CST 3613 Java Application Development
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HW 2

Homework 2

THE MATRIX AS A 2-DIMENSIONAL NUMERIC ARRAY

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General

- Some of the problems, we are going to solve in our course are much easier to understand if we could represent them graphical.
- Later in our course, we will explore Processing which is a graphical software program that will allow us to visualize our problem solving.
  - https://processing.org/
- To get a solid foundation of how we will use Processing to solve these more complex problems, we need a good understanding of vectors and matrices.
- In this exercise, we explore matrices as natural extension of a 2D array and build a collection of tools that we can use when we get to Processing.

Understanding 2-Dimensional Arrays

- In computer science, we use 2-Dimensional arrays a lot.
- We often refer to them as matrices when they contain numeric values.
- Matrices can be square or rectangular.
  - A 2D array in which the **length of the array is different from the length of the individual arrays** in the 2D array is a **non-square (rectangular) array**. This may be specified as (row x column) 3 x 2, 2 x 4 etc.
  - A 2D array in which the **length of the array is same as the length of each individual array** in the 2D array is a **square array**. This may be specified as (row x column) 3 x 3, 4 x 4 etc.
- A 2D array in which the **length of the array is different from the length of the individual arrays and the individual arrays have different lengths** is a **ragged array**.

	0	1
0	1	2
1	4	5
2	7	8

Rectangular

	0	1	2
0	1	2	3
1	4	5	6

Rectangular

	0	1	2
0	1	2	3
1	4	5	6
2	7	8	9

Square

Not A Matrix

	0	1	2	3
0	1	2	3	
1	4	5		
2	7	8	9	10

Ragged

Describing Matrices

- A matrix consist of rows and columns and its dimensions are written as a combinations of rows and columns.

	0	1	2
0	1	2	3
1	4	5	6
2	7	8	9

3 x 3

V0

0	1	2
1	2	3

V1

0	1	2
4	5	6

V2

0	1	2
7	8	9

We can think of the rows in the matrix as individual vectors.

…we might, for practical purposes, think of the columns as vectors but mathematically this is not true.

	0	1
0	1	2
1	4	5
2	7	8

3 x 2

0	1
1	2

V0

0	1
4	5

V1

0	1
7	8

V2

Rows and Columns

	0	1
0	1	2
1	4	5
2	7	8

A

The columns in A are represented by the indices:
[0][0] [1][0] [2][0] …Column 0
[0][1] [1][1] [2][1] …Column 1

	0	1	2
0	1	2	3
1	4	5	6

B

The columns in B are represented by the indices:
[0][0] [1][0] …Column 0
[0][1] [1][1] …Column 1
[0][2] [1][2] …Column 2

Notice the relationship between indices when specifying a column.

	0	1	2
0	1	2	3
1	4	5	6
2	7	8	9

C

The columns in C are represented by the indices:
[0][0] [1][0] [2][0] …Column 0
[0][1] [1][1] [2][1] …Column 1
[0][2] [1][2] [2][2] …Column 2

…and of course, the rows in any matrix are just the individual arrays in the 2D Array.

Main Diagonal and Transpose

- <u>Square</u> matrices have a main diagonal.
- The sum of the values on the main diagonal is known as the Trace of the matrix.

	0	1	2
0	1	2	3
1	4	5	6
2	7	8	9

The main diagonal for this matrix is represented by the indices:
[0][0]   [1][1]   [2][2] …main diagonal

The Trace of the matrix is 15.

- If we switch the rows with the columns, we create a Transpose of a matrix

	0	1	2
0	1	2	3
1	4	5	6
2	7	8	9

	0	1	2
0	1	4	7
1	2	5	8
2	3	6	9

	0	1
0	1	2
1	4	5
2	7	8

	0	1	2
0	1	4	7
1	2	5	8

Identical and Symmetric Matrices

- Two matrices are identical if they have the same elements at the same indices.
- A matrix is symmetric if it is equal to its transpose.

	0	1	2
0	1	2	3
1	4	5	6
2	7	8	9

A

	0	1	2
0	1	2	3
1	4	5	6
2	7	8	9

B

Identical

	0	1	2
0	1	7	3
1	7	4	5
2	3	5	0

A

	0	1	2
0	1	7	3
1	7	4	5
2	3	5	0

A^T

Symmetric

Preliminary Tasks

Create a static method that:

1. Returns true if a matrix is square.
2. Returns the dimensions of a matrix.
3. Prints a matrix to the console in matrix form.
4. Returns a square integer matrix of a specified length whose elements are random integers between 1 and 9.
5. Returns a rectangular integer matrix with a specified dimensions whose elements are random integers between 1 and 9.
6. Returns a specified row from a matrix.
7. Returns a specified column from a matrix.
8. Returns the main diagonal from a square matrix.
9. Returns the Transpose of a matrix.
10. Returns the Trace of a square matrix.
11. Return true if two matrices are identical.
12. Return true if a matrix is symmetric.

