	0.92	-99.217406	1.175664e+21
3	random	10	0.58	-107.761580	2.288735e+17
10	latin_cube_u	10	0.92	-85.010612	1.738664e+27
18	latin_cube_q	15	0.54	-109.339265	4.725151e+16
32	None	15	0.98	-148.209376	6.213567e-01
4	random	15	0.96	-114.548049	2.583864e+14
11	latin_cube_u	15	0.84	-89.469352	2.012841e+25
25	uniform	15	0.86	-101.824457	8.670726e+19
5	random	25	0.50	-136.284052	9.385196e+04
26	uniform	25	0.56	-103.710749	1.314769e+19
12	latin_cube_u	25	0.72	-111.012472	8.866479e+15
19	latin_cube_q	25	0.54	-130.834266	2.184014e+07
33	None	25	0.98	-148.209391	6.213478e-01
27	uniform	50	0.76	-121.055775	3.854786e+11
6	random	50	0.58	-174.041446	3.755049e-12
13	latin_cube_u	50	0.50	-132.497099	4.140913e+06
20	latin_cube_q	50	0.88	-150.771129	4.794972e-02
34	None	50	0.98	-148.209384	6.213518e-01

/tmp/ipykernel_54422/2508230348.py:83: FutureWarning: In future versions
`DataFrame.to_latex` is expected to utilise the base implementation of
`Styler.to_latex` for formatting and rendering. The arguments signature may
therefore change. It is recommended instead to use `DataFrame.style.to_latex`
which also contains additional functionality.

```

print(summary.to_latex())

\begin{tabular}{llrrrr}
\toprule
{} & discretizing & number\_of\_nodes & accuracy & log-likelihood &
likelihood-ratio \\
\midrule
0 & random & 3 & 0.96 & -43.065071 &
2.863734e+45 \\
14 & latin\_cube\_q & 3 & 0.94 & -38.422619 &
2.972515e+47 \\
21 & uniform & 3 & 0.62 & -43.734166 &
1.466723e+45 \\
7 & latin\_cube\_u & 3 & 0.78 & -16.165127 &
1.378573e+57 \\
28 & None & 3 & 0.98 & -148.209396 &
6.213444e-01 \\
22 & uniform & 5 & -1.00 & -47.376320 &
3.842209e+43 \\
15 & latin\_cube\_q & 5 & 0.92 & -64.804632 &
1.036481e+36 \\

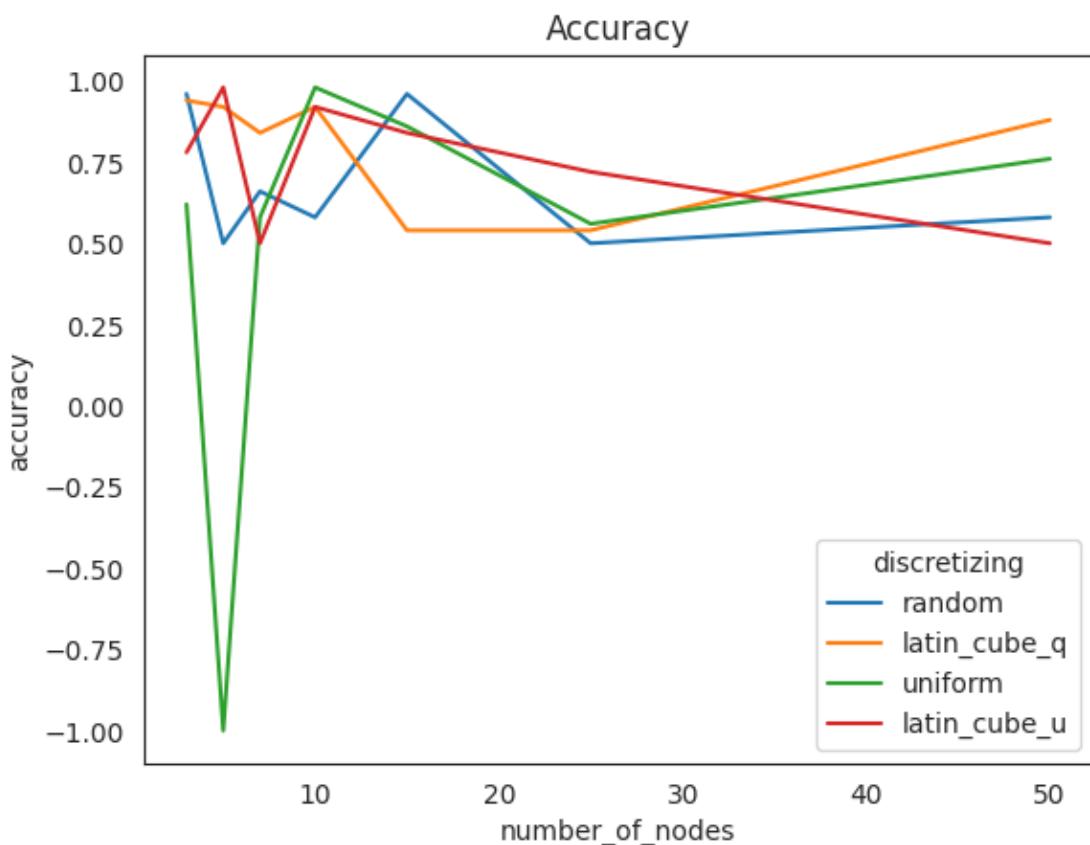
```

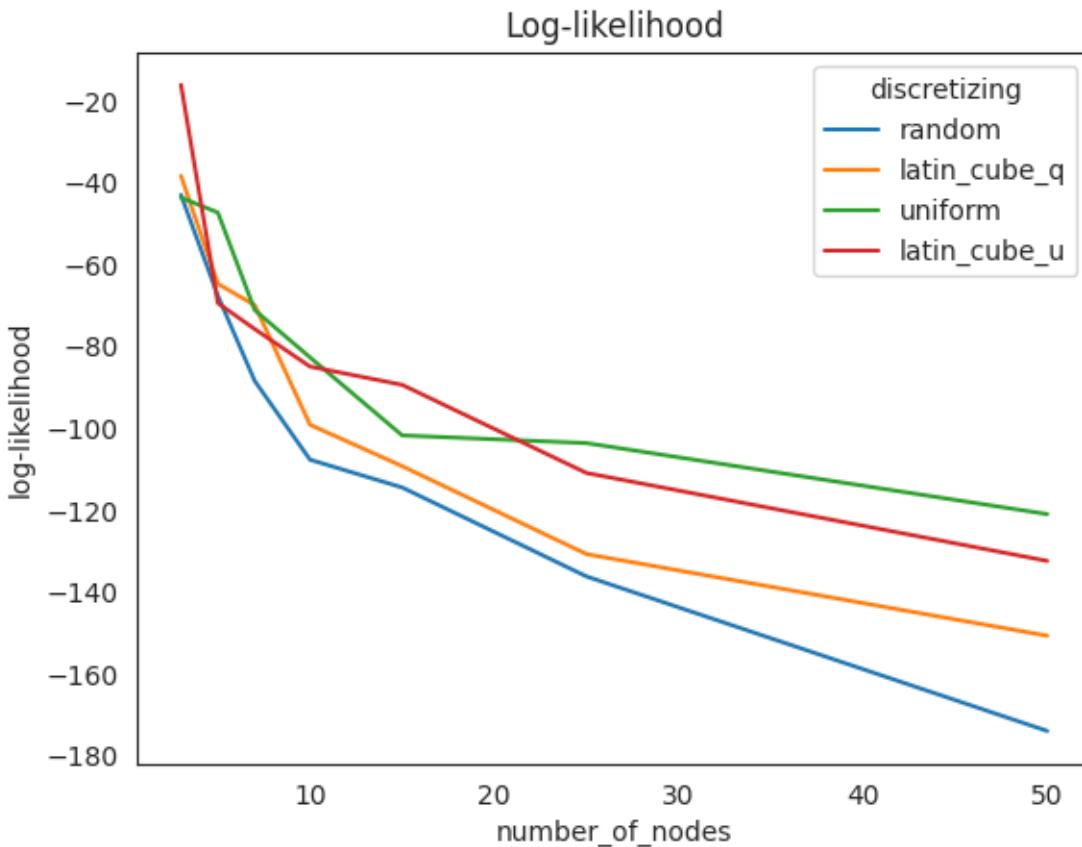
8 & latin_cube_u &	5 &	0.98 &	-69.495502 &
9.513541e+33 \\			
1 & random &	5 &	0.50 &	-67.705601 &
5.697529e+34 \\			
29 & None &	5 &	0.98 &	-148.209380 &
6.213542e-01 \\			
30 & None &	7 &	0.98 &	-148.209383 &
6.213524e-01 \\			
9 & latin_cube_u &	7 &	0.50 &	-75.845488 &
1.661798e+31 \\			
23 & uniform &	7 &	0.58 &	-71.202538 &
1.725783e+33 \\			
2 & random &	7 &	0.66 &	-88.517243 &
5.215608e+25 \\			
16 & latin_cube_q &	7 &	0.84 &	-69.994853 &
5.774000e+33 \\			
24 & uniform &	10 &	0.98 &	-82.761885 &
1.647500e+28 \\			
31 & None &	10 &	0.98 &	-148.209411 &
6.213350e-01 \\			
17 & latin_cube_q &	10 &	0.92 &	-99.217406 &
1.175664e+21 \\			
3 & random &	10 &	0.58 &	-107.761580 &
2.288735e+17 \\			
10 & latin_cube_u &	10 &	0.92 &	-85.010612 &
1.738664e+27 \\			
18 & latin_cube_q &	15 &	0.54 &	-109.339265 &
4.725151e+16 \\			
32 & None &	15 &	0.98 &	-148.209376 &
6.213567e-01 \\			
4 & random &	15 &	0.96 &	-114.548049 &
2.583864e+14 \\			
11 & latin_cube_u &	15 &	0.84 &	-89.469352 &
2.012841e+25 \\			
25 & uniform &	15 &	0.86 &	-101.824457 &
8.670726e+19 \\			
5 & random &	25 &	0.50 &	-136.284052 &
9.385196e+04 \\			
26 & uniform &	25 &	0.56 &	-103.710749 &
1.314769e+19 \\			
12 & latin_cube_u &	25 &	0.72 &	-111.012472 &
8.866479e+15 \\			
19 & latin_cube_q &	25 &	0.54 &	-130.834266 &
2.184014e+07 \\			
33 & None &	25 &	0.98 &	-148.209391 &
6.213478e-01 \\			
27 & uniform &	50 &	0.76 &	-121.055775 &
3.854786e+11 \\			

```

6 &           random &
3.755049e-12 \\
13 & latin\_cube\_u &
4.140913e+06 \\
20 & latin\_cube\_q &
4.794972e-02 \\
34 &           None &
6.213518e-01 \\
\bottomrule
\end{tabular}

```





Fitting a model with 80 free scalar parameters with only 50 data points will result in a degenerate solution.

Fitting a model with 155 free scalar parameters with only 50 data points will result in a degenerate solution.

Fitting a model with 80 free scalar parameters with only 50 data points will result in a degenerate solution.

Fitting a model with 146 free scalar parameters with only 50 data points will result in a degenerate solution.

Fitting a model with 80 free scalar parameters with only 50 data points will result in a degenerate solution.

Fitting a model with 155 free scalar parameters with only 50 data points will result in a degenerate solution.

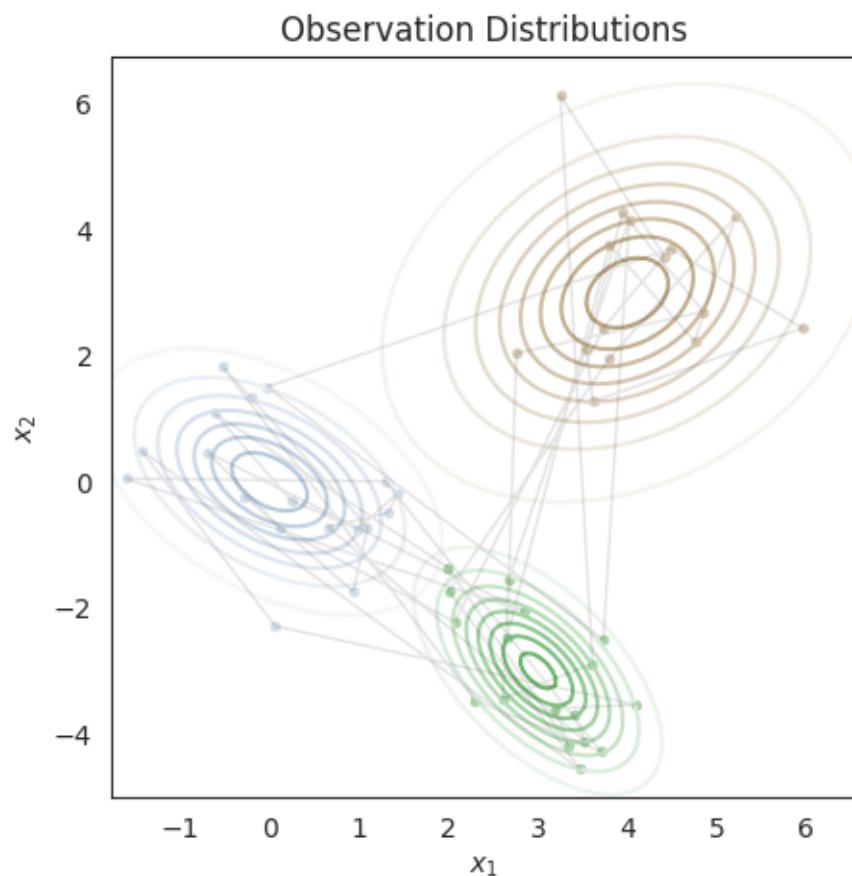
Fitting a model with 74 free scalar parameters with only 50 data points will result in a degenerate solution.

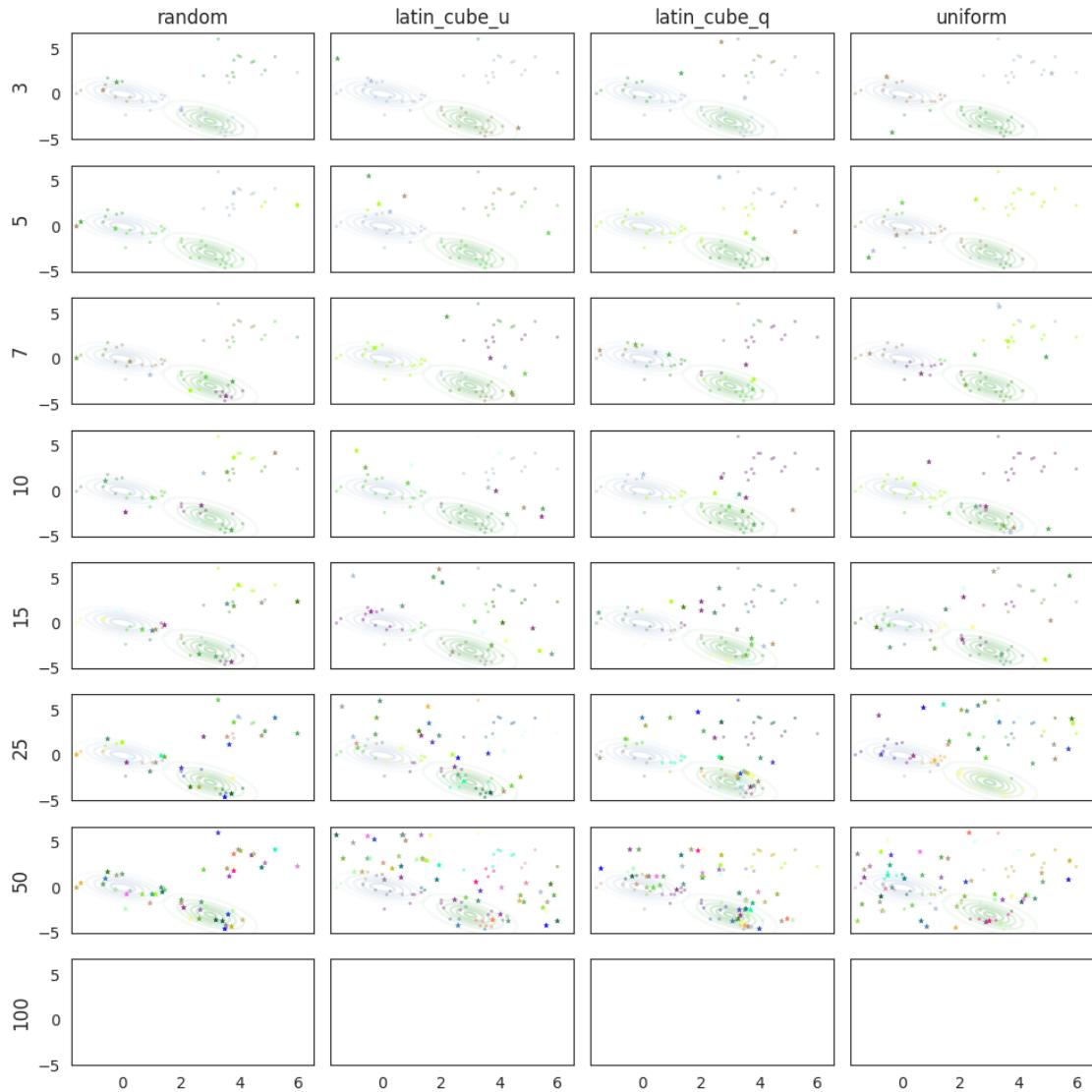
Fitting a model with 152 free scalar parameters with only 50 data points will result in a degenerate solution.

```
/home/kabalce/.local/lib/python3.10/site-
packages/sklearn/cluster/_kmeans.py:870: FutureWarning: The default value of
`n_init` will change from 10 to 'auto' in 1.4. Set the value of `n_init`
explicitly to suppress the warning
```

```
warnings.warn(
/home/kabalce/.local/lib/python3.10/site-
packages/sklearn/cluster/_kmeans.py:870: FutureWarning: The default value of
`n_init` will change from 10 to 'auto' in 1.4. Set the value of `n_init`
explicitly to suppress the warning
    warnings.warn(
Recognized 2 instead of 3 states!

/home/kabalce/.local/lib/python3.10/site-
packages/sklearn/cluster/_kmeans.py:870: FutureWarning: The default value of
`n_init` will change from 10 to 'auto' in 1.4. Set the value of `n_init`
explicitly to suppress the warning
    warnings.warn(
/home/kabalce/.local/lib/python3.10/site-
packages/sklearn/cluster/_kmeans.py:870: FutureWarning: The default value of
`n_init` will change from 10 to 'auto' in 1.4. Set the value of `n_init`
explicitly to suppress the warning
    warnings.warn(
/home/kabalce/.local/lib/python3.10/site-
packages/sklearn/cluster/_kmeans.py:870: FutureWarning: The default value of
`n_init` will change from 10 to 'auto' in 1.4. Set the value of `n_init`
explicitly to suppress the warning
    warnings.warn(
/home/kabalce/.local/lib/python3.10/site-
packages/sklearn/cluster/_kmeans.py:870: FutureWarning: The default value of
`n_init` will change from 10 to 'auto' in 1.4. Set the value of `n_init`
explicitly to suppress the warning
    warnings.warn(
/home/kabalce/.local/lib/python3.10/site-
packages/sklearn/cluster/_kmeans.py:870: FutureWarning: The default value of
`n_init` will change from 10 to 'auto' in 1.4. Set the value of `n_init`
explicitly to suppress the warning
    warnings.warn(
/home/kabalce/.local/lib/python3.10/site-
```





	discretizing	number_of_nodes	accuracy	log-likelihood	likelihood-ratio
0	random	3	0.34	-44.709748	1.982924e+56
14	latin_cube_q	3	0.54	-43.695599	5.466951e+56
21	uniform	3	0.84	-40.341293	1.564956e+58
7	latin_cube_u	3	0.68	-33.057795	2.278689e+61
28	None	3	0.98	-171.025298	2.748906e+01
22	uniform	5	-1.00	-63.451926	1.437744e+48
15	latin_cube_q	5	0.78	-56.282642	1.867506e+51
8	latin_cube_u	5	0.58	-46.100324	4.936130e+55
1	random	5	0.60	-45.247299	1.158379e+56
29	None	5	0.02	-171.025303	2.748891e+01
30	None	7	0.00	-171.025316	2.748856e+01
9	latin_cube_u	7	0.62	-66.932729	4.425763e+46

