

Uncertainty in Coastal Ocean Modeling

Clint Dawson



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Motivating Problem

- Coastal Ocean Modeling
- Hurricane Storm Surge
- The Stochastic Inverse Problem
- Approximation

2

A Case Study: The Idealized Inlet

- Parameter Domain
- Data Domain
- The Condition of the Stochastic Inverse Problem
- Results

3

A Case Study: Hurricane Gustav

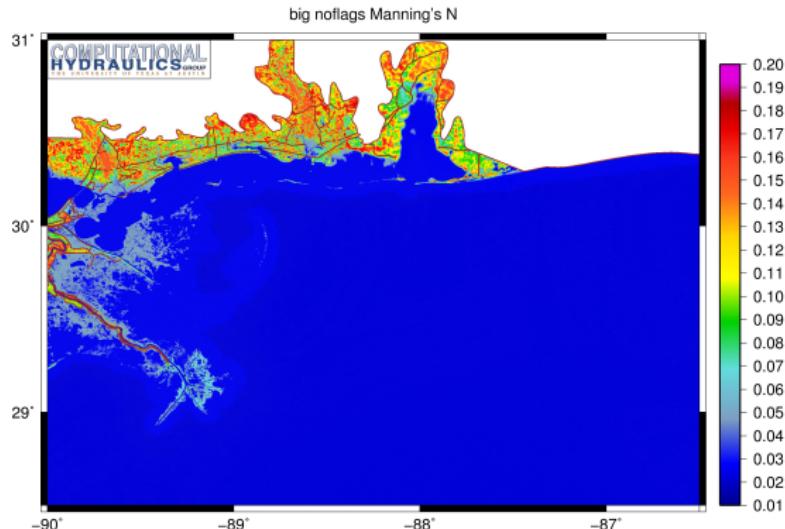
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Motivation

Coastal Ocean Modeling

- Oil transport
- Sediment transport
- Dredging feasibility
- **Hurricane storm surge**



Hurricane Storm Surge

Hurricane Katrina -- 2005

- Category 5
- 1833 fatalities
- \$108 billion in damage
- Costliest US hurricane

(Wik, 2012b, Craft, 2011, Blake et al., 2007)



Hurricane Ike -- 2008

- Category 4
- 195 fatalities
- \$37.6 billion in damage
- 3rd Costliest US hurricane

(Wik, 2012a, Craft, 2011, Blake et al., 2007)



Advanced Circulation (ADCIRC) Model

- Finite element in space, finite-difference in time
- Validated on Hurricane Ike, Katrina, Gustav, Rita...
- Fortran, MPI, C/C++
- storm surge
- tidal and wind driven circulation
- oil movement
- near shore coastal development studies

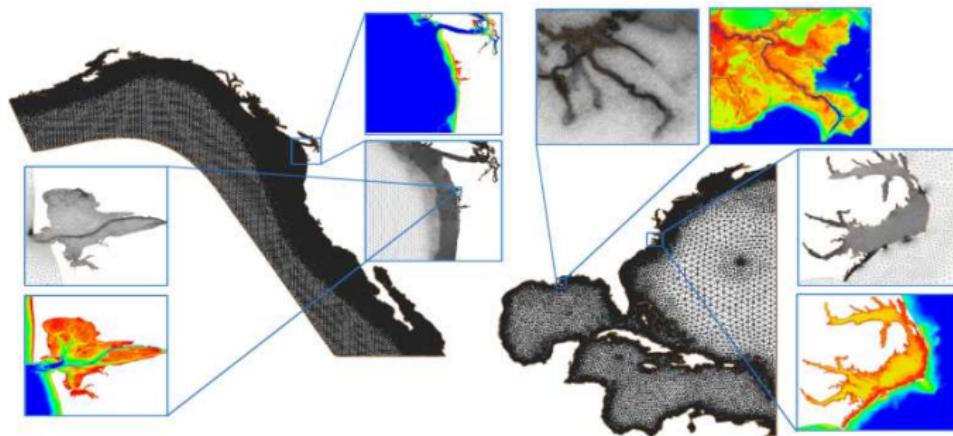


Figure : ADCIRC Coastal Circulation and Storm Surge Model adcirc.org



Shallow Water Equations

“Storm surge is primarily a competition between wind forcing and frictional resistance.” (Dawson et al., 2011)

$$\frac{dU}{dt} - fV = -g \frac{\partial[\zeta + P_s/g\rho_0 - \alpha\eta]}{\partial x} + \frac{\tau_{sx}}{\rho_0 H} - \frac{\tau_{bx}}{H\rho_0} + \frac{M_x}{H} - \frac{D_x}{H} - \frac{B_x}{H}$$
$$\frac{dV}{dt} + fU = -g \frac{\partial[\zeta + P_s/g\rho_0 - \alpha\eta]}{\partial y} + \frac{\tau_{sy}}{\rho_0 H} - \frac{\tau_{by}}{H\rho_0} + \frac{M_y}{H} - \frac{D_y}{H} - \frac{B_y}{H}$$

	Bottom stress components		Applied free-surface stress
ζ	Free-surface elevation relative to the geoid	τ_{sx}, τ_{sy}	Gravitational acceleration
H	$\zeta + h$ = water depth	g	Bathymetric depth
P_s	Atmospheric pressure at the free surface	ρ_0	Reference density of water
α	Effective earth elasticity factor	η	Newtonian equilibrium tide potential
f	Coriolis coefficient	U, V	Depth-averaged horizontal velocities
M_x, M_y	Vert.-integrated lateral stress gradient	D_x, D_y	Momentum Dispersion
B_x, B_y	Vert.-integrated baroclinic pressure gradient		

(Luettich and Westerink, 2004)



Modeling bottom friction with a spatially varying **Manning's n** value

Bottom friction

$$\frac{\tau_{bx}}{\rho_0} = C_d U \sqrt{U^2 + V^2}; \quad \frac{\tau_{by}}{\rho_0} = C_d V \sqrt{U^2 + V^2}$$

Drag Coefficient

$$C_d(t) = \frac{g n^2}{(h + \eta(t))^{1/3}} = \frac{g n^2}{H^{1/3}}$$

(Luettich and Westerink, 2010, Yen, 2002, 2004, Leopold et al., 1964, Chow, 2008).



NOAA C-CAP Land Coverage and Classification

The Manning's n coefficient is a highly variable spatial parameter often defined on a sub-grid scale, e.g. using land classification data defined pixel-by-pixel from high resolution imagery.



<http://www.csc.noaa.gov/crs/lca/gulfcoast.html>

Developed - High Intensity
Developed - Medium Intensity
Developed - Low Intensity
Developed - Open Space
Cultivated
Pasture / Hay
Grassland
Deciduous Forest
Evergreen Forest
Mixed Forest
Scrub / Shrub
Palustrine Forested Wetland
Palustrine Scrub / Shrub Wetland
Palustrine Emergent Wetland
Estuarine Forested Wetland
Estuarine Scrub / Shrub Wetland
Estuarine Emergent Wetland
Unconsolidated Shore
Bare Land
Water
Palustrine Aquatic Bed
Estuarine Aquatic Bed
Tundra
Snow / Ice

Table: Manning's n values for LA-GAP classification

2	0.120	High Intensity Developed
3	0.100	Medium Intensity Developed
4	0.070	Low Intensity Developed
5	0.035	Developed Open Space
6	0.100	Cultivated Land
7	0.055	Pasture/Hay
8	0.035	Grassland
9	0.160	Deciduous Forest
10	0.180	Evergreen Forest
11	0.170	Mixed Forest
12	0.080	Scrub/Shrub
13	0.150	Palustrine Forested Wetland
14	0.075	Palustrine Scrub/Shrub Wetland
15	0.070	Palustrine Emergent Wetland
16	0.150	Estuarine Forested Wetland
17	0.070	Estuarine Scrub/Shrub Wetland
18	0.050	Estuarine Emergent Wetland
19	0.030	Unconsolidated Shore
20	0.030	Bare Land
21	0.025	Open Water
22	0.035	Palustrine Aquatic Bed
23	0.030	Estuarine Aquatic Bed

<http://www.csc.noaa.gov/crs/lca/gulfcoast.html>



The Stochastic Inverse Problem for ADCIRC

Given

- $\Lambda \subset \mathbb{R}^m$, a compact **parameter domain** with a specified metric
(e.g. a finite dimensional representation of possible Manning's n fields),
- $M(Y, \lambda)$ with a solution $Y = Y(\lambda)$, a deterministic **physics based model**
(e.g. the Advanced Circulation Model of Oceanic, Coastal and Estuarine Waters (ADCIRC)),
- $Q : \Lambda \rightarrow \mathcal{D}$, a map composed of **quantities of interest** (QoI)
 $Q(\lambda) = (q_1(\lambda), \dots, q_d(\lambda)) \in \mathcal{D} \subset \mathbb{R}^d$ where $q_i(\lambda) := q_i(Y(\lambda))$, for all $\lambda \in \Lambda$ where $d \leq m$
(e.g. maximum water elevation at d observation stations),
- and an observed (or assumed) **probability density** $p_{\mathcal{D}}$ on $\mathcal{B}_{\mathcal{D}}$ for the data domain \mathcal{D} .