Matrix Operations

	0	1	2
0	1	2	3
1	4	5	6
2	7	8	9

A

	0	1	2
0	6	5	4
1	3	2	1
2	7	1	1

B

A+B

	0	1	2
0	7	7	7
1	7	7	7
2	14	9	10

	0	1	2
0	1	2	3
1	4	5	6
2	7	8	9

A

	0	1	2
0	6	5	4
1	3	2	1
2	7	1	1

B

A-B

	0	1	2
0	-5	-3	-1
1	1	3	5
2	0	7	8

…if matrices have the same dimensions they can be added or subtracted.

Matrix Operations

	0	1	2
0	1	2	3
1	4	5	6
2	7	8	9

A

	0	1	2
0	3	6	9
1	12	15	18
2	21	24	27

3A

…We can scale a matrix by multiplying each element by some scalar (a value)

	0	1	2
0	3	6	9
1	9	12	15
2	18	21	27

A

	0	1	2
0	1	2	3
1	3	4	5
2	6	7	8

1/3A

…there is no such thing as division of a matrix, but we can scale a matrix by the reciprocal.

Matrix Operations

	0	1	2
0	1	2	3
1	4	5	6
2	7	8	9

A

	0	1	2
0	2	3	1
1	1	2	3
2	3	1	2

B

	0	1	2
0	13	10	13
1	31	28	31
2	49	46	49

AB

	0	1	2
0	1 x 2 2 x 1 3 x 3 **13**	1 x 3 2 x 2 3 x 1 **10**	1 x 1 2 x 3 3 x 2 **13**
1	4 x 2 5 x 1 6 x 3 **31**	4 x 3 5 x 2 6 x 1 **28**	4 x 1 5 x 3 6 x 2 **31**
2	7 x 2 8 x 1 9 x 3 **49**	7 x 3 8 x 2 9 x 1 **46**	7 x 1 8 x 3 9 x 2 **49**

…We can multiply matrices, but the process has some specific rules.

Matrices must have compatible dimensions and involves rows being multiplied by columns

	0	1	2
0	R1C1	R1C2	R1C3
1	R2C1	R2C2	R2C3
2	R3C1	R3C2	R3C3

Matrix Operations

	0	1
0	1	2
1	4	5
2	7	8

3 X 2

	0	1	2
0	2	3	1
1	1	2	3

2 X 3

	0	1	2
0	4	7	7
1	13	22	19
2	22	37	31

AB

Matrices must have compatible dimensions and involves rows being multiplied by columns

- For matrix multiplication, the number of columns in the first matrix must be equal to the number of rows in the second matrix.
- The resulting matrix has the number of rows of the first and the number of columns of the second matrix.

3 X 2 2 X 3

3 X 3

	0	1	2
0	1 x 2 2 x 1 **4**	1 x 3 2 x 2 **7**	1 x 1 2 x 3 **7**
1	4 x 2 5 x 1 **13**	4 x 3 5 x 2 **22**	4 x 1 5 x 3 **19**
2	7 x 2 8 x 1 **22**	7 x 3 8 x 2 **37**	7 x 1 8 x 3 **31**

Matrix Operations

	0	1	2
0	1	2	3
1	4	5	1

2 X 3

	0
0	1
1	4
2	7

3 X 1

	0
0	30
1	31

AB

	0
0	1 x 1 2 x 4 3 x 7 **30**
1	4 x 1 5 x 4 1 x 7 **31**

Matrices must have compatible dimensions and involves rows being multiplied by columns

- For matrix multiplication, the number of columns in the first matrix must be equal to the number of rows in the second matrix.
- The resulting matrix has the number of rows of the first and the number of columns of the second matrix.

2 X 3 **3 X 1**

2 X 1

Matrix Tasks

Create a static method that:

1. Returns true if two matrices a compatible for a specified operation.
2. Returns the matrix that results from adding two matrices.
3. Returns the matrix that results from subtracting two matrices.
4. Returns the matrix that results from scaling a matrix by a specified scalar.
5. Returns the matrix that results from multiplying two matrices.