23	uniform	7	0.14	-63.728834	1.089988e+48
2	random	7	0.48	-73.544052	5.952874e+43
16	latin_cube_q	7	0.40	-82.909050	5.099879e+39
24	uniform	10	0.60	-77.386543	1.276304e+42
31	None	10	0.98	-171.025284	2.748944e+01
17	latin_cube_q	10	0.72	-63.124447	1.994821e+48
3	random	10	0.24	-87.656168	4.424997e+37
10	latin_cube_u	10	0.66	-56.833444	1.076594e+51
18	latin_cube_q	15	0.74	-89.826495	5.050703e+36
32	None	15	0.02	-171.025437	2.748523e+01
4	random	15	0.56	-104.287417	2.648823e+30
11	latin_cube_u	15	0.52	-82.836092	5.485864e+39
25	uniform	15	0.68	-100.865630	8.111780e+31
5	random	25	0.26	-128.809525	5.932491e+19
26	uniform	25	0.14	-90.426496	2.771883e+36
12	latin_cube_u	25	0.40	-108.514498	3.865942e+28
19	latin_cube_q	25	0.28	-96.862755	4.441637e+33
33	None	25	0.98	-171.025408	2.748602e+01
27	uniform	50	0.38	-129.130863	4.302113e+19
6	random	50	0.36	-163.852540	3.583018e+04
13	latin_cube_u	50	0.70	-102.244439	2.043184e+31
20	latin_cube_q	50	0.48	-120.436896	2.566987e+23
34	None	50	0.00	-171.025327	2.748825e+01

/tmp/ipykernel_54422/2508230348.py:83: FutureWarning: In future versions
`DataFrame.to_latex` is expected to utilise the base implementation of
`Styler.to_latex` for formatting and rendering. The arguments signature may
therefore change. It is recommended instead to use `DataFrame.style.to_latex`
which also contains additional functionality.

```

print(summary.to_latex())

\begin{tabular}{llrrrr}
\toprule
{} & discretizing & number\_of\_nodes & accuracy & log-likelihood &
likelihood-ratio \\
\midrule
0 & random & 3 & 0.34 & -44.709748 &
1.982924e+56 \\
14 & latin\_cube\_q & 3 & 0.54 & -43.695599 &
5.466951e+56 \\
21 & uniform & 3 & 0.84 & -40.341293 &
1.564956e+58 \\
7 & latin\_cube\_u & 3 & 0.68 & -33.057795 &
2.278689e+61 \\
28 & None & 3 & 0.98 & -171.025298 &
2.748906e+01 \\
22 & uniform & 5 & -1.00 & -63.451926 &
1.437744e+48 \\
15 & latin\_cube\_q & 5 & 0.78 & -56.282642 &

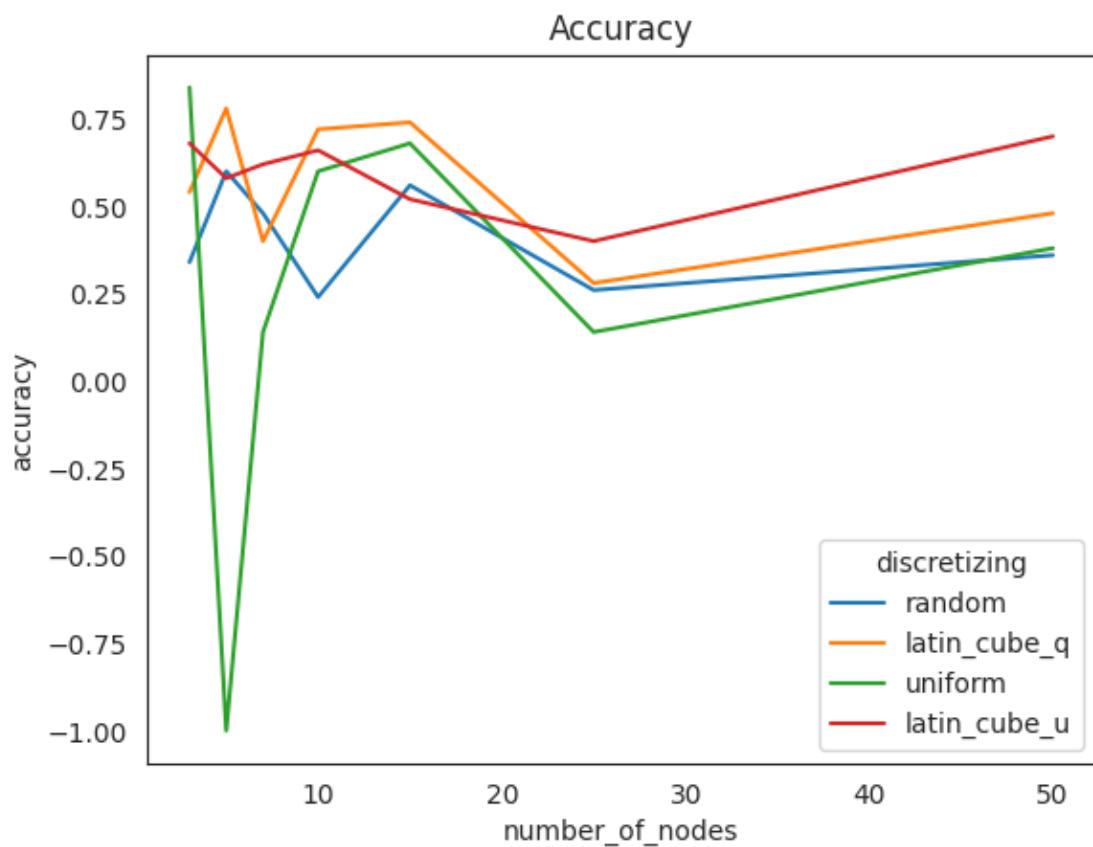
```

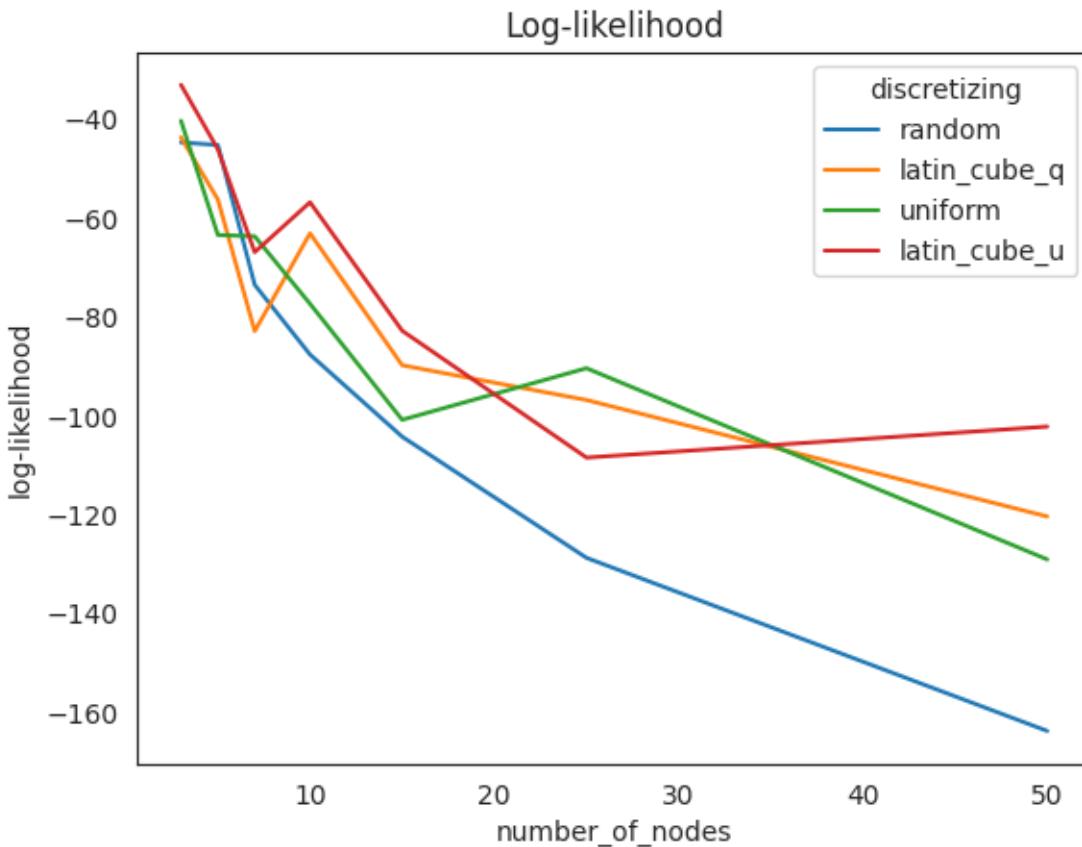
1.867506e+51 \\			
8 & latin_cube_u &	5 &	0.58 &	-46.100324 &
4.936130e+55 \\			
1 & random &	5 &	0.60 &	-45.247299 &
1.158379e+56 \\			
29 & None &	5 &	0.02 &	-171.025303 &
2.748891e+01 \\			
30 & None &	7 &	0.00 &	-171.025316 &
2.748856e+01 \\			
9 & latin_cube_u &	7 &	0.62 &	-66.932729 &
4.425763e+46 \\			
23 & uniform &	7 &	0.14 &	-63.728834 &
1.089988e+48 \\			
2 & random &	7 &	0.48 &	-73.544052 &
5.952874e+43 \\			
16 & latin_cube_q &	7 &	0.40 &	-82.909050 &
5.099879e+39 \\			
24 & uniform &	10 &	0.60 &	-77.386543 &
1.276304e+42 \\			
31 & None &	10 &	0.98 &	-171.025284 &
2.748944e+01 \\			
17 & latin_cube_q &	10 &	0.72 &	-63.124447 &
1.994821e+48 \\			
3 & random &	10 &	0.24 &	-87.656168 &
4.424997e+37 \\			
10 & latin_cube_u &	10 &	0.66 &	-56.833444 &
1.076594e+51 \\			
18 & latin_cube_q &	15 &	0.74 &	-89.826495 &
5.050703e+36 \\			
32 & None &	15 &	0.02 &	-171.025437 &
2.748523e+01 \\			
4 & random &	15 &	0.56 &	-104.287417 &
2.648823e+30 \\			
11 & latin_cube_u &	15 &	0.52 &	-82.836092 &
5.485864e+39 \\			
25 & uniform &	15 &	0.68 &	-100.865630 &
8.111780e+31 \\			
5 & random &	25 &	0.26 &	-128.809525 &
5.932491e+19 \\			
26 & uniform &	25 &	0.14 &	-90.426496 &
2.771883e+36 \\			
12 & latin_cube_u &	25 &	0.40 &	-108.514498 &
3.865942e+28 \\			
19 & latin_cube_q &	25 &	0.28 &	-96.862755 &
4.441637e+33 \\			
33 & None &	25 &	0.98 &	-171.025408 &
2.748602e+01 \\			
27 & uniform &	50 &	0.38 &	-129.130863 &

```

4.302113e+19 \\
6 &      random &      50 &      0.36 &      -163.852540 &
3.583018e+04 \\
13 &      latin\_cube\_u &      50 &      0.70 &      -102.244439 &
2.043184e+31 \\
20 &      latin\_cube\_q &      50 &      0.48 &      -120.436896 &
2.566987e+23 \\
34 &      None &      50 &      0.00 &      -171.025327 &
2.748825e+01 \\
\bottomrule
\end{tabular}

```





Fitting a model with 101 free scalar parameters with only 100 data points will result in a degenerate solution.

Fitting a model with 201 free scalar parameters with only 100 data points will result in a degenerate solution.

```
sample_size = 100
Recognized 1 instead of 2 states!
Recognized 1 instead of 2 states!
```

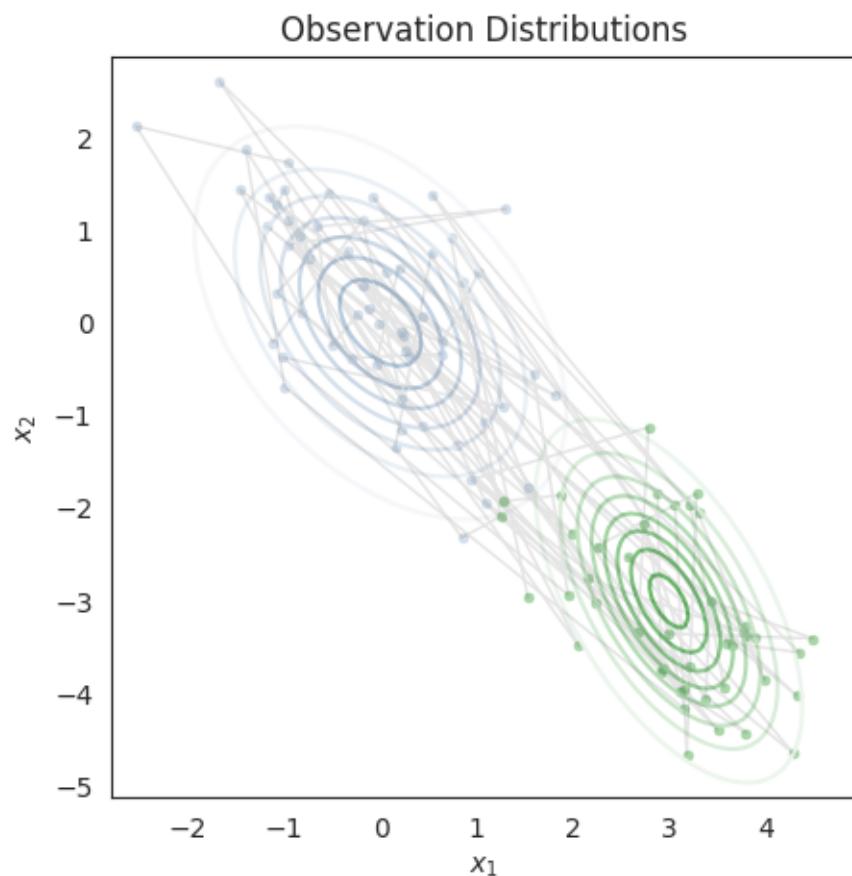
Fitting a model with 191 free scalar parameters with only 100 data points will result in a degenerate solution.

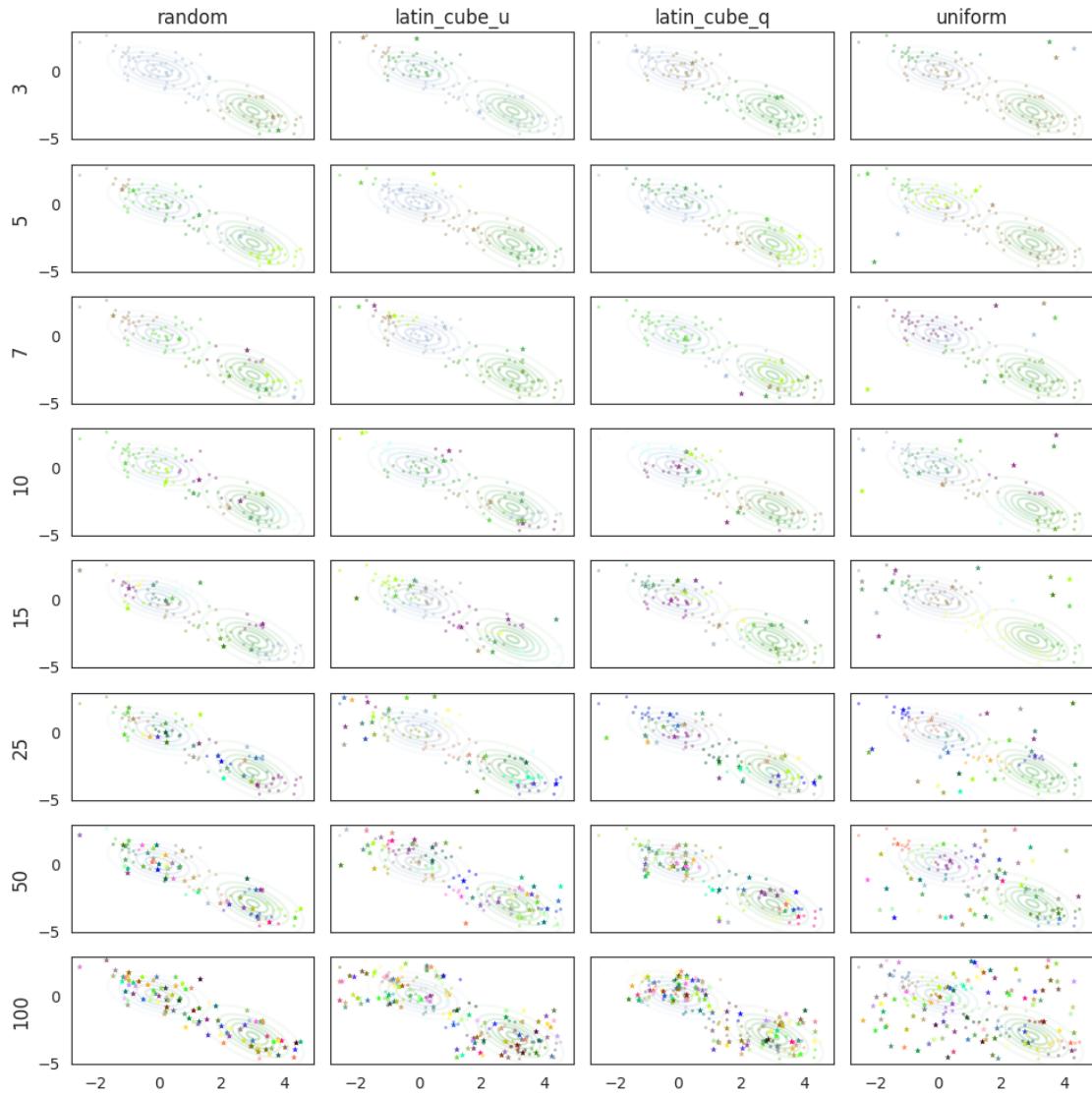
Fitting a model with 101 free scalar parameters with only 100 data points will result in a degenerate solution.

Fitting a model with 199 free scalar parameters with only 100 data points will result in a degenerate solution.