Assume $\mathcal{D} := Q(\Lambda)$ and that the Jacobian of $Q(\lambda)$ has full rank a.e. in Λ .



The Stochastic Inverse Problem for ADCIRC

Determine

- the **probability density** ρ_Λ on \mathcal{B}_Λ for the parameter domain Λ .

Use ρ_Λ to

- compute **probabilities**, $(P(A))$, of arbitrary measurable events $A \in \mathcal{B}_\Lambda$
(Borel σ -algebra on Λ)
(e.g. the probability of sets of Manning's n fields),
- identify highly probable regions of Λ (**parameter estimation**)
(e.g. the most probable set of Manning's n fields),
- etc.

This information is useful for forward uncertainty quantification, parameter estimation, inverse sensitivity analysis, and identification of inverse sets.



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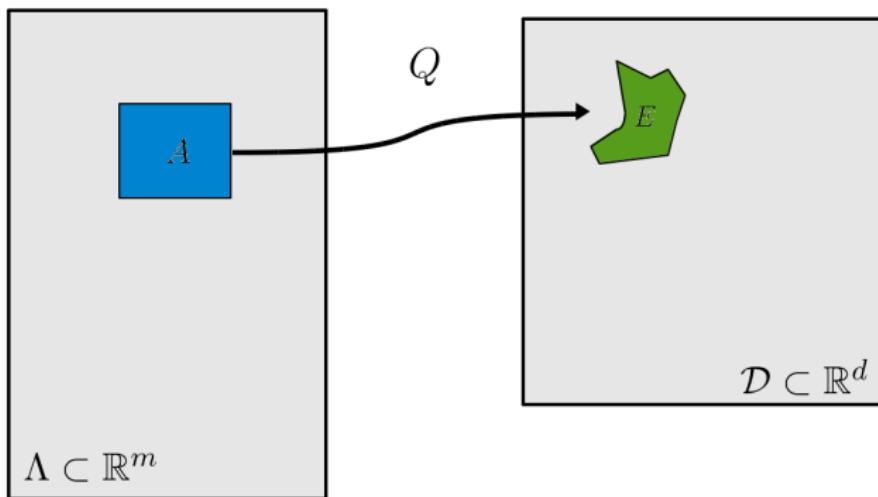
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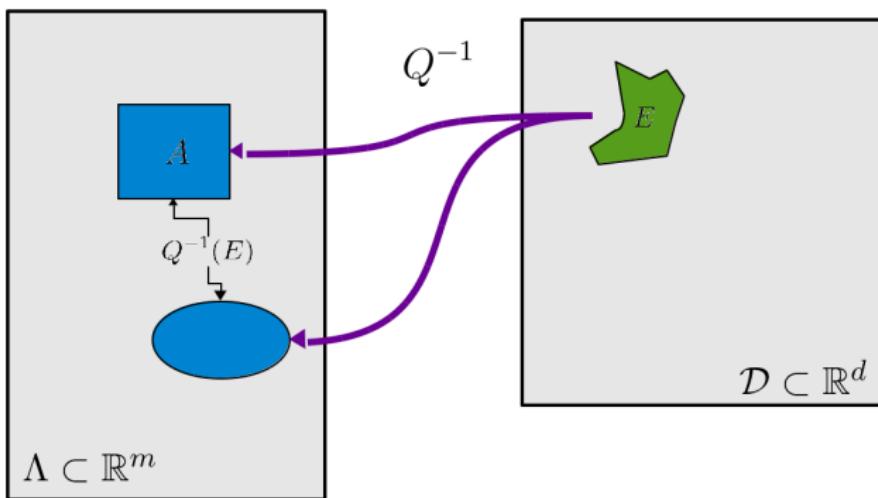
Forward Problem

Given a QoI map $Q : \Lambda \rightarrow \mathcal{D}$, a set of parameters $A \in \mathcal{B}_\Lambda$, and a measure μ_Λ . Determine the set of data $E \in \mathcal{B}_{\mathcal{D}}$ such that $Q(A) = E$ where the push-forward measure is $\mu_{\mathcal{D}}(E) = \int_E d\mu_{\mathcal{D}} = \int_{Q^{-1}(E)} d\mu_\Lambda = \mu_\Lambda(Q^{-1}(E))$.



Inverse Problem

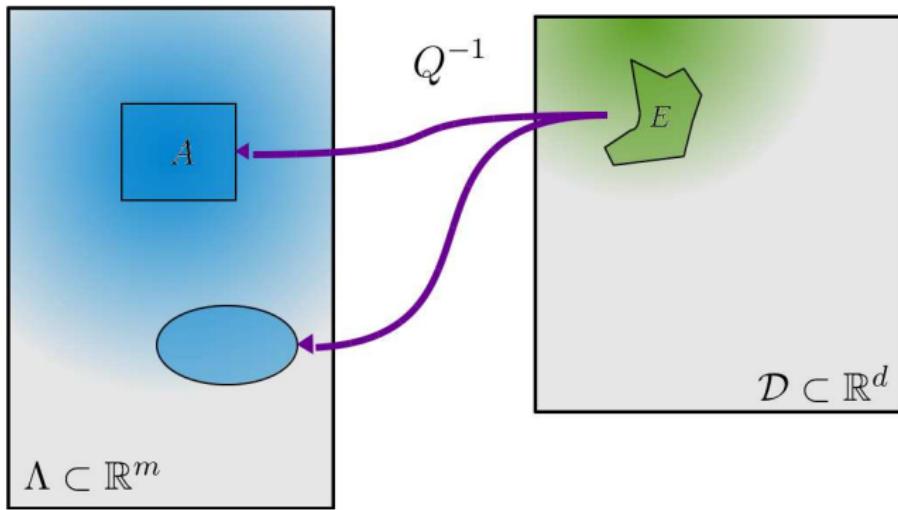
Given a QoI map $Q : \Lambda \rightarrow \mathcal{D}$, a set of data $E \in \mathcal{B}_{\mathcal{D}}$, and a measure μ_{Λ} . Determine the set of parameters $A \in \mathcal{B}_{\Lambda}$ such that $Q^{-1}(E) = A$ where $\mu_{\Lambda}(A) = \int_A d\mu_{\Lambda} = \int_{Q(A)} d\mu_{\mathcal{D}} = \mu_{\mathcal{D}}(Q(A))$.



Inverse Stochastic Problem

Given a QoI map $Q : \Lambda \rightarrow \mathcal{D}$, a set of parameters $A \in \mathcal{B}_\Lambda$, a measure μ_Λ , and a density $\rho_{\mathcal{D}}$ on the data space. Determine and use ρ_Λ to compute probabilities of events.

$$P_\Lambda(A) = \int_A \rho_\Lambda d\mu_\Lambda = \int_{Q(A)} \rho_{\mathcal{D}} d\mu_{\mathcal{D}} = P_{\mathcal{D}}(Q(A)).$$



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Sample Based Approximation of P_Λ

Theorem

Given

- $Q : \Lambda \rightarrow \mathcal{D}$, a QoI map;
- μ_Λ , the measure on the parameter space Λ ;
- $\rho_{\mathcal{D}}$, a probability density on $(\mathcal{D}, \mathcal{B}_{\mathcal{D}})$, absolutely continuous with respect to $\mu_{\mathcal{D}}(\cdot) = \mu_\Lambda(Q^{-1}(\cdot))$;
- and an event $A \in \mathcal{B}_\Lambda$

there exists a sequence of simple function approximations to probability densities, $\rho_{\Lambda,N}$ and $\rho_{\mathcal{D},M}$, requiring only calculations of volumes such that

$$P_{\Lambda,N}(A) \rightarrow P_\Lambda(A) \text{ as } N, M \rightarrow \infty$$

(Butler et al., 2014).



A Case Study: The Idealized Inlet



The Stochastic Inverse Problem for ADCIRC

Given

- $\Lambda \subset \mathbb{R}^m$, a compact **parameter domain** with a specified metric
(e.g. a finite dimensional representation of possible Manning's n fields),
- $M(Y, \lambda)$ with a solution $Y = Y(\lambda)$, a deterministic **physics based model**
(e.g. the Advanced Circulation Model of Oceanic, Coastal and Estuarine Waters (ADCIRC)),
- $Q : \Lambda \rightarrow \mathcal{D}$, a map composed of **quantities of interest** (QoI)
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(e.g. maximum water elevation at d observation stations),
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The Stochastic Inverse Problem for ADCIRC

Determine

- the **probability density** ρ_Λ on \mathcal{B}_Λ for the parameter domain Λ .

Use ρ_Λ to

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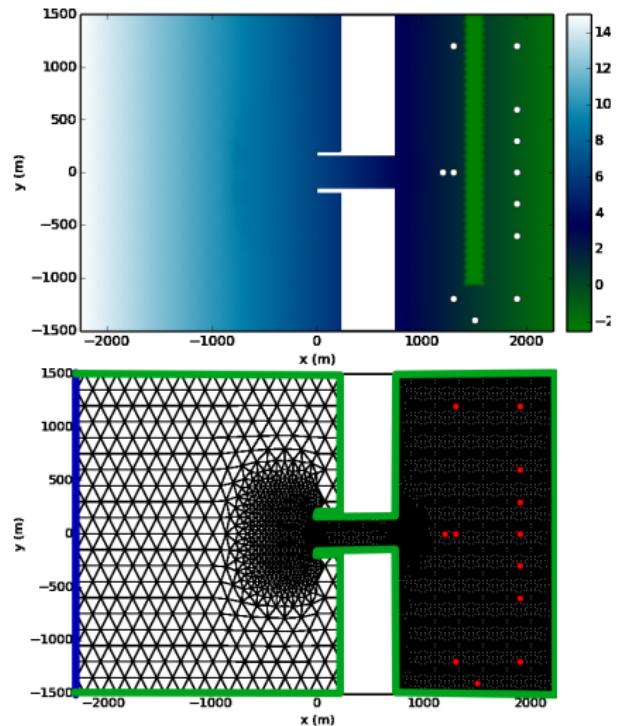
This information is useful for forward uncertainty quantification, parameter estimation, inverse sensitivity analysis, and identification of inverse sets.



$M(Y, \lambda)$ with a solution $Y = Y(\lambda)$, a deterministic **physics based model**
(e.g. the Advanced Circulation Model of Oceanic, Coastal and Estuarine Waters
(ADCIRC))



Computational Domain



- Open ocean boundary with M_2 tidal amplitude of $1.2[m/s]$
- Land boundary ($\mathbf{u} \cdot \mathbf{n} = 0$) and no slip
- Earthen jetty between $x = 1420[m], 1580[m]$, and $y = 1500[m], -1050[m]$.



Define $\Lambda \subset \mathbb{R}^m$, a compact **parameter domain** with a specified metric
(e.g. a finite dimensional representation of possible Manning's n fields)

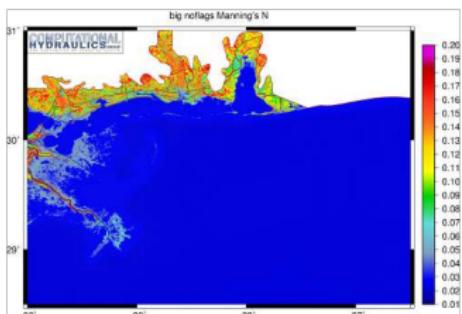


Parameter Domain

Mapping Manning's n onto a computational mesh



spatial average
→
Manning's n table

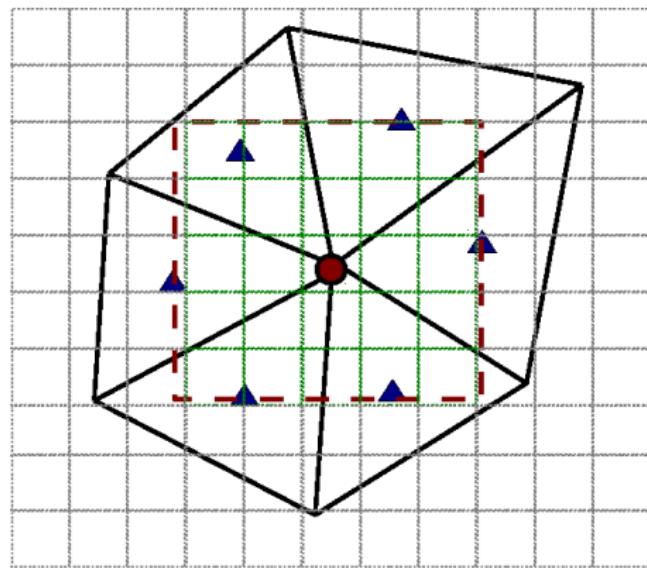


A Mesoscale representation of Manning's n

We can construct a set of land classification meshes $\{\mathbf{b}_i\}_{i=1}^m$ such that

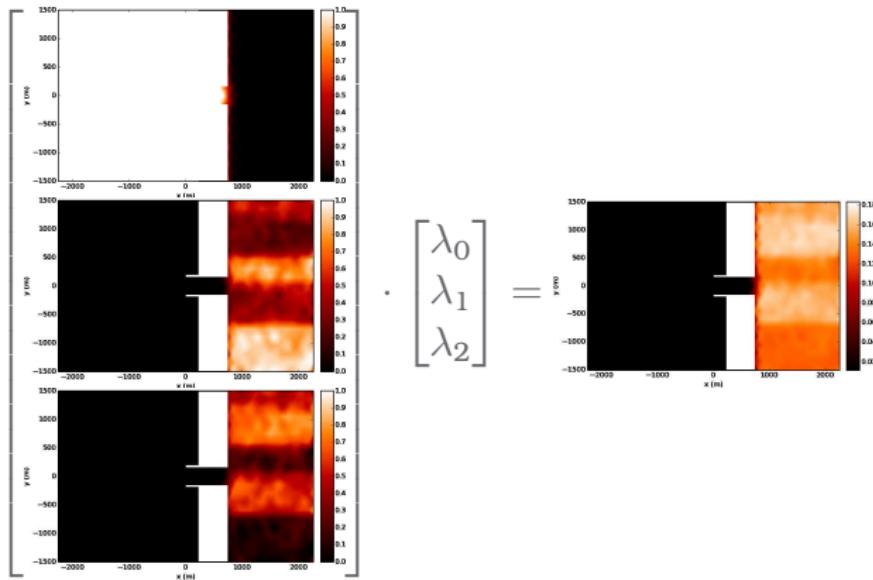
$$\mathbf{n} = [\mathbf{b}_1 \dots \mathbf{b}_m] \cdot \lambda$$

$$\mathbf{n} = \{n_k\}_{k=1}^{N_{nodes}}$$



A Mesoscale representation of Manning's n

Let $\lambda = \{\lambda_i\}_{i=1}^m \in \mathbb{R}^{+m}$, where λ_i is the Manning's n value associated with the i^{th} land classification and m is the total number of land classifications.



A Mesoscale representation of Manning's n

We construct a set of land classification meshes $\{\mathbf{b}_i\}_{i=1}^m$ such that

$$\mathbf{n} = [\mathbf{b}_1 \dots \mathbf{b}_m] \cdot \lambda$$

$$\mathbf{n} = \{n_k\}_{k=1}^{N_{nodes}}.$$

The Manning's n value at the k^{th} node in the mesh is n_k .

Parameter	Range of Manning's n values	Land Classification
λ_0	0.012	Water
λ_1	[0.07, 0.15]	Wetland
λ_2	[0.1, 0.2]	Forest

So the compact finite dimensional parameter domain is
 $\Lambda = [0.07, 0.15] \times [0.1, 0.2] \subset \mathbb{R}^2$.

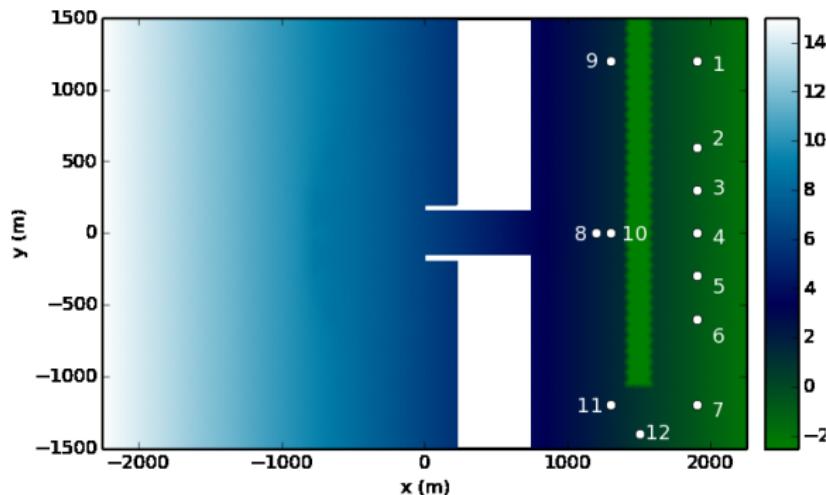


Quantities of interest (QoI) $\mathbf{Q}(\lambda) = (q_1(\lambda), \dots, q_d(\lambda)) \in \mathcal{D} \subset \mathbb{R}^d$ where
 $q_i(\lambda) := q_i(Y(\lambda))$, for all $\lambda \in \Lambda$
(e.g. maximum water elevation at d observation stations)



Data Domain

Observation Stations $q_i(\lambda) := q_i(Y(\lambda))$ is the maximum water elevation at the i^{th} observation station.



Data Domain

Choose a set of QoI to define \mathcal{D} and define $\rho_{\mathcal{D}}$.

Prescribe an observed (or assumed) **probability density** $\rho_{\mathcal{D}}$ on $\mathcal{B}_{\mathcal{D}}$.

Assume $\mathcal{D} := Q(\Lambda)$ and that the components of Q are **geometrically distinct**.