Fitting a model with 185 free scalar parameters with only 100 data points will result in a degenerate solution.

```
/home/kabalce/.local/lib/python3.10/site-
packages/sklearn/cluster/_kmeans.py:870: FutureWarning: The default value of
`n_init` will change from 10 to 'auto' in 1.4. Set the value of `n_init`
explicitly to suppress the warning
```



	discretizing	number_of_nodes	accuracy	log-likelihood	likelihood-ratio
0	random	3	-1.00	-83.842206	6.183897e+98
24	uniform	3	-1.00	-50.034046	2.978238e+113
32	None	3	0.96	-328.338920	4.052177e-08
8	latin_cube_u	3	0.57	-93.767105	3.026447e+94
16	latin_cube_q	3	0.57	-101.923693	8.681050e+90
33	None	5	0.96	-328.339848	4.048419e-08
17	latin_cube_q	5	0.86	-149.922809	1.238285e+70
9	latin_cube_u	5	-1.00	-128.137169	3.582648e+79
25	uniform	5	-1.00	-107.338213	3.864346e+88
1	random	5	0.96	-149.168526	2.632703e+70
34	None	7	0.96	-328.346897	4.019981e-08
2	random	7	0.53	-170.344387	1.674335e+61

10	latin_cube_u	7	0.83	-141.581027	5.195303e+73
26	uniform	7	0.57	-112.102580	3.295623e+86
18	latin_cube_q	7	0.86	-128.795174	1.855394e+79
27	uniform	10	0.50	-183.007479	5.300678e+55
35	None	10	0.98	-328.413837	3.759693e-08
19	latin_cube_q	10	0.83	-180.932723	4.220718e+56
3	random	10	0.78	-188.303752	2.655762e+53
11	latin_cube_u	10	0.84	-181.665201	2.028967e+56
28	uniform	15	0.55	-163.150957	2.227968e+64
20	latin_cube_q	15	0.53	-223.852209	9.675878e+37
36	None	15	0.96	-328.339599	4.049425e-08
12	latin_cube_u	15	0.51	-212.631548	7.223734e+42
4	random	15	0.54	-238.749126	3.281267e+31
37	None	25	0.96	-328.339221	4.050956e-08
21	latin_cube_q	25	0.57	-258.719065	6.969589e+22
13	latin_cube_u	25	0.81	-244.285910	1.292549e+29
5	random	25	0.50	-274.530444	9.471367e+15
29	uniform	25	0.54	-221.037613	1.614559e+39
6	random	50	0.53	-345.001679	2.350413e-15
14	latin_cube_u	50	0.51	-309.285140	7.632027e+00
30	uniform	50	0.55	-270.179103	7.348114e+17
22	latin_cube_q	50	0.55	-320.597792	9.324332e-05
38	None	50	0.98	-328.412939	3.763070e-08
15	latin_cube_u	100	0.63	-353.408114	5.251405e-19
7	random	100	0.57	-454.916866	4.321006e-63
23	latin_cube_q	100	0.51	-351.518581	3.474476e-18
31	uniform	100	0.74	-305.815175	2.452600e+02
39	None	100	0.96	-328.338575	4.053576e-08

/tmp/ipykernel_54422/2508230348.py:83: FutureWarning: In future versions
`DataFrame.to_latex` is expected to utilise the base implementation of
`Styler.to_latex` for formatting and rendering. The arguments signature may
therefore change. It is recommended instead to use `DataFrame.style.to_latex`
which also contains additional functionality.

```

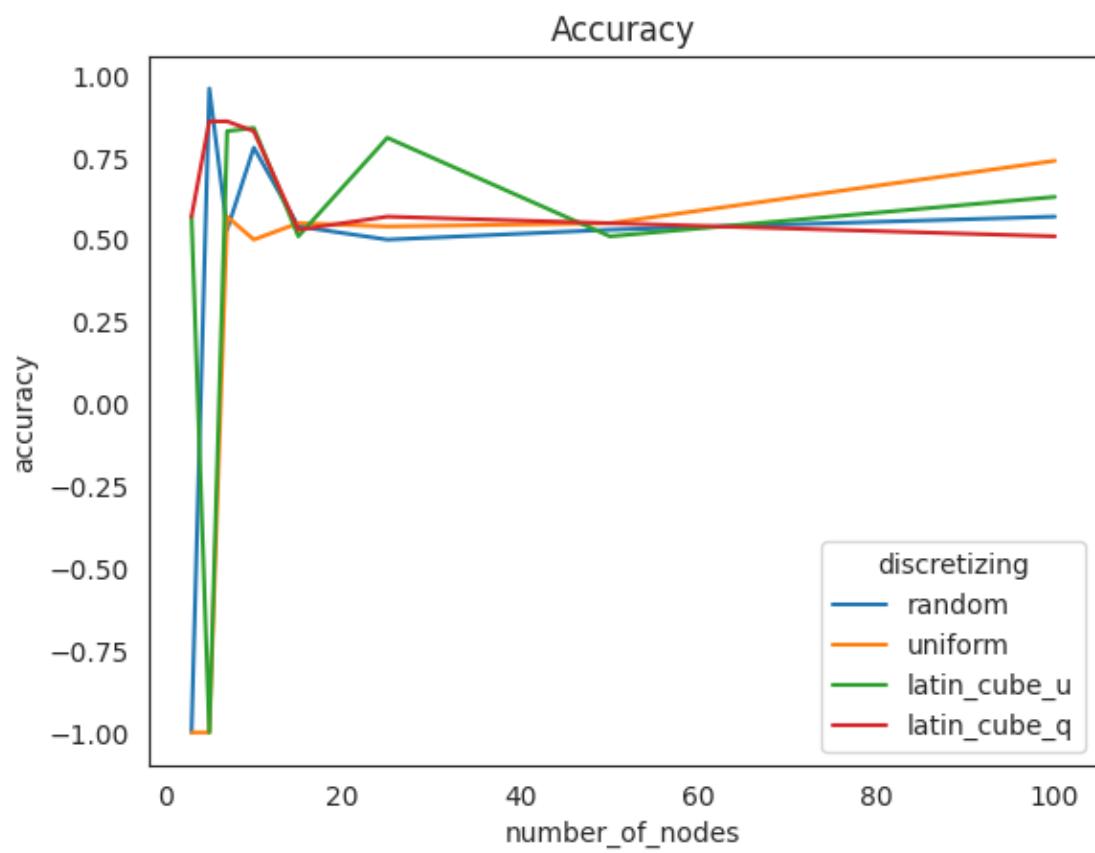
print(summary.to_latex())

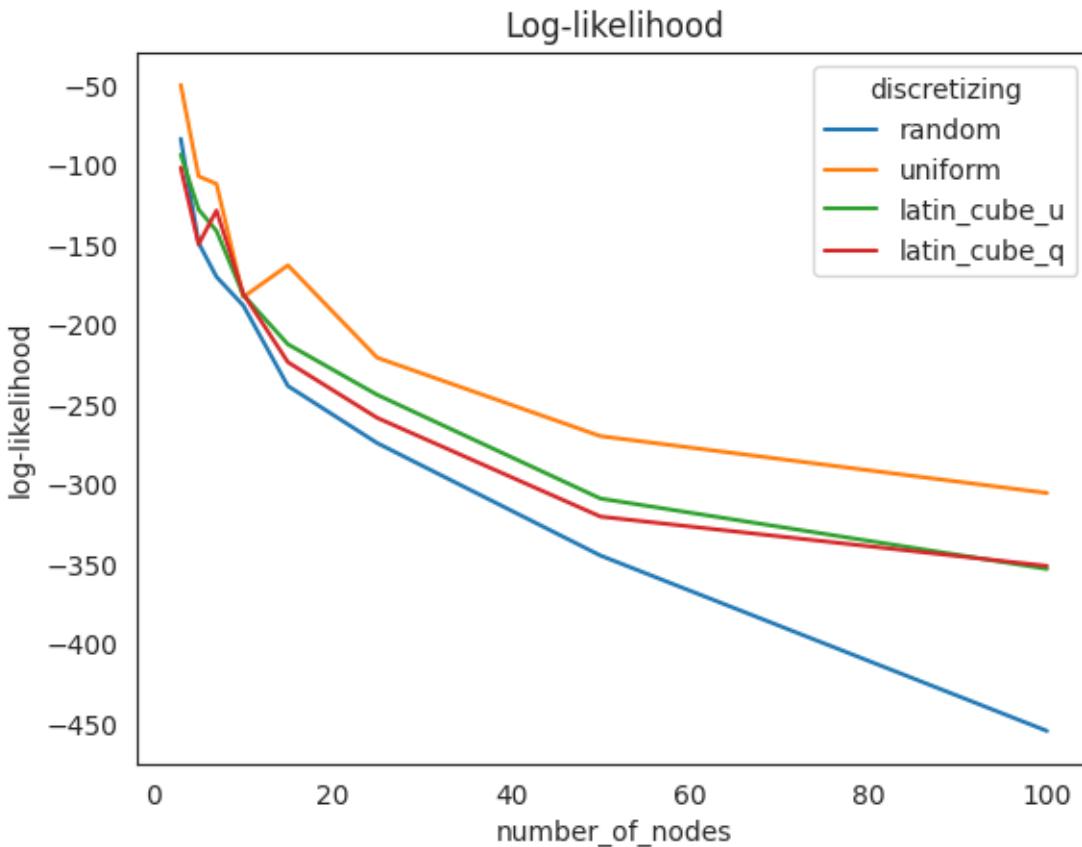
\begin{tabular}{llrrrr}
\toprule
{} & discretizing & number\_of\_nodes & accuracy & log-likelihood & \\
& likelihood-ratio \\
\midrule
0 & random & 3 & -1.00 & -83.842206 & \\
6.183897e+98 \\
24 & uniform & 3 & -1.00 & -50.034046 & \\
2.978238e+113 \\
32 & None & 3 & 0.96 & -328.338920 & \\
4.052177e-08 \\
8 & latin\_cube\_u & 3 & 0.57 & -93.767105 & \\
3.026447e+94 \\

```

16 & latin_cube_q &	3 &	0.57 &	-101.923693 &
8.681050e+90 \\			
33 & None &	5 &	0.96 &	-328.339848 &
4.048419e-08 \\			
17 & latin_cube_q &	5 &	0.86 &	-149.922809 &
1.238285e+70 \\			
9 & latin_cube_u &	5 &	-1.00 &	-128.137169 &
3.582648e+79 \\			
25 & uniform &	5 &	-1.00 &	-107.338213 &
3.864346e+88 \\			
1 & random &	5 &	0.96 &	-149.168526 &
2.632703e+70 \\			
34 & None &	7 &	0.96 &	-328.346897 &
4.019981e-08 \\			
2 & random &	7 &	0.53 &	-170.344387 &
1.674335e+61 \\			
10 & latin_cube_u &	7 &	0.83 &	-141.581027 &
5.195303e+73 \\			
26 & uniform &	7 &	0.57 &	-112.102580 &
3.295623e+86 \\			
18 & latin_cube_q &	7 &	0.86 &	-128.795174 &
1.855394e+79 \\			
27 & uniform &	10 &	0.50 &	-183.007479 &
5.300678e+55 \\			
35 & None &	10 &	0.98 &	-328.413837 &
3.759693e-08 \\			
19 & latin_cube_q &	10 &	0.83 &	-180.932723 &
4.220718e+56 \\			
3 & random &	10 &	0.78 &	-188.303752 &
2.655762e+53 \\			
11 & latin_cube_u &	10 &	0.84 &	-181.665201 &
2.028967e+56 \\			
28 & uniform &	15 &	0.55 &	-163.150957 &
2.227968e+64 \\			
20 & latin_cube_q &	15 &	0.53 &	-223.852209 &
9.675878e+37 \\			
36 & None &	15 &	0.96 &	-328.339599 &
4.049425e-08 \\			
12 & latin_cube_u &	15 &	0.51 &	-212.631548 &
7.223734e+42 \\			
4 & random &	15 &	0.54 &	-238.749126 &
3.281267e+31 \\			
37 & None &	25 &	0.96 &	-328.339221 &
4.050956e-08 \\			
21 & latin_cube_q &	25 &	0.57 &	-258.719065 &
6.969589e+22 \\			
13 & latin_cube_u &	25 &	0.81 &	-244.285910 &
1.292549e+29 \\			

5 & random &	25 &	0.50 &	-274.530444 &
9.471367e+15 \\			
29 & uniform &	25 &	0.54 &	-221.037613 &
1.614559e+39 \\			
6 & random &	50 &	0.53 &	-345.001679 &
2.350413e-15 \\			
14 & latin_cube_u &	50 &	0.51 &	-309.285140 &
7.632027e+00 \\			
30 & uniform &	50 &	0.55 &	-270.179103 &
7.348114e+17 \\			
22 & latin_cube_q &	50 &	0.55 &	-320.597792 &
9.324332e-05 \\			
38 & None &	50 &	0.98 &	-328.412939 &
3.763070e-08 \\			
15 & latin_cube_u &	100 &	0.63 &	-353.408114 &
5.251405e-19 \\			
7 & random &	100 &	0.57 &	-454.916866 &
4.321006e-63 \\			
23 & latin_cube_q &	100 &	0.51 &	-351.518581 &
3.474476e-18 \\			
31 & uniform &	100 &	0.74 &	-305.815175 &
2.452600e+02 \\			
39 & None &	100 &	0.96 &	-328.338575 &
4.053576e-08 \\			
\bottomrule			
\end{tabular}			





Fitting a model with 155 free scalar parameters with only 100 data points will result in a degenerate solution.

Fitting a model with 305 free scalar parameters with only 100 data points will result in a degenerate solution.

Fitting a model with 131 free scalar parameters with only 100 data points will result in a degenerate solution.

Fitting a model with 296 free scalar parameters with only 100 data points will result in a degenerate solution.

Fitting a model with 152 free scalar parameters with only 100 data points will result in a degenerate solution.

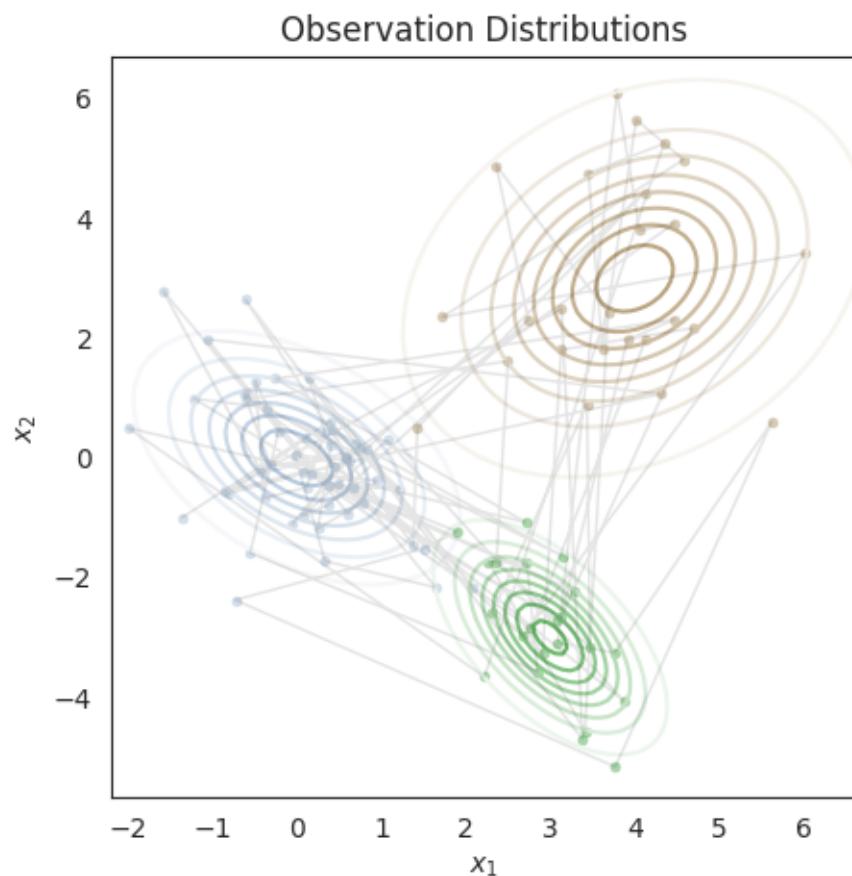
Fitting a model with 296 free scalar parameters with only 100 data points will result in a degenerate solution.

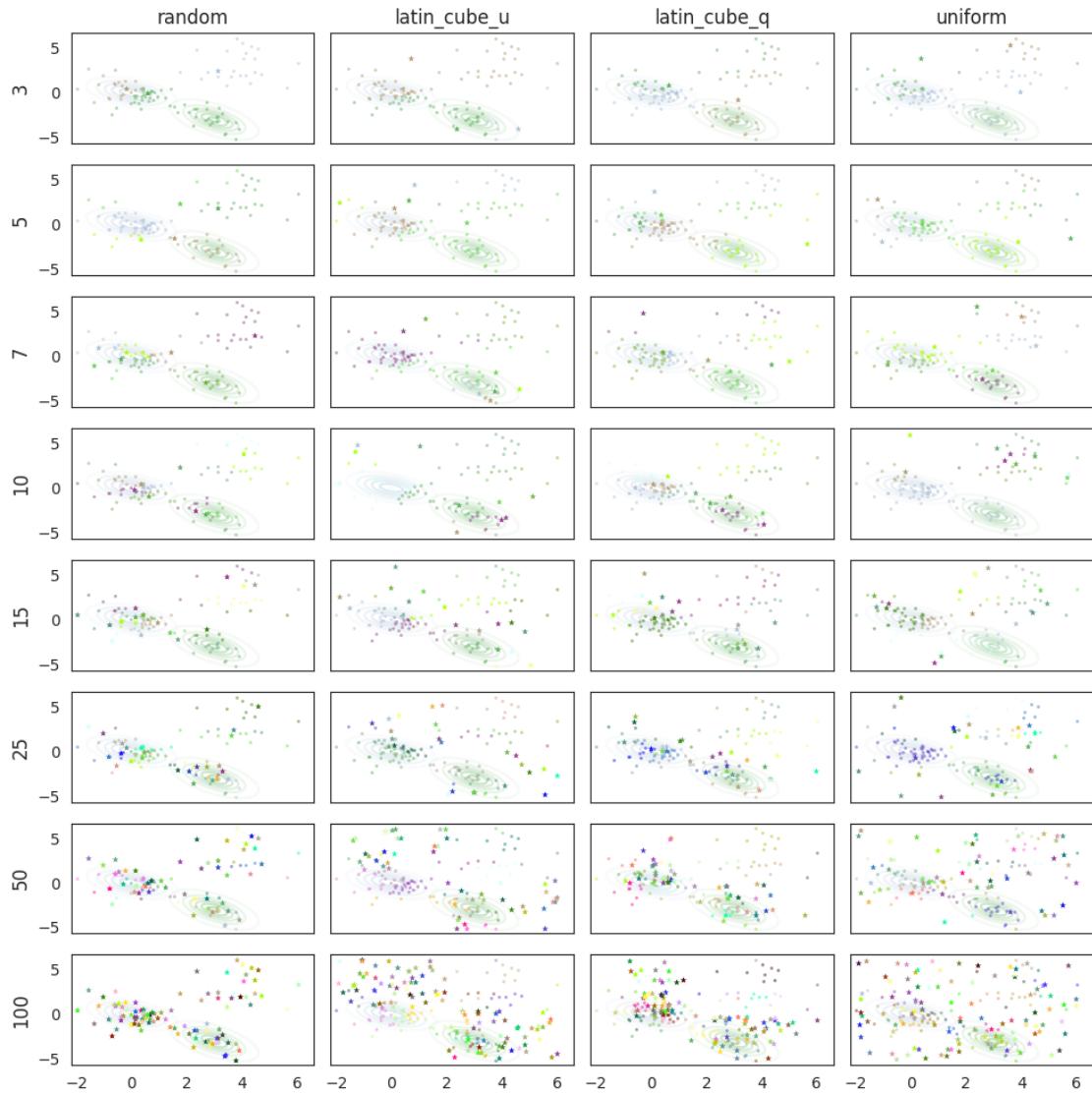
Fitting a model with 155 free scalar parameters with only 100 data points will result in a degenerate solution.