Geometrically Distinct QoI

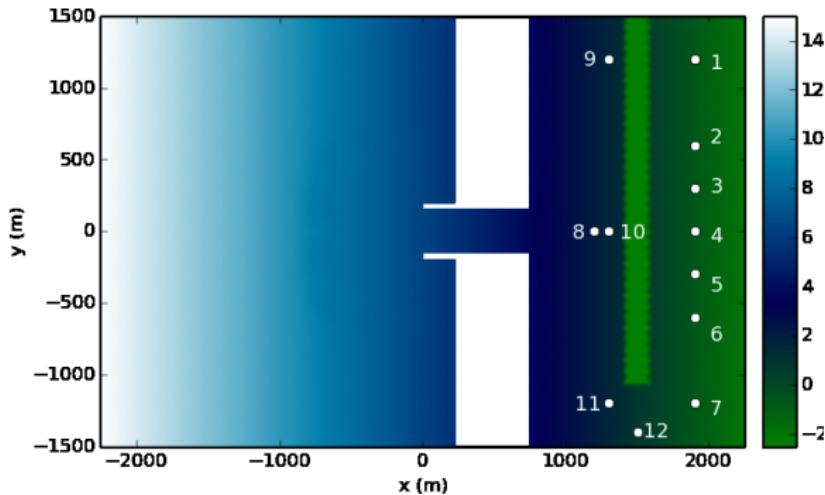
The Jacobian of $Q(\lambda)$ has full rank a.e. in Λ .

Recall that $Q(\lambda) \in \mathbb{R}^d$ and $d \leq m$ so there are at most m **geometrically distinct** QoI.



Observation Stations

$q_i(\lambda) := q_i(Y(\lambda))$ is the maximum water elevation at the i^{th} observation station.



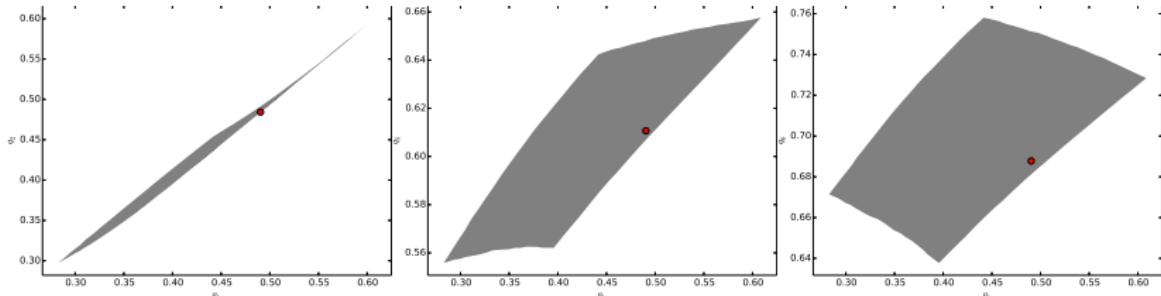
Choosing QoI for a well-conditioned inverse problem

Restrict Q to $Q_{\mathcal{L}} : \mathcal{L} \rightarrow \mathcal{D}$ then the accuracy of our solution is dependent on

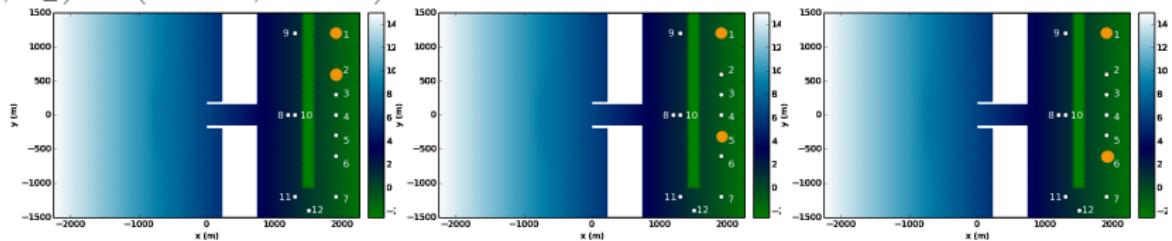
$$\max_i(\text{skew}(J_{Q_{\mathcal{L}}}(\lambda^{(i)}))). \quad (2.1)$$

Large skewness implies less precision in the predictions using the solution of the inverse problem.

Choosing QoI for a well-conditioned inverse problem



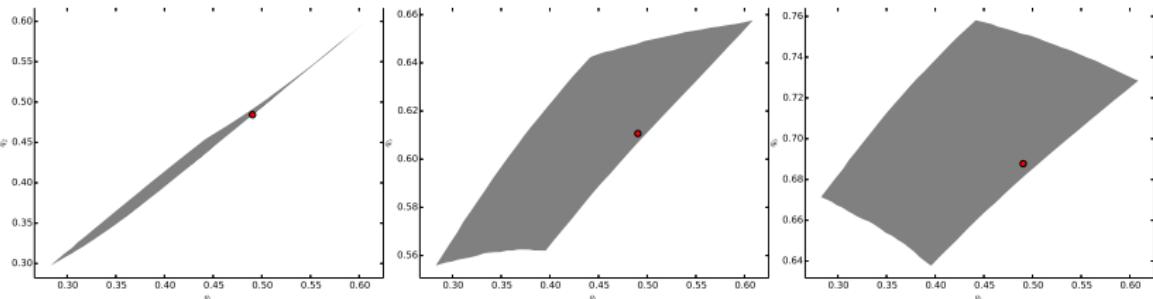
The domain $\mathcal{D} := Q(\Lambda) = \{q_1(\Lambda), q_n(\Lambda)\}$ for $n = 2, 5, 6$. $Q_{\text{reference}}$ for $(\lambda_1, \lambda_2) = (0.107, 0.106)$ is marked in red.



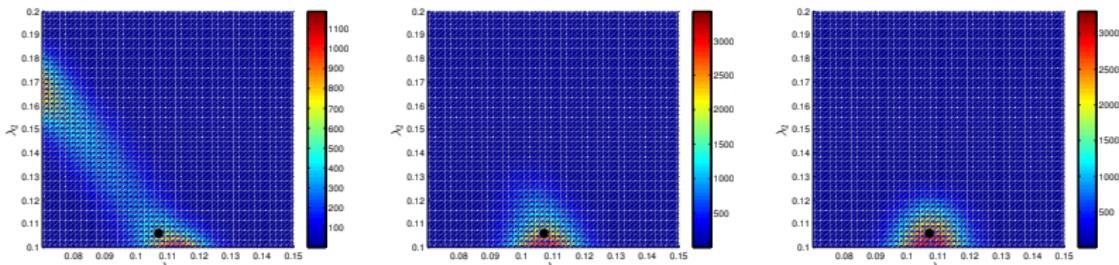
The observation stations. Columns: $Q = (q_1, q_2)$, (q_1, q_5) , and (q_1, q_6) for the first, second, and third columns respectively.



Choosing QoI for a well-conditioned inverse problem



The domain $\mathcal{D} := Q(\Lambda) = \{q_1(\Lambda), q_n(\Lambda)\}$ for $n = 2, 5, 6$. The truth $Q_{\text{reference}}$ for $(\lambda_1, \lambda_2) = (0.107, 0.106)$ is marked in red.



The approximation of ρ_λ where $\lambda_{\text{reference}} = (\lambda_1, \lambda_2) = (0.107, 0.106)$.
Columns: $Q = (q_1, q_2)$, (q_1, q_5) , and (q_1, q_6) for the first, second, and third columns respectively.



Forward Prediction Problem

- Define the inverse problem (Choose $\Lambda, \mathcal{D}, \rho_{\mathcal{D}}$)
- Solve inverse problem
- Define the forward problem (Choose Prediction Space)
- Solve the forward problem

Prediction of time of inundation at $(x_1, y_1) = (1593.75, 1087.5)$ and $(x_2, y_2) = (1593.75, 1012.5)$ for regions in Λ containing 95% of the total probability

Prediction Location (x, y)	Rol in Prediction Space (q_1, q_2)	Rol in Prediction Space (q_1, q_6)	Inundation t
(x_1, y_1)	(16:36:50, 19:08:56)	(18:01:36, 19:08:56)	18:36:58
(x_2, y_2)	(26:35:38, 27:18:52)	(27:14:24, 27:19:14)	27:17:08



Forward Prediction Problem

- Define and solve the inverse problem (Choose $\Lambda, \mathcal{D}, \rho_{\mathcal{D}}$)
- Define and solve the forward problem (Choose Prediction Space)

Prediction of time of inundation at $(x_1, y_1) = (1593.75, 1087.5)$ and $(x_2, y_2) = (1593.75, 1012.5)$ for regions in Λ containing 95% of the total probability

Prediction Location (x, y)	Rol in Prediction Space (q_1, q_2)	Rol in Prediction Space (q_1, q_6)	Inundation t
(x_1, y_1)	9126s	4040s	18:36:58
(x_2, y_2)	2594s	290s	27:17:08



A Case Study: Hurricane Gustav



The Stochastic Inverse Problem for ADCIRC

Recall that we require

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Assume $\mathcal{D} := Q(\Lambda)$ and that the Jacobian of $Q(\lambda)$ has full rank a.e. in Λ .



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Hurricane Gustav

- August 25, 2008 - September 7, 2008
- Highest winds: 155 mph (250 km/h)
- Fatalities: 153
- Damage: \$6.61 billion (2008 USD)



Computational Domain

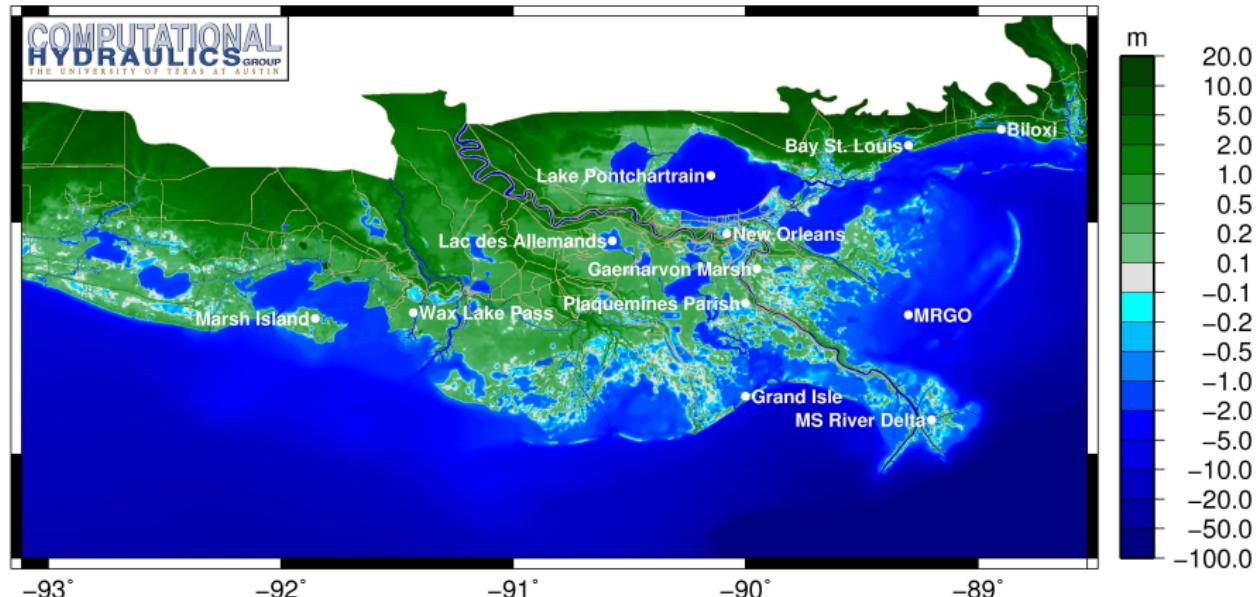
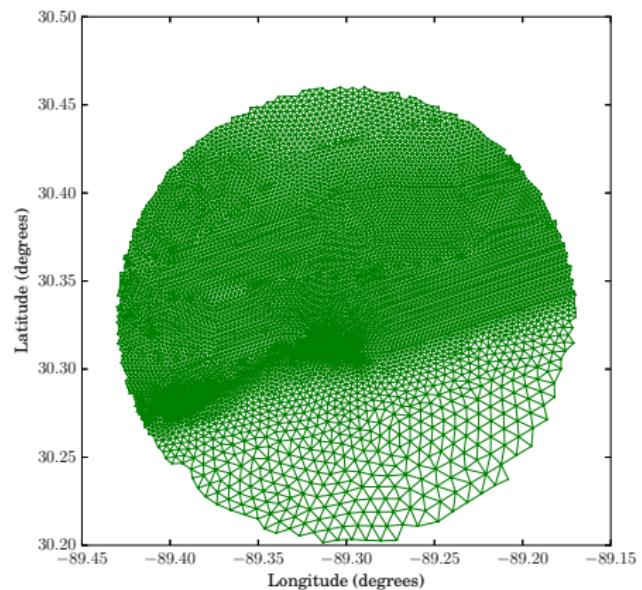
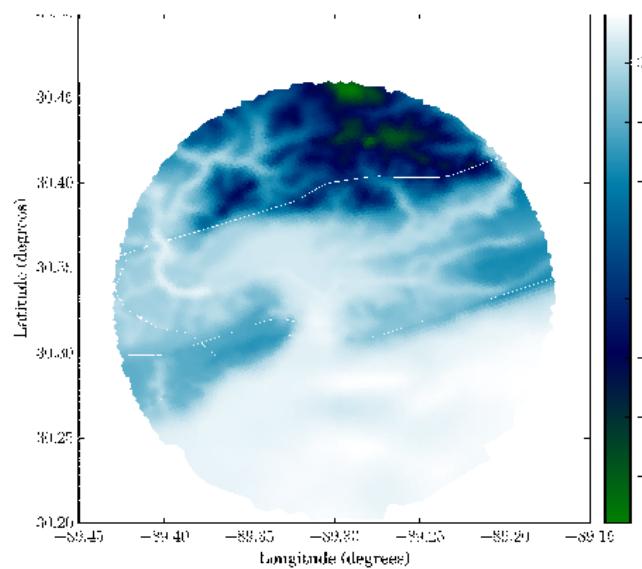


Figure : Bathymetry/topography (m) of the full domain mesh in southeastern Louisiana.



Subdomain Model of Bay St. Louis



Subdomain Model of Bay St. Louis

Subdomain ADCIRC

- ① Create full domain ADCIRC model.
- ② Create subdomain ADCIRC model.
- ③ Generate full domain control file to control output of subdomain boundary forcing data.
- ④ Run ADCIRC on the full domain.
- ⑤ Extract subdomain boundary forcing data (elevation, velocity, wet/dry status) from the full domain.
- ⑥ Run ADCIRC on the subdomain

Simon (2011), Altuntas (2012), Altuntas and Simon (2013).



Define $\Lambda \subset \mathbb{R}^m$, a compact **parameter domain** with a specified metric
(e.g. a finite dimensional representation of possible Manning's n fields)

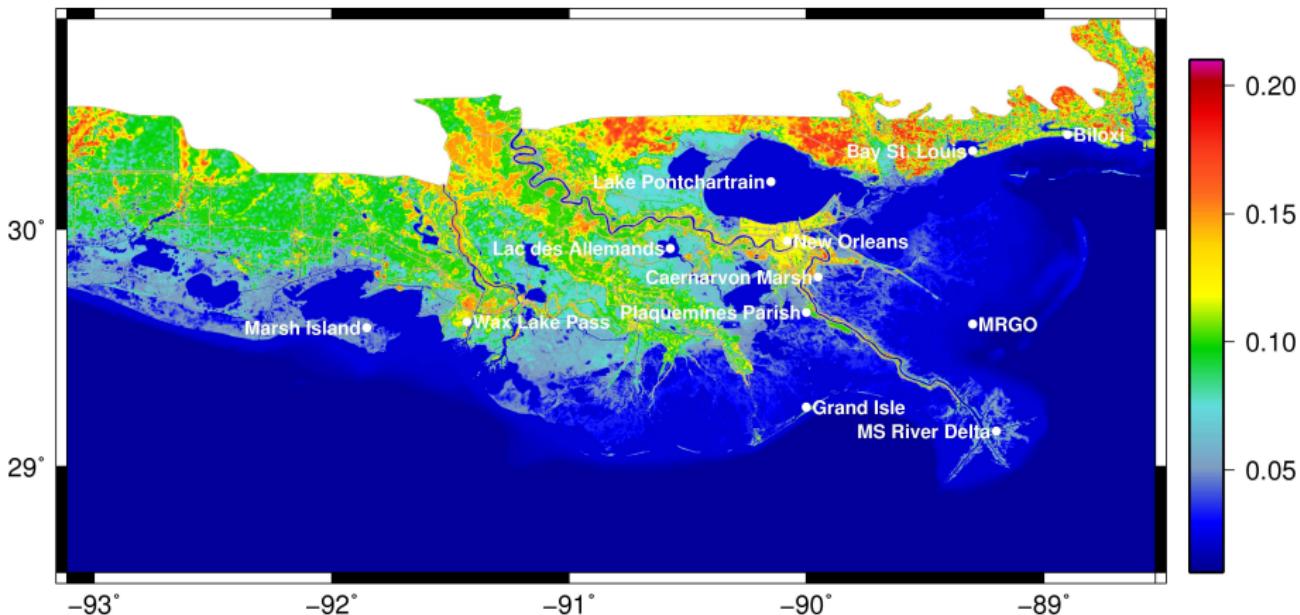


Figure : Manning's n values of the full domain mesh in southeastern Louisiana. (23 different land cover classifications)