Fitting a model with 305 free scalar parameters with only 100 data points will result in a degenerate solution.

```
/home/kabalce/.local/lib/python3.10/site-
packages/sklearn/cluster/_kmeans.py:870: FutureWarning: The default value of
`n_init` will change from 10 to 'auto' in 1.4. Set the value of `n_init`
explicitly to suppress the warning
```

```
    warnings.warn(
/home/kabalce/.local/lib/python3.10/site-
packages/sklearn/cluster/_kmeans.py:870: FutureWarning: The default value of
`n_init` will change from 10 to 'auto' in 1.4. Set the value of `n_init`
explicitly to suppress the warning
    warnings.warn(
/home/kabalce/.local/lib/python3.10/site-
packages/sklearn/cluster/_kmeans.py:870: FutureWarning: The default value of
`n_init` will change from 10 to 'auto' in 1.4. Set the value of `n_init`
explicitly to suppress the warning
    warnings.warn(
/home/kabalce/.local/lib/python3.10/site-
packages/sklearn/cluster/_kmeans.py:870: FutureWarning: The default value of
`n_init` will change from 10 to 'auto' in 1.4. Set the value of `n_init`
explicitly to suppress the warning
    warnings.warn(
/home/kabalce/.local/lib/python3.10/site-
packages/sklearn/cluster/_kmeans.py:870: FutureWarning: The default value of
`n_init` will change from 10 to 'auto' in 1.4. Set the value of `n_init`
explicitly to suppress the warning
    warnings.warn(
/home/kabalce/.local/lib/python3.10/site-
packages/sklearn/cluster/_kmeans.py:870: FutureWarning: The default value of
`n_init` will change from 10 to 'auto' in 1.4. Set the value of `n_init`
explicitly to suppress the warning
    warnings.warn(
/home/kabalce/.local/lib/python3.10/site-
packages/sklearn/cluster/_kmeans.py:870: FutureWarning: The default value of
`n_init` will change from 10 to 'auto' in 1.4. Set the value of `n_init`
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    warnings.warn(
/home/kabalce/.local/lib/python3.10/site-
packages/sklearn/cluster/_kmeans.py:870: FutureWarning: The default value of
`n_init` will change from 10 to 'auto' in 1.4. Set the value of `n_init`
explicitly to suppress the warning
    warnings.warn(
/home/kabalce/.local/lib/python3.10/site-
```





	discretizing	number_of_nodes	accuracy	log-likelihood	likelihood-ratio
0	random	3	0.79	-91.632695	2.458106e+115
24	uniform	3	0.65	-76.178359	1.265708e+122
32	None	3	0.97	-354.109901	2.501489e+01
8	latin_cube_u	3	0.18	-79.068765	7.031471e+120
16	latin_cube_q	3	0.26	-98.069630	3.936182e+112
33	None	5	0.01	-354.109688	2.502022e+01
17	latin_cube_q	5	0.50	-142.375407	2.255905e+93
9	latin_cube_u	5	0.20	-98.944679	1.640765e+112
25	uniform	5	0.47	-122.030859	1.544705e+102
1	random	5	0.73	-123.294552	4.365467e+101
34	None	7	0.97	-354.109781	2.501788e+01
2	random	7	0.43	-173.820935	4.973949e+79

10	latin_cube_u	7	0.10	-122.523159	9.441532e+101
26	uniform	7	0.08	-155.434348	4.807227e+87
18	latin_cube_q	7	0.61	-154.381015	1.378324e+88
27	uniform	10	0.56	-107.937743	2.038958e+108
35	None	10	0.01	-354.109031	2.503664e+01
19	latin_cube_q	10	0.37	-201.133060	6.842206e+67
3	random	10	0.10	-204.069871	3.628737e+66
11	latin_cube_u	10	0.85	-146.643015	3.161711e+91
28	uniform	15	0.64	-194.260106	6.608188e+70
20	latin_cube_q	15	0.42	-201.549644	4.511037e+67
36	None	15	0.01	-354.108933	2.503910e+01
12	latin_cube_u	15	0.23	-195.211019	2.553324e+70
4	random	15	0.38	-233.196857	8.129546e+53
37	None	25	0.97	-354.110132	2.500911e+01
21	latin_cube_q	25	0.38	-235.517151	7.986846e+52
13	latin_cube_u	25	0.79	-195.975570	1.188682e+70
5	random	25	0.48	-269.227217	1.829279e+38
29	uniform	25	0.40	-202.425922	1.878078e+67
6	random	50	0.41	-321.190721	4.952410e+15
14	latin_cube_u	50	0.57	-225.189937	2.440211e+57
30	uniform	50	0.68	-259.225041	4.038029e+42
22	latin_cube_q	50	0.44	-283.990784	7.088333e+31
38	None	50	0.02	-372.065796	3.981551e-07
15	latin_cube_u	100	0.28	-281.580855	7.891559e+32
7	random	100	0.24	-392.094825	7.971786e-16
23	latin_cube_q	100	0.49	-304.053759	1.371844e+23
31	uniform	100	0.50	-308.309270	1.946077e+21
39	None	100	0.01	-354.109914	2.501457e+01

/tmp/ipykernel_54422/2508230348.py:83: FutureWarning: In future versions
`DataFrame.to_latex` is expected to utilise the base implementation of
`Styler.to_latex` for formatting and rendering. The arguments signature may
therefore change. It is recommended instead to use `DataFrame.style.to_latex`
which also contains additional functionality.

```

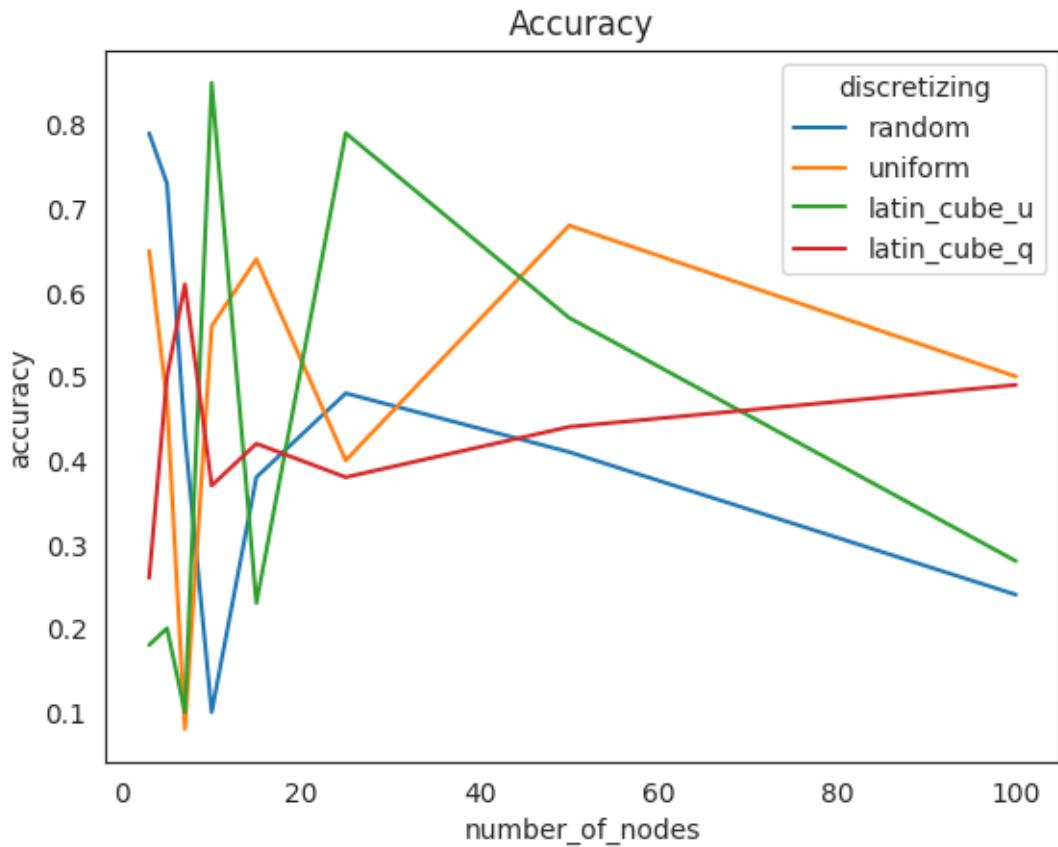
print(summary.to_latex())

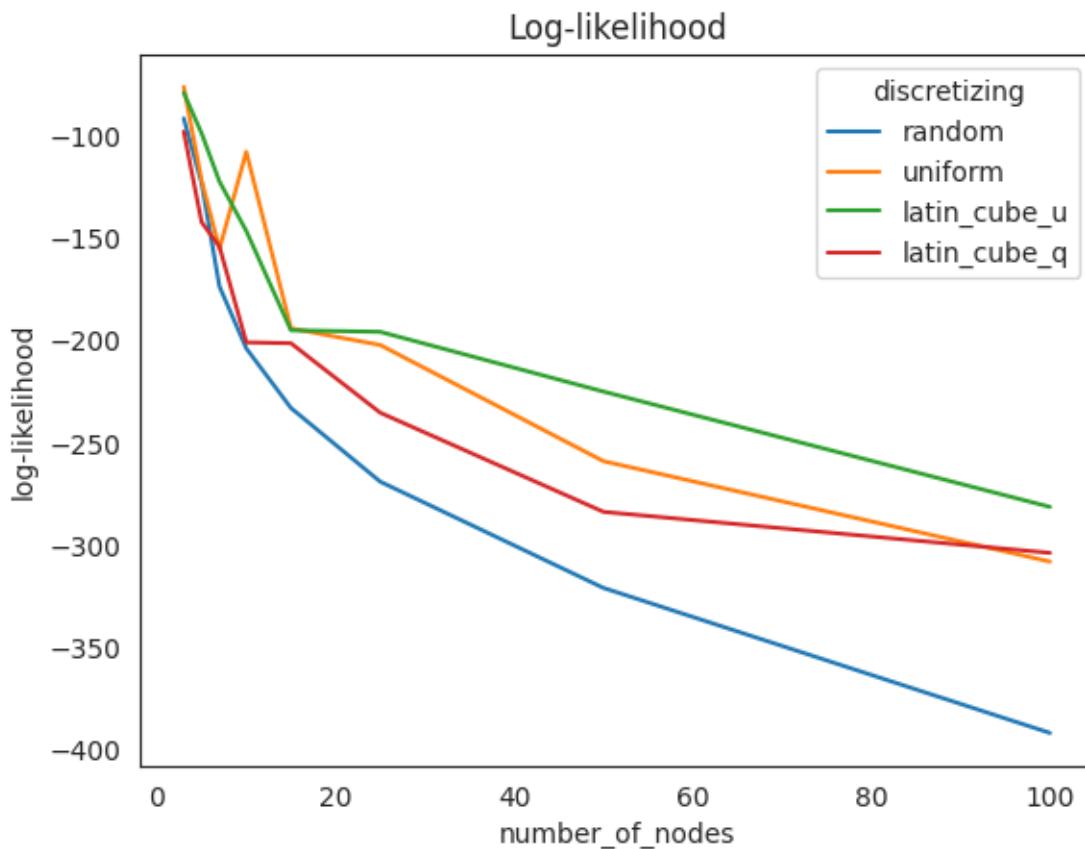
\begin{tabular}{llrrrr}
\toprule
{} & discretizing & number\_of\_nodes & accuracy & log-likelihood & \\
& likelihood-ratio & \\
\midrule
0 & random & 3 & 0.79 & -91.632695 & \\
2.458106e+115 & \\
24 & uniform & 3 & 0.65 & -76.178359 & \\
1.265708e+122 & \\
32 & None & 3 & 0.97 & -354.109901 & \\
2.501489e+01 & \\
8 & latin\_cube\_u & 3 & 0.18 & -79.068765 & \\
7.031471e+120 &

```

16 & latin_cube_q &	3 &	0.26 &	-98.069630 &
3.936182e+112 \\			
33 & None &	5 &	0.01 &	-354.109688 &
2.502022e+01 \\			
17 & latin_cube_q &	5 &	0.50 &	-142.375407 &
2.255905e+93 \\			
9 & latin_cube_u &	5 &	0.20 &	-98.944679 &
1.640765e+112 \\			
25 & uniform &	5 &	0.47 &	-122.030859 &
1.544705e+102 \\			
1 & random &	5 &	0.73 &	-123.294552 &
4.365467e+101 \\			
34 & None &	7 &	0.97 &	-354.109781 &
2.501788e+01 \\			
2 & random &	7 &	0.43 &	-173.820935 &
4.973949e+79 \\			
10 & latin_cube_u &	7 &	0.10 &	-122.523159 &
9.441532e+101 \\			
26 & uniform &	7 &	0.08 &	-155.434348 &
4.807227e+87 \\			
18 & latin_cube_q &	7 &	0.61 &	-154.381015 &
1.378324e+88 \\			
27 & uniform &	10 &	0.56 &	-107.937743 &
2.038958e+108 \\			
35 & None &	10 &	0.01 &	-354.109031 &
2.503664e+01 \\			
19 & latin_cube_q &	10 &	0.37 &	-201.133060 &
6.842206e+67 \\			
3 & random &	10 &	0.10 &	-204.069871 &
3.628737e+66 \\			
11 & latin_cube_u &	10 &	0.85 &	-146.643015 &
3.161711e+91 \\			
28 & uniform &	15 &	0.64 &	-194.260106 &
6.608188e+70 \\			
20 & latin_cube_q &	15 &	0.42 &	-201.549644 &
4.511037e+67 \\			
36 & None &	15 &	0.01 &	-354.108933 &
2.503910e+01 \\			
12 & latin_cube_u &	15 &	0.23 &	-195.211019 &
2.553324e+70 \\			
4 & random &	15 &	0.38 &	-233.196857 &
8.129546e+53 \\			
37 & None &	25 &	0.97 &	-354.110132 &
2.500911e+01 \\			
21 & latin_cube_q &	25 &	0.38 &	-235.517151 &
7.986846e+52 \\			
13 & latin_cube_u &	25 &	0.79 &	-195.975570 &
1.188682e+70 \\			

5 & random &	25 & 0.48 &	-269.227217 &
1.829279e+38 \\		
29 & uniform &	25 & 0.40 &	-202.425922 &
1.878078e+67 \\		
6 & random &	50 & 0.41 &	-321.190721 &
4.952410e+15 \\		
14 & latin_cube_u &	50 & 0.57 &	-225.189937 &
2.440211e+57 \\		
30 & uniform &	50 & 0.68 &	-259.225041 &
4.038029e+42 \\		
22 & latin_cube_q &	50 & 0.44 &	-283.990784 &
7.088333e+31 \\		
38 & None &	50 & 0.02 &	-372.065796 &
3.981551e-07 \\		
15 & latin_cube_u &	100 & 0.28 &	-281.580855 &
7.891559e+32 \\		
7 & random &	100 & 0.24 &	-392.094825 &
7.971786e-16 \\		
23 & latin_cube_q &	100 & 0.49 &	-304.053759 &
1.371844e+23 \\		
31 & uniform &	100 & 0.50 &	-308.309270 &
1.946077e+21 \\		
39 & None &	100 & 0.01 &	-354.109914 &
2.501457e+01 \\		
\bottomrule		
\end{tabular}		





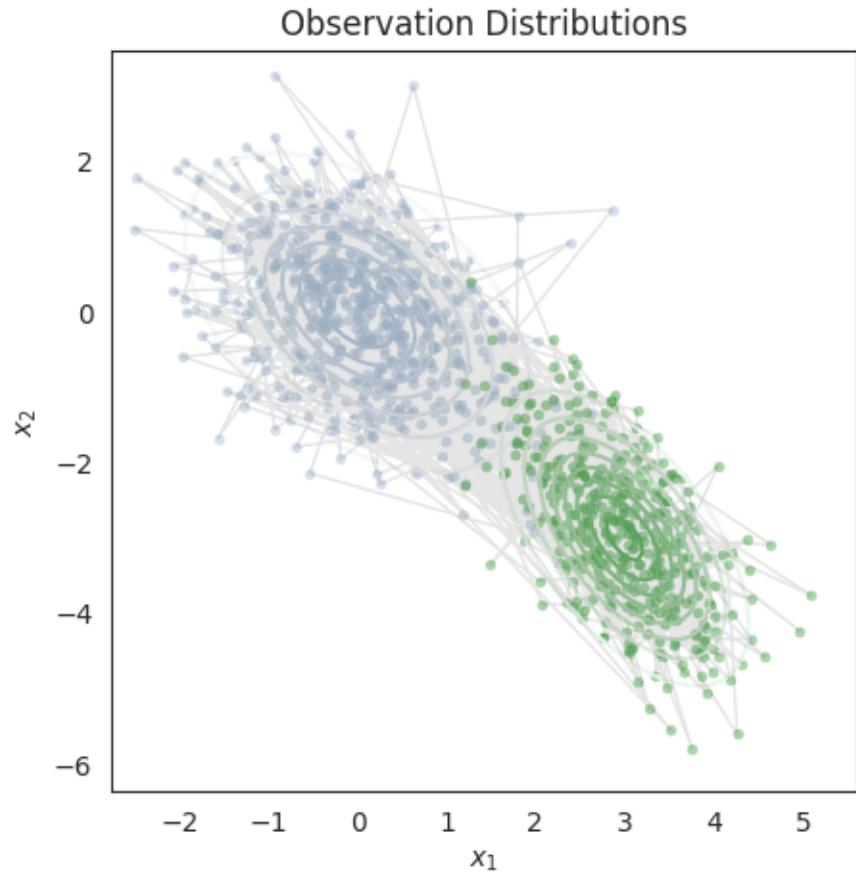
```

sample_size = 1000
Recognized 1 instead of 2 states!
Recognized 1 instead of 2 states!

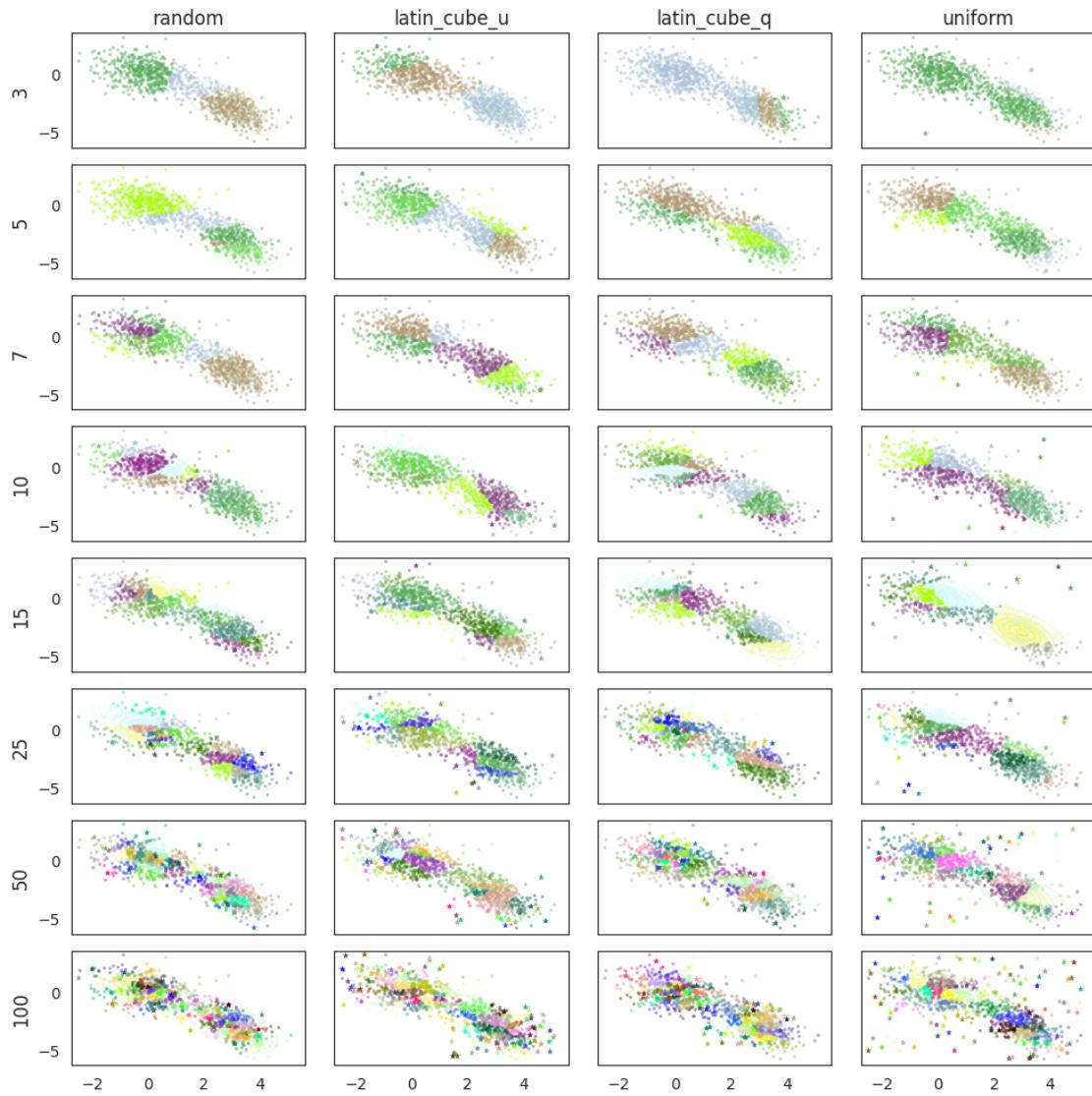
/home/kabalce/.local/lib/python3.10/site-
packages/sklearn/cluster/_kmeans.py:870: FutureWarning: The default value of
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/home/kabalce/.local/lib/python3.10/site-
packages/sklearn/cluster/_kmeans.py:870: FutureWarning: The default value of
`n_init` will change from 10 to 'auto' in 1.4. Set the value of `n_init`
explicitly to suppress the warning
    warnings.warn(