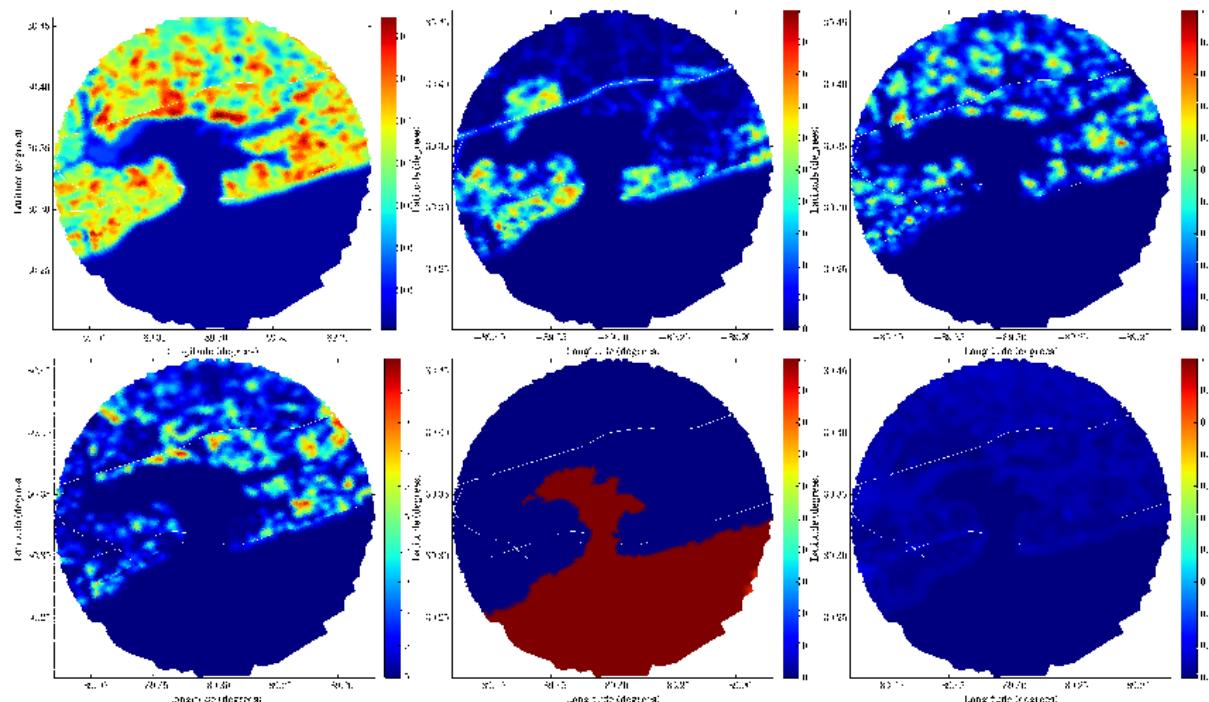


Dominant land cover classifications

Percentage	C-CAP Class	Description	Manning's <i>n</i>
23.95	21	Open water	0.025
13.45	10	Evergreen forest	0.180
12.57	13	Palustrine forested wetland	0.150
12.33	4	Low-intensity developed	0.120



Parameter Domain



The mesoscale representation of Manning's n values from basis vectors defining a linear mapping.



Parameter Domain

Parameter	Range of Manning's n values	Land Classification
λ_1	[0.0396, 0.21]	4
λ_2	[0.0594, 0.315]	10
λ_3	[0.0495, 0.2625]	13
λ_4	[0.00825, 0.04375]	21

Table : Manning's n ranges for subdomain.

So the compact finite dimensional parameter domain is $\Lambda \subset \mathbb{R}^4$.



Quantities of interest (QoI) $\mathbf{Q}(\lambda) = (q_1(\lambda), \dots, q_d(\lambda)) \in \mathcal{D} \subset \mathbb{R}^d$ where
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Data Domain

Observation Stations $q_i(\lambda) := q_i(Y(\lambda))$ is the maximum water elevation at the i^{th} observation station.

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Geometrically Distinct QoI

The Jacobian of $Q(\lambda)$ has full rank a.e. in Λ .

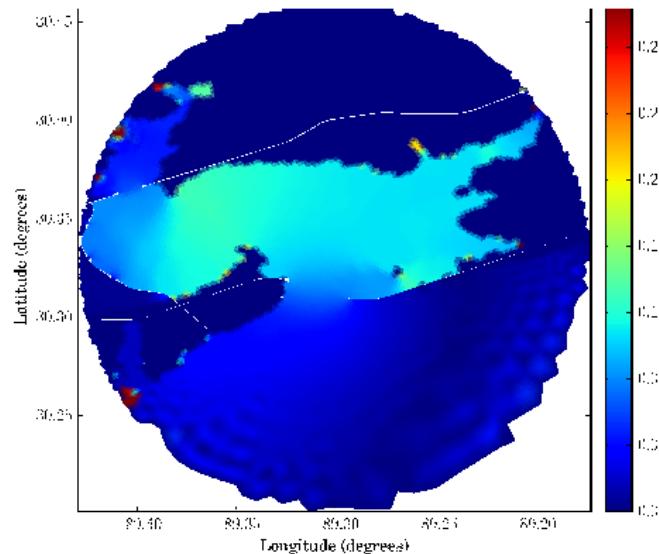
Recall that $Q(\lambda) \in \mathbb{R}^d$ and $d \leq m$ so there are at most m **geometrically distinct** QoI.



Which observation stations should we choose?



Preliminary Study



Maximum difference in subdomain water elevation over a set of 80 preliminary model evaluations. Values shown are $\left(\max_k(Y_i^{(k)}) - \min_k(Y_i^{(k)}) \right)$ in [m] where $Y_i^{(k)} = Y_i(\lambda^{(k)})$ is the maximum water elevation at a finite element node i for the k^{th} parameter value $\lambda^{(k)} = (\lambda_1^{(k)}, \lambda_2^{(k)}, \lambda_3^{(k)}, \lambda_4^{(k)})$. Note that to show subtle variations in the bay the color scale has been reduced.



Preliminary Study

	Number of Stations	Number of Data Spaces $\binom{n}{4}$
All	1970	$6.26 \cdot 10^{11}$
Reduced	194	$5.72 \cdot 10^7$

Table : Number of possible data spaces for subdomain 16.



Optimal Choice of QoI Based on Skewness

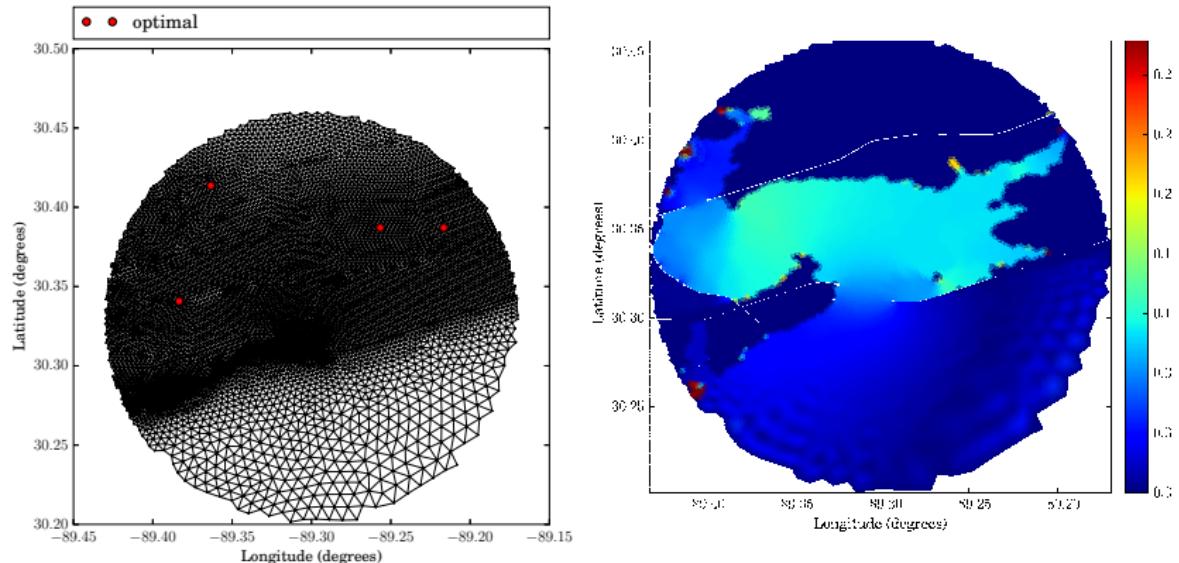


Figure : Left: Optimal maximum water elevation observation stations. Right: Maximum difference in subdomain water elevation..



Data Domain

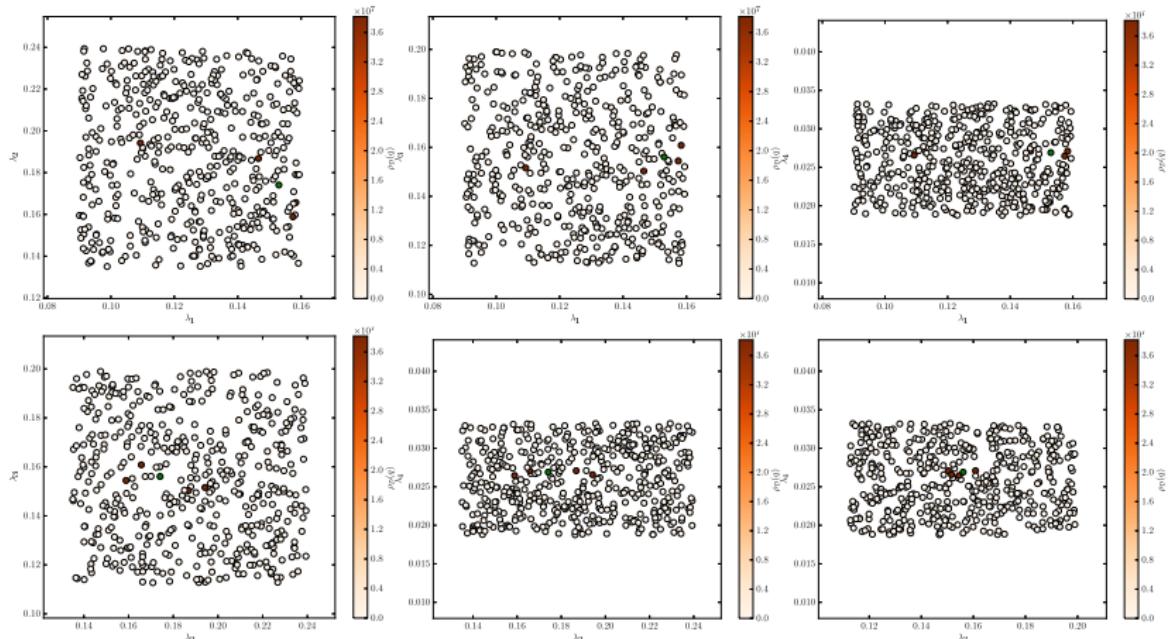
Choose a set of QoI to define \mathcal{D} and define $\rho_{\mathcal{D}}$.

Prescribe an observed (or assumed) **probability density** $\rho_{\mathcal{D}}$ on $\mathcal{B}_{\mathcal{D}}$.

Here, $\rho_{\mathcal{D}}$ is defined as a uniform density on a small rectangular box centered at the reference QoI values associated with $\lambda_{ref} = (0.1523, 0.1751, 0.1561, 0.0269)$.



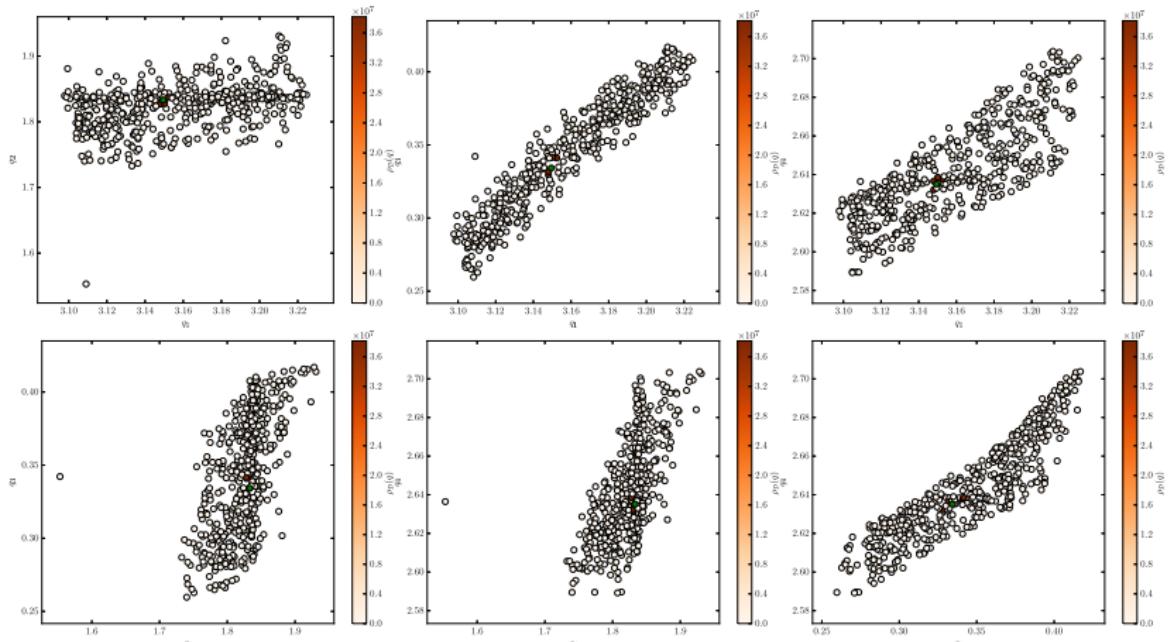
Uniform i.i.d. samples in the parameter space



The uniform samples in the parameter space where $\lambda = (\lambda_1, \lambda_2, \lambda_3, \lambda_4)$ and $\lambda_{ref} = (0.1523, 0.1751, 0.1561, 0.0269)$ is marked in green. The samples are colored by $\rho_{\mathcal{D}}(Q)$. Top: Left to right, (λ_1, λ_2) , (λ_1, λ_3) , (λ_1, λ_4) . Bottom: Left to right, (λ_2, λ_3) , (λ_2, λ_4) , (λ_3, λ_4) .



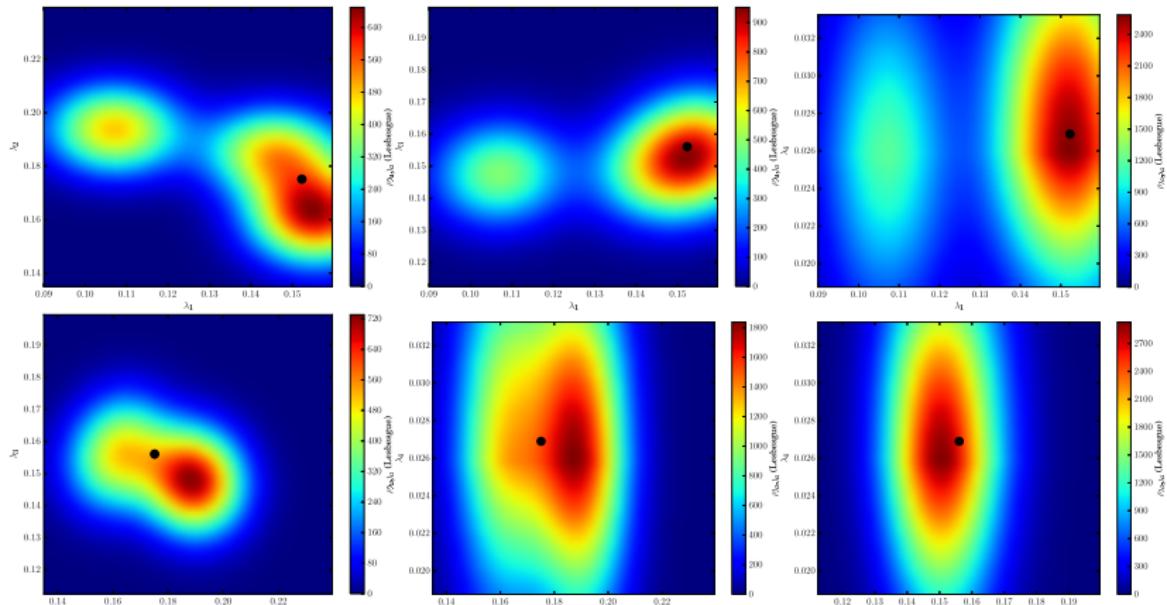
Uniform i.i.d. samples in the data space using optimal QoI



The uniform samples in the data space for the optimal stations where $Q = (q_1, q_2, q_3, q_4)$ for the optimal set of stations and Q_{ref} is marked in green. The samples are colored by $\rho_D(Q)$. Top: Left to right, (q_1, q_2) , (q_1, q_3) , (q_1, q_4) . Bottom: Left to right, (q_2, q_3) , (q_2, q_4) , (q_3, q_4) .



Approximated ρ_Λ using optimal QoI



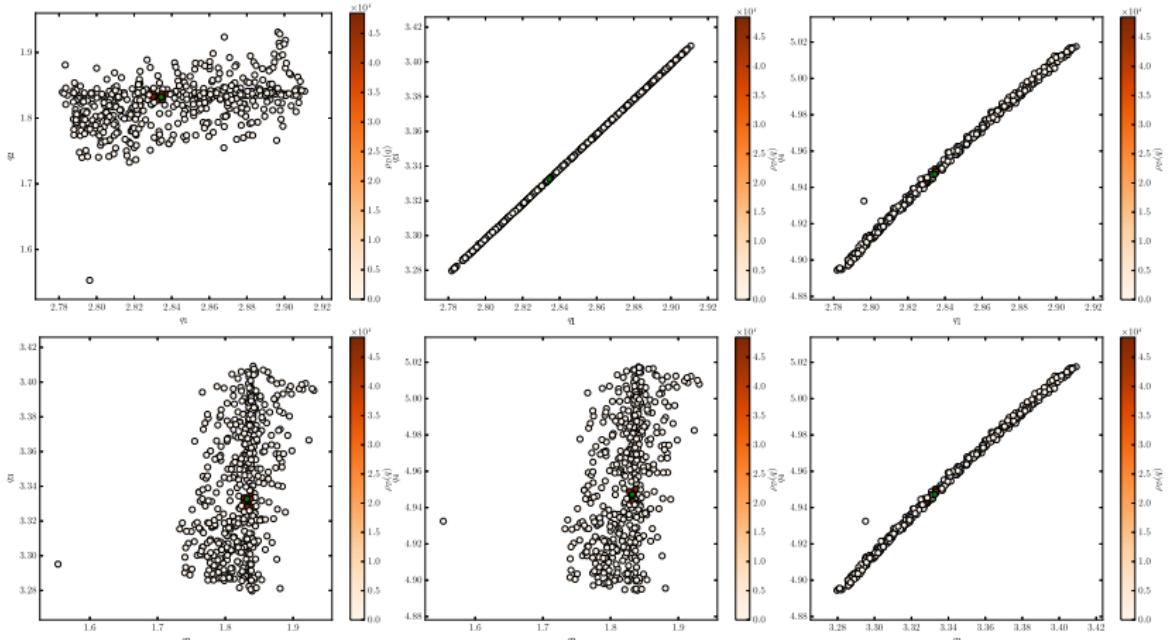
Plots the marginals of ρ_Λ using uniform samples using optimal stations. The reference value is illustrated by a black circle. Top: Left to right, $(q_1, q_2), (q_1, q_3), (q_1, q_4)$. Bottom: Left to right, $(q_2, q_3), (q_2, q_4), (q_3, q_4)$.