```

```
`n_init` will change from 10 to 'auto' in 1.4. Set the value of `n_init`  
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    warnings.warn(  
/home/kabalce/.local/lib/python3.10/site-  
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/home/kabalce/.local/lib/python3.10/site-  
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packages/sklearn/cluster/_kmeans.py:870: FutureWarning: The default value of  
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/home/kabalce/.local/lib/python3.10/site-  
packages/sklearn/cluster/_kmeans.py:870: FutureWarning: The default value of  
`n_init` will change from 10 to 'auto' in 1.4. Set the value of `n_init`  
explicitly to suppress the warning
```



```
/tmp/ipykernel_54422/2508230348.py:60: RuntimeWarning: overflow encountered in exp
    'likelihood-ratio': np.exp(models[discretizing, number_of_nodes]['ll'] -
target_ll)
```



	discretizing	number_of_nodes	accuracy	log-likelihood	likelihood-ratio
0	random	3	0.956	-962.252612	inf
24	uniform	3	0.629	-321.227909	inf
32	None	3	0.974	-3099.383847	7.223287e-54
8	latin_cube_u	3	-1.000	-1011.464923	inf
16	latin_cube_q	3	-1.000	-584.768157	inf
33	None	5	0.974	-3099.383870	7.223122e-54
17	latin_cube_q	5	0.902	-1349.483449	inf
9	latin_cube_u	5	0.851	-1384.692713	inf
25	uniform	5	0.516	-1401.124204	inf
1	random	5	0.906	-1281.301558	inf
34	None	7	0.974	-3099.384075	7.221636e-54
2	random	7	0.504	-1708.690169	inf

10	latin_cube_u	7	0.928	-1618.375773	inf
26	uniform	7	0.501	-1474.265017	inf
18	latin_cube_q	7	0.501	-1771.781968	inf
27	uniform	10	0.601	-1584.872611	inf
35	None	10	0.974	-3099.383675	7.224525e-54
19	latin_cube_q	10	0.952	-2041.212472	inf
3	random	10	0.554	-1978.065757	inf
11	latin_cube_u	10	0.511	-1714.662907	inf
28	uniform	15	0.962	-1849.087166	inf
20	latin_cube_q	15	0.937	-2414.534446	1.927749e+244
36	None	15	0.974	-3099.383862	7.223174e-54
12	latin_cube_u	15	0.506	-2040.981182	inf
4	random	15	0.955	-2498.246308	8.500624e+207
37	None	25	0.974	-3099.383817	7.223503e-54
21	latin_cube_q	25	0.910	-2796.105824	3.721401e+78
13	latin_cube_u	25	0.932	-2387.708830	8.615244e+255
5	random	25	0.510	-2899.335356	5.478864e+33
29	uniform	25	0.505	-2433.930774	7.266618e+235
6	random	50	0.512	-3701.055493	3.597982e-315
14	latin_cube_u	50	0.921	-2853.981712	2.726013e+53
30	uniform	50	0.961	-2696.102063	1.004126e+122
22	latin_cube_q	50	0.536	-3534.096825	1.162217e-242
38	None	50	0.974	-3099.383847	7.223284e-54
15	latin_cube_u	100	0.894	-3647.038117	1.036352e-291
7	random	100	0.504	-4387.627320	0.000000e+00
23	latin_cube_q	100	0.509	-4082.638256	0.000000e+00
31	uniform	100	0.915	-3251.591863	5.696831e-120
39	None	100	0.974	-3099.383954	7.222511e-54

/tmp/ipykernel_54422/2508230348.py:83: FutureWarning: In future versions
`DataFrame.to_latex` is expected to utilise the base implementation of
`Styler.to_latex` for formatting and rendering. The arguments signature may
therefore change. It is recommended instead to use `DataFrame.style.to_latex`
which also contains additional functionality.

```

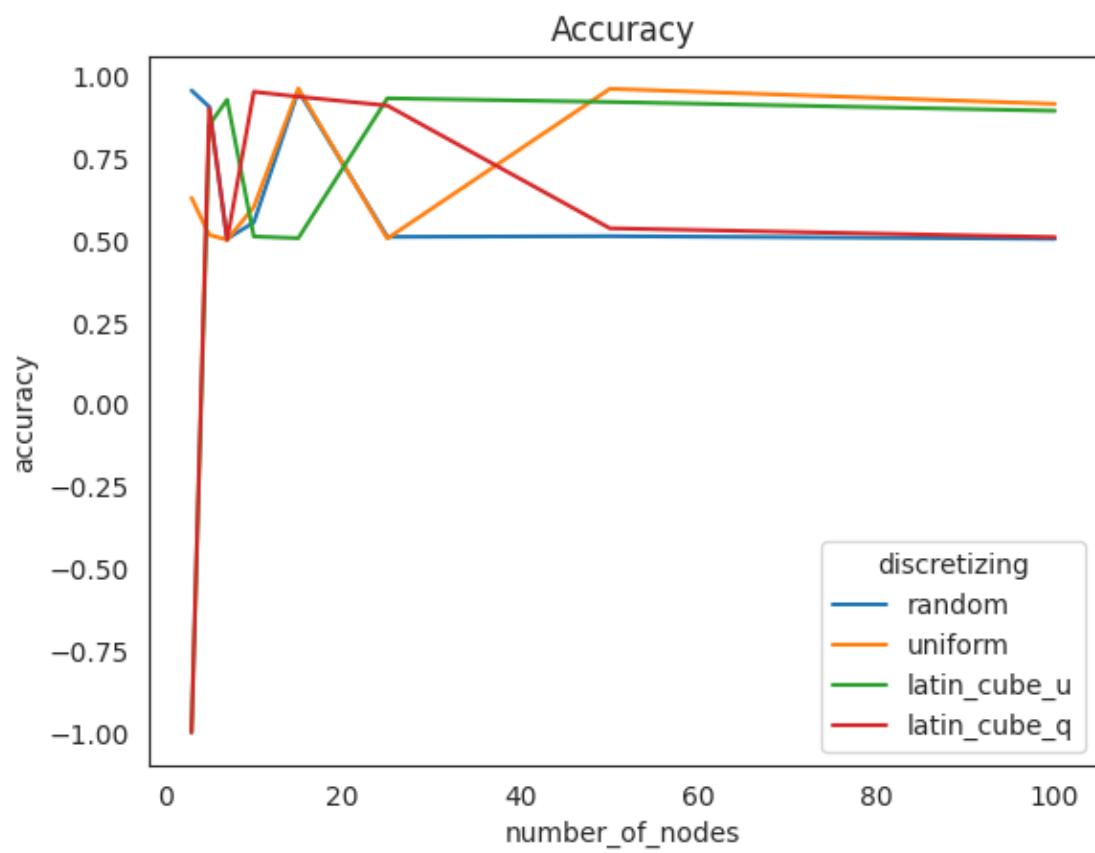
print(summary.to_latex())

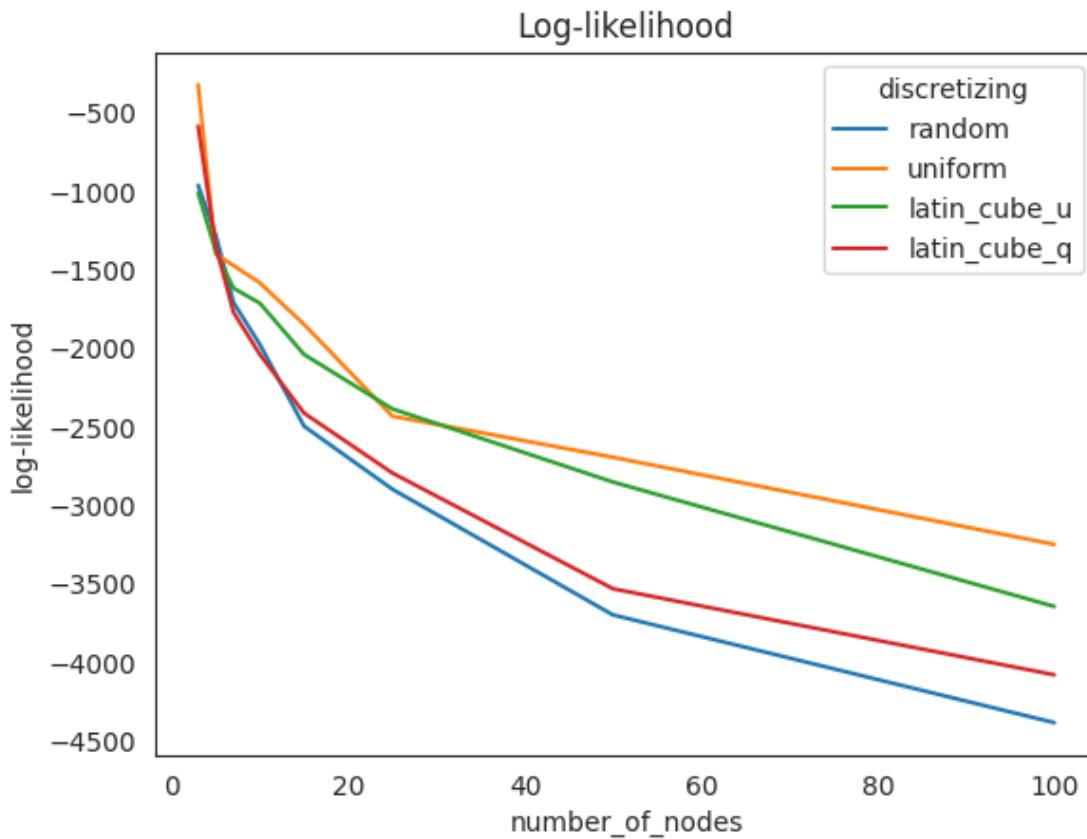
\begin{tabular}{llrrrr}
\toprule
{} & discretizing & number\_of\_nodes & accuracy & log-likelihood & \\
& likelihood-ratio \\
\midrule
0 & random & 3 & 0.956 & -962.252612 & \\
& inf \\ 
24 & uniform & 3 & 0.629 & -321.227909 & \\
& inf \\ 
32 & None & 3 & 0.974 & -3099.383847 & \\
7.223287e-54 \\ 
8 & latin\_cube\_u & 3 & -1.000 & -1011.464923 & \\
& inf \\
\bottomrule
\end{tabular}

```

16 & latin_cube_q &	3 &	-1.000 &	-584.768157 &
inf \\			
33 & None &	5 &	0.974 &	-3099.383870 &
7.223122e-54 \\			
17 & latin_cube_q &	5 &	0.902 &	-1349.483449 &
inf \\			
9 & latin_cube_u &	5 &	0.851 &	-1384.692713 &
inf \\			
25 & uniform &	5 &	0.516 &	-1401.124204 &
inf \\			
1 & random &	5 &	0.906 &	-1281.301558 &
inf \\			
34 & None &	7 &	0.974 &	-3099.384075 &
7.221636e-54 \\			
2 & random &	7 &	0.504 &	-1708.690169 &
inf \\			
10 & latin_cube_u &	7 &	0.928 &	-1618.375773 &
inf \\			
26 & uniform &	7 &	0.501 &	-1474.265017 &
inf \\			
18 & latin_cube_q &	7 &	0.501 &	-1771.781968 &
inf \\			
27 & uniform &	10 &	0.601 &	-1584.872611 &
inf \\			
35 & None &	10 &	0.974 &	-3099.383675 &
7.224525e-54 \\			
19 & latin_cube_q &	10 &	0.952 &	-2041.212472 &
inf \\			
3 & random &	10 &	0.554 &	-1978.065757 &
inf \\			
11 & latin_cube_u &	10 &	0.511 &	-1714.662907 &
inf \\			
28 & uniform &	15 &	0.962 &	-1849.087166 &
inf \\			
20 & latin_cube_q &	15 &	0.937 &	-2414.534446 &
1.927749e+244 \\			
36 & None &	15 &	0.974 &	-3099.383862 &
7.223174e-54 \\			
12 & latin_cube_u &	15 &	0.506 &	-2040.981182 &
inf \\			
4 & random &	15 &	0.955 &	-2498.246308 &
8.500624e+207 \\			
37 & None &	25 &	0.974 &	-3099.383817 &
7.223503e-54 \\			
21 & latin_cube_q &	25 &	0.910 &	-2796.105824 &
3.721401e+78 \\			
13 & latin_cube_u &	25 &	0.932 &	-2387.708830 &
8.615244e+255 \\			

5 & random &	25 & 0.510 & -2899.335356 &
5.478864e+33 \\	
29 & uniform &	25 & 0.505 & -2433.930774 &
7.266618e+235 \\	
6 & random &	50 & 0.512 & -3701.055493 &
3.597982e-315 \\	
14 & latin_cube_u &	50 & 0.921 & -2853.981712 &
2.726013e+53 \\	
30 & uniform &	50 & 0.961 & -2696.102063 &
1.004126e+122 \\	
22 & latin_cube_q &	50 & 0.536 & -3534.096825 &
1.162217e-242 \\	
38 & None &	50 & 0.974 & -3099.383847 &
7.223284e-54 \\	
15 & latin_cube_u &	100 & 0.894 & -3647.038117 &
1.036352e-291 \\	
7 & random &	100 & 0.504 & -4387.627320 &
0.000000e+00 \\	
23 & latin_cube_q &	100 & 0.509 & -4082.638256 &
0.000000e+00 \\	
31 & uniform &	100 & 0.915 & -3251.591863 &
5.696831e-120 \\	
39 & None &	100 & 0.974 & -3099.383954 &
7.222511e-54 \\	
\bottomrule	
\end{tabular}	





```

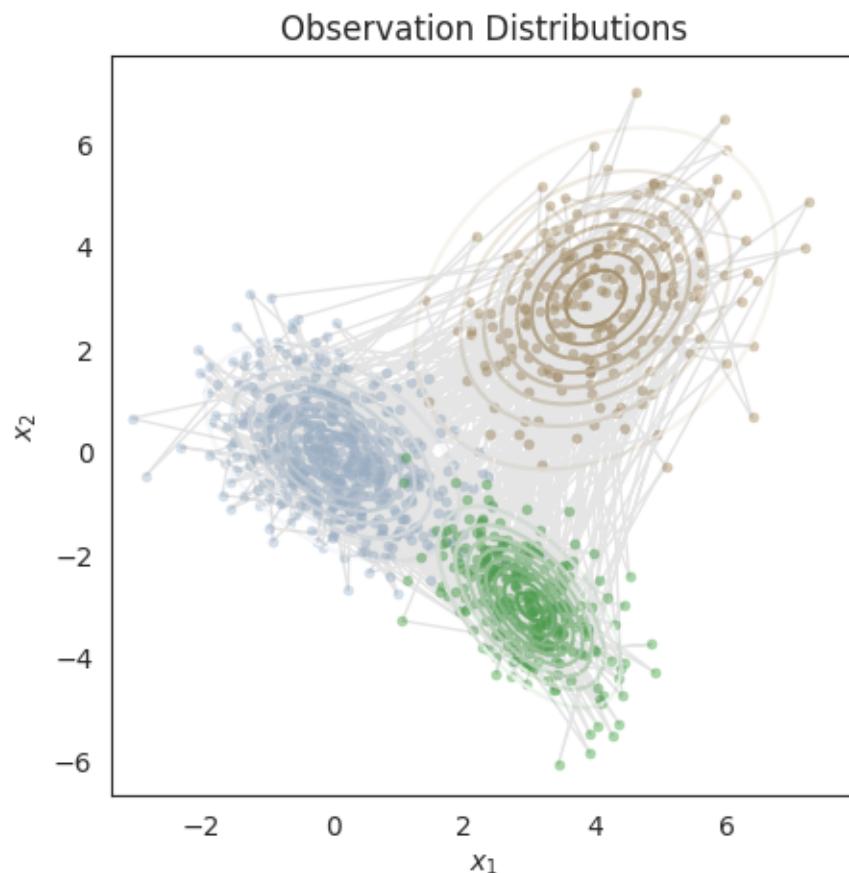
Recognized 2 instead of 3 states!
Recognized 1 instead of 3 states!

/home/kabalce/.local/lib/python3.10/site-
packages/sklearn/cluster/_kmeans.py:870: FutureWarning: The default value of
`n_init` will change from 10 to 'auto' in 1.4. Set the value of `n_init`
explicitly to suppress the warning
    warnings.warn(
/home/kabalce/.local/lib/python3.10/site-
packages/sklearn/cluster/_kmeans.py:870: FutureWarning: The default value of
`n_init` will change from 10 to 'auto' in 1.4. Set the value of `n_init`
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    warnings.warn(
/home/kabalce/.local/lib/python3.10/site-
packages/sklearn/cluster/_kmeans.py:870: FutureWarning: The default value of
`n_init` will change from 10 to 'auto' in 1.4. Set the value of `n_init`
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    warnings.warn(
/home/kabalce/.local/lib/python3.10/site-
packages/sklearn/cluster/_kmeans.py:870: FutureWarning: The default value of
`n_init` will change from 10 to 'auto' in 1.4. Set the value of `n_init`
```

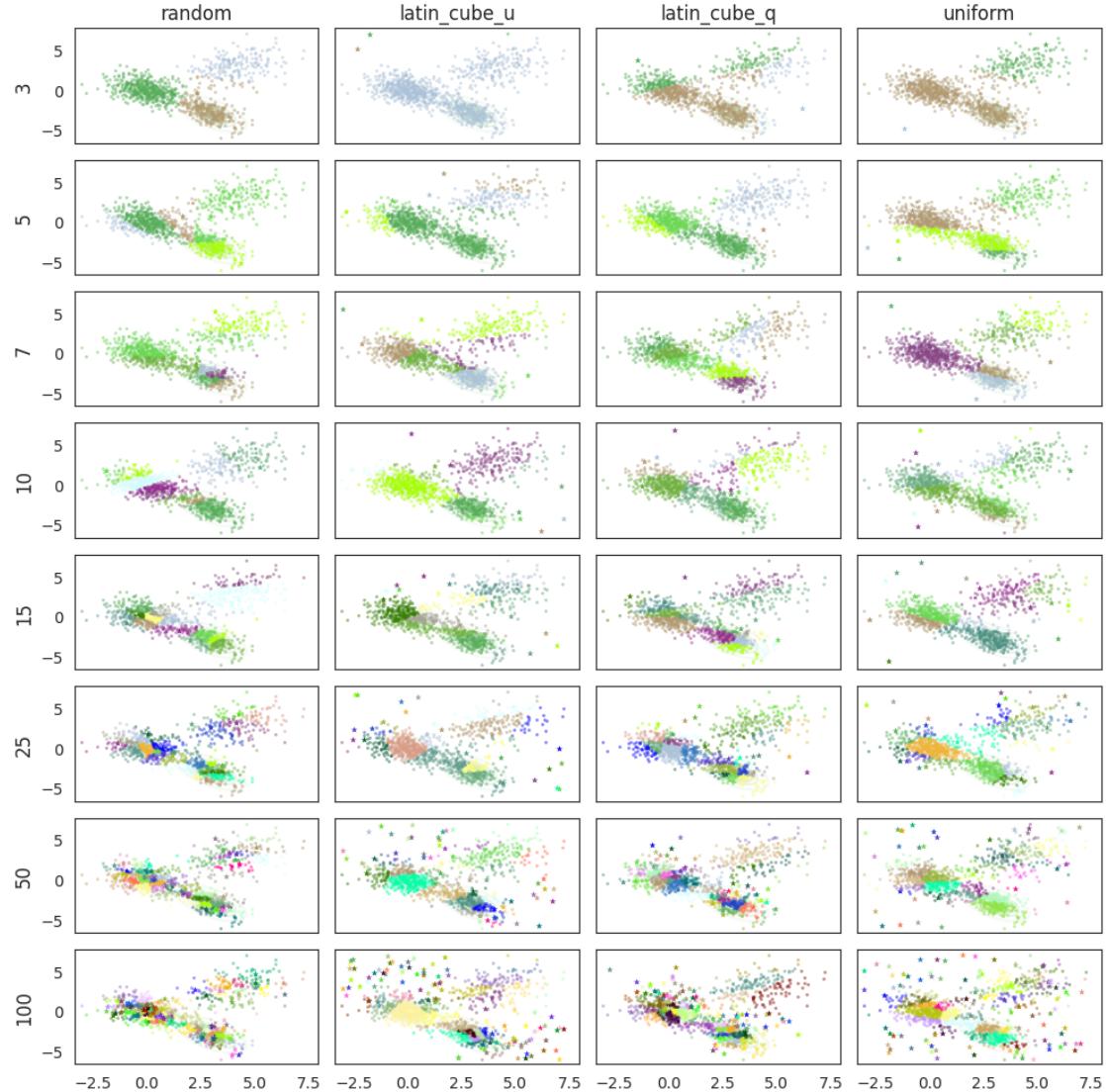
```

explicitly to suppress the warning
    warnings.warn(
/home/kabalce/.local/lib/python3.10/site-
packages/sklearn/cluster/_kmeans.py:870: FutureWarning: The default value of
`n_init` will change from 10 to 'auto' in 1.4. Set the value of `n_init`
explicitly to suppress the warning
    warnings.warn(
/home/kabalce/.local/lib/python3.10/site-
packages/sklearn/cluster/_kmeans.py:870: FutureWarning: The default value of
`n_init` will change from 10 to 'auto' in 1.4. Set the value of `n_init`
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    warnings.warn(
/home/kabalce/.local/lib/python3.10/site-
packages/sklearn/cluster/_kmeans.py:870: FutureWarning: The default value of
`n_init` will change from 10 to 'auto' in 1.4. Set the value of `n_init`
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    warnings.warn(
/home/kabalce/.local/lib/python3.10/site-
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`n_init` will change from 10 to 'auto' in 1.4. Set the value of `n_init`
explicitly to suppress the warning
    warnings.warn(

```



```
/tmp/ipykernel_54422/2508230348.py:60: RuntimeWarning: overflow encountered in exp
  'likelihood-ratio': np.exp(models[discretizing, number_of_nodes]['ll'] -
target_ll)
```



	discretizing	number_of_nodes	accuracy	log-likelihood	likelihood-ratio
0	random	3	0.710	-956.667759	inf
24	uniform	3	0.512	-469.801312	inf
32	None	3	0.979	-3609.715042	7.768523e-43
8	latin_cube_u	3	-1.000	-7.912357	inf
16	latin_cube_q	3	-1.000	-802.398491	inf

33	None	5	0.979	-3609.714824	7.770213e-43
17	latin_cube_q	5	0.470	-1290.415821	inf
9	latin_cube_u	5	0.372	-853.260509	inf
25	uniform	5	0.411	-1207.974354	inf
1	random	5	0.607	-1377.520133	inf
34	None	7	0.979	-3609.714683	7.771307e-43
2	random	7	0.209	-1581.876708	inf
10	latin_cube_u	7	0.149	-1565.741668	inf
26	uniform	7	0.707	-1327.152042	inf
18	latin_cube_q	7	0.160	-1805.401631	inf
27	uniform	10	0.526	-1476.118838	inf
35	None	10	0.979	-3609.714157	7.775400e-43
19	latin_cube_q	10	0.485	-1712.940540	inf
3	random	10	0.640	-2025.263714	inf
11	latin_cube_u	10	0.637	-1306.533377	inf
28	uniform	15	0.672	-1667.319908	inf
20	latin_cube_q	15	0.194	-2235.702066	inf
36	None	15	0.979	-3609.714188	7.775160e-43
12	latin_cube_u	15	0.149	-1910.151060	inf
4	random	15	0.603	-2542.242102	inf
37	None	25	0.979	-3609.714461	7.773034e-43
21	latin_cube_q	25	0.623	-2714.853469	inf
13	latin_cube_u	25	0.649	-1971.406066	inf
5	random	25	0.343	-3074.974881	1.333643e+190
29	uniform	25	0.500	-2151.454213	inf
6	random	50	0.250	-3665.499764	4.606183e-67
14	latin_cube_u	50	0.105	-2448.876150	inf
30	uniform	50	0.233	-2686.465555	inf
22	latin_cube_q	50	0.511	-3250.420404	8.511715e+113
38	None	50	0.979	-3609.714124	7.775655e-43
15	latin_cube_u	100	0.483	-2761.234077	inf
7	random	100	0.405	-4251.240512	1.902153e-321
23	latin_cube_q	100	0.455	-3565.216715	1.643298e-23
31	uniform	100	0.367	-3152.340535	3.353998e+156
39	None	100	0.979	-3609.715444	7.765402e-43

/tmp/ipykernel_54422/2508230348.py:83: FutureWarning: In future versions
`DataFrame.to_latex` is expected to utilise the base implementation of
`Styler.to_latex` for formatting and rendering. The arguments signature may
therefore change. It is recommended instead to use `DataFrame.style.to_latex`
which also contains additional functionality.

```

print(summary.to_latex())
\begin{tabular}{llrrrr}
\toprule
{} & discretizing & number\_of\_nodes & accuracy & log-likelihood & \\
& likelihood-ratio \\
\midrule
0 & random & 3 & 0.710 & -956.667759 &

```

```

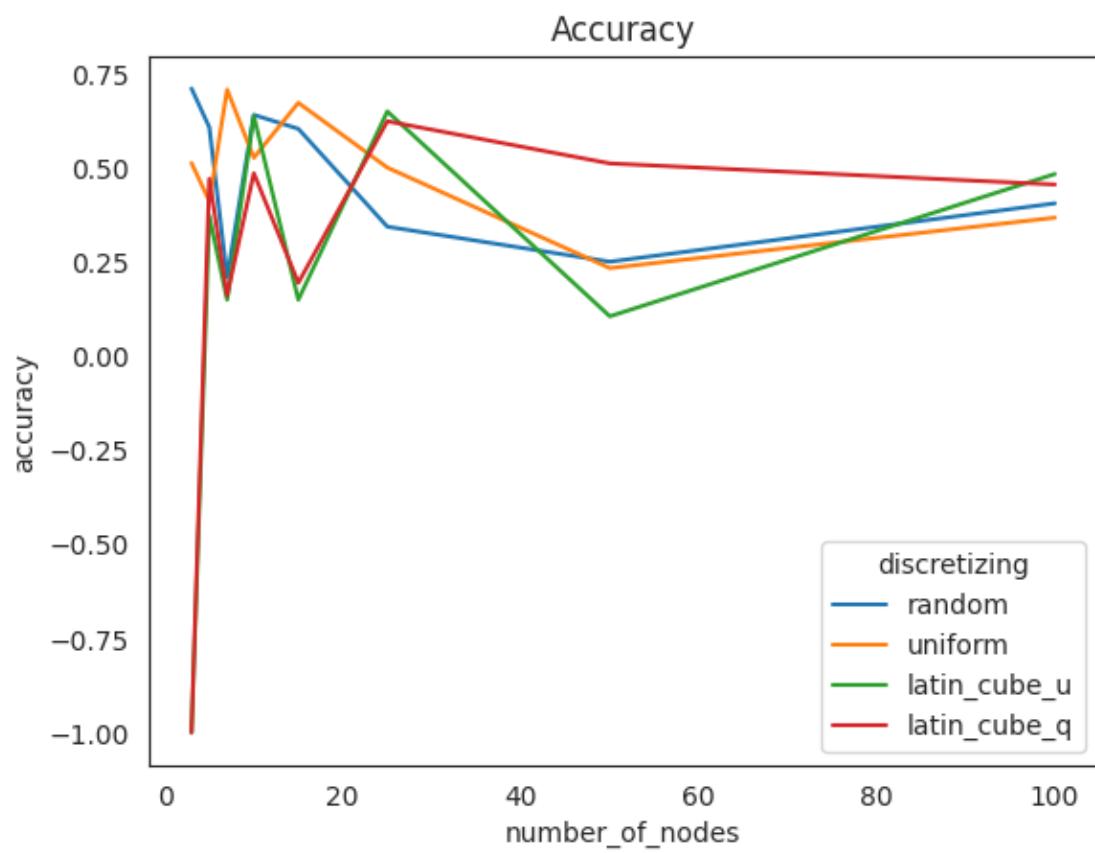
inf \\
24 & uniform & 3 & 0.512 & -469.801312 &
inf \\
32 & None & 3 & 0.979 & -3609.715042 &
7.768523e-43 \\
8 & latin\_cube\_u & 3 & -1.000 & -7.912357 &
inf \\
16 & latin\_cube\_q & 3 & -1.000 & -802.398491 &
inf \\
33 & None & 5 & 0.979 & -3609.714824 &
7.770213e-43 \\
17 & latin\_cube\_q & 5 & 0.470 & -1290.415821 &
inf \\
9 & latin\_cube\_u & 5 & 0.372 & -853.260509 &
inf \\
25 & uniform & 5 & 0.411 & -1207.974354 &
inf \\
1 & random & 5 & 0.607 & -1377.520133 &
inf \\
34 & None & 7 & 0.979 & -3609.714683 &
7.771307e-43 \\
2 & random & 7 & 0.209 & -1581.876708 &
inf \\
10 & latin\_cube\_u & 7 & 0.149 & -1565.741668 &
inf \\
26 & uniform & 7 & 0.707 & -1327.152042 &
inf \\
18 & latin\_cube\_q & 7 & 0.160 & -1805.401631 &
inf \\
27 & uniform & 10 & 0.526 & -1476.118838 &
inf \\
35 & None & 10 & 0.979 & -3609.714157 &
7.775400e-43 \\
19 & latin\_cube\_q & 10 & 0.485 & -1712.940540 &
inf \\
3 & random & 10 & 0.640 & -2025.263714 &
inf \\
11 & latin\_cube\_u & 10 & 0.637 & -1306.533377 &
inf \\
28 & uniform & 15 & 0.672 & -1667.319908 &
inf \\
20 & latin\_cube\_q & 15 & 0.194 & -2235.702066 &
inf \\
36 & None & 15 & 0.979 & -3609.714188 &
7.775160e-43 \\
12 & latin\_cube\_u & 15 & 0.149 & -1910.151060 &
inf \\
4 & random & 15 & 0.603 & -2542.242102 &

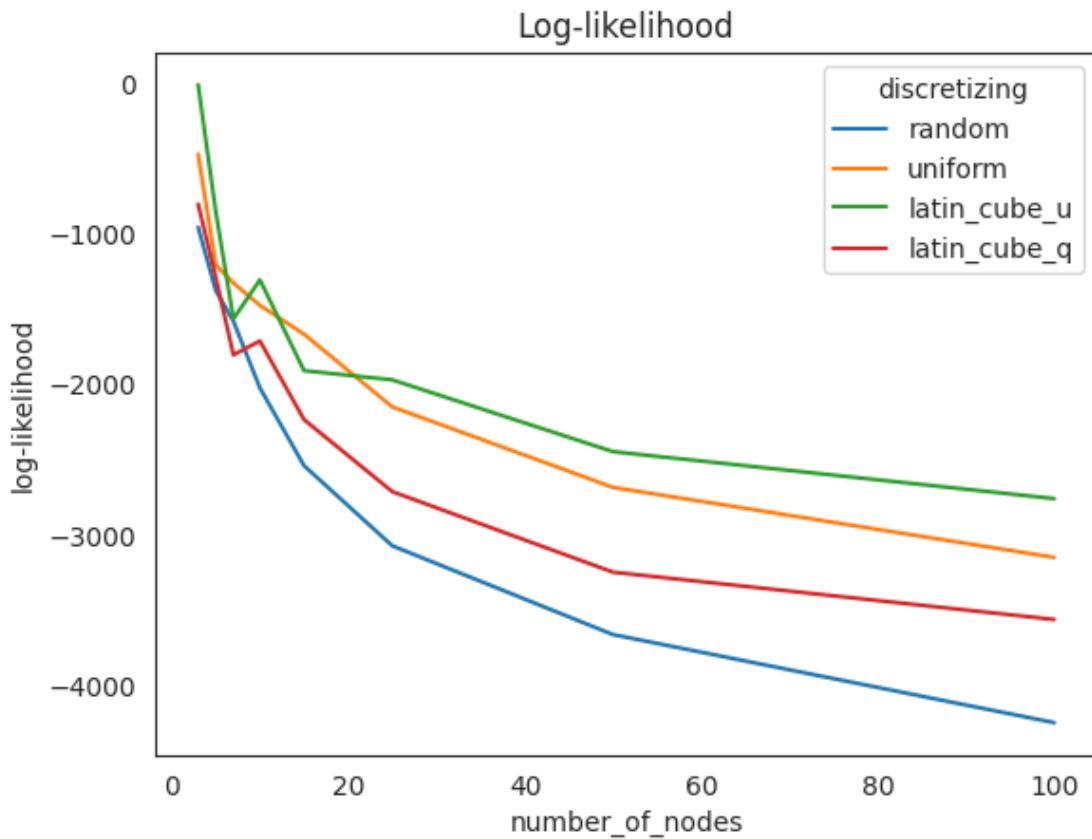
```

```

inf \\
37 & None & 25 & 0.979 & -3609.714461 &
7.773034e-43 \\
21 & latin\_cube\_q & 25 & 0.623 & -2714.853469 &
inf \\
13 & latin\_cube\_u & 25 & 0.649 & -1971.406066 &
inf \\
5 & random & 25 & 0.343 & -3074.974881 &
1.333643e+190 \\
29 & uniform & 25 & 0.500 & -2151.454213 &
inf \\
6 & random & 50 & 0.250 & -3665.499764 &
4.606183e-67 \\
14 & latin\_cube\_u & 50 & 0.105 & -2448.876150 &
inf \\
30 & uniform & 50 & 0.233 & -2686.465555 &
inf \\
22 & latin\_cube\_q & 50 & 0.511 & -3250.420404 &
8.511715e+113 \\
38 & None & 50 & 0.979 & -3609.714124 &
7.775655e-43 \\
15 & latin\_cube\_u & 100 & 0.483 & -2761.234077 &
inf \\
7 & random & 100 & 0.405 & -4251.240512 &
1.902153e-321 \\
23 & latin\_cube\_q & 100 & 0.455 & -3565.216715 &
1.643298e-23 \\
31 & uniform & 100 & 0.367 & -3152.340535 &
3.353998e+156 \\
39 & None & 100 & 0.979 & -3609.715444 &
7.765402e-43 \\
\bottomrule
\end{tabular}

```





```

sample_size = 10000

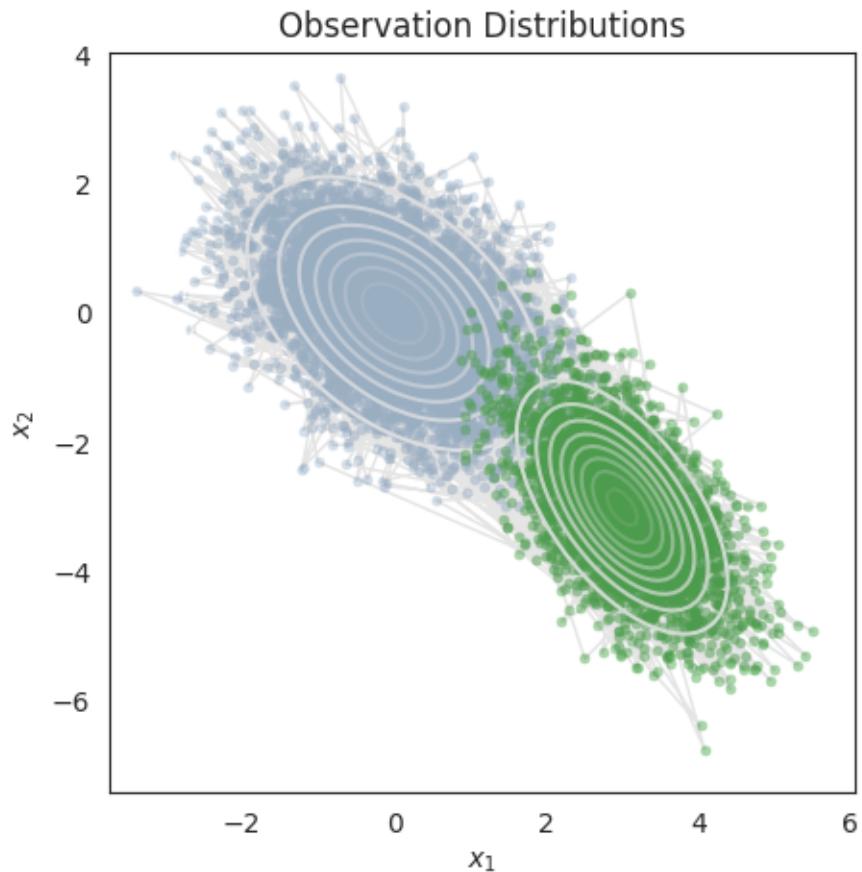
/home/kabalce/.local/lib/python3.10/site-
packages/sklearn/cluster/_kmeans.py:870: FutureWarning: The default value of
`n_init` will change from 10 to 'auto' in 1.4. Set the value of `n_init`
explicitly to suppress the warning
    warnings.warn(
/home/kabalce/.local/lib/python3.10/site-
packages/sklearn/cluster/_kmeans.py:870: FutureWarning: The default value of
`n_init` will change from 10 to 'auto' in 1.4. Set the value of `n_init`
explicitly to suppress the warning
    warnings.warn(
/home/kabalce/.local/lib/python3.10/site-
packages/sklearn/cluster/_kmeans.py:870: FutureWarning: The default value of
`n_init` will change from 10 to 'auto' in 1.4. Set the value of `n_init`
explicitly to suppress the warning
    warnings.warn(
/home/kabalce/.local/lib/python3.10/site-
packages/sklearn/cluster/_kmeans.py:870: FutureWarning: The default value of
`n_init` will change from 10 to 'auto' in 1.4. Set the value of `n_init`
explicitly to suppress the warning
    warnings.warn(

```

```

warnings.warn(
/home/kabalce/.local/lib/python3.10/site-
packages/sklearn/cluster/_kmeans.py:870: FutureWarning: The default value of
`n_init` will change from 10 to 'auto' in 1.4. Set the value of `n_init`
explicitly to suppress the warning
    warnings.warn(
/home/kabalce/.local/lib/python3.10/site-
packages/sklearn/cluster/_kmeans.py:870: FutureWarning: The default value of
`n_init` will change from 10 to 'auto' in 1.4. Set the value of `n_init`
explicitly to suppress the warning
    warnings.warn(
/home/kabalce/.local/lib/python3.10/site-
packages/sklearn/cluster/_kmeans.py:870: FutureWarning: The default value of
`n_init` will change from 10 to 'auto' in 1.4. Set the value of `n_init`
explicitly to suppress the warning
    warnings.warn(
/home/kabalce/.local/lib/python3.10/site-
packages/sklearn/cluster/_kmeans.py:870: FutureWarning: The default value of
`n_init` will change from 10 to 'auto' in 1.4. Set the value of `n_init`
explicitly to suppress the warning
    warnings.warn(

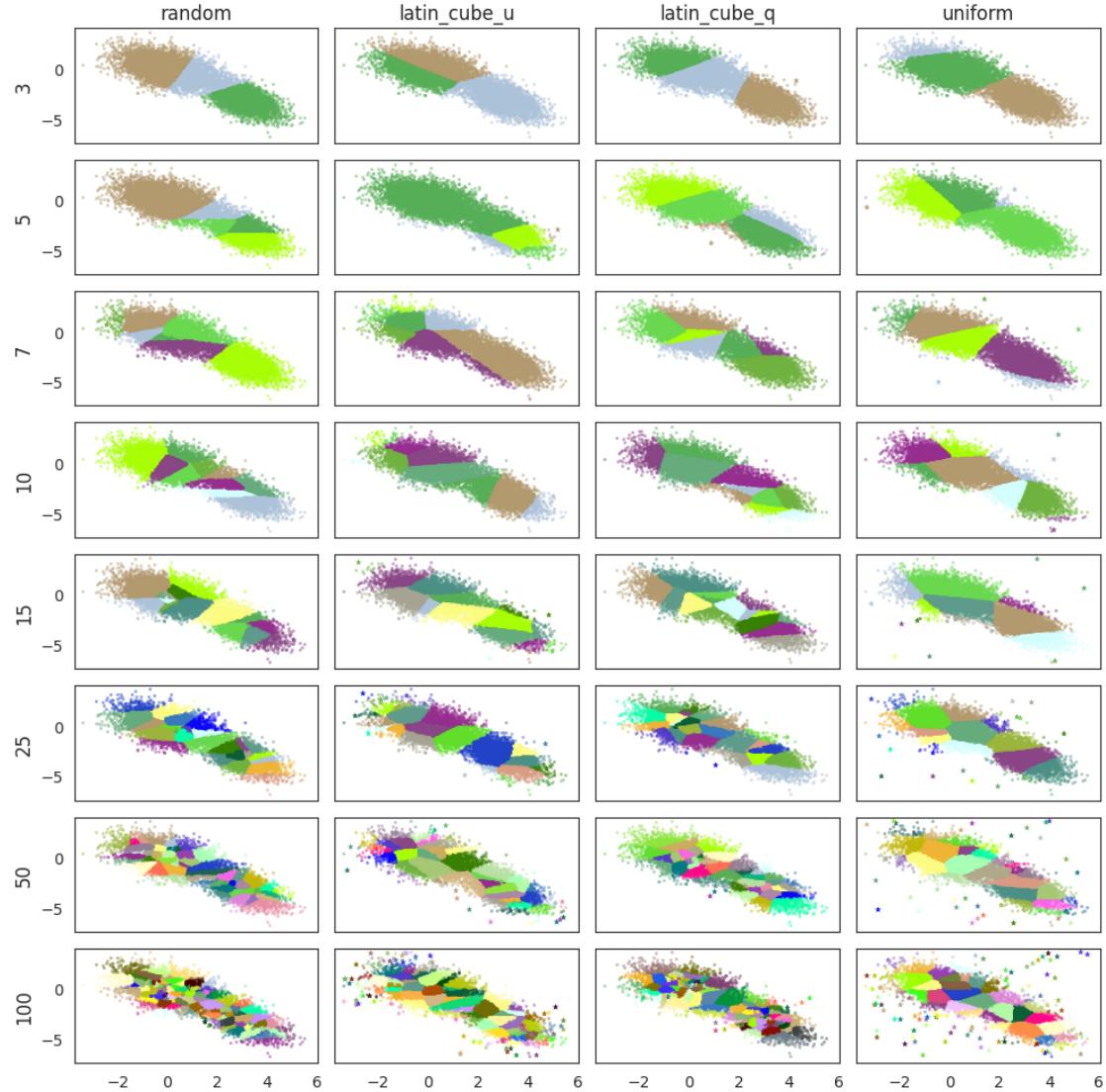
```



```

/tmp/ipykernel_54422/2508230348.py:60: RuntimeWarning: overflow encountered in
exp
    'likelihood-ratio': np.exp(models[discretizing, number_of_nodes]['ll'] -
target_ll)

```



	discretizing	number_of_nodes	accuracy	log-likelihood	likelihood-ratio
0	random	3	0.5167	-10653.845848	inf
24	uniform	3	0.5376	-8420.761914	inf
32	None	3	0.9785	-31238.463701	0.000000e+00
8	latin_cube_u	3	0.9638	-10225.405687	inf
16	latin_cube_q	3	0.5955	-10802.429404	inf

33	None	5	0.9784	-31238.464025	0.000000e+00
17	latin_cube_q	5	0.9746	-12755.709066	inf
9	latin_cube_u	5	0.8037	-6236.228979	inf
25	uniform	5	0.9477	-10488.945330	inf
1	random	5	0.5054	-12923.918276	inf
34	None	7	0.9785	-31238.463733	0.000000e+00
2	random	7	0.5887	-16590.904729	inf
10	latin_cube_u	7	0.8647	-13697.927303	inf
26	uniform	7	0.9674	-11951.778522	inf
18	latin_cube_q	7	0.9683	-17549.851860	inf
27	uniform	10	0.5067	-15738.118260	inf
35	None	10	0.9785	-31238.463722	0.000000e+00
19	latin_cube_q	10	0.9631	-18777.664443	inf
3	random	10	0.9679	-20638.595393	inf
11	latin_cube_u	10	0.5024	-17284.636475	inf
28	uniform	15	0.9733	-14440.207391	inf
20	latin_cube_q	15	0.9096	-24503.466741	inf
36	None	15	0.9785	-31238.463767	0.000000e+00
12	latin_cube_u	15	0.9531	-20772.747994	inf
4	random	15	0.5070	-24750.073196	inf
37	None	25	0.9784	-31238.463803	0.000000e+00
21	latin_cube_q	25	0.9668	-28989.460375	inf
13	latin_cube_u	25	0.9743	-22187.978081	inf
5	random	25	0.5064	-30762.212391	0.000000e+00
29	uniform	25	0.9699	-20019.398572	inf
6	random	50	0.9687	-37022.365478	0.000000e+00
14	latin_cube_u	50	0.9679	-30315.747051	1.445741e-163
30	uniform	50	0.5021	-24968.589975	inf
22	latin_cube_q	50	0.5177	-36312.866561	0.000000e+00
38	None	50	0.9784	-31238.464062	0.000000e+00
15	latin_cube_u	100	0.9692	-35216.275955	0.000000e+00
7	random	100	0.9686	-43866.580363	0.000000e+00
23	latin_cube_q	100	0.5036	-42650.353290	0.000000e+00
31	uniform	100	0.9685	-29644.652189	4.100963e+128
39	None	100	0.9784	-31238.463724	0.000000e+00

/tmp/ipykernel_54422/2508230348.py:83: FutureWarning: In future versions
`DataFrame.to_latex` is expected to utilise the base implementation of
`Styler.to_latex` for formatting and rendering. The arguments signature may
therefore change. It is recommended instead to use `DataFrame.style.to_latex`
which also contains additional functionality.

```

print(summary.to_latex())

\begin{tabular}{llrrrr}
\toprule
{} & discretizing & number\_of\_nodes & accuracy & log-likelihood &
likelihood-ratio \\
\midrule
0 & random & 0.5167 & -10653.845848 &

```

```

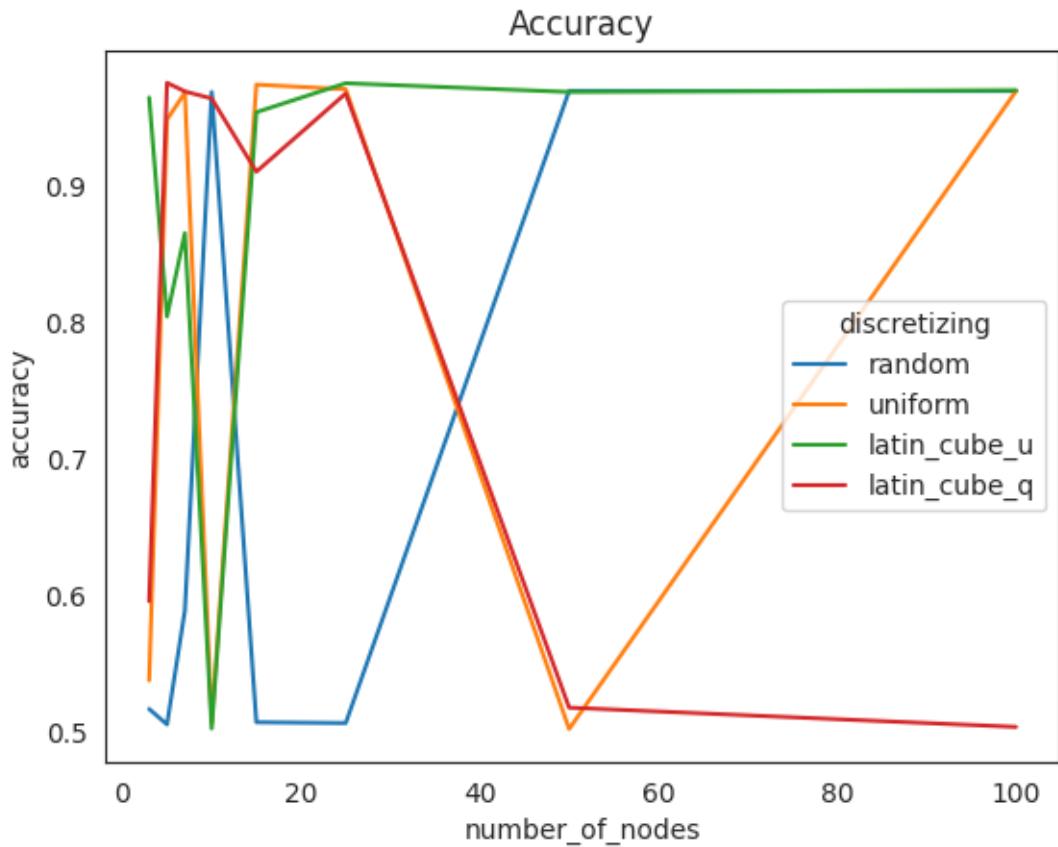
inf \\
24 &      uniform &            3 &    0.5376 &   -8420.761914 &
inf \\
32 &      None &            3 &    0.9785 &   -31238.463701 &
0.000000e+00 \\
8 &      latin\_cube\_u &            3 &    0.9638 &   -10225.405687 &
inf \\
16 &      latin\_cube\_q &            3 &    0.5955 &   -10802.429404 &
inf \\
33 &      None &            5 &    0.9784 &   -31238.464025 &
0.000000e+00 \\
17 &      latin\_cube\_q &            5 &    0.9746 &   -12755.709066 &
inf \\
9 &      latin\_cube\_u &            5 &    0.8037 &   -6236.228979 &
inf \\
25 &      uniform &            5 &    0.9477 &   -10488.945330 &
inf \\
1 &      random &            5 &    0.5054 &   -12923.918276 &
inf \\
34 &      None &            7 &    0.9785 &   -31238.463733 &
0.000000e+00 \\
2 &      random &            7 &    0.5887 &   -16590.904729 &
inf \\
10 &      latin\_cube\_u &            7 &    0.8647 &   -13697.927303 &
inf \\
26 &      uniform &            7 &    0.9674 &   -11951.778522 &
inf \\
18 &      latin\_cube\_q &            7 &    0.9683 &   -17549.851860 &
inf \\
27 &      uniform &            10 &   0.5067 &   -15738.118260 &
inf \\
35 &      None &            10 &   0.9785 &   -31238.463722 &
0.000000e+00 \\
19 &      latin\_cube\_q &            10 &   0.9631 &   -18777.664443 &
inf \\
3 &      random &            10 &   0.9679 &   -20638.595393 &
inf \\
11 &      latin\_cube\_u &            10 &   0.5024 &   -17284.636475 &
inf \\
28 &      uniform &            15 &   0.9733 &   -14440.207391 &
inf \\
20 &      latin\_cube\_q &            15 &   0.9096 &   -24503.466741 &
inf \\
36 &      None &            15 &   0.9785 &   -31238.463767 &
0.000000e+00 \\
12 &      latin\_cube\_u &            15 &   0.9531 &   -20772.747994 &
inf \\
4 &      random &            15 &   0.5070 &   -24750.073196 &

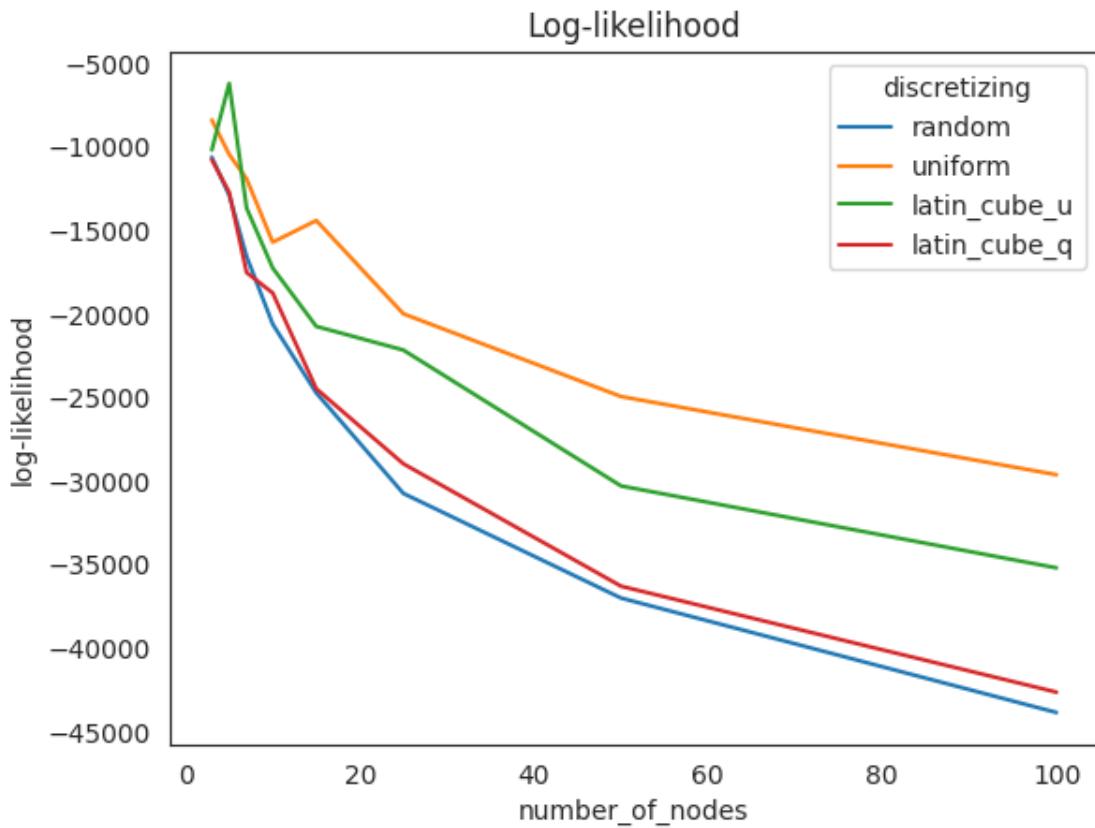
```

```

inf \\
37 & None & 25 & 0.9784 & -31238.463803 &
0.000000e+00 \\
21 & latin\_cube\_q & 25 & 0.9668 & -28989.460375 &
inf \\
13 & latin\_cube\_u & 25 & 0.9743 & -22187.978081 &
inf \\
5 & random & 25 & 0.5064 & -30762.212391 &
0.000000e+00 \\
29 & uniform & 25 & 0.9699 & -20019.398572 &
inf \\
6 & random & 50 & 0.9687 & -37022.365478 &
0.000000e+00 \\
14 & latin\_cube\_u & 50 & 0.9679 & -30315.747051 &
1.445741e-163 \\
30 & uniform & 50 & 0.5021 & -24968.589975 &
inf \\
22 & latin\_cube\_q & 50 & 0.5177 & -36312.866561 &
0.000000e+00 \\
38 & None & 50 & 0.9784 & -31238.464062 &
0.000000e+00 \\
15 & latin\_cube\_u & 100 & 0.9692 & -35216.275955 &
0.000000e+00 \\
7 & random & 100 & 0.9686 & -43866.580363 &
0.000000e+00 \\
23 & latin\_cube\_q & 100 & 0.5036 & -42650.353290 &
0.000000e+00 \\
31 & uniform & 100 & 0.9685 & -29644.652189 &
4.100963e+128 \\
39 & None & 100 & 0.9784 & -31238.463724 &
0.000000e+00 \\
\bottomrule
\end{tabular}

```





```

Recognized 1 instead of 3 states!
Recognized 2 instead of 3 states!

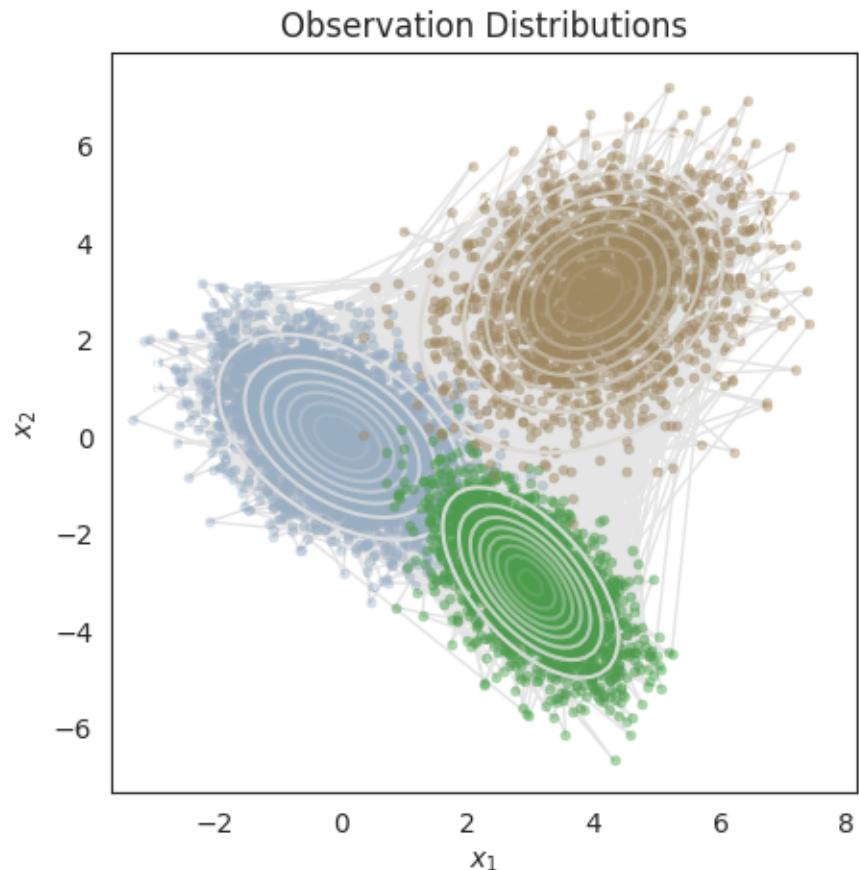
/home/kabalce/.local/lib/python3.10/site-
packages/sklearn/cluster/_kmeans.py:870: FutureWarning: The default value of
`n_init` will change from 10 to 'auto' in 1.4. Set the value of `n_init`
explicitly to suppress the warning
    warnings.warn(
/home/kabalce/.local/lib/python3.10/site-
packages/sklearn/cluster/_kmeans.py:870: FutureWarning: The default value of
`n_init` will change from 10 to 'auto' in 1.4. Set the value of `n_init`
explicitly to suppress the warning
    warnings.warn(
/home/kabalce/.local/lib/python3.10/site-
packages/sklearn/cluster/_kmeans.py:870: FutureWarning: The default value of
`n_init` will change from 10 to 'auto' in 1.4. Set the value of `n_init`
explicitly to suppress the warning
    warnings.warn(
/home/kabalce/.local/lib/python3.10/site-
packages/sklearn/cluster/_kmeans.py:870: FutureWarning: The default value of
`n_init` will change from 10 to 'auto' in 1.4. Set the value of `n_init`
```

```

```

explicitly to suppress the warning
 warnings.warn(
/home/kabalce/.local/lib/python3.10/site-
packages/sklearn/cluster/_kmeans.py:870: FutureWarning: The default value of
`n_init` will change from 10 to 'auto' in 1.4. Set the value of `n_init`
explicitly to suppress the warning
 warnings.warn(
/home/kabalce/.local/lib/python3.10/site-
packages/sklearn/cluster/_kmeans.py:870: FutureWarning: The default value of
`n_init` will change from 10 to 'auto' in 1.4. Set the value of `n_init`
explicitly to suppress the warning
 warnings.warn(
/home/kabalce/.local/lib/python3.10/site-
packages/sklearn/cluster/_kmeans.py:870: FutureWarning: The default value of
`n_init` will change from 10 to 'auto' in 1.4. Set the value of `n_init`
explicitly to suppress the warning
 warnings.warn(
/home/kabalce/.local/lib/python3.10/site-
packages/sklearn/cluster/_kmeans.py:870: FutureWarning: The default value of
`n_init` will change from 10 to 'auto' in 1.4. Set the value of `n_init`
explicitly to suppress the warning
 warnings.warn(

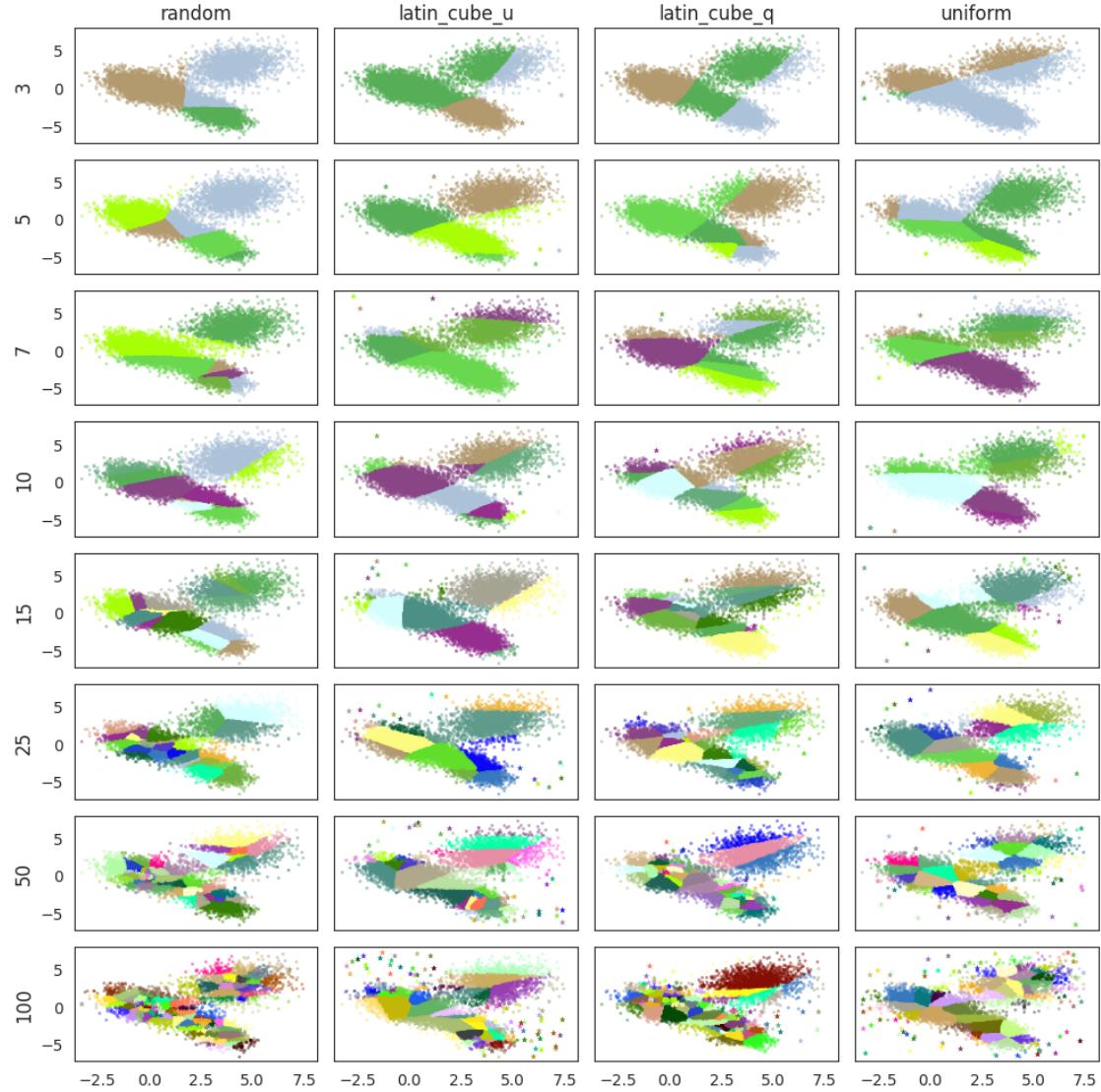
```



```

/tmp/ipykernel_54422/2508230348.py:60: RuntimeWarning: overflow encountered in
exp
 'likelihood-ratio': np.exp(models[discretizing, number_of_nodes]['ll'] -
target_ll)

```



|    | discretizing | number_of_nodes | accuracy | log-likelihood | likelihood-ratio |
|----|--------------|-----------------|----------|----------------|------------------|
| 0  | random       | 3               | 0.3929   | -9734.235502   | inf              |
| 24 | uniform      | 3               | -1.0000  | -6190.607674   | inf              |
| 32 | None         | 3               | 0.9798   | -36014.665851  | 0.0              |
| 8  | latin_cube_u | 3               | 0.7212   | -8020.677525   | inf              |
| 16 | latin_cube_q | 3               | 0.0780   | -10353.676142  | inf              |

|    |              |     |         |               |     |
|----|--------------|-----|---------|---------------|-----|
| 33 | None         | 5   | 0.9798  | -36014.666302 | 0.0 |
| 17 | latin_cube_q | 5   | 0.6425  | -12613.357425 | inf |
| 9  | latin_cube_u | 5   | 0.5992  | -10394.234514 | inf |
| 25 | uniform      | 5   | 0.5624  | -13247.434116 | inf |
| 1  | random       | 5   | 0.7036  | -13493.710351 | inf |
| 34 | None         | 7   | 0.9798  | -36014.665961 | 0.0 |
| 2  | random       | 7   | 0.3458  | -15966.347533 | inf |
| 10 | latin_cube_u | 7   | 0.6660  | -11941.825172 | inf |
| 26 | uniform      | 7   | 0.0816  | -12806.300765 | inf |
| 18 | latin_cube_q | 7   | 0.7290  | -15560.221864 | inf |
| 27 | uniform      | 10  | -1.0000 | -14301.671022 | inf |
| 35 | None         | 10  | 0.5782  | -38879.399410 | 0.0 |
| 19 | latin_cube_q | 10  | 0.6861  | -19070.000079 | inf |
| 3  | random       | 10  | 0.1064  | -19524.343464 | inf |
| 11 | latin_cube_u | 10  | 0.5547  | -14192.603193 | inf |
| 28 | uniform      | 15  | 0.0779  | -16770.941861 | inf |
| 20 | latin_cube_q | 15  | 0.9518  | -21762.900989 | inf |
| 36 | None         | 15  | 0.9798  | -36014.665804 | 0.0 |
| 12 | latin_cube_u | 15  | 0.0168  | -14603.931347 | inf |
| 4  | random       | 15  | 0.3157  | -25756.033016 | inf |
| 37 | None         | 25  | 0.9798  | -36014.665950 | 0.0 |
| 21 | latin_cube_q | 25  | 0.7114  | -26832.018039 | inf |
| 13 | latin_cube_u | 25  | 0.6224  | -18210.540645 | inf |
| 5  | random       | 25  | 0.7486  | -30566.408924 | inf |
| 29 | uniform      | 25  | 0.2896  | -22132.505126 | inf |
| 6  | random       | 50  | 0.1568  | -36470.680878 | 0.0 |
| 14 | latin_cube_u | 50  | 0.7406  | -25227.859146 | inf |
| 30 | uniform      | 50  | 0.3937  | -29850.697868 | inf |
| 22 | latin_cube_q | 50  | 0.5480  | -33461.231511 | inf |
| 38 | None         | 50  | 0.0096  | -36014.666122 | 0.0 |
| 15 | latin_cube_u | 100 | 0.6954  | -27766.208585 | inf |
| 7  | random       | 100 | 0.7436  | -43778.138035 | 0.0 |
| 23 | latin_cube_q | 100 | 0.7044  | -36204.275877 | 0.0 |
| 31 | uniform      | 100 | 0.9473  | -30346.423885 | inf |
| 39 | None         | 100 | 0.9798  | -36014.666352 | 0.0 |

/tmp/ipykernel\_54422/2508230348.py:83: FutureWarning: In future versions  
`DataFrame.to\_latex` is expected to utilise the base implementation of  
`Styler.to\_latex` for formatting and rendering. The arguments signature may  
therefore change. It is recommended instead to use `DataFrame.style.to\_latex`  
which also contains additional functionality.

```

print(summary.to_latex())

\begin{tabular}{llrrrr}
\toprule
{} & discretizing & number_of_nodes & accuracy & log-likelihood & \\
& likelihood-ratio \\
\midrule
0 & random & 3 & 0.3929 & -9734.235502 &

```

```

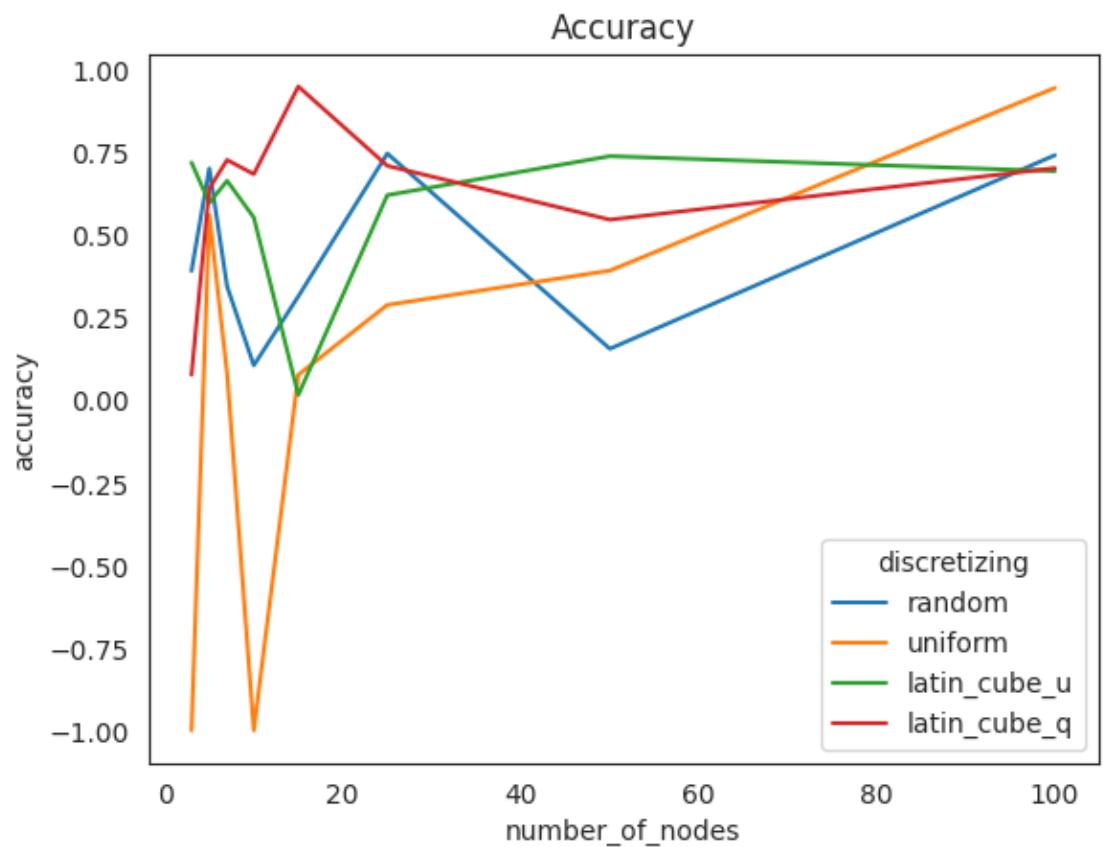
inf \\
24 & uniform & 3 & -1.0000 & -6190.607674 &
inf \\
32 & None & 3 & 0.9798 & -36014.665851 &
0.0 \\
8 & latin_cube_u & 3 & 0.7212 & -8020.677525 &
inf \\
16 & latin_cube_q & 3 & 0.0780 & -10353.676142 &
inf \\
33 & None & 5 & 0.9798 & -36014.666302 &
0.0 \\
17 & latin_cube_q & 5 & 0.6425 & -12613.357425 &
inf \\
9 & latin_cube_u & 5 & 0.5992 & -10394.234514 &
inf \\
25 & uniform & 5 & 0.5624 & -13247.434116 &
inf \\
1 & random & 5 & 0.7036 & -13493.710351 &
inf \\
34 & None & 7 & 0.9798 & -36014.665961 &
0.0 \\
2 & random & 7 & 0.3458 & -15966.347533 &
inf \\
10 & latin_cube_u & 7 & 0.6660 & -11941.825172 &
inf \\
26 & uniform & 7 & 0.0816 & -12806.300765 &
inf \\
18 & latin_cube_q & 7 & 0.7290 & -15560.221864 &
inf \\
27 & uniform & 10 & -1.0000 & -14301.671022 &
inf \\
35 & None & 10 & 0.5782 & -38879.399410 &
0.0 \\
19 & latin_cube_q & 10 & 0.6861 & -19070.000079 &
inf \\
3 & random & 10 & 0.1064 & -19524.343464 &
inf \\
11 & latin_cube_u & 10 & 0.5547 & -14192.603193 &
inf \\
28 & uniform & 15 & 0.0779 & -16770.941861 &
inf \\
20 & latin_cube_q & 15 & 0.9518 & -21762.900989 &
inf \\
36 & None & 15 & 0.9798 & -36014.665804 &
0.0 \\
12 & latin_cube_u & 15 & 0.0168 & -14603.931347 &
inf \\
4 & random & 15 & 0.3157 & -25756.033016 &

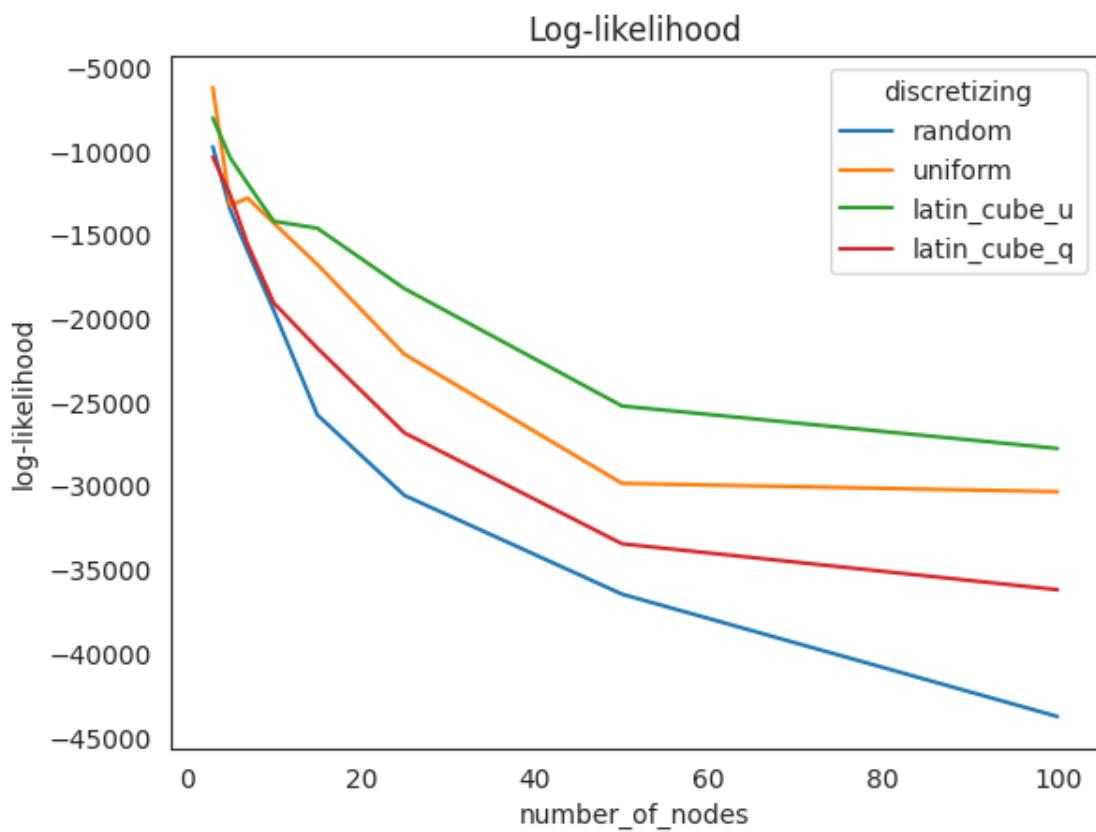
```

```

inf \\
37 & None & 25 & 0.9798 & -36014.665950 &
0.0 \\
21 & latin_cube_q & 25 & 0.7114 & -26832.018039 &
inf \\
13 & latin_cube_u & 25 & 0.6224 & -18210.540645 &
inf \\
5 & random & 25 & 0.7486 & -30566.408924 &
inf \\
29 & uniform & 25 & 0.2896 & -22132.505126 &
inf \\
6 & random & 50 & 0.1568 & -36470.680878 &
0.0 \\
14 & latin_cube_u & 50 & 0.7406 & -25227.859146 &
inf \\
30 & uniform & 50 & 0.3937 & -29850.697868 &
inf \\
22 & latin_cube_q & 50 & 0.5480 & -33461.231511 &
inf \\
38 & None & 50 & 0.0096 & -36014.666122 &
0.0 \\
15 & latin_cube_u & 100 & 0.6954 & -27766.208585 &
inf \\
7 & random & 100 & 0.7436 & -43778.138035 &
0.0 \\
23 & latin_cube_q & 100 & 0.7044 & -36204.275877 &
0.0 \\
31 & uniform & 100 & 0.9473 & -30346.423885 &
inf \\
39 & None & 100 & 0.9798 & -36014.666352 &
0.0 \\
\bottomrule
\end{tabular}

```





```
[14]: "Done"
```

```
[14]: 'Done'
```

```
[]:
```