What if we choose a suboptimal set of stations?



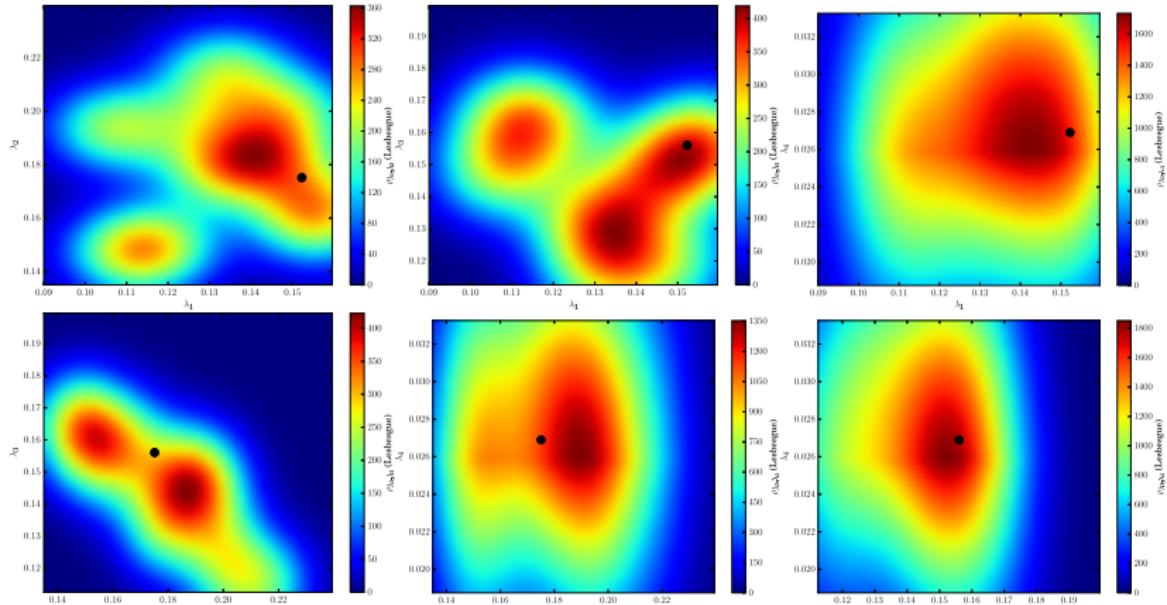
Uniform i.i.d. samples in the data space using suboptimal QoI



The uniform samples in the data space for the suboptimal stations and Q_{ref} is marked in green. Top: Left to right, (q_1, q_2) , (q_1, q_3) , (q_1, q_4) . Bottom: Left to right, (q_2, q_3) , (q_2, q_4) , (q_3, q_4) .



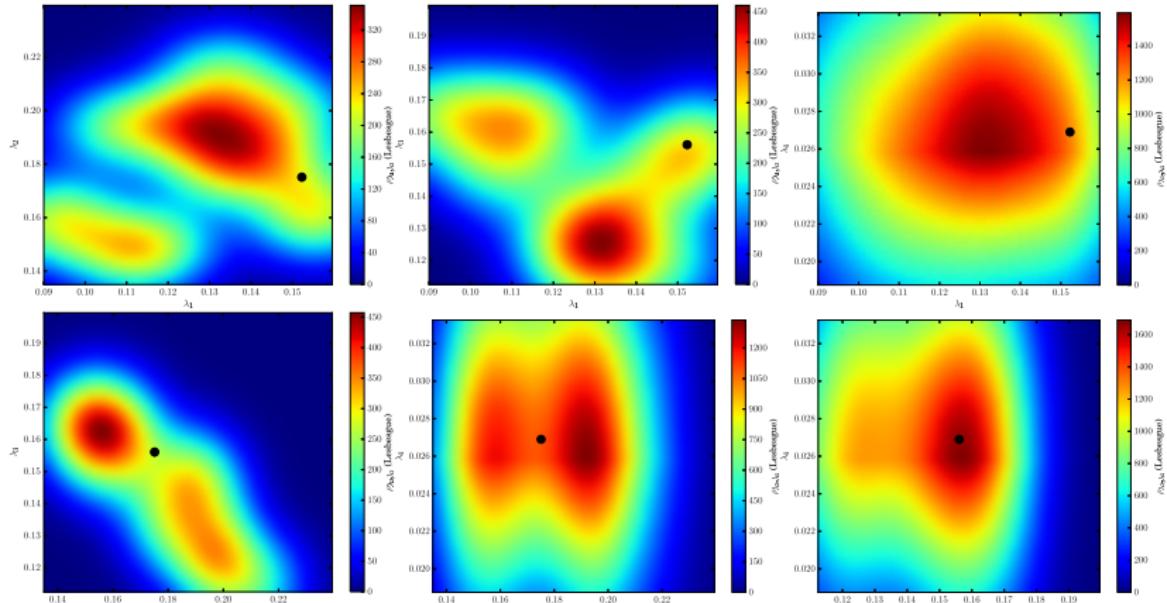
Approximated ρ_Λ using suboptimal QoI



Plots the marginals of ρ_Λ using uniform samples using suboptimal stations. The reference value is illustrated by a black circle. Top: Left to right, (q_1, q_2) , (q_1, q_3) , (q_1, q_4) . Bottom: Left to right, (q_2, q_3) , (q_2, q_4) , (q_3, q_4) .



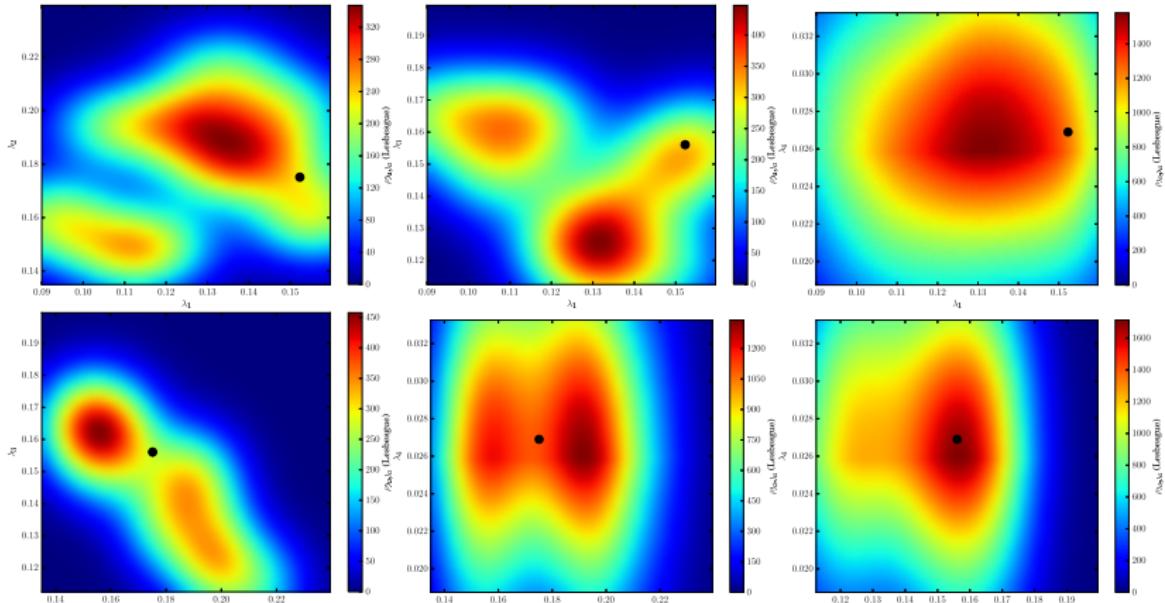
Approximated ρ_Λ using 3 QoI



Plots the marginals of ρ_Λ using uniform samples using 3 stations. The reference value is illustrated by a black circle. Top: Left to right, (q_1, q_2) , (q_1, q_3) , (q_1, q_4) . Bottom: Left to right, (q_2, q_3) , (q_2, q_4) , (q_3, q_4) .



Approximated ρ_Λ using 2 QoI



Plots the marginals of ρ_Λ using uniform samples using 2 stations. The reference value is illustrated by a black circle. Top: Left to right, (q_1, q_2) , (q_1, q_3) , (q_1, q_4) . Bottom: Left to right, (q_2, q_3) , (q_2, q_4) , (q_3, q_4) .



Conclusion and Future Work

Summary

- QoI choice influences the condition of the stochastic inverse problem.
- The stochastic inverse problem influences experimental design.
- Adaptive sampling identifies regions of interest (high probability) for stochastic inverse problems.

Next Steps

- Additional physical domains, forcing, and parameters.
- Using field data to construct $\rho_{\mathcal{D}}$.
- Using surrogate models.



Thanks!

Questions?